Research Article

Active Vibration Control of an Axially Translating Robot Arm with Rotating-Prismatic Joint Using Self-Sensing Actuator

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Active vibration control of an axially translating robot arm with rotating-prismatic joint using self-sensing actuator is investigated. The equations of the system are derived by Lagrange's equation with the assumed mode method. The displacement and velocity control law is used to configure the self-sensing actuator, which provides the active damping and stiffness effect to the structure. The numerical simulations reveal that the tip deflection of the arm can be effectively reduced by the self-sensing actuator. The amplitude of sensor voltage is inversely proportional to the length of axially translating arm. And higher feedback control gain results in lower sensor voltages and vibration amplitudes.

1. Introduction

Dynamics of elastic beam sliding in a prismatic joint is an important problem in many engineering applications, such as textile machines, automotive belts, band saws, robot arms, and elevator cables. The axial movement of the structures can lead to transverse vibration and will exert great effect on product quality and the safety and reliability of work processes. Therefore, they have received great attention.

Many research papers deal with the beam axially translating through a stationary prismatic joint. Wang and Wei [1] used Newton's approach in deriving the governing equation of an axially moving beam and utilized Galerkin's approximation technique in discretizing the elastic displacements. Stylianou and Tabarrok [2] employed finite element method to analyse an axially moving beam. They made the number of elements fixed while the sizes of the elements change with time. Fung et al. [3] applied Hamilton's principle to derive nonlinear partial differential equations of an axially moving beam with a tip mass. Zhu and Ni [4] studied the axially translating cantilever beam with an arbitrarily varying length. The dynamic stability of the continuous media relative to the inertial and moving coordinate systems is studied from the energy standpoint. Wang et al. [5, 6] used Hamilton's principle to investigate the axially translating elastic

Bernoulli-Euler and viscoelastic cantilever beam with time-variant velocity, respectively. Chang et al. [7] investigated the vibration and stability of an axially moving cantilever Rayleigh beam. The finite element method with variable-domain elements is used to derive the equations of motion and two kinds of axial motions including constant-speed extension deployment and back-and-forth periodical motion are considered. Ghaleh and Malaek [8] studied dynamics of deploying arms carrying tip mass by applying assumed modes method and employing the numerical solutions to find the arm responses.

The problem becomes more difficult if the beam is translating through a rotating-prismatic joint. For a translating and rotating beam, the interaction between rigid and flexible body motions is highly demanded. Gaultier and Cleghorn [9] used finite element method to analyze the vibration of a spatially translating and rotating beam to model elastic link manipulators. The internal and external damping effects were also included in the model. Yuh and Young [10] analyzed the dynamics of a beam experiencing a combination of rotational and translational motions. An approximation scheme was developed by using assumed modes method. The validity of the approximate model was evaluated by a series of experimental work. Dynamic response of the elastic beam undergoing various motions was investigated by computer

simulation. Al-Bedoor and Khulief [11] analyzed the dynamics of an elastic arm sliding in a rotating-prismatic joint by using finite element technique. In the finite element model all the inertia coupling terms were considered. Time dependent boundary conditions were used in order to account for the prismatic joint. The effect of an end mass was also taken into account in the model. And they [12] presented a dynamic model for the vibrational motion of an elastic beam with prismatic and revolute joints. The Lagrangian approach in conjunction with the assumed modes technique is employed in deriving the equations of motion. Basher [13] investigated the dynamic modeling of a single-link flexible beam having both rotational and translation motions and the effects of higher-order dynamics on the response of the beam. An analytical model of the beam, characterized by an infinite number of modes, was developed using Euler-Bernoulli beam equation and modal expansion method. The infinite-dimensional transcendental transfer function for the manipulator was formulated without modal approximation that was conveniently transformed into state-space form. Farid and Salimi [14] proposed an inverse dynamic approach to determine the required actuating torque and force for a planar flexible-link manipulator with revolute-prismatic joints such that its end-point follows a given trajectory. The formulation includes all of the nonlinear terms due to large rotation of the links. Kalyoncu [15] investigated the dynamic response of a flexible robot manipulator with rotating-prismatic joint. The tip end of the flexible robot manipulator traces a multi-straight-line path under the action of an external driving torque and an axial force.

Recently, piezoelectric materials have been more and more often applied to improve structural behavior. Such materials can actuate forces by electrical excitation, making them suitable as actuators for vibration control. Besides, they can sense deformation and generate a real-time voltage signal. Therefore they can be used as sensor as well. In most cases, a piezoelectric element performs a single transducer function: either sensing or actuation. Sun and Huang [16] developed an analytical formulation for modeling the behavior of laminated composite beams with integrated piezoelectric sensor and actuator. And the analytical solutions for active vibration control and suppression of smart laminated composite beams are presented. Raja et al. [17] derived a coupled finite element procedure to model the piezoelectric actuation in a smart sandwich beam. Chen et al. [18] considered a slender laminated composite beam with piezoelectric layers subjected to axial periodic compressive loads. The top and bottom piezoelectric layers act as actuator and sensor, respectively, and the influence of the feedback control gain on the response of the beam is evaluated. Lin and Nien [19] investigated the modeling and active vibration control of a smart beam using piezoelectric damping-modal actuators/sensors. Hu and Ng [20] developed an approach for active vibration control of flexible structures with integrated piezoelectric actuators using a robust vibration control method. Qiu et al. [21] used piezoelectric ceramic patches as sensors and actuators to suppress the vibration of the smart flexible clamped plate. Kumar and Narayanan [22] investigated the optimal placement of collocated piezoelectric

FIGURE 1: An axially translating robot arm with rotating-prismatic joint.

actuator-sensor pairs on flexible beams using a model-based linear quadratic regulator (LQR) controller. Song and Li [23] applied the piezoelectric actuator/sensor pairs to control the vibration of supersonic beams.

The concept of combining sensing and actuating features in the same device leads to the so-called self-sensing techniques [24]. In the self-sensing actuator configuration, the piezoelectric element is used simultaneously as both a sensor and an actuator. A self-sensing based system needs half the number of transducers normally necessary, making this approach appreciable for applications. Yellin and Shen [25] used the self-sensing technique to form a self-sensing active constrained layer damping treatment, which can improve the damping ability of the classical passive constrained layer. Gao and Liao [26] studied the active vibration control of simply supported beams with enhanced self-sensing active constrained layer damping treatments.

In this paper, we aim to study the active vibration control of an axially translating robot arm with rotating-prismatic joint using self-sensing actuator. Lagrange's equation with the assumed mode method is employed to derive the equations of motion. And the displacement and velocity control law is used to configure the self-sensing actuator.

2. Formulation of the Equations of Motion

An axially translating robot arm with rotating-prismatic joint is displayed in Figure 1. A tip mass m_{tip} is considered to be concentrated at the free end of the arm. The flexible arm slides in the prismatic joint under the action of axial force F . The sliding motion of the arm is assumed to be frictionless. The initial length of the arm is L_0 and the variation in the length of the arm is denoted by S . Torque T rotates prismatic joint about Y -axis and the prismatic joint is assumed to be rigid. The robot arm experiences a combination of rotational and translational gross motion. XYZ is the global reference frame, while xyz is the rotating and translating reference frame. Angle between the rotating and translating reference frame xyz and the global reference frame XYZ is denoted by θ . dm on the arm denotes an infinitesimal mass as seen in Figure 1.

The displacement of dm from the undeformed position in z direction is denoted by w . The piezoelectric patch as the self-sensing actuator is bonded on the top surface of the arm and positioned by the coordinates $x_1(t)$ and $x_2(t)$ in the reference frame xyz . The length of the piezoelectric patch is L_p . It is obvious that when the arm is axially translating, $x_1(t)$ and $x_2(t)$ change with time and can be expressed as $x_1(t) = x_1(0) + S$ and $x_2(t) = x_1(t) + L_p$, respectively. In this paper, we assume $0 \leq x_1(0) < x_2(0) \leq L_0$ and $0 \leq x_1(t) < x_2(t) \leq L(t)$ during the arm axial movement. h and h_p are the thicknesses of the arm and piezoelectric patch, respectively. The width of the arm and the piezoelectric patch are both b .

Euler-Bernoulli beam theory is employed to analyze the arm. Based on the Kirchhoff assumptions, the normal stress and strain of the arm in the axial direction are expressed as

$$\sigma_x = E\varepsilon_x, \quad (1a)$$

$$\varepsilon_x = -z \frac{\partial^2 w}{\partial x^2}, \quad (1b)$$

where E is the modulus of elasticity of the arm. Let the piezoelectric patch be polarized along the z direction. The axial stress, strain, and the electrical displacement of the piezoelectric patch can be expressed as

$$\sigma_x^p = c_{11}\varepsilon_x^p - e_{31}E_z, \quad (2a)$$

$$\varepsilon_x^p = -z \frac{\partial^2 w}{\partial x^2}, \quad (2b)$$

$$D_z = e_{31}\varepsilon_x^p + \epsilon_{33}E_z, \quad (2c)$$

where c_{11} is the elastic stiffness, D_z is the electrical displacement and $E_z = V_0(t)/h_p$ is the electrical field in the z direction [23], and $V_0(t)$ is the external applied voltage. ϵ_{33} is the dielectric constant and e_{31} is the piezoelectric constant.

The global position of an arbitrary material point p on the arm can be expressed as

$$R_p = A \cdot r_p, \quad (3)$$

where A is the rotational transformation matrix from the moving coordinate system xz to the fixed reference frame XZ and r_p is the location of the point p in the rotating coordinate system xz ; they can be written as

$$A = \begin{bmatrix} \cos \theta & -\sin \theta \\ \sin \theta & \cos \theta \end{bmatrix}, \quad (4a)$$

$$r_p = [x \quad w]^T. \quad (4b)$$

The velocity of an arbitrary material point p is

$$\dot{R}_p = A \cdot \dot{r}_p + \dot{\theta} \frac{dA}{d\theta} r_p, \quad (5)$$

where $\dot{\theta}$ is the angular velocity of the prismatic joint, and

$$\dot{r}_p = \begin{bmatrix} \dot{x} & \frac{dw}{dt} \end{bmatrix}^T = \begin{bmatrix} v & \frac{dw}{dt} \end{bmatrix}. \quad (6)$$

Derivative of w with respect to time is written as

$$\frac{dw}{dt} = \dot{w} + \dot{x}w'. \quad (7)$$

Substituting (4a), (4b), (6), and (7) into (5), we will arrive at

$$\dot{R}_p = \begin{bmatrix} (\dot{x} - w\dot{\theta}) \cos \theta - (x\dot{\theta} + \dot{w} + \dot{x}w') \sin \theta \\ (\dot{x} - w\dot{\theta}) \sin \theta + (x\dot{\theta} + \dot{w} + \dot{x}w') \cos \theta \end{bmatrix}. \quad (8)$$

The total kinetic energy of the system can be written as

$$K = \frac{1}{2} \rho A_b \int_0^L \dot{R}_p^T \dot{R}_p dx + \frac{1}{2} \rho_p A_p \int_{x_1}^{x_1+L_p} \dot{R}_p^T \dot{R}_p dx + \frac{1}{2} m_{\text{tip}} \dot{R}_p^T \dot{R}_p \Big|_{x=L} + \frac{1}{2} (J_b + J_p) \dot{\theta}^2, \quad (9)$$

where ρ and ρ_p are the mass density of the arm and the piezoelectric patch, $A_b = bh$ and $A_p = bh_p$ are the cross-sectional area of the arm and the piezoelectric patch, respectively, and J_b and J_p are the rotational inertia of the arm and the piezoelectric patch, respectively. By substituting for the value of $\dot{R}_p$ from (8) and making use of some trigonometric properties, the total kinetic energy of the system can be expressed as

$$K = \frac{1}{2} \rho A_b \int_0^L \bar{V}^2 dx + \frac{1}{2} \rho_p A_p \int_{x_1}^{x_1+L_p} \bar{V}^2 dx + \frac{1}{2} m_{\text{tip}} \bar{V}^2 \Big|_{x=L} + \frac{1}{2} (J_b + J_p) \dot{\theta}^2, \quad (10)$$

where

$$\bar{V}^2 = (\dot{x} - w\dot{\theta})^2 + (x\dot{\theta} + \dot{w} + \dot{x}w')^2. \quad (11)$$

The strain energy of the arm and the strain energy and electric potential energy of the piezoelectric patch can be written as

$$U_s = \frac{1}{2} \int_0^L \int_{-h/2}^{h/2} b \sigma_x \varepsilon_x dz dx + \frac{1}{2} \int_{x_1}^{x_1+L_p} \int_{h/2}^{h/2+h_p} b (\sigma_x^p \varepsilon_x^p - D_z E_z) dz dx. \quad (12)$$

Substituting (1a), (1b), (2a), (2b), and (2c) into (12), we can obtain

$$U_s = \frac{1}{2} \int_0^L EI_b (w'')^2 dx + \frac{1}{2} \int_{x_1}^{x_1+L_p} \left[EI_p (w'')^2 + 2B_1 V_0(t) w'' - B_2 V_0^2(t) \right] dx, \quad (13)$$

where

$$EI_b = \frac{Eb h^3}{12}, \quad EI_p = \frac{1}{3} c_{11} b \left[\left(\frac{h}{2} + h_p \right)^3 - \left(\frac{h}{2} \right)^3 \right], \quad (14)$$

$$B_1 = \frac{e_{31} b (h + h_p)}{2}, \quad B_2 = \frac{b \epsilon_{33}}{h_p}.$$

The gravitational potential energy for the arm, including the piezoelectric patch and the end mass, can be written as

$$U_g = \frac{1}{2} \rho A_b g L^2 \sin \theta + \rho_p A_p g L_p \left(x_1 + \frac{L_p}{2} \right) \sin \theta + m_{\text{tip}} g L \sin \theta. \quad (15)$$

The component of absolute acceleration in x direction can be written as

$$a_x = \ddot{x} - \dot{\theta}^2 x - \ddot{\theta} w. \quad (16)$$

And the component of gravitational acceleration in x direction is

$$g_x = -g \sin \theta. \quad (17)$$

Thus the axial inertial force is

$$F_x = \rho A_b \int_x^L (-a_x + g_x) dx + m_{\text{tip}} (-a_x|_{x=L} + g_x). \quad (18)$$

Because the piezoelectric patch is very thin, axial inertial force of the piezoelectric patch is neglected. Now, the potential energy due to axial shortening can be written as

$$U_x = \frac{1}{2} \int_0^L F_x (w')^2 dx. \quad (19)$$

The total potential energy of the system can be expressed as

$$U = U_s + U_g + U_x. \quad (20)$$

Lagrange's equation with the assumed mode method is used to determine the equation of motion of the structural system. To use the assumed mode method, the displacements of the arm can be expressed in terms of the generalized coordinates and displacement shape functions. According to [5, 8], the transverse displacement is written as

$$w(x, t) = \frac{1}{\sqrt{L(t)}} \sum_{i=1}^n \phi_i(x, t) \cdot q_i(t) = \frac{1}{\sqrt{L(t)}} [\phi] \cdot \{q\}^T, \quad (21)$$

where $[\phi] = [\phi_1, \dots, \phi_n]$, $\{q\} = \{q_1, \dots, q_n\}$, and

$$\begin{aligned} \phi_i &= \left[\cosh \left(\lambda_i \frac{x}{L} \right) - \cos \left(\lambda_i \frac{x}{L} \right) \right] \\ &\quad - \frac{\cosh(\lambda_i) + \cos(\lambda_i)}{\sinh(\lambda_i) + \sin(\lambda_i)} \left[\sinh \left(\lambda_i \frac{x}{L} \right) - \sin \left(\lambda_i \frac{x}{L} \right) \right], \\ &\quad i = 1, 2, \dots, n, \end{aligned} \quad (22)$$

where λ_i is obtained from the following characteristic equation:

$$1 + \cosh(\lambda_i) \cos(\lambda_i) = 0 \quad i = 1, 2, \dots, n. \quad (23)$$

Lagrange's equation is written as

$$\frac{d}{dt} \left(\frac{\partial \ell}{\partial \dot{q}} \right) - \frac{\partial \ell}{\partial q} = F, \quad \ell = K - U. \quad (24)$$

Substituting the energy expressions into (24), the equation of motion of the whole structural system is gotten:

$$[M] \{\ddot{q}^T\} + [C] \{\dot{q}^T\} + [K] \{q^T\} + \{Q\} + [F_V] V_0(t) = 0, \quad (25)$$

where

$$[M] = \rho A_b \int_0^L [\phi]^T [\phi] dx + \rho_p A_p \int_{x_1}^{x_1+L_p} [\phi]^T [\phi] dx + m_{\text{tip}} ([\phi]^T [\phi])|_{x=L},$$

$$[C] = \rho A_b \int_0^L [C_1] dx + \rho_p A_p \int_{x_1}^{x_1+L_p} [C_1] dx,$$

$$\begin{aligned} [K] &= EI_b \int_0^L \frac{d^2[\phi]^T}{dx^2} \frac{d^2[\phi]}{dx^2} dx \\ &\quad + EI_p \int_{x_1}^{x_1+L_p} \frac{d^2[\phi]^T}{dx^2} \frac{d^2[\phi]}{dx^2} dx \\ &\quad + \rho A_b \int_0^L [K_1] dx + \int_0^L F_x \frac{d[\phi]^T}{dx} \frac{d[\phi]}{dx} dx \\ &\quad + \rho_p A_p \int_{x_1}^{x_1+L_p} [K_1] dx + m_{\text{tip}} [K_1]|_{x=L}, \end{aligned}$$

$$[Q] = \rho A_b \int_0^L ([Q_1] + [Q_2]) dx + \rho_p A_p \int_{x_1}^{x_1+L_p} [Q_1] dx + m_{\text{tip}} [Q_1]|_{x=L},$$

$$F_V = B_1 \sqrt{L} \int_{x_1}^{x_1+L_p} \frac{d^2[\phi]^T}{dx^2} dx,$$

$$[C_1] = \frac{\dot{L}}{L} \left[\left(\frac{d[\phi]^T}{dx} [\phi] + [\phi]^T \frac{d[\phi]}{dx} \right) (L-x) - [\phi]^T [\phi] \right],$$

$$\begin{aligned} [K_1] &= \left(\frac{3\dot{L}^2}{4L^2} - \frac{\ddot{L}}{2L} - \dot{\theta}^2 \right) [\phi]^T [\phi] \\ &\quad + \left[\frac{\dot{L}^2}{2L^2} (9L+5x) + \frac{\ddot{L}}{L} (2L-x) \right] [\phi]^T \frac{d[\phi]}{dx} \\ &\quad - \frac{\dot{L}^2}{2L^2} (L-x) \frac{d[\phi]^T}{dx} [\phi] \\ &\quad + \frac{\dot{L}^2}{L^2} (2L-x)(L-x) [\phi]^T \frac{d^2[\phi]}{dx^2} \\ &\quad - \frac{\dot{L}^2}{L} (2L-x) \frac{d[\phi]^T}{dx} \frac{d[\phi]}{dx}, \end{aligned}$$

$$\begin{aligned} F_x &= \rho A_b \left[\frac{\dot{\theta}^2}{2} (L^2 - x^2) - (g \sin \theta + \ddot{L})(L-x) \right] \\ &\quad + m_{\text{tip}} (\dot{\theta}^2 L - g \sin \theta - \ddot{L}), \end{aligned}$$

FIGURE 2: Tip deflection of the axially translating beam when $L(t) = 1.8 + 0.3t$.

FIGURE 3: Tip deflection of the axially translating and rotating beam when $L(t) = 1.8 + 0.1t$, $\dot{\theta} = 0.785$ rad/s.

$$[Q_1] = \sqrt{L} \left[(2\dot{L}\dot{\theta} + x\ddot{\theta}) [\phi]^T - \dot{L}\dot{\theta}x \frac{d[\phi]^T}{dx} \right],$$

$$[Q_2] = \frac{3}{2\sqrt{L}} \ddot{\theta} \left[\rho A_b \int_0^L [\phi] dx + m_{\text{tip}} [\phi] \Big|_{x=L} \right] \times \left(\{q\} \frac{d[\phi]^T}{dx} \frac{d[\phi]}{dx} \{q\}^T \right).$$

(26)

FIGURE 4: Tip trajectory of the arm when $L(t) = 1 + 0.2t$, $\dot{\theta} = 0.2\pi$ rad/s.

Equation (25) characterizes the piezoelectric actuator driven under the external applied voltage $V_0(t)$. The matrix $[Q_2]$ represents the nonlinear inertia coupling between the beam reference motion and the local elastic deformations. Clearly, when the prismatic joint rotates with a uniform angular velocity we need to solve a linear problem.

3. Active Vibration Control

When the arm moves with deformation, the charge Q_s developed by structural deformation on the piezoelectric patch can be found by integrating the electric displacement D_z in (2b) over the electrode area A_p [23, 26]:

$$Q_s(t) = \int_{A_p} D_z dA_p = e_{31}b \int_{x_1(t)}^{x_2(t)} \left(-z_p \frac{\partial^2 w}{\partial x^2} \right) dx, \quad (27)$$

FIGURE 5: Tip deflection (a), sensor voltage (b), and control voltage (c) of the arm under different control gain when $L(t) = 1 + 0.2t$, $\dot{\theta} = 0.2\pi$ rad/s.

where z_p is the z coordinate of the mid-surface of the piezoelectric patch from the neutral surface of the arm. The output sensing voltage $V_s(t)$ can be obtained by dividing the charge by the sensor capacitance C_p [26]:

$$V_s(t) = \frac{Q_s(t)}{C_p}, \quad (28)$$

where

$$C_p = \frac{\epsilon_{33} A_p}{h_p}, \quad A_p = bL_p. \quad (29)$$

Substituting (27) into (28) we will get

$$V_s(t) = \frac{e_{31} h_p}{\epsilon_{33} L_p} \int_{x_1(t)}^{x_2(t)} \left(-z_p \frac{\partial^2 w}{\partial x^2} \right) dx. \quad (30)$$

FIGURE 6: Tip trajectory of the arm when $L(t) = 2 - 0.2t$, $\dot{\theta} = 0.2\pi$ rad/s.

The displacement and velocity feedback control law is considered and the feedback control voltage exerted on the piezoelectric patch is written as

$$\begin{aligned} V_0(t) &= -G_d \cdot V_s(t) - G_v \cdot \dot{V}_s(t) \\ &= \frac{e_{31}h_p z_p}{\epsilon_{33}L_p} \int_{x_1(t)}^{x_2(t)} \left[G_d \frac{\partial^2 w}{\partial x^2} + G_v \frac{d}{dt} \left(\frac{\partial^2 w}{\partial x^2} \right) \right] dx \\ &= \frac{e_{31}h_p (h + h_p)}{2\epsilon_{33}L_p} \\ &\quad \times \int_{x_1(t)}^{x_2(t)} \left\{ G_d \frac{\partial^2 w}{\partial x^2} + G_v \left[\frac{\partial}{\partial t} \left(\frac{\partial^2 w}{\partial x^2} \right) + \dot{L} \frac{\partial^3 w}{\partial x^3} \right] \right\} dx, \end{aligned} \quad (31)$$

where G_d and G_v are the displacement and velocity feedback control gains, respectively, for the piezoelectric patch. Substituting (21) into (31) results in

$$V_0(t) = [S_1] \{\dot{q}\}^T + [S_2] \{q\}^T, \quad (32)$$

where

$$\begin{aligned} [S_1] &= \frac{e_{31}h_p (h + h_p)}{2\epsilon_{33}L_p \sqrt{L}} G_v \int_{x_1}^{x_1+L_p} \frac{d^2 [\phi]}{dx^2} dx, \\ [S_2] &= \frac{e_{31}h_p (h + h_p)}{2\epsilon_{33}L_p \sqrt{L}} \\ &\quad \times \int_{x_1}^{x_1+L_p} \left\{ G_d \frac{d^2 [\phi]}{dx^2} \right. \end{aligned}$$

$$\begin{aligned} &+ G_v \frac{\dot{L}}{L} \left(\frac{d^3 [\phi]}{dx^3} (2L - x) \right. \\ &\quad \left. \left. - \frac{5}{2} \frac{d^2 [\phi]}{dx^2} \right) \right\} dx. \end{aligned} \quad (33)$$

Substituting (32) into (25), we can obtain

$$\begin{aligned} [M] \{\ddot{q}^T\} + ([C] + [F_V] [S_1]) \{\dot{q}^T\} \\ + ([K] + [F_V] [S_2]) \{q^T\} + \{Q\} = 0. \end{aligned} \quad (34)$$

The controlled response of the system can be gotten by solving (34).

4. Numerical Simulation

In order to validate the equations of motion in this paper, comparisons with the results available in the open literature are made. A uniform beam similar to that of [11, 12] is considered: mass per unit length $\rho A = 4.015$ kg/m, flexural rigidity $EI = 756.65$ Nm², translating velocity $v = 0.3$ m/s, initial length $L_0 = 1.8$ m, and initial tip deflection $w(L_0, 0) = -0.005$ m. Tip deflections of the axially translating beam are shown in Figure 2. It is observed that the tip deflection obtained by the present work is in good agreement with that of [11]. The tip deflections of the translating and rotating beam with $L(t) = 1.8 + 0.1t$ and $\dot{\theta} = 0.785$ rad/s are plotted in Figure 3, which reveals a reasonable agreement between our work and [12].

The following numerical examples are presented to study the active control of the axially translating arm with rotating-prismatic joint. The material of the arm is steel, where $E = 209$ GPa, $\rho = 7800$ kg/m³. The piezoelectric material is PZT-5H, where $c_{11} = 49$ GPa, $\rho_p = 7500$ kg/m³, $e_{31} = -6.5$ C/m², and $\epsilon_{33} = 1.3 \times 10^{-8}$ F/m. The dimensions of the system are

FIGURE 7: Tip deflection (a), sensor voltage, (b) and control voltage (c) of the arm under different control gain when $L(t) = 2 - 0.2t$, $\dot{\theta} = 0.2\pi$ rad/s.

FIGURE 8: Tip trajectory of the arm when $L(t) = 1 + \sin(0.2\pi t)$, $\dot{\theta} = 0.2\pi$ rad/s.

$L_p = 0.5$ m, $b = 0.2$ m, $h = 0.01$ m, and $h_p = 0.001$ m. The tip mass is $m_{\text{tip}} = 1$ kg. The equations of motion are solved by Newmark method.

First, the arm extending at a constant axial velocity is considered. The variation in the length of the arm is expressed as $S = vt$, and the constant velocity $v = 0.2$ m/s. The angular velocity and acceleration are $\dot{\theta} = 0.2\pi$ rad/s and $\ddot{\theta} = 0$, respectively. The initial conditions are $L_0 = 1$ m, $\theta(0) = 0$, $x_1(0) = 0$, $w(L_0, 0) = 0.001$ m/s, and $\dot{w}(L_0, 0) = 0$. Figure 4 shows the tip trajectory of the arm. The tip responses of the arm and the sensor and control voltages under different control gains are plotted in Figure 5. It is observed that the self-sensing actuator can reduce the tip deflection of the arm significantly. Moreover, higher feedback control gain results in lower vibration and sensor voltage amplitudes. But the feedback control gain cannot be increased enough to a certain value beyond which the actuator will provide a larger actuating force and it will have a side effect on the vibration system [23]. It is also observed that the sensor voltage amplitude will be reduced with the extension of the arm. This is because the elastic strain of the arm section where the piezoelectric patch is placed decreases during the extension of the arm.

Next, the arm retracting at a constant axial velocity is evaluated. The variation in the length of the arm is expressed as $S = vt$, and the constant velocity $v = -0.2$ m/s. We set $L_0 = 2$ m, $x_1(0) = 1$ m, and other parameters are the same as previous. The tip trajectory of the arm is displayed in Figure 6. Figure 7 shows the time history of tip deflection of the arm and the sensor and control voltages under different control gains. Clearly, the self-sensing actuator can reduce the tip deflection of the arm and higher feedback control gain results in lower vibration and sensor voltage amplitudes. It is also observed that the sensor voltage amplitude will be increased with the retraction of the arm in the absence of active control. This is because the elastic strain of the arm section where the piezoelectric patch is placed increases during the retraction of the arm.

The arm travelling with harmonic length variation is also considered. The variation in the length of the arm is expressed as $S = \sin(0.2\pi t)$. The tip trajectory of the arm is displayed in Figure 8. In this simulation, the arm experiences an extension and subsequent retraction mode. It can be observed from Figure 9(a) that the self-sensing actuator works well and increasing the control gain can enhance the performance of the controller. We can also find that the amplitude of the sensor voltage is inversely proportional to the length of the arm in the absence of active control. This is because the elastic strain of the arm section where the piezoelectric patch is placed decreases during the arm extension mode while it increases during the arm retraction mode.

5. Conclusions

In the present paper, active vibration control of an axially translating robot arm with rotating-prismatic joint using self-sensing actuator is investigated. The equations of the system are derived by Lagrange's equation with the assumed mode method. When the prismatic joint rotates with a uniform angular velocity, we only need to solve a linear problem. Some numerical examples are employed to evaluate the effect of the self-sensing actuator. It is found that the tip deflection of the arm can be effectively reduced by the self-sensing actuator. We can also find that the amplitude of sensor voltage is inversely proportional to the length of the arm. This is because the elastic strain of the arm section where the piezoelectric patch is placed decreases during the arm in extension mode while it increases during the arm in retraction mode. Furthermore, higher feedback control gain results in lower vibration and sensor voltage amplitudes.

In the further research work, the shear and torsional deformations in the Timoshenko beam theory should be considered. Further studies are also needed to account for nonlinear dynamic formulations involving large motion.

FIGURE 9: Tip deflection (a), sensor voltage, (b) and control voltage (c) of the arm under different control gain when $L(t) = 1 + \sin(0.2\pi t)$, $\dot{\theta} = 0.2\pi$ rad/s.

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

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