Research Article

Model Updating of Spindle Systems Based on the Identification of Joint Dynamics

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Received 1 February 2015; Accepted 28 March 2015

Academic Editor: Yumin He

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In order to simulate the cutting performance of a spindle mounted in the machine tool, the finite element (FE) model of spindles is required to be coupled with machine tool. However, the unknown joint dynamics (e.g., bolts) between the spindle and machine tool column limit the accuracy of the model. In this paper, an FE model updating method is proposed based on the identification of joint dynamics in both translational and rotational degrees-of-freedom (DOF). The receptance coupling (RC) technique is enhanced to estimate frequency response functions (FRFs) corresponding to rotational DOFs. The joint stiffness is identified through the iteration process by minimizing the difference between the simulated FRF and the measured FRF of the assembly. The proposed method is verified with a machine-tool spindle system. The good agreement between simulation and experiment shows the effectiveness of the method.

1. Introduction

Apart from excessive tool wear/breakage, chatter is still the biggest obstacle in increasing material removal rates in high-speed machining. In order to predict chatter vibrations, the accurate prediction or measurement of the machine-tool dynamics is critical. Among all the components of a machine tool, the spindle-tool system is usually the most flexible part that limits the overall dynamic stiffness. Therefore, a full understanding of the dynamic response of the spindle-tool system is required in order to avoid or suppress the chatter problem.

In the past, many researchers have investigated the dynamic modeling of high-speed spindles with consideration of the nonlinear behavior of bearings. Recently, Abele et al. reviewed the historical development, recent challenges, and future trends of the machine-tool spindle unit in detail [1]. Undoubtedly, these studies have promoted the development of high-speed spindle technology to a new level. However, due to the nonlinear behavior of machine joints, it is still a challenge to accurately predict the dynamic performance of the whole machine-tool spindle system at the design stage. Once the spindle is installed on the machine tool, the dynamic characteristics of the spindle change. Most existed

work mainly considered the spindle-bearing system as an independent system, while the effects of the machine-tool column on the spindle dynamics were neglected. In order to simulate the cutting performance of a real high-speed spindle mounted on the machine tool, the effects of the machine-tool column on the spindle system must be included in the model. Cao and Altintas first considered the effects of the machine-tool column on the spindle dynamics, and a coupled finite element (FE) model of the machine-tool spindle system was presented in [2]. However, the stiffness of bolted joints values was estimated by experience and this procedure is very time-consuming. Therefore, there is a critical need to develop a scientific and systematic tool that can assist industrial engineers in achieving a reliable FE model which can accurately represent the dynamic characteristics of the machine-tool spindle system.

While finite element analysis (FEA) is a well-established and accepted technique for the dynamic analysis of structural systems at the design stage, typically the joint dynamics in the machine are not taken into account [3]. In the machine-tool spindle system, the unknown joint dynamics are the main obstacles to obtain an FE model with high accuracy. Usually very little is known about the dynamic properties of the joints in an assembly. The less-than-rigid bolted

joints in the assembly are often inappropriately modeled as rigid connections, which results in mismatch between the finite element analysis and experimental measurements. Inaccuracies in FE model lead to the development of model updating techniques. Nowadays, FE model updating has become a viable approach to identify and correct uncertain modeling parameters [4]. A number of model updating methods have been proposed in recent years as shown in the text by Friswell and Mottershead [5]. FE model updating methods are generally classified into two broad categories: direct methods and iterative methods. The early algorithms that directly solve updated global system matrices are referred to as direct methods, which usually make use of the measured eigenvalues and eigenvectors [6–8]. However, the inherent coordinate incompatibility between the number of measurement degree-of-freedom (DOF) and the number in the analytical FE model is the main obstacle that restricts the successful application of these methods. On the other hand, the iterative methods are formulated in line with the discrete nature of the FE model and introduce changes to a predefined number of updated parameters (e.g., joint dynamics) on an elemental basis. Iterative methods use either modal parameters (eigenvalue and eigenvector) or frequency response functions (FRFs), which leads to modal-based or FRF-based model updating approaches. Although good experience has been made with modal-based model updating methods [9–11], the number of updated parameters is usually limited as such formulations are often confined to a few of updating eigenvalues only and nonunique solutions emerge due to the underdetermined nature of the sensitivity matrix.

Differently, the FRF-based model updating methods make use of the measured FRFs directly which circumvent the need to identify the modal parameters from the experiments [12–17]. These methods are able to produce highly accurate updated system matrices through iteration. The idea of using measured FRFs directly for model updating purposes is not a new one and probably started to crystallize with the paper by Natke [18]. Later, many researchers further developed this type of updating approach. Chen et al. [19] and Adhikari and Woodhouse [20, 21] respectively studied the use of the FRFs and modes in identifying linear damping models. Lu and Tu [22] proposed a two-step model updating procedure for lightly damped structures using neural networks. In the first step, mass and stiffness were updated using natural and antiresonance frequencies. In the second step, damping ratios were updated. Lin and Zhu [23] extended the response function method which is based on FRF data to update damping coefficients in addition to mass and stiffness matrices. Arora et al. [24] proposed a damped model updating method using complex updating parameters. Esfandiari et al. [25] used the FRF-based structural updating to identify structure damage. Pradhan and Modak [26] proposed a method for damping matrix identification using frequency response data, and Wang et al. [27] established a method of identifying joint dynamic properties using partially measured FRF. Sracic et al. [28] calibrated nonlinear system models by comparing measured and computed nonlinear frequency responses. Additionally, Gang et al. [29] proposed a new iterative model updating method using incomplete FRF data. However,

the majority of the mentioned studies update the FE model only considering the FRFs of the translational DOFs, while neglecting the FRFs of the rotational DOFs. Nevertheless, the FRFs of rotational DOFs are indispensable for the estimation of rotational stiffness and damping parameters at the joints. Due to the infeasibility of accurate measurements of angular displacements and applied moments, the direct measurement of rotational DOF dynamics through experiment is very difficult.

Receptance Coupling (RC) method divides complex systems into several simple substructures, the FRFs of which can be respectively obtained by analytical or experimental methods. The system response can be gained by composing the subsystems, according to the coupling relationship between the subsystems, that is, equilibrium condition and compatibility condition on the common border. In 2000, Schmitz introduced the RC technique into kinetic analysis of the machine-tool field to predict the FRFs at the tool tip in the spindle-tool system. Compared with the conventional methods which measure the tool tip dynamic response after each tool change, the RC technique can largely improve the efficiency [30–33]. Later on, Altintas et al. improved the RC technique with the consideration of the influence of the rotational DOFs on the structural dynamic behavior and systematically studied the dynamic performance of the spindle-tool holder-tool system [34–37].

In this paper, the FRF-based iteration algorithm is introduced and improved. The RC technique is enhanced to predict the FRFs of the rotational DOFs, and then a general FE model updating method is proposed. Next, the coupled model of a machine-tool spindle system is updated using the proposed method. The experimental results show that the updated model can represent the dynamic behavior of the spindle mounted in the machine-tool column fairly accurately when the dynamics of the joints are largely unknown.

The rest of the paper is organized as follows. In Section 2, a general FE model updating method is proposed. In Section 3, the method is applied to a coupled model of the machine-tool spindle system. Finally, conclusions are given in Section 4.

2. Methods

2.1. The FRF-Based Iterative Algorithm. The principle of FRF-based iterative algorithm can be described in the form of

$$\mathbf{S} \cdot \Delta \mathbf{u} = \boldsymbol{\varepsilon}, \quad (1)$$

where $\mathbf{S}$ is the sensitivity matrix, $\Delta \mathbf{u}$ is the vector of updating parameters, and $\boldsymbol{\varepsilon}$ denotes the residual, that is, the difference between the analytical and measured dynamic FRFs. Such a system of equations is often overdetermined and solved by using least square method. After solving (1), the updated parameter changes and the FE model is updated. Following an eigen-solution of the updated FE model, a new residual is obtained. This process of solving and updating the system is repeated until the residual is zero or smaller than a defined threshold. Unlike direct methods, the connectivity patterns of the modified mass and stiffness matrices remain meaningful.

Equation (1) can be derived in the form of

$$\{\varepsilon_{H_j}(\{u^0\})\}_v + \sum_{i=1}^{N_u} [H^a(\omega_j)] \frac{\partial \{\varepsilon_{F_j}\}_v}{\partial u_i} \Delta u_i = 0, \quad (2)$$

where $\{\varepsilon_{H_j}(\{u^0\})\}_v$ is the residual FRF, $[H^a(\omega_j)]$ is the analytical FRF matrix, $\{\varepsilon_{F_j}\}_v$ is the force residual, and u_i is the updated parameter. The detailed derivation can be seen in [14].

If the updated parameters are k_{pp} (the local stiffness of p th DOF) and k_{pq} (the relative stiffness between the p th and the q th DOFs), then (2) is specifically obtained as

$$\begin{bmatrix} -H_{1p}^a(\omega_j) H_{pv}^x(\omega_j) & (H_{1q}^a(\omega_j) - H_{1p}^a(\omega_j)) (H_{pv}^x(\omega_j) - H_{qv}^x(\omega_j)) \\ -H_{2p}^a(\omega_j) H_{pv}^x(\omega_j) & (H_{2q}^a(\omega_j) - H_{2p}^a(\omega_j)) (H_{pv}^x(\omega_j) - H_{qv}^x(\omega_j)) \\ \vdots & \vdots \\ -H_{Np}^a(\omega_j) H_{pv}^x(\omega_j) & (H_{Nq}^a(\omega_j) - H_{Np}^a(\omega_j)) (H_{pv}^x(\omega_j) - H_{qv}^x(\omega_j)) \end{bmatrix} \begin{Bmatrix} \Delta k_{pp} \\ \Delta k_{pq} \end{Bmatrix} = - \begin{Bmatrix} \varepsilon_{H_{1v}}(\omega_j) \\ \varepsilon_{H_{2v}}(\omega_j) \\ \vdots \\ \varepsilon_{H_{Nv}}(\omega_j) \end{Bmatrix}. \quad (3)$$

In (3), each row of the sensitivity matrix contains the same experimental information $(H_{pv}^x(\omega_j), H_{qv}^x(\omega_j))$, so it is unnecessary to use all the rows during the iteration. If the v th DOF is excited, the w th DOF is measured and N_p

frequency points $(\omega_1, \omega_2, \dots, \omega_{N_p})$ are chosen from FRFs; the over-determined equation is then given as follows:

$$\begin{bmatrix} -H_{wp}^a(\omega_1) H_{pv}^x(\omega_1) & (H_{wq}^a(\omega_1) - H_{wp}^a(\omega_1)) (H_{pv}^x(\omega_1) - H_{qv}^x(\omega_1)) \\ -H_{wp}^a(\omega_2) H_{pv}^x(\omega_2) & (H_{wq}^a(\omega_2) - H_{wp}^a(\omega_2)) (H_{pv}^x(\omega_2) - H_{qv}^x(\omega_2)) \\ \vdots & \vdots \\ -H_{wp}^a(\omega_{N_p}) H_{pv}^x(\omega_{N_p}) & (H_{wq}^a(\omega_{N_p}) - H_{wp}^a(\omega_{N_p})) (H_{pv}^x(\omega_{N_p}) - H_{qv}^x(\omega_{N_p})) \end{bmatrix} \begin{Bmatrix} \Delta k_{pp} \\ \Delta k_{pq} \end{Bmatrix} = - \begin{Bmatrix} \varepsilon_{H_{wv}}(\omega_1) \\ \varepsilon_{H_{wv}}(\omega_2) \\ \vdots \\ \varepsilon_{H_{wv}}(\omega_{N_p}) \end{Bmatrix}, \quad (4)$$

where $H_{wq}^a(\omega)$ and $H_{wp}^a(\omega)$ are analytical FRFs, $H_{pv}^x(\omega)$ and $H_{qv}^x(\omega)$ are experimental FRFs, and the FRF residual $\varepsilon_{H_{wv}}(\omega) = H_{wv}^a(\omega) - H_{wv}^x(\omega)$. From (4), it can be seen that only the experimental FRFs related to the joint points (the p th and the q th DOFs) are needed for the iterative algorithm, which decreases the number of measured DOFs largely.

2.2. FRFs Estimation of the Rotational DOFs. Figure 1 presents a representative RC model with two substructures, including the substructure A and substructure B, and the joint J.

A force F_1 is applied to the node 1, but no force is applied to nodes 2 and 3 of the overall structure. If only the responses X_1 and X_2 corresponding to nodes 1 and 2 are considered, the FRFs G_{11} and G_{21} of the overall structure can be obtained as

$$\begin{aligned} G_{11} &= \frac{X_1}{F_1} = H_{A,11} - H_{A,12} H_2^{-1} H_{A,21}, \\ G_{21} &= \frac{X_2}{F_1} = H_{A,21} - H_{A,22} H_2^{-1} H_{A,21}, \end{aligned} \quad (5)$$

where $H_{A,ij}$ is the receptance matrix of substructure A, with response at region i and excitation at region j , $H_2 = (H_{A,22} + H_{B,22} + H^J)$, and $H^J = (M^J s^2 + C^J s + K^J)^{-1}$ is the receptance matrix of the joint surface J.

Similarly, if the external force F_2 is just applied to joint 2, the FRFs can be gained as

$$\begin{aligned} G_{12} &= \frac{X_1}{F_2} = H_{A,12} - H_{A,12} H_2^{-1} H_{A,22}, \\ G_{22} &= \frac{X_2}{F_2} = H_{A,22} - H_{A,22} H_2^{-1} H_{A,22}. \end{aligned} \quad (6)$$

For Timoshenko beam elements, the motion of nodes at a specific surface is composed of translational and rotational DOFs. The input force vector F consists of the force f and moment M , and the output response X is composed of the translational displacement x and the rotational displacement θ . The input force and output response satisfy the following relationship:

$$\begin{Bmatrix} x \\ \theta \end{Bmatrix} = \begin{bmatrix} h_{ij,ff} & h_{ij,fM} \\ h_{ij,Mf} & h_{ij,MM} \end{bmatrix} \begin{Bmatrix} f \\ M \end{Bmatrix} \longrightarrow X = H_{ij} \cdot F, \quad (7)$$

where $h_{ij,ff}$ is the receptance between translational DOFs with response at region i and excitation at region j , $h_{ij,Mf}$ and $h_{ij,fM}$ are receptances between translational and rotational DOFs, and $h_{ij,MM}$ is the receptance between rotational DOFs.

FIGURE 1: RC model.

By substituting (7) into (5) and (6) and letting

$$\mathbf{H}_2 = \begin{bmatrix} h_{2,ff} & h_{2,fM} \\ h_{2,Mf} & h_{2,MM} \end{bmatrix}, \quad (8)$$

the systematic receptance matrix can be obtained as

$$\begin{aligned} & \begin{bmatrix} g_{11,ff} & g_{11,fM} \\ g_{11,Mf} & g_{11,MM} \end{bmatrix} \\ &= \begin{bmatrix} h_{A11,ff} & h_{A11,fM} \\ h_{A11,Mf} & h_{A11,MM} \end{bmatrix} - \begin{bmatrix} h_{A12,ff} & h_{A12,fM} \\ h_{A12,Mf} & h_{A12,MM} \end{bmatrix} \\ & \cdot \begin{bmatrix} h_{2,ff} & h_{2,fM} \\ h_{2,Mf} & h_{2,MM} \end{bmatrix}^{-1} \begin{bmatrix} h_{A21,ff} & h_{A21,fM} \\ h_{A21,Mf} & h_{A21,MM} \end{bmatrix}, \\ & \begin{bmatrix} g_{21,ff} & g_{21,fM} \\ g_{21,Mf} & g_{21,MM} \end{bmatrix} \\ &= \begin{bmatrix} h_{A21,ff} & h_{A21,fM} \\ h_{A21,Mf} & h_{A21,MM} \end{bmatrix} - \begin{bmatrix} h_{A22,ff} & h_{A22,fM} \\ h_{A22,Mf} & h_{A22,MM} \end{bmatrix} \\ & \cdot \begin{bmatrix} h_{2,ff} & h_{2,fM} \\ h_{2,Mf} & h_{2,MM} \end{bmatrix}^{-1} \begin{bmatrix} h_{A21,ff} & h_{A21,fM} \\ h_{A21,Mf} & h_{A21,MM} \end{bmatrix}, \\ & \begin{bmatrix} g_{12,ff} & g_{12,fM} \\ g_{12,Mf} & g_{12,MM} \end{bmatrix} \\ &= \begin{bmatrix} h_{A12,ff} & h_{A12,fM} \\ h_{A12,Mf} & h_{A12,MM} \end{bmatrix} - \begin{bmatrix} h_{A12,ff} & h_{A12,fM} \\ h_{A12,Mf} & h_{A12,MM} \end{bmatrix} \\ & \cdot \begin{bmatrix} h_{2,ff} & h_{2,fM} \\ h_{2,Mf} & h_{2,MM} \end{bmatrix}^{-1} \begin{bmatrix} h_{A22,ff} & h_{A22,fM} \\ h_{A22,Mf} & h_{A22,MM} \end{bmatrix}, \\ & \begin{bmatrix} g_{22,ff} & g_{22,fM} \\ g_{22,Mf} & g_{22,MM} \end{bmatrix} \\ &= \begin{bmatrix} h_{A22,ff} & h_{A22,fM} \\ h_{A22,Mf} & h_{A22,MM} \end{bmatrix} - \begin{bmatrix} h_{A22,ff} & h_{A22,fM} \\ h_{A22,Mf} & h_{A22,MM} \end{bmatrix} \\ & \cdot \begin{bmatrix} h_{2,ff} & h_{2,fM} \\ h_{2,Mf} & h_{2,MM} \end{bmatrix}^{-1} \begin{bmatrix} h_{A22,ff} & h_{A22,fM} \\ h_{A22,Mf} & h_{A22,MM} \end{bmatrix}, \end{aligned}$$

where

$$\begin{aligned} & \begin{bmatrix} h_{2,ff} & h_{2,fM} \\ h_{2,Mf} & h_{2,MM} \end{bmatrix}^{-1} = \left(\begin{bmatrix} h_{A22,ff} & h_{A22,fM} \\ h_{A22,Mf} & h_{A22,MM} \end{bmatrix} \right. \\ & \quad + \begin{bmatrix} h_{B22,ff} & h_{B22,fM} \\ h_{B22,Mf} & h_{B22,MM} \end{bmatrix} \\ & \quad \left. + \begin{bmatrix} h_{ff}^J & h_{fM}^J \\ h_{Mf}^J & h_{MM}^J \end{bmatrix} \right)^{-1}. \end{aligned} \quad (10)$$

By respectively extracting the first element of the receptance matrices $\mathbf{G}_{11}$, $\mathbf{G}_{12}$, and $\mathbf{G}_{22}$, following equation set can be gained:

$$\begin{aligned} g_{11,ff} &= h_{A11,ff} + \frac{1}{(h_{2,fM} \cdot h_{2,Mf} - h_{2,ff} \cdot h_{2,MM})} \\ & \cdot [h_{A21,ff} (h_{A12,ff} \cdot h_{2,MM} - h_{A12,fM} \cdot h_{2,Mf}) \\ & \quad + h_{A21,Mf} (h_{A12,fM} \cdot h_{2,ff} - h_{A12,ff} \cdot h_{2,fM})], \\ g_{12,ff} &= h_{A12,ff} + \frac{1}{(h_{2,fM} \cdot h_{2,Mf} - h_{2,ff} \cdot h_{2,MM})} \\ & \cdot [h_{A22,ff} (h_{A12,ff} \cdot h_{2,MM} - h_{A12,fM} \cdot h_{2,Mf}) \\ & \quad + h_{A22,Mf} (h_{A12,fM} \cdot h_{2,ff} - h_{A12,ff} \cdot h_{2,fM})], \\ g_{22,ff} &= h_{A22,ff} + \frac{1}{(h_{2,fM} \cdot h_{2,Mf} - h_{2,ff} \cdot h_{2,MM})} \\ & \cdot [h_{A22,ff} (h_{A22,ff} \cdot h_{2,MM} - h_{A22,fM} \cdot h_{2,Mf}) \\ & \quad + h_{A22,Mf} (h_{A22,fM} \cdot h_{2,ff} - h_{A22,ff} \cdot h_{2,fM})], \\ h_{2,fM} &= h_{2,Mf}. \end{aligned} \quad (11)$$

As (11) is composed of four equations with four unknowns, that is, $h_{2,ff}$, $h_{2,fM}$, $h_{2,Mf}$, and $h_{2,MM}$, the analytical solutions of the FRFs $\mathbf{H}_{A11}$, $\mathbf{H}_{A12}$, $\mathbf{H}_{A21}$, and $\mathbf{H}_{A22}$ of the substructure A at free state can all be gained by using the FE model. The FRFs of the overall structures $g_{11,ff}$, $g_{12,ff}$, and $g_{22,ff}$ can be gained by using the experimental method. Therefore, the receptance matrix $\mathbf{H}_2$ which includes the coupling characteristics of the joint surface can be obtained by solving (11).

The FRFs of each rotational DOF can be estimated by using (9). For instance, the coupled FRF $g_{21,Mf}$ of

FIGURE 2: The flowchart of the proposed method.

the translational and rotational DOF between the node 1 and 2 can be solved by the following equation:

$$\begin{aligned}
 g_{21,Mf} = & h_{A21,Mf} + \frac{1}{(h_{2,fM} \cdot h_{2,Mf} - h_{2,ff} \cdot h_{2,MM})} \\
 & \cdot [h_{A21,ff} (h_{A22,Mf} \cdot h_{2,MM} - h_{A22,MM} \cdot h_{2,Mf}) \\
 & + h_{A21,Mf} (h_{A22,MM} \cdot h_{2,ff} - h_{A22,Mf} \cdot h_{2,fM})] \cdot
 \end{aligned} \quad (12)$$

2.3. The Proposed Model Updating Method. The proposed method is based on the assumption that the analytical FE model of a mechanical structure ($\mathbf{M}^a, \mathbf{K}^a, \mathbf{C}^a$) can be modified with an appropriate set of design parameter changes ($\Delta\mathbf{M}, \Delta\mathbf{K}, \Delta\mathbf{C}$) and these changes lead to an updated FE model whose predicted FRFs are identical to the corresponding measurements. On the basis of the FRF-based iterative algorithm, a method for finite element model updating and joint stiffness identification is proposed, as shown in Figure 2.

The FE model updating process starts from analytical modeling of the mechanical structure. The mass, stiffness, and damping matrices are used to represent the properties of each element and then assembled by nodes to obtain the systematical matrices ($\mathbf{M}^a, \mathbf{K}^a, \mathbf{C}^a$). For the complex systems, the joint dynamics between each substructure are usually unknown and initial dynamic parameters are given by experience. Analytical FRFs ($\mathbf{H}^a(\omega_j)$) of every node can then be predicted with the FE model. The experimental FRFs with high quality are the base to update FE models successfully. FRFs corresponding to translational DOFs can be measured directly with modal tests, while FRFs corresponding to rotational DOFs are calculated with the RC method.

After data is prepared, the analytical FRFs are compared with experimental data. The residual between the analytical and measured FRFs is calculated. As the initial FE model is usually not accurate, the 2-norm of the residual ($\|\epsilon\|$) is larger than the threshold e . Then the iterative procedure starts. The equation $\mathbf{S} \cdot \Delta\mathbf{u} = \epsilon$ is solved by using least square method. After the solution, the vector of updating parameters ($\Delta\mathbf{u}$) is obtained and the FE model is updated by adding $\Delta\mathbf{M}$, $\Delta\mathbf{K}$, and $\Delta\mathbf{C}$ to the original systematical matrices. Following an eigen-solution of the updated FE model, new analytical FRFs are obtained and the 2-norm of the residual FRFs is refreshed. This process of solving and updating the system is repeated until the 2-norm of the residual is zero or smaller than the defined threshold. Then the iterative process stops and a reliable FE model can be obtained. In the FE model updating procedure, the unknown dynamic characteristics of the joints are identified and the FE model is updated simultaneously.

3. Application

The coupled model of the machine-tool spindle system was presented by Cao and Altintas [2]. As shown in Figure 3, six bolts are used to fix the front part of the spindle, while a fastening ring is used between the rear part of the spindle and the spindle-head. The bolted connection was modeled by translational and rotational springs to represent the joint stiffness. The detailed procedure can be referred to Cao and Altintas's work [2].

In the coupled model of the machine-tool spindle system, the mass parameter of the joint surface is ignored and the damping parameter is determined by the experimental modal analysis method. Thus only the stiffness parameter

FIGURE 3: Coupled modeling of the machine-tool spindle system.

of the joint surface needs to be identified. The stiffness parameter of the joint surface between the spindle and the spindle-head is expressed as equivalent springs. In the following parts, the joint stiffness between the spindle and the spindle-head will be estimated systematically by using the proposed FE model updating method. Without considering the joint dynamics between the spindle and machine tool, the global stiffness matrix of the machine-tool spindle system is

$$\mathbf{K}^a = \begin{bmatrix} \mathbf{K}^S & 0 \\ 0 & \mathbf{K}^H \end{bmatrix}, \quad (13)$$

where $\mathbf{K}^S$ and $\mathbf{K}^H$ are the stiffness matrices of the spindle and the spindle-head, respectively, and there is no coupling relationship between $\mathbf{K}^S$ and $\mathbf{K}^H$ by now.

If the node i in the spindle substructure and the node j in the spindle-head substructure are connected and they have the same joint stiffness $\mathbf{K}^J$, then the global stiffness matrix can be rewritten as

$\mathbf{K}^x$

$$= \begin{bmatrix} \ddots & & & & & & \\ \dots & \mathbf{K}_i^S + \mathbf{K}^J & & & & & \\ \ddots & \vdots & \ddots & & & & \\ & & & \ddots & & & \\ & & & & \ddots & & \\ & & & & & \mathbf{K}_j^H + \mathbf{K}^J & \\ & & & & & \vdots & \ddots \end{bmatrix} \begin{matrix} i \\ \\ \\ \\ j \\ j \\ j \end{matrix}, \quad (14)$$

where $\mathbf{K}_i^S$ is the stiffness matrix of the node i in the spindle substructure, $\mathbf{K}_j^H$ is the stiffness term of the node j in

the spindle-head substructure, and $\mathbf{K}^J$ is the joint stiffness matrix which is expressed as

$$\mathbf{K}^J = \begin{bmatrix} k_{11}^J & k_{12}^J \\ k_{21}^J & k_{22}^J \end{bmatrix}. \quad (15)$$

The joint damping can be added in the same manner as the joint stiffness. However, due to the complexity of the nonlinear damping, the damping matrix is ignored when modeling the system. The modal damping ratios are obtained by experimental modal analysis.

In the coupled model, the two ends of the equivalent spring separately connect with the 40th node (substructure of the spindle) and the 51st node (substructure of the spindle-head); that is, $i = 40$, $j = 51$. Set the residual FRF $\varepsilon_{H_{1,1}}(\omega) = H_{1,1}^a(\omega) - H_{1,1}^x(\omega)$ of the translational DOF at the spindle nose, that is, the first node of the FE model, as the objective function of correcting algorithm. The parameters that need to be updated are listed as follows:

$$\Delta \mathbf{u} = \begin{Bmatrix} \Delta k_{2i-1,2i-1} \\ \Delta k_{2i,2i-1} \\ \Delta k_{2i,2i} \end{Bmatrix} = \begin{Bmatrix} \Delta k_{79,79} \\ \Delta k_{80,79} \\ \Delta k_{80,80} \end{Bmatrix} = \begin{Bmatrix} k_{11}^J \\ k_{21}^J \\ k_{22}^J \end{Bmatrix}, \quad (16)$$

where the stiffness parameter at the joint surface k_{11}^J is corresponding to the local stiffness of the translational DOF 79, k_{21}^J is corresponding to coupled stiffness between the rotational DOF 80 and translational DOF 79, and k_{22}^J is corresponding to the local stiffness of the rotational DOF 80. According to (4), the overdetermined equation of the updating algorithm of the spindle model is gained as

$$\begin{bmatrix} -H_{1,79}^a(\omega_1) H_{79,1}^x(\omega_1) & (H_{1,79}^a(\omega_1) - H_{1,80}^a(\omega_1))(H_{80,1}^x(\omega_1) - H_{79,1}^x(\omega_1)) & -H_{1,80}^a(\omega_1) H_{80,1}^x(\omega_1) \\ -H_{1,79}^a(\omega_2) H_{79,1}^x(\omega_2) & (H_{1,79}^a(\omega_2) - H_{1,80}^a(\omega_2))(H_{80,1}^x(\omega_2) - H_{79,1}^x(\omega_2)) & -H_{1,80}^a(\omega_2) H_{80,1}^x(\omega_2) \\ \vdots & \vdots & \vdots \\ -H_{1,79}^a(\omega_{N_p}) H_{79,1}^x(\omega_{N_p}) & (H_{1,79}^a(\omega_{N_p}) - H_{1,80}^a(\omega_{N_p}))(H_{80,1}^x(\omega_{N_p}) - H_{79,1}^x(\omega_{N_p})) & -H_{1,80}^a(\omega_{N_p}) H_{80,1}^x(\omega_{N_p}) \end{bmatrix} \quad (17)$$

$$\cdot \begin{Bmatrix} \Delta k_{79,79} \\ \Delta k_{80,79} \\ \Delta k_{80,80} \end{Bmatrix} = - \begin{Bmatrix} \varepsilon_{H_{1,1}}(\omega_1) \\ \varepsilon_{H_{1,1}}(\omega_2) \\ \vdots \\ \varepsilon_{H_{1,1}}(\omega_{N_p}) \end{Bmatrix},$$

where $H_{1,1}^x(\omega)$ is the experimental FRF of translational DOF at the free end (response DOF 1 and excitation DOF 1), $H_{79,1}^x(\omega)$ is the experimental FRF of the translational DOF at joint surface (response DOF 79 and excitation DOF 1), and $H_{80,1}^x(\omega)$ denotes the rotational DOF at the joint surface (response DOF 80 and excitation DOF 1). The frequency points $(\omega_1, \omega_2, \dots, \omega_{N_p})$ are chosen randomly from the frequency region of the FRF.

Utilize the RC technique to estimate the FRF of the rotational DOF $H_{80,1}^x(\omega)$, as shown in Figure 4. Point 1 locates the front end of the spindle and point 2 denotes the joint point between the spindle and the spindle-head. Three groups of hammer modal tests were conducted, respectively, to measure the origin FRF $g_{11,ff}$ of point 1, the FRF $g_{12,ff}$ between the measured point 1 and measured point 2, and the origin FRF $g_{22,ff}$ of point 2. The FRF matrices $h_{2,ff}$, $h_{2,ffM}$, $h_{2,Mf}$, and $h_{2,MM}$ can be obtained by solving (11), and then FRFs with respect to each rotational DOF can be calculated by using (9). In the FE model, point 1 is corresponding to the 1st node, while point 2 is corresponding to the 40th node. Thus, $H_{80,1}^x(\omega)$ is equivalent to the coupled FRF $g_{21,Mf}$ between point 1 and point 2; that is, excitation is the force f applied at point 1 (the 1st node) of the model and the response is the rotational displacement θ at point 2 (the 40th node). Lastly, $g_{21,Mf}$ can be obtained by using (12).

By using least squares algorithm to solve the over-determined equation (17) of the spindle model correcting algorithm, the residual error stabilized after 30 iterations. The variation tendencies of the updated parameters, including the translational stiffness k_{11}^J , coupled stiffness k_{12}^J , and the rotational stiffness k_{22}^J of the joint surface, are respectively depicted in Figures 5(a), 5(b), and 5(c).

Then the updated coupled model of the machine-tool spindle system is applied to simulate the FRFs at the spindle nose and the tool tip and the simulated results are compared with experimental results so as to verify the validity of the updated model. Figures 6(a) and 6(b), respectively, present the FRF tests procedure at the Weiss spindle nose and tool

tip. In this experiment, the output oil pressure of the bearing preload hydraulic oil pump was set to 100 psi, and the tool used was a simulation tool without cutter tooth. A Kistler 9722 hammer (sensitivity 2.13 mv/N) was used to excite the spindle nose and tool tip, and an accelerometer PCB353B11 (sensitivity 5.325 mv/g) was used to collect the vibration response signals. After obtaining the input and output signals, the experimental FRFs were calculated by utilizing the modal analysis software CutPro-MalTF.

The FRF at the spindle nose in radial direction is simulated and compared with the measured value, as shown in Figure 7. It can be seen that the FRF obtained by the updated model agrees well with the experimental result.

When the tool holder and the tool are installed on the spindle, the mode of the tool dominates the FRF. In the modeling, the Timoshenko beam elements were still used to build the FE model of the tool, assuming that the connection between the tool and tool holder was rigid. In this way, as shown in Figure 8, the FRF of the system is relatively clean. The simulated FRF has satisfactory agreements with the experimental result at the main modes. Thus, the updated FE model can be considered reliable to describe the dynamic behaviors of the spindle installed on the machine-tool and the proposed method achieves the purpose of updating the coupled model of the machine-tool spindle system.

4. Conclusions

In this work, a general FE model updating method has been proposed to update mechanical structure with unknown joints. The FRF-based iterative algorithm is improved to lower the requirement for the experimental FRFs and the RC technique is used to predict the FRFs of the rotational DOFs. The translational stiffness, rotational stiffness, and coupled stiffness of joints are identified after iterations, and the coupled model of the machine-tool spindle system is updated. The results show that the simulated FRFs at both the spindle nose and the tool tip match very well with measurements. With the updated model, it is possible to

FIGURE 4: The schematic diagram of the spindle modal test.

FIGURE 5: Iterative procedure of the updated parameters.

simulate the cutting performance of a high-speed spindle mounted on the machine tool.

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

Acknowledgments

The authors wish to express their heartfelt gratitude to Professor Yusuf Altintas from Manufacturing Automation Laboratory (MAL), The University of British Columbia. All the experiments of this paper were carried out in MAL. This work is jointly supported by National Natural Science

FIGURE 6: FRFs measurement.

FIGURE 7: Comparison of the FRF at the spindle nose between simulation and measurement.

FIGURE 8: Comparison of the FRF at the tool tip between simulation and measurement.

Foundation of China (no. 51421004), the National Science and Technology Major Project (2014ZX04001-191-01), and the Fundamental Research Funds for the Central University (CXTD2014001).

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