Research Article

Effects of the van der Waals Force on the Dynamics Performance for a Micro Resonant Pressure Sensor

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The micro resonant pressure sensor outputs the frequency signals where the distortion does not take place in a long distance transmission. As the dimensions of the sensor decrease, the effects of the van der Waals forces should be considered. Here, a coupled dynamic model of the micro resonant pressure sensor is proposed and its coupled dynamic equation is given in which the van der Waals force is considered. By the equation, the effects of the van der Waals force on the natural frequencies and vibration amplitudes of the micro resonant pressure sensor are investigated. Results show that the natural frequency and the vibrating amplitudes of the micro resonant pressure sensor are affected significantly by van der Waals force for a small clearance between the film and the base plate, a small initial tension stress of the film, and some other conditions.

1. Introduction

Microelectromechanical Systems (MEMS) have advantages such as compact structure, low cost, small power loss, high response speed, and high accuracy [1, 2]. The micro resonant sensor is more attractive because it outputs the frequency signals where distortion error does not occur and it is suitable for the distant range transmission.

In 1990-1991, the micro resonant pressure sensors with heat excitation and electromagnetic excitation were developed [3, 4]. In this year, a micro resonant pressure sensor with electrostatic excitation was proposed which can achieve an accuracy of 100 ppm at the temperature range of the automobile operation [5]. A new alloy based on transition metals was developed to obtain large amplitude of the resonant longitudinal magnetoelastic waves which can be used to improve performance of the micro resonant sensor [6]. A type of NEMS double Si₃N₄ resonant beams pressure sensor was presented and the sensitivity of the sensor is getting to 498.24 Hz/kPa [7]. An electrothermally excited dual beams silicon resonant pressure sensor with temperature compensation was proposed and the experimental results indicate that the maxim residual error is 1.8 kPa in the working temperature range from -40 to 60°C [8]. The non-linear dynamics of a resonant silicon bridge pressure sensor

with electrothermal excitation was investigated in which the measured pressure, the heating effect of the electrothermal excitation, and the residual internal force in the bridge were considered [9]. The sensitivity of the micro resonant pressure sensor with docks was investigated [10]. A novel resonant pressure sensor with an improved micromechanical double-ended tuning fork resonator packaged in dry air at atmospheric pressure was presented in which the fundamental frequency of the resonant pressure sensor is approximately 34.55 kHz with a pressure sensitivity of 20.77 Hz/kPa [11]. A new stress isolation method based resonant pressure sensor was presented to minimize thermal stresses arising from device packaging due to thermal mismatches between the silicon sensor body and its housing materials [12].

In a word, a number of studies about micro resonant pressure sensor have been done. However, in a micro resonant pressure sensor, as the clearance between the resonant film and basement is small enough, the effects of the van der Waals force will become obvious.

The influence of surface effects on the pull-in instability of a cantilever nanoactuator was investigated incorporating the influence of the Casimir attraction and van der Waals force [13, 14]. Besides it, the effects of the molecular forces on the free vibration of electromechanical integrated electrostatic harmonic actuator were studied as well [15].

FIGURE 1: Multifields coupled dynamic model of a micro film.

Above-mentioned studies mainly focus on dynamics performance of the micro ring (see [15]) and micro beam (other references) which is only controlled by one partial differential equation. In our micro resonant pressure sensor, the resonator is the micro film in which the dynamics performance is controlled by two partial differential equations. So, the effect problem of the van der Waals force on the dynamics performance of a micro resonant pressure sensor is more complicated and has not been resolved yet.

In this paper, a coupled dynamic model of the micro resonant pressure sensor is proposed and its coupled dynamic equation is given in which the van der Waals force is considered. Using these equations, the effects of the van der Waals force on the natural frequencies and vibration amplitudes of the micro resonant pressure sensor are investigated. Results show that the natural frequency and the vibrating amplitudes of the micro resonant pressure sensor are affected significantly by van der Waals force for a small clearance between the film and the base plate, a small initial tension stress of the film, and some other conditions. These results can be used to improve design about dynamics performance for the micro resonant pressure sensor.

2. Vibration Equations

Figure 1 illustrates a micro film in the micro resonant pressure sensor. The electrostatic force and the van der Waals force are applied to the micro film. Its boundary condition is that two sides are fixed and two sides are free. Here, the dynamics partial differential equation of the film under uniform tension is not applicable. Two dynamics partial differential equations of the orthotropic film can be used. The vibration equations of the orthotropic micro film are [16]

$$\rho h \frac{\partial^2 w}{\partial t^2} - \left(\sigma_{0x} + \frac{\partial^2 \varphi}{\partial y^2} \right) h \frac{\partial^2 w}{\partial x^2} - \left(\sigma_{y0} + \frac{\partial^2 \varphi}{\partial x^2} \right) h \frac{\partial^2 w}{\partial y^2} = q, \quad (1a)$$

$$\frac{1}{E_1} \frac{\partial^4 \varphi}{\partial y^4} + \frac{1}{E_2} \frac{\partial^4 \varphi}{\partial x^4} = \left(\frac{\partial^2 w}{\partial x \partial y} \right)^2 - \frac{\partial^2 w}{\partial x^2} \frac{\partial^2 w}{\partial y^2}, \quad (1b)$$

where ρ is the mass density of the film, t is the time, v is the initial clearance between the micro film and base plate, U is the voltage between the micro film and base plate, w is the transverse displacement of the film, h is the thickness of the film, φ is the stress function, σ_{0x} is the initial tension stress in x direction of the film, σ_{y0} is the initial tension stress in y direction of the film, x is the coordinate in the film length direction, y is the coordinate in the film width direction, E is the modulus of elasticity of the micro film material, and $q(x, y, t)$ is the transverse load per unit area on the film.

The electrostatic force per unit area is

$$q_e = \frac{F_e}{A} = \frac{\epsilon_0 \epsilon_r}{2} \left(\frac{U}{v - w} \right)^2, \quad (2)$$

where F_e is the electrostatic force applied to the micro film, A is the area of the micro film, ϵ_0 is permittivity constant of free space, and ϵ_r is relative dielectric constant of the insulating layer.

The van der Waals force per unit area between the film and fixed plate is

$$q_{r3} = \frac{A}{6\pi(v - w)^3}, \quad (3)$$

where A is the Hamaker constant: $A = 10^{-19}$ J.

The damping force per unit area from air is

$$q_c = c_a \frac{\partial w}{\partial t}. \quad (4)$$

Here, c_a is the damping coefficient of the gas.

Thus, the total force per unit area on the film is

$$q(x, y, t) = q_e + q_{r3} + q_c. \quad (5)$$

The displacement w of the micro film consists of a static component w_s and a dynamic one Δw :

$$w = w_s + \Delta w. \quad (6)$$

The load $q(x, y, t)$ of static (q_s) and dynamic (Δq) components is

$$q(x, y, t) = q_s + \Delta q, \quad (7)$$

$$q_s = \frac{U_0^2 \epsilon_0 \epsilon_r}{2(\nu - w_s)^2} + \frac{A}{6\pi(\nu - w_s)^3}. \quad (8)$$

From $\Delta q = (dq/dw)\Delta w$, we know that

$$\Delta q = \frac{U_0^2 \epsilon_0 \epsilon_r}{(\nu - w_s)^3} \Delta w + \frac{A}{2\pi(\nu - w_s)^4} \Delta w - c_a \frac{\partial \Delta w}{\partial t}, \quad (9)$$

where $c_a = \mu b^3/(\nu - w_s)^3$ and μ is the gas viscosity: $\mu = 1.86 \times 10^{-5} \text{ N}\cdot\text{S}\cdot\text{m}^{-2}$.

Substituting (6) and (7) into (1a) and (1b) yields the following equations:

$$\left(\sigma_{0x} + \frac{\partial^2 \varphi}{\partial y^2} \right) h \frac{\partial^2 w_s}{\partial x^2} - \left(\sigma_{y0} + \frac{\partial^2 \varphi}{\partial x^2} \right) h \frac{\partial^2 w_s}{\partial y^2} = q_s, \quad (10a)$$

$$\frac{1}{E_1} \frac{\partial^4 \varphi}{\partial y^4} + \frac{1}{E_2} \frac{\partial^4 \varphi}{\partial x^4} = \left(\frac{\partial^2 w_s}{\partial x \partial y} \right)^2 - \frac{\partial^2 w_s}{\partial x^2} \frac{\partial^2 w_s}{\partial y^2}, \quad (10b)$$

$$\rho h \frac{\partial^2 \Delta w}{\partial t^2} - \left(\sigma_{0x} + \frac{\partial^2 \Delta \varphi}{\partial y^2} \right) h \frac{\partial^2 \Delta w}{\partial x^2} - \left(\sigma_{y0} + \frac{\partial^2 \Delta \varphi}{\partial x^2} \right) h \frac{\partial^2 \Delta w}{\partial y^2} = \Delta q, \quad (11a)$$

$$\frac{1}{E_1} \frac{\partial^4 \Delta \varphi}{\partial y^4} + \frac{1}{E_2} \frac{\partial^4 \Delta \varphi}{\partial x^4} = \left(\frac{\partial^2 \Delta w}{\partial x \partial y} \right)^2 - \frac{\partial^2 \Delta w}{\partial x^2} \frac{\partial^2 \Delta w}{\partial y^2}. \quad (11b)$$

3. Free Vibration

For a micro pressure sensor, two ends of the film are fixed and other two ends of the film are free; the boundary conditions are

$$\begin{aligned} \Delta w(0, y, t) = \Delta w(a, y, t) = 0, \\ \left. \frac{\partial \Delta w}{\partial y} \right|_{y=0} = \left. \frac{\partial \Delta w}{\partial y} \right|_{y=b} = 0. \end{aligned} \quad (12)$$

For the boundary conditions, the solutions of (11a) and (11b) can be given as

$$\Delta w(x, y, t) = W(x, y) T(t), \quad (13a)$$

$$\Delta \varphi(x, y, t) = \phi(x, y) T^2(t). \quad (13b)$$

Here,

$$W(x, y) = A \sin \frac{m\pi x}{a} \cos \frac{n\pi y}{b}. \quad (13c)$$

Thus

$$\Delta w(x, y, t) = A \sin \frac{m\pi x}{a} \cos \frac{n\pi y}{b} T(t). \quad (13d)$$

Substituting (13b) and (13d) into (11b) yields

$$\begin{aligned} \frac{1}{E_1} \frac{\partial^4 \phi}{\partial y^4} + \frac{1}{E_2} \frac{\partial^4 \phi}{\partial x^4} \\ = \frac{m^2 n^2 \pi^4}{2a^2 b^2} \left(\cos \frac{2m\pi x}{a} - \cos \frac{2n\pi y}{b} \right). \end{aligned} \quad (14)$$

Letting $\phi(x, y) = \alpha \cos(2m\pi x/a) - \beta \cos(2n\pi y/b)$, and substituting it into (14), yields

$$\begin{aligned} \alpha &= \frac{E_2 n^2 a^2}{32 m^2 b^2}, \\ \beta &= \frac{E_1 m^2 b^2}{32 n^2 a^2}. \end{aligned} \quad (15)$$

Substituting it into (13b) yields

$$\begin{aligned} \Delta \varphi(x, y, t) \\ = \left(\frac{E_2 n^2 a^2}{32 m^2 b^2} \cos \frac{2m\pi x}{a} - \frac{E_1 m^2 b^2}{32 n^2 a^2} \cos \frac{2n\pi y}{b} \right) T^2(t). \end{aligned} \quad (16)$$

Letting $E = U_0^2 \epsilon_0 \epsilon_r / (\nu - w_0)^3 + A / 2\pi(\nu - w_0)^4$ (here, w_0 is the average static displacement of the micro film) and substituting (13b), (13d), and (9) into (11a) and using Galerkin method yield

$$\begin{aligned} \ddot{T} + \left[\frac{\pi^2}{\rho} \left(\frac{m^2}{a^2} \sigma_{0x} + \frac{n^2}{b^2} \sigma_{y0} \right) - \frac{E}{\rho h} \right] T + \frac{C_a}{\rho h} \dot{T} \\ + \frac{3\pi^4}{16\rho} \left(\frac{E_1 m^4}{a^4} - \frac{E_2 n^4}{b^4} \right) T^3 = 0. \end{aligned} \quad (17)$$

Letting $\varepsilon = h^2/ab$, (17) can be changed into the following form:

$$\ddot{T} + \omega_0^2 T = \varepsilon (\alpha_1 T^3 + \alpha_2 \dot{T}), \quad (18)$$

where $\omega_0^2 = (\pi^2/\rho)((m^2/a^2)\sigma_{0x} + (n^2/b^2)\sigma_{y0}) - E/\rho h$, $\alpha_1 = -(3\pi^4 ab/16\rho h)(E_1 m^4/a^4 - E_2 n^4/b^4)$, and $\alpha_2 = -abC_a/\rho h^2$.

Letting $f(T, \dot{T}) = (\alpha_1 T^3 + \alpha_2 \dot{T})$ and $\psi = \omega_0 t + \theta$, if $\varepsilon = 0$, then (18) can be changed into the following form:

$$\frac{dT}{dt} = -a\omega_0 \sin \psi. \quad (19)$$

Letting $T = a \cos \psi$, here a and ψ are the function of the time. Thus

$$\frac{dT}{dt} = \frac{da}{dt} \cos \psi - a \left(\omega_0 + \frac{d\theta}{dt} \right) \sin \psi. \quad (20)$$

From (19) and (20), we can give

$$\frac{da}{dt} \cos \psi - a \frac{d\theta}{dt} \sin \psi = 0, \quad (21)$$

$$\frac{d^2 T}{dt^2} = -\frac{da}{dt} \omega_0 \sin \psi - a\omega_0 \left(\omega_0 + \frac{d\theta}{dt} \right) \cos \psi. \quad (22)$$

Substituting $T = a \cos \psi$ and (22) into (18) yields

$$\begin{aligned} & -\frac{da}{dt} \omega_0 \sin \psi - a\omega_0 \frac{d\theta}{dt} \cos \psi \\ & = \varepsilon f(a \cos \psi, -a\omega_0 \sin \psi). \end{aligned} \quad (23)$$

From (21) and (23), we can give

$$\frac{da}{dt} = -\frac{\varepsilon}{\omega_0} f(a \cos \psi, -a\omega_0 \sin \psi) \sin \psi, \quad (24a)$$

$$\frac{d\theta}{dt} = -\frac{\varepsilon}{a\omega_0} f(a \cos \psi, -a\omega_0 \sin \psi) \cos \psi. \quad (24b)$$

The right parts of (24a) and (24b) can be written in Fourier series:

$$\begin{aligned} & f(a \cos \psi, -a\omega_0 \sin \psi) \sin \psi \\ & = A_0(a) + \sum_{n=1}^{\infty} (A_n \cos n\psi + B_n \sin n\psi), \end{aligned} \quad (25a)$$

$$\begin{aligned} & f(a \cos \psi, -a\omega_0 \sin \psi) \cos \psi \\ & = C_0(a) + \sum_{n=1}^{\infty} (C_n \cos n\psi + D_n \sin n\psi), \end{aligned} \quad (25b)$$

TABLE 1: Parameters of the micro film.

a (mm)	b (mm)	h (μm)	ε_0 ($\text{C}^2 \cdot \text{N}^{-1} \cdot \text{m}^{-2}$)	ν (μm)	E (GPa)	ρ (kg/m^3)
2	1	5	8.85×10^{-12}	0.5	190	2330

where

$$\begin{aligned} A_0(a) &= \frac{1}{2\pi} \int_0^{2\pi} \sin \psi f(a \cos \psi, -a\omega_0 \sin \psi) d\psi \\ &= -\frac{1}{2} \alpha_2 a \omega_0, \\ C_0(a) &= \frac{1}{2\pi} \int_0^{2\pi} \cos \psi f(a \cos \psi, -a\omega_0 \sin \psi) d\psi \\ &= \frac{3}{8} \alpha_1 a^3. \end{aligned} \quad (26)$$

In (25a) and (25b), only the average values $A_0(a)$ and $C_0(a)$ are kept. Substituting (25a) and (25b) into (24a) and (24b) yields

$$\frac{da}{dt} = -\frac{\varepsilon}{\omega_0} A_0(a) = \frac{1}{2} \varepsilon \alpha_2 a, \quad (27a)$$

$$\frac{d\theta}{dt} = -\frac{\varepsilon}{a\omega_0} C_0(a) = -\frac{3}{8} \frac{\varepsilon \alpha_1 a^2}{\omega_0}, \quad (27b)$$

$$\frac{d\psi}{dt} = \omega_0 - \frac{\varepsilon}{a\omega_0} C_0(a) = \omega_0 - \frac{3}{8} \frac{\varepsilon \alpha_1 a^2}{\omega_0}. \quad (27c)$$

From (27a), (27b), and (27c), we know that

$$a = a_0 e^{\alpha_2 \varepsilon t / 2}, \quad (28a)$$

$$\psi = \left(\omega_0 - \frac{3\alpha_1 a_0^2 \varepsilon}{8\omega_0} \right) t + \psi_0, \quad (28b)$$

where a_0 is the initial displacement and ψ_0 is the initial phase. Letting $\psi_0 = 0$, we obtain

$$T(t) = a_0 e^{\alpha_2 \varepsilon t / 2} \cos \left[\left(\omega_0 - \frac{3\alpha_1 a_0^2 \varepsilon}{8\omega_0} \right) t \right], \quad (29a)$$

$$\omega = \omega_0 - \frac{3\alpha_1 a_0^2 \varepsilon}{8\omega_0}. \quad (29b)$$

Substituting (29a) and (29b) into (13d) yields

$$\begin{aligned} w(x, y, t) &= \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} \sin \frac{m\pi x}{a} \cos \frac{n\pi y}{b} \\ &\cdot \left\{ a_0 e^{\alpha_2 \varepsilon t / 2} \cos \left[\left(\omega_0 - \frac{3\alpha_1 a_0^2 \varepsilon}{8\omega_0} \right) t \right] \right\}. \end{aligned} \quad (30)$$

4. Results and Discussions

Above equations are utilized for the free vibration analysis of the micro resonant pressure sensor. The parameters of the numerical example are shown in Table 1 (here,

TABLE 2: Natural frequency of the micro pressure sensor for various clearances (rad/s).

	ω	ω_f	η (%)		ω	ω_f	η (%)
$\nu = 5.0 \times 10^{-7}$ m							
Mode (1, 1)	28163.51	24990.48	11.27	Mode (3, 3)	207730.60	207324.24	0.196
Mode (2, 2)	129142.32	128487.67	0.507	Mode (4, 4)	283224.23	282926.33	0.105
$\nu = 5.5 \times 10^{-7}$ m							
Mode (1, 1)	49598.20	48671.80	1.868	Mode (3, 3)	211704.51	211489.39	0.102
Mode (2, 2)	135442.01	135105.52	0.248	Mode (4, 4)	286151.65	285992.53	0.056
$\nu = 6.0 \times 10^{-7}$ m							
Mode (1, 1)	58272.73	57813.02	0.789	Mode (3, 3)	213903.08	213728.30	0.082
Mode (2, 2)	138853.40	138661.11	0.138	Mode (4, 4)	287782.02	287689.29	0.032

TABLE 3: Natural frequency of the micro pressure sensor for various initial tension stresses (rad/s).

	ω	ω_f	η (%)		ω	ω_f	η (%)
$\sigma_{0x} = 4.5 \times 10^3$ KN/m							
Mode (1, 1)	16238.77	9748.79	39.966	Mode (3, 3)	195925.07	195494.18	0.220
Mode (2, 2)	120663.99	119963.07	0.581	Mode (4, 4)	267851.08	267536.05	0.118
$\sigma_{0x} = 5.0 \times 10^3$ KN/m							
Mode (1, 1)	28163.51	24990.48	11.27	Mode (3, 3)	207730.60	207324.24	0.196
Mode (2, 2)	129142.32	128487.67	0.507	Mode (4, 4)	283224.23	282926.33	0.105
$\sigma_{0x} = 5.5 \times 10^3$ KN/m							
Mode (1, 1)	36368.50	33970.71	6.594	Mode (3, 3)	218900.35	218514.77	0.176
Mode (2, 2)	137097.33	136480.84	0.450	Mode (4, 4)	297804.85	297521.54	0.095

$a_0 = 2 \times 10^{-7}$ m, $U_0 = 0.4$ V, and $\sigma_{y0} = 0$). Natural frequencies of the micro resonant pressure sensor under various clearances are shown in Table 2 (here, $\sigma_{0x} = 5.0 \times 10^3$ KN/m). Natural frequencies of the micro resonant pressure sensor under various initial tension stresses are shown in Table 3 (here, $\nu = 5.0 \times 10^{-7}$ m). ω is the natural frequency of the sensor without considering the van der Waals force, ω_f is the natural frequency of the sensor with considering the van der Waals force, and η is the relative error between them. From Tables 2 and 3, the following observations are worth noting.

(1) Without considering the van der Waals force, the natural frequencies of the micro sensor are larger than those in the case of considering the van der Waals force. It is because the van der Waals force system is equivalent to a soft spring system. The van der Waals force can cause decrease of the natural frequencies of the micro sensor.

(2) The deviation between the natural frequencies of the micro sensor with and without the van der Waals force decreases with increasing the order number of the mode. For mode (1, 1), the relative error between the natural frequencies is 11.27% ($\nu = 5.0 \times 10^{-7}$ m). For mode (4, 4), the relative error

between the natural frequencies is 0.1% ($\nu = 5.0 \times 10^{-7}$ m). It shows that influence of the van der Waals force on the natural frequencies decreases with increasing the order number of the mode.

(3) At an initial tension stress, the natural frequency of the sensor increases significantly with increasing the clearance between the film and the base plate.

If the clearance grows, the deviation between the natural frequencies of the sensor with and without the van der Waals force drops. At $\nu = 5.0 \times 10^{-7}$ m, the relative error between the natural frequencies with and without the van der Waals force is 11.27% for mode (1, 1). At $\nu = 6.0 \times 10^{-7}$ m, the relative error between the natural frequencies with and without the van der Waals force is 0.79%.

(4) At a constant clearance between the film and the base plate, the natural frequency of the sensor increases significantly with increasing the film tension. As the film tension grows, the deviation between the natural frequencies of the sensor with and without the van der Waals force decreases. At $\sigma_{0x} = 4.5 \times 10^3$ KN/m, the relative error between the natural frequencies with and without the van der Waals force is 39.97% for mode (1, 1). At $F = \sigma_{0x} = 5.5 \times 10^3$ KN/m,

FIGURE 2: Effects of the van der Waals force on vibrating amplitude of the film center for various a .

the relative error between the natural frequencies with and without the van der Waals force is 6.59% for mode (1, 1).

Hence, the effects of the van der Waals force on the natural frequency of the micro resonant pressure sensor should be considered for a small clearance between the film and the base plate, a small initial tension stress of the film, and a low order mode of the vibrations.

The effects of the van der Waals force on the vibrating amplitudes of the film center are investigated for mode (1, 1)

and various system parameters (see Figures 2–6). They show the following.

(1) Due to the effects of the damping, the free vibration of the film center is periodic vibration with the amplitude decay. As the length a of the micro film grows, the vibrating amplitudes of the film center drop more rapidly with the time. When the van der Waals force is not considered, the vibrating amplitudes of the film center drop more rapidly with time compared to the case when the van der Waals force

FIGURE 3: Effects of the van der Waals force on vibrating amplitude of the film center for various b .

is considered. As the length a of the micro film grows, the effects of the van der Waals force on the vibrating amplitudes of the film center become large.

(2) As the width b of the micro film grows, the vibrating amplitudes of the film center drop. However, the amplitude decay does not occur. When the van der Waals force is not considered, the vibrating amplitudes of the film center are larger than those in the case when the van der Waals force is

considered. As the width b of the micro film grows, the effects of the van der Waals force on the vibrating amplitudes of the film center become small.

(3) As the thickness h of the micro film grows, the vibrating amplitudes of the film center drop more slowly with time. When the van der Waals force is not considered, the vibrating amplitudes of the film center drop first more rapidly and then more slowly with time compared to those in the case

FIGURE 4: Effects of the van der Waals force on vibrating amplitude of the film center for various h .

when the van der Waals force is considered. As the thickness h of the micro film grows, the effects of the van der Waals force on the vibrating amplitudes of the film center first become small and then become large.

(4) As the initial tension stress of the micro film grows, the vibrating amplitudes of the film center drop more slowly with time. When the van der Waals force is not considered, the vibrating amplitudes of the film center drop first more

rapidly and then more slowly with time compared to those in the case when the van der Waals force is considered. As the initial tension stress of the micro film grows, the effects of the van der Waals force on the vibrating amplitudes of the film center first become small and then become large.

(5) As the voltage between the micro film and the back plate grows, the vibrating amplitudes of the film center drop more rapidly with time. When the van der Waals force is

FIGURE 5: Effects of the van der Waals force on vibrating amplitude of the film center for various σ_{0x} .

not considered, the vibrating amplitudes of the film center drop more rapidly with time compared to those in the case when the van der Waals force is considered. As the voltage grows, the effects of the van der Waals force on the vibrating amplitudes of the film center become large.

In a word, the effects of the van der Waals force on the vibrating amplitudes of the film become large under some conditions which include a large voltage between the micro film and the back plate, a large film length and thickness,

a small film width, and a small initial tension stress of the micro film.

5. Conclusions

In this paper, a coupled dynamic model of the micro resonant pressure sensor is proposed and its coupled dynamic equation is given in which the van der Waals force is considered. By the equation, the effects of the van der Waals force on the natural

FIGURE 6: Effects of the van der Waals force on vibrating amplitude of the film center for various U_0 .

frequencies and vibration amplitudes of the micro resonant pressure sensor are investigated. Results show the following.

(1) The effects of the van der Waals force on the natural frequency of the micro resonant pressure sensor should be considered for a small clearance between the film and the base plate, a small initial tension stress of the film, and a low order mode of the vibrations.

(2) The effects of the van der Waals force on the vibrating amplitudes of the film should be considered for a large voltage

between the micro film and the back plate, a large film length and thickness, a small film width, and a small initial tension stress of the micro film.

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

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