Research Article

An Enhanced Plane Wave Expansion Method to Solve Piezoelectric Phononic Crystal with Resonant Shunting Circuits

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An enhanced plane wave expansion (PWE) method is proposed to solve piezoelectric phononic crystal (PPC) connected with resonant shunting circuits (PPC-C), which is named as PWE-PPC-C. The resonant shunting circuits can not only bring about the locally resonant (LR) band gap for the PPC-C but also conveniently tune frequency and bandwidth of band gaps through adjusting circuit parameters. However, thus far, more than one-dimensional PPC-C has been studied just by Finite Element method. Compared with other methods, the PWE has great advantages in solving more than one-dimensional PC as well as various lattice types. Nevertheless, the conventional PWE cannot accurately solve coupling between the structure and resonant shunting circuits of the PPC-C since only taking one-way coupling from displacements to electrical parameters into consideration. A two-dimensional PPC-C model of orthorhombic lattice is established to demonstrate the whole solving process of PWE-PPC-C. The PWE-PPC-C method is validated by Transfer Matrix method as well as Finite Element method. The dependence of band gaps on circuit parameters has been investigated in detail by PWE-PPC-C. Its advantage in solving various lattice types is further illustrated by calculating the PPC-C of triangular and hexagonal lattices, respectively.

1. Introduction

The propagation of elastic or acoustic waves in phononic crystals (PCs) has attracted growing attention during the past two decades [1, 2], both because of its amazing physical properties and because of its potential applications. One of the most attractive properties of PCs is the band gaps, in which elastic or acoustic waves are attenuated significantly. According to the generation mechanisms of Bragg scattering and locally resonant (LR), the band gaps can be divided into Bragg band gap and LR band gap [3].

The PC has the potential to be used as a vibration isolator because of the band gap, but one of main difficulties of isolating mechanical vibration lies in how to attenuate

low-frequency vibration. Compared to the Bragg band gap, the LR band gap has a lower frequency and can be generated by a smaller PC, which enables the PC to isolate low-frequency vibration for the high-precision machine [3–5]. Yet a unit cell of conventional LR PCs commonly consists of a heavy core and a soft coat, as a mass-spring oscillator, to obtain a low resonance frequency. Therefore, a lower frequency LR band gap implies a heavier PC. By replacing the heavy mass-spring oscillator, a light inductor-capacitor oscillator can be constituted by the equivalent capacitance of piezoelectric phononic crystal (PPC) and the inductance of the resonant shunting circuit. The beam-like piezoelectric phononic crystal connected with resonant shunting circuits (PPC-C) was studied to obtain LR band gaps for controlling

the propagation of vibration [6–10]. Furthermore, the PPC-C has another significant advantage that its band gaps can be tuned just by circuit parameters, but reconfiguring the structure is not needed [11–16]. This property will further enhance the application flexibility of the PPC-C.

The PPCs have been researched by Transfer Matrix method [10], plane wave expansion (PWE) method [17, 18], Finite Difference Time Domain method [19], and Finite Element method [20, 21]. Nevertheless, among these methods, only Transfer Matrix method [6–9, 11–13] and Finite Element methods [15, 16, 22] have been used to study the PPC-C. Transfer Matrix method is just applicable to one-dimensional PPC-C, such as beam [7] and circle plate [23]. Up to now, two-dimensional plate-like PPC-C have been researched merely by Finite Element method [15, 16, 22]. Although the utilization of Finite Element method becomes popular nowadays, it is useful to have another method to handle this kind of problem.

Two-dimensional and three-dimensional PPCs were investigated by the methods of Finite Difference Time Domain [19] and conventional PWE [17, 18]. However, without resonant shunting circuits are connected with piezoelectric patches of the PPC. Under this circumstance, we only need to consider two kinds of electric boundary conditions for the piezoelectric patches, that is, short and open circuits. When electrodes of one piezoelectric patch are shorted, electric field is equal to zero in a certain direction, but when these electrodes are open, normal component of electric displacement is equal to zero. Electric field or electric displacement can be expressed directly by displacements. Hence the band structure can be obtained by solving governing equation in terms of displacements. Yet for the PPC with circuits (PPC-C), electrical parameters, such as electric potential and electric displacement, are not contained in the equations of motion. Only one-way coupling, that is, from displacements to electrical parameters, can be taken into account. Firstly, displacements are always obtained from the equations of motion and then get electrical parameters according to circuit equation. Hence the influence of the circuit on structure movement is not taken into consideration. Thus the PPC-C cannot be solved accurately by conventional PWE.

The PWE method is one of the most extensively used methods to calculate band structure because of its convenience [24]. Kushwaha et al. [25] firstly obtain the band structure of a PC by PWE. PWE method includes three steps as follows. Firstly, all parameters such as modulus, density, and Poisson's ratio are expanded as the Fourier series in the reciprocal space. Secondly, equations of motion are transformed into the standard eigenvalue problem based on the Bloch theorem. Finally, band structure of the dispersion relations between the frequency and wave vector is obtained by solving the eigenvalue equation. It is convenient that wave vector just needs to traverse along the boundary of the irreducible region of the first Brillouin zone. Compared with Finite Element method, the PWE shows great advantages in dealing with scatterers of different geometries and different arrangements, because it does not require meshing and reconstructing the Finite Element matrix. But the conventional PWE has a disadvantage of the slow convergence

rate, especially for systems of either very high or very low filling ratios or of large elastic mismatch. The fictitious band gaps always appear as redundant lines in the band structure. Nevertheless, an improved PWE has a good convergence and can provide much more accurate numerical results [26, 27].

The main aim of the paper is to propose an enhanced PWE method to solve the PPC-C, which is abbreviated as PWE-PPC-C. A two-dimensional PPC-C model is established to demonstrate solving process of the PWE-PPC-C. The model is composed of two portions: elastic plate and elastic-piezoelectric composite plate. In general, the conventional PWE does not directly utilize the continuity conditions of forces and bending moment between two portions of the PPC. Moreover, the voltage appears in the expression of bending moment of elastic-piezoelectric composite plate but does not exist in that of elastic plate. To deal with this voltage mismatch, the voltage is included in equations of motion of these two portions of the PPC to ensure the continuity of bending moment at the interface between these two portions. And the voltage is introduced by multiplying an infinitely small quantity for the elastic plate. Then band structure is obtained by solving the equations of motion and circuit equation. The paper is organized as follows: Section 2 presents governing equations of a PPC-C. The equations of motion are expanded by PWE-PPC-C and Bloch theorem in Section 3. Section 4 describes two approaches to solve the PPC with two kinds of common shunting circuits, respectively. The validity and correctness of the PWE-PPC-C are verified by comparisons with Transfer Matrix method and Finite Element method in Section 5. Finally, some conclusions are given in Section 6.

2. Governing Equations of a Two-Dimensional PPC-C

In order to demonstrate solving process of the PWE-PPC-C, we propose a two-dimensional PPC-C which consists of an isotropic elastic plate and rectangular piezoelectric patches. These piezoelectric patches are bonded on the upper and lower surfaces of the plate and are arrayed as orthorhombic system. Its lattice and arrangement are shown in Figures 1 and 2(a). With The same polarization direction, each pair of parallel piezoelectric patches is connected by a resonant shunting circuit, which can be simplified by a complex impedance Z . Each unit cell of the PPC can be divided into two portions: the elastic plate as a matrix and the elastic-piezoelectric composite plate as a scatterer. Subscripts (1, 2, 3) of physical quantities correspond to coordinate system (x, y, z) .

Based on the classical theory of plate [28], equation of motion of a thin plate with flexural vibration is given by

$$\frac{\partial Q_{31}}{\partial x} + \frac{\partial Q_{32}}{\partial y} = \rho_0 h_0 \ddot{u}_3, \quad (1)$$

where Q_{3i} ($i = 1, 2$), u_3 , ρ_0 , and h_0 represent shear force, deflection, density, and thickness of the plate, respectively. A superimposed dot denotes the derivative with respect to time.

FIGURE 1: Unit cell of the two-dimensional PPC-C of orthogonal lattice: (a) top view and (b) side view.

FIGURE 2: Wigner-Seitz unit cell and its first Brillouin zone: (a) orthogonal lattice, (b) triangular lattice, and (c) hexagonal lattice.

The shear force-bending moment M_{ij} ($i, j = 1, 2$) relations are

$$\begin{aligned} Q_{31} &= \frac{\partial M_{11}}{\partial x} + \frac{\partial M_{21}}{\partial y}, \\ Q_{32} &= \frac{\partial M_{12}}{\partial x} + \frac{\partial M_{22}}{\partial y}. \end{aligned} \quad (2)$$

According to Kirchhoff assumption of thin plate, the strain-displacement relations with a compact matrix notation are

$$\begin{aligned} S_1 &= -z \frac{\partial^2 u_3}{\partial x^2}, \\ S_2 &= -z \frac{\partial^2 u_3}{\partial y^2}, \\ S_6 &= 2S_{12} = -2z \frac{\partial^2 u_3}{\partial x \partial y}. \end{aligned} \quad (3)$$

The governing equations of two portions and circuits of the PPC-C are presented, respectively, as follows.

2.1. Elastic Plate. For the isotropic elastic material, nontrivial stresses are

$$\begin{aligned} T_1 &= (\lambda + 2G) S_1 + \lambda S_2 + \lambda S_3, \\ T_2 &= \lambda S_1 + (\lambda + 2G) S_2 + \lambda S_3, \\ T_3 &= \lambda S_1 + \lambda S_2 + (\lambda + 2G) S_3, \\ T_6 &= T_{12} = GS_6, \end{aligned} \quad (4)$$

where $\lambda = Y_1 \mu_1 / [(1 + \mu_1)(1 - 2\mu_1)]$ and $G = Y_1 / [2(1 + \mu_1)]$. Y_1 and μ_1 are Young's modulus and Poisson's ratio of the isotropic elastic material, respectively. Stress relaxation of the thin shell, that is, $T_3 = 0$, is performed on (4) and then yields

$$\begin{aligned} T_1 &= \bar{Y}_1 (S_1 + \mu_1 S_2), \\ T_2 &= \bar{Y}_1 (\mu_1 S_1 + S_2), \\ T_6 &= T_{12} = GS_6, \end{aligned} \quad (5)$$

where $\bar{Y}_1 = Y_1/(1 - \mu_1^2)$. The bending moments can be written as

$$\begin{aligned} M_{11} &= \int_{-h/2}^{h/2} T_1 z \, dz = -D_e \frac{\partial^2 u_3}{\partial x^2} - v_1 \frac{\partial^2 u_3}{\partial y^2}, \\ M_{22} &= \int_{-h/2}^{h/2} T_2 z \, dz = -v_1 \frac{\partial^2 u_3}{\partial x^2} - D_e \frac{\partial^2 u_3}{\partial y^2}, \\ M_{12} &= M_{21} = \int_{-h/2}^{h/2} T_{12} z \, dz = -\gamma_1 \frac{\partial^2 u_3}{\partial x \partial y}, \end{aligned} \quad (6)$$

where flexural stiffness of the elastic plate $D_e = \bar{Y}_1 h^3/12$, $v_1 = D_e \mu_1$, and $\gamma_1 = D_e(1 - \mu_1)$. Substituting (6) into (1)–(3), equation of motion (1) can be expressed in terms of displacement as

$$\begin{aligned} -\eta_1 \frac{\partial^2 u_3}{\partial t^2} &= \frac{\partial^2}{\partial x^2} \left(D_e \frac{\partial^2 u_3}{\partial x^2} + v_1 \frac{\partial^2 u_3}{\partial y^2} \right) \\ &+ \frac{\partial^2}{\partial y^2} \left(D_e \frac{\partial^2 u_3}{\partial y^2} + v_1 \frac{\partial^2 u_3}{\partial x^2} \right) \\ &+ 2 \frac{\partial^2}{\partial x \partial y} \left(\gamma_1 \frac{\partial^2 u_3}{\partial x \partial y} \right), \end{aligned} \quad (7)$$

where $\eta_1 = \rho_1 h$ is the area density of the elastic plate with volume density ρ_1 and thickness h .

2.2. Elastic-Piezoelectric Composite Plate. The constitutive equations of the piezoelectric patches are [29]

$$\begin{aligned} S_1 &= s_{11} T_1 + s_{12} T_2 + d_{31} E_3, \\ S_2 &= s_{12} T_1 + s_{11} T_2 + d_{31} E_3, \\ S_6 &= s_{66} T_6, \\ D_3 &= d_{31} T_1 + d_{31} T_2 + \varepsilon_{33} E_3, \end{aligned} \quad (8)$$

where D_3 is electric displacement in z direction; s_{11} , s_{12} , and $s_{66} = 2(s_{11} - s_{12})$ are compliance constants; and d_{31} and ε_{33} are piezoelectric and dielectric constants, respectively.

With components in other directions being ignored, the electric field in z direction is approximately equal to [30]

$$E_3 = \begin{cases} -\frac{V}{h_p}, & \frac{h}{2} \leq z \leq \frac{h}{2} + h_p, \\ \frac{V}{h_p}, & -\left(\frac{h}{2} + h_p\right) \leq z \leq -\frac{h}{2}, \end{cases} \quad (9)$$

where V is the voltage on the two outer electrodes of the elastic-piezoelectric composite plate and h_p is the thickness of the piezoelectric patches. Using (8) yields

$$\begin{aligned} T_1 &= \bar{Y}_2 [S_1 + \mu_2 S_2 - d_{31} (1 + \mu_2) E_3], \\ T_2 &= \bar{Y}_2 [\mu_2 S_1 + S_2 - d_{31} (1 + \mu_2) E_3], \\ T_6 &= \frac{Y_2}{2(1 + \mu_2)} S_6, \\ D_3 &= \frac{d_{31} Y_2}{1 - \mu_2} (S_1 + S_2) + \bar{\varepsilon}_{33} E_3, \end{aligned} \quad (10)$$

where $Y_2 = 1/s_{11}$ and $\mu_2 = -s_{12}/s_{11}$ are Young's modulus and Poisson's ratio of the piezoelectric material under condition of constant electric field. $\bar{Y}_2 = Y_2/(1 - \mu_2^2)$ and $\bar{\varepsilon}_{33} = \varepsilon_{33}(1 - k_{31}^2)$ with $k_{31}^2 = 2d_{31}^2 Y_2 / [(1 - \mu_2)\varepsilon_{33}]$ are equivalent Young's modulus and equivalent two-dimensional dielectric permittivity. From (3), (9), and (10), the bending moments of the composite plate are given by

$$\begin{aligned} M_{11} &= \int_{-(h/2+h_p)}^{h/2+h_p} T_1 z \, dz = -D_s \frac{\partial^2 u_3}{\partial x^2} - v_2 \frac{\partial^2 u_3}{\partial y^2} + \tau_2 V, \\ M_{22} &= \int_{-(h/2+h_p)}^{h/2+h_p} T_2 z \, dz = -v_2 \frac{\partial^2 u_3}{\partial x^2} - D_s \frac{\partial^2 u_3}{\partial y^2} + \tau_2 V, \\ M_{12} &= M_{21} = \int_{-(h/2+h_p)}^{h/2+h_p} T_{12} z \, dz = -\gamma_2 \frac{\partial^2 u_3}{\partial x \partial y}, \end{aligned} \quad (11)$$

where flexural stiffness of the composite plate $D_s = D_e + D_p$, $v_2 = \mu_1 D_e + \mu_2 D_p$, $\tau_2 = \bar{Y}_2 d_{31} (1 + \mu_2) (h_p + h)$, $\gamma_2 = (1 - \mu_1) D_e + (1 - \mu_2) D_p$, and $D_p = 2\bar{Y}_2 [(h/2 + h_p)^3 - (h/2)^3]/3$. Substituting (11) into (2), equation of motion (1) can be rewritten as

$$\begin{aligned} -\eta_2 \frac{\partial^2 u_3}{\partial t^2} &= \frac{\partial^2}{\partial x^2} \left(D_s \frac{\partial^2 u_3}{\partial x^2} + v_2 \frac{\partial^2 u_3}{\partial y^2} - \tau_2 V \right) \\ &+ \frac{\partial^2}{\partial y^2} \left(D_s \frac{\partial^2 u_3}{\partial y^2} + v_2 \frac{\partial^2 u_3}{\partial x^2} - \tau_2 V \right) \\ &+ 2 \frac{\partial^2}{\partial x \partial y} \left(\gamma_2 \frac{\partial^2 u_3}{\partial x \partial y} \right), \end{aligned} \quad (12)$$

where $\eta_2 = \rho_1 h + 2\rho_2 h_p$ is the area density of the composite plate. ρ_2 denotes volume density of piezoelectric patch. Although the voltage is a constant on electrodes of a pair of piezoelectric patches, the voltage is retained in the differential equation of motion to ensure that the bending moments meet the continuity condition between two portions of the unit cell.

2.3. Resonant Shunting Circuits. The electric current I flowing out from one of two outer electrodes of the elastic-piezoelectric composite plate is

$$I = -\dot{Q}_e = A_p D_3 \left(\frac{h}{2} + h_p \right) s, \quad (13)$$

where Laplace operator $s = i\omega$ with imaginary unit i and angular frequency ω . $A_p = l_1 l_2$ is the area of the electrode. l_1 and l_2 are length and width of a piezoelectric patch, respectively. The total current flowing to the impedance Z is twice the current output from one electrode since two piezoelectric patches in one unit cell are connected in parallel. Therefore the voltage and current satisfy Ohm's law

$$2I = \frac{V}{Z}. \quad (14)$$

Using (13) and (14) yields the voltage

$$V = 2ZA_p D_3 \left(\frac{h}{2} + h_p \right) s. \quad (15)$$

3. Expansion by PWE-PPC-C Method and Bloch Theorem

The voltage multiplying an infinitesimal quantity τ_1 is introduced into equation of motion of elastic plate in order that equations of motion have a unified form for two portions of the PPC. From (7) and (12), the unified form of equations of motion is

$$\begin{aligned} -\eta \frac{\partial^2 u_3}{\partial t^2} &= \frac{\partial^2}{\partial x^2} \left(D \frac{\partial^2 u_3}{\partial x^2} + v \frac{\partial^2 u_3}{\partial y^2} - \tau V \right) \\ &+ \frac{\partial^2}{\partial y^2} \left(D \frac{\partial^2 u_3}{\partial y^2} + v \frac{\partial^2 u_3}{\partial x^2} - \tau V \right) \\ &+ 2 \frac{\partial^2}{\partial x \partial y} \left(\gamma \frac{\partial^2 u_3}{\partial x \partial y} \right), \end{aligned} \quad (16)$$

where η , D , v , γ , and τ represent η_1 , D_e , v_1 , γ_1 , and τ_1 for elastic plate and η_2 , D_s , v_2 , γ_2 , and τ_2 for composite plate, respectively.

Due to the periodicity, η , D , v , γ , and τ can be expanded as Fourier series:

$$\begin{aligned} \eta(\mathbf{r}) &= \sum_{\mathbf{G}} \eta(\mathbf{G}) e^{-i\mathbf{G} \cdot \mathbf{r}}, \\ D(\mathbf{r}) &= \sum_{\mathbf{G}} D(\mathbf{G}) e^{-i\mathbf{G} \cdot \mathbf{r}}, \\ v(\mathbf{r}) &= \sum_{\mathbf{G}} v(\mathbf{G}) e^{-i\mathbf{G} \cdot \mathbf{r}}, \\ \gamma(\mathbf{r}) &= \sum_{\mathbf{G}} \gamma(\mathbf{G}) e^{-i\mathbf{G} \cdot \mathbf{r}}, \\ \tau(\mathbf{r}) &= \sum_{\mathbf{G}} \tau(\mathbf{G}) e^{-i\mathbf{G} \cdot \mathbf{r}}, \end{aligned} \quad (17)$$

where $\mathbf{r} = n_1 \mathbf{a}_1 + n_2 \mathbf{a}_2$ denotes two-dimensional lattice vector in primitive space, in which $\mathbf{a}_1 = a_1 \mathbf{i}$ and $\mathbf{a}_2 = a_2 \mathbf{j}$ are basis vectors. a_1 and a_2 are lattice constants of orthorhombic lattice along x -axis and y -axis, respectively. n_1 and n_2 are both integers. $\mathbf{G} = n_1 \mathbf{b}_1 + n_2 \mathbf{b}_2$ are the two-dimensional reciprocal

lattice vectors. $\mathbf{b}_1$ and $\mathbf{b}_2$ are basis vectors of reciprocal space along x -axis and y -axis, respectively, which can be written as $\mathbf{b}_1 = 2\pi \mathbf{i}/a_1$ and $\mathbf{b}_2 = 2\pi \mathbf{j}/a_2$.

Let $g(\mathbf{r}) = \sum_{\mathbf{G}} g(\mathbf{G}) e^{-i\mathbf{G} \cdot \mathbf{r}}$, where g can be any of the parameters η , D , v , γ , and τ . As a Fourier coefficient, $g(\mathbf{G})$ is defined by

$$g(\mathbf{G}) = \frac{1}{A} \iint_A g(\mathbf{r}) e^{i\mathbf{G} \cdot \mathbf{r}} d^2 \mathbf{r}, \quad (18)$$

where $A = a_1 a_2$ denotes the area of a unit cell. The integration gives

$$g(\mathbf{G}) = \begin{cases} g_2 f + g_1 (1 - f), & \mathbf{G} = 0, \\ (g_2 - g_1) P(\mathbf{G}) = \Delta g P(\mathbf{G}), & \mathbf{G} \neq 0, \end{cases} \quad (19)$$

where g_1 and g_2 represent any one kind of parameters, among η , D , v , γ , and τ , of elastic and composite portions, respectively. $f = A_p/A$ is defined as filling ratio of the PPC. $P(\mathbf{G}) = (1/A) \iint_{A_p} e^{i\mathbf{G} \cdot \mathbf{r}} d^2 \mathbf{r}$ is called structure factor, which depends only on the geometry of the piezoelectric patches but has nothing to do with its arrangement. For the elastic-piezoelectric composite plate, the structure factor is

$$P(\mathbf{G}) = f p(G_x, l_1) p(G_y, l_2), \quad (20)$$

where subscripts x and y denote components of the vector on the x -axis and y -axis, respectively. Function $p(a, b)$ is defined as

$$p(a, b) = \begin{cases} 1, & a = 0, \\ \frac{\sin(ab/2)}{ab/2}, & a \neq 0. \end{cases} \quad (21)$$

To satisfy the Bloch theorem, solutions of (16), the displacement and the voltage, are expanded as follows:

$$\begin{aligned} u(\mathbf{r}, t) &= e^{i\omega t} \sum_{\mathbf{K}} u_{\mathbf{K}}(\mathbf{G}) e^{-i(\mathbf{K} + \mathbf{G}) \cdot \mathbf{r}}, \\ V(\mathbf{r}, t) &= e^{i\omega t} \sum_{\mathbf{G}} u_4(\mathbf{G}) e^{-i(\mathbf{K} + \mathbf{G}) \cdot \mathbf{r}}, \end{aligned} \quad (22)$$

where $\mathbf{K}$ is Bloch wave vector. Substituting (17) and (22) into (16) gives

$$\begin{aligned}
 \omega^2 \sum_{\mathbf{G}'} \eta(\mathbf{G} - \mathbf{G}') u_{\mathbf{K}}(\mathbf{G}') &= \sum_{\mathbf{G}'} D(\mathbf{G} - \mathbf{G}') (\mathbf{K} + \mathbf{G}')_x^2 \\
 &\cdot (\mathbf{K} + \mathbf{G})_x^2 u_{\mathbf{K}}(\mathbf{G}') + \sum_{\mathbf{G}'} v(\mathbf{G} - \mathbf{G}') (\mathbf{K} + \mathbf{G}')_y^2 \\
 &\cdot (\mathbf{K} + \mathbf{G})_x^2 u_{\mathbf{K}}(\mathbf{G}') + \sum_{\mathbf{G}'} \tau(\mathbf{G} - \mathbf{G}') (\mathbf{K} + \mathbf{G})_x^2 \\
 &\cdot u_4(\mathbf{G}') + \sum_{\mathbf{G}'} D(\mathbf{G} - \mathbf{G}') (\mathbf{K} + \mathbf{G}')_y^2 (\mathbf{K} + \mathbf{G})_y^2 \\
 &\cdot u_{\mathbf{K}}(\mathbf{G}') + \sum_{\mathbf{G}'} v(\mathbf{G} - \mathbf{G}') (\mathbf{K} + \mathbf{G}')_x^2 (\mathbf{K} + \mathbf{G})_y^2 \\
 &\cdot u_{\mathbf{K}}(\mathbf{G}') + \sum_{\mathbf{G}'} \tau(\mathbf{G} - \mathbf{G}') (\mathbf{K} + \mathbf{G})_y^2 u_4(\mathbf{G}') \\
 &+ 2 \sum_{\mathbf{G}'} \gamma(\mathbf{G} - \mathbf{G}') (\mathbf{K} + \mathbf{G}')_x (\mathbf{K} + \mathbf{G}')_y (\mathbf{K} + \mathbf{G})_x \\
 &\cdot (\mathbf{K} + \mathbf{G})_y u_{\mathbf{K}}(\mathbf{G}').
 \end{aligned} \quad (23)$$

4. Two Kinds of Resonant Shunting Circuits

4.1. Pure Inductive Circuit. The complex impedance $Z(\omega) = i\omega L$ if each resonant shunting circuit only consists of a single inductor L . Then (10) and (15) yield

$$(\chi_1 + \omega^2 \chi_2) V = -\omega^2 \chi_3 \left(\frac{\partial^2 u_3}{\partial x^2} + \frac{\partial^2 u_3}{\partial y^2} \right), \quad (24)$$

where $\chi_1 = 1$, $\chi_2 = -LC_p$, and $\chi_3 = -LA_p d_{31} Y_2 (h + 2h_p) / (1 - \mu_2)$ with equivalent static capacitance $C_p = 2\bar{\epsilon}_{33} A_p / h_p$ of the two parallel piezoelectric patches. Hence estimated oscillation frequency $f_r = 1/2\pi \sqrt{LC_p}$ for the pure inductive resonant shunting circuit. Expanding χ_j ($j = 1, 2, 3$) as (17) and substituting (22) into (24) yield

$$\begin{aligned}
 &\left[\sum_{\mathbf{G}'} \chi_1 (\mathbf{G} - \mathbf{G}') + \omega^2 \sum_{\mathbf{G}'} \chi_2 (\mathbf{G} - \mathbf{G}') \right] u_4(\mathbf{G}') \\
 &= \omega^2 \sum_{\mathbf{G}'} \chi_3 (\mathbf{G} - \mathbf{G}') \left[(\mathbf{K} + \mathbf{G}')_x^2 + (\mathbf{K} + \mathbf{G}')_y^2 \right] \\
 &\cdot u_{\mathbf{K}}(\mathbf{G}').
 \end{aligned} \quad (25)$$

Then (23) and (25) can be written as matrix form

$$\begin{aligned}
 \mathbf{H} u_{\mathbf{K}}(\mathbf{G}') + \mathbf{E} u_4(\mathbf{G}') &= \omega^2 \mathbf{F} u_{\mathbf{K}}(\mathbf{G}'), \\
 (\mathbf{P} + \omega^2 \mathbf{Q}) u_4(\mathbf{G}') &= \omega^2 \mathbf{W} u_{\mathbf{K}}(\mathbf{G}'),
 \end{aligned} \quad (26)$$

where $\mathbf{H}$, $\mathbf{E}$, $\mathbf{F}$, $\mathbf{P}$, $\mathbf{Q}$, and $\mathbf{W}$ are matrices of $(n_1 + n_2 + 1) \times (n_1 + n_2 + 1)$ and

$$\begin{aligned}
 \mathbf{H} &= \sum_{\mathbf{G}'} D(\mathbf{G} - \mathbf{G}') \\
 &\cdot \left[(\mathbf{K} + \mathbf{G}')_x^2 (\mathbf{K} + \mathbf{G})_x^2 + (\mathbf{K} + \mathbf{G}')_y^2 (\mathbf{K} + \mathbf{G})_y^2 \right] \\
 &+ \sum_{\mathbf{G}'} v(\mathbf{G} - \mathbf{G}') \left[(\mathbf{K} + \mathbf{G}')_y^2 (\mathbf{K} + \mathbf{G})_x^2 + (\mathbf{K} + \mathbf{G}')_x^2 \right. \\
 &\cdot (\mathbf{K} + \mathbf{G})_y^2 \left. \right] + 2 \sum_{\mathbf{G}'} \gamma(\mathbf{G} - \mathbf{G}') (\mathbf{K} + \mathbf{G}')_x (\mathbf{K} + \mathbf{G}')_y \\
 &\cdot (\mathbf{K} + \mathbf{G})_x (\mathbf{K} + \mathbf{G})_y, \\
 \mathbf{E} &= \sum_{\mathbf{G}'} -\tau(\mathbf{G} - \mathbf{G}') \left[(\mathbf{K} + \mathbf{G})_x^2 + (\mathbf{K} + \mathbf{G})_y^2 \right], \\
 \mathbf{F} &= \sum_{\mathbf{G}'} \varepsilon(\mathbf{G} - \mathbf{G}'), \\
 \mathbf{P} &= \sum_{\mathbf{G}'} \chi_1 (\mathbf{G} - \mathbf{G}'), \\
 \mathbf{Q} &= \sum_{\mathbf{G}'} \chi_2 (\mathbf{G} - \mathbf{G}'), \\
 \mathbf{W} &= \sum_{\mathbf{G}'} \chi_3 (\mathbf{G} - \mathbf{G}') \left[(\mathbf{K} + \mathbf{G}')_x^2 + (\mathbf{K} + \mathbf{G}')_y^2 \right].
 \end{aligned} \quad (27)$$

In order to facilitate solving eigenvalues, (26) is further simplified into standard form

$$\begin{bmatrix} \mathbf{F} & \mathbf{0} \\ \mathbf{W} & -\mathbf{Q} \end{bmatrix}^{-1} \begin{bmatrix} \mathbf{H} & \mathbf{E} \\ \mathbf{0} & \mathbf{P} \end{bmatrix} \begin{bmatrix} u_{\mathbf{K}}(\mathbf{G}') \\ u_4(\mathbf{G}') \end{bmatrix} = \omega^2 \begin{bmatrix} u_{\mathbf{K}}(\mathbf{G}') \\ u_4(\mathbf{G}') \end{bmatrix}, \quad (28)$$

where $\mathbf{0}$ represents zero matrix with same order as $\mathbf{H}$.

4.2. Resistive-Inductive Circuit. Complex impedance $Z(\omega) = R_L + i\omega L$ if each resonant shunting circuit is composed of a resistor and an inductor in series. For the resistive-inductive circuit, when $R_L < 2\sqrt{L/C}$, the oscillation frequency can be estimated as

$$f_r = \frac{1}{2\pi} \sqrt{\frac{1}{LC_p} - \frac{R_L^2}{4L^2}}. \quad (29)$$

Equations (10) and (15) give

$$\begin{aligned}
 &(\chi_1 + \omega \chi_4 + \omega^2 \chi_2) V \\
 &= -(\omega \chi_5 + \omega^2 \chi_3) \left(\frac{\partial^2 u_3}{\partial x^2} + \frac{\partial^2 u_3}{\partial y^2} \right),
 \end{aligned} \quad (30)$$

FIGURE 3: Dispersion relation of the one-dimensional PPC-C calculated by (a) Transfer Matrix method and (b) PWE-PPC-C, where $a = 80$ mm, $R_L = 0$, and $L = 0.419$ H.

where $\chi_4 = iR_L C_p$ and $\chi_5 = iR_L A_p d_{31} Y_2(h + 2h_p)/(1 - \mu_2)$. Expanding χ_j ($j = 1, 2, 3, 4, 5$) as (17) and substituting (22) into (30) yield

$$\begin{aligned} & \left[\sum_{\mathbf{G}'} \chi_1 (\mathbf{G} - \mathbf{G}') + \omega \sum_{\mathbf{G}'} \chi_4 (\mathbf{G} - \mathbf{G}') \right. \\ & \quad \left. + \omega^2 \sum_{\mathbf{G}'} \chi_2 (\mathbf{G} - \mathbf{G}') \right] u_4(\mathbf{G}') = \left[\omega \sum_{\mathbf{G}'} \chi_5 (\mathbf{G} - \mathbf{G}') \right. \\ & \quad \left. + \omega^2 \sum_{\mathbf{G}'} \chi_3 (\mathbf{G} - \mathbf{G}') \right] \left[(\mathbf{K} + \mathbf{G}')_x^2 + (\mathbf{K} + \mathbf{G}')_y^2 \right] \\ & \quad \cdot u_{\mathbf{K}}(\mathbf{G}'). \end{aligned} \quad (31)$$

Equations (23) and (31) may be expressed as matrix form

$$\begin{aligned} \mathbf{H} u_{\mathbf{K}}(\mathbf{G}') + \mathbf{E} u_4(\mathbf{G}') &= \omega^2 \mathbf{F} u_{\mathbf{K}}(\mathbf{G}'), \\ (\mathbf{P} + \omega \mathbf{S} + \omega^2 \mathbf{Q}) u_4(\mathbf{G}') &= (\omega \mathbf{T} + \omega^2 \mathbf{W}) u_{\mathbf{K}}(\mathbf{G}'), \end{aligned} \quad (32)$$

where $\mathbf{S} = \sum_{\mathbf{G}'} \chi_4 (\mathbf{G} - \mathbf{G}')$ and $\mathbf{T} = \sum_{\mathbf{G}'} \chi_5 (\mathbf{G} - \mathbf{G}')$. Equation (32) can also be written as standard form [31]

$$\begin{aligned} & \begin{bmatrix} \mathbf{0} & \mathbf{0} & \mathbf{F} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{W} & -\mathbf{Q} \\ \mathbf{I} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{I} & \mathbf{0} & \mathbf{0} \end{bmatrix}^{-1} \begin{bmatrix} \mathbf{H} & \mathbf{E} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{P} & -\mathbf{T} & \mathbf{S} \\ \mathbf{0} & \mathbf{0} & \mathbf{I} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{I} \end{bmatrix} \begin{bmatrix} \mathbf{Z}_1 \\ \mathbf{Z}_2 \\ \mathbf{Z}_3 \\ \mathbf{Z}_4 \end{bmatrix} \\ &= \omega \begin{bmatrix} \mathbf{Z}_1 \\ \mathbf{Z}_2 \\ \mathbf{Z}_3 \\ \mathbf{Z}_4 \end{bmatrix}, \end{aligned} \quad (33)$$

where $\mathbf{Z}_1 = u_{\mathbf{K}}(\mathbf{G}')$, $\mathbf{Z}_2 = u_4(\mathbf{G}')$, $\mathbf{Z}_3 = \omega \mathbf{Z}_1$, and $\mathbf{Z}_4 = \omega \mathbf{Z}_2$.

In order to improve the convergence rate and the accuracy of the numerical calculation, technique of the improved PWE [26] is adopted by PWE-PPC-C. Namely, a parameter matrix $[g]$ is replaced by $[1/g]^{-1}$, where the index -1 means inverse matrix. All parameters η , D , v , γ , and τ are converted in the

TABLE 1: Geometrical parameters of unit cell of the PPC.

a_1 (mm)	a_2 (mm)	l_1 (mm)	l_2 (mm)	h (mm)	h_p (mm)
80	60	40	30	5	0.2

same way. It can be noted that the parameter τ_1 cannot be equal to zero. The band structure is obtained when $\mathbf{K}$ has been traversed along the boundary of the irreducible region of the first Brillouin zone as shown in Figure 2(a).

5. Numerical Results and Discussion

For the two-dimensional PPC, geometrical parameters of unit cell are listed in Table 1. Epoxy is chosen as the material of the elastic plate with density $\rho_1 = 1180$ kg/m³, Poisson's ratio $\mu_1 = 0.37$, and Young's modulus $Y_1 = 4.35$ GPa. PZT-5H is considered as the material of the piezoelectric patches. Its density $\rho_2 = 7500$ kg/m³ and other material parameters are given as

$$\begin{aligned} s_{11} &= 16.5, \\ s_{12} &= -4.78 \times 10^{-12} \text{ m}^2/\text{N}, \\ d_{31} &= -274 \times 10^{-12} \text{ C/N}, \\ \varepsilon_{33} &= 3400\varepsilon_0, \end{aligned} \quad (34)$$

where ε_0 is permittivity of free space. The infinitesimal $\tau_1 = 1.0 \times 10^{-10}$ or less can ensure the convergence.

The effectiveness of the PWE-PPC-C method is verified by other computational methods. Firstly, we compare PWE-PPC-C with Transfer Matrix method in solving a one-dimensional PPC-C. As we know, Transfer Matrix method can accurately solve the one-dimensional PPC-C and obtain its dispersion relation. With same structure and circuit parameters as [7], the PPC-C is solved by Transfer Matrix method and PWE-PPC-C, respectively. In PWE-PPC-C, 101 reciprocal vectors ($n_1 = 50$) are employed to ensure the convergence sufficiently. The dispersion relation curves calculated by these two methods are shown in Figures 3(a) and 3(b), respectively. For results of Transfer Matrix method, the first LR band gap locates at 422.3–424.5 Hz and the Bragg

TABLE 2: Comparisons between PWE-PPC-C and other methods.

Models	Methods	LR band gaps	Bragg band gaps
One-dimensional PPC-C	Transfer Matrix method [7]	422.3–424.5 Hz	708–987 Hz
	PWE-PPC-C	422.3–424.2 Hz	708–986 Hz
Two-dimensional PPC-C	Finite Element method [22]	556–563 Hz	752–948 Hz
	PWE-PPC-C	559–566 Hz	754–949 Hz

FIGURE 4: Band structure of the two-dimensional PPC-C of tetragonal lattice, where $a = 80$ mm and $L = 0.2$ H. FIGURE 5: Band structure of the two-dimensional PPC-C of orthogonal lattice, where $R_L = 50 \Omega$ and $L = 0.25$ H.

band gap lies in 708–987 Hz; for those of PWE-PPC-C, the first LR band gap is 422.3–424.2 Hz and the Bragg band gap is 708–986 Hz. The comparisons are listed in Table 2. They show that results of PWE-PPC-C are consistent with those of Transfer Matrix method. Yet the voltage mismatch between two portions of the PPC-C leads to calculation error which has not been completely eliminated since τ_1 cannot be zero. So the fictitious band gaps still appear as redundant lines in the band structure of PWE-PPC-C.

The PWE-PPC-C inherits the advantage of the PWE in effectively solving more than one-dimensional PC. As the next step, PWE-PPC-C is further verified by Finite Element method in solving two-dimensional PPC-C. Figure 4 illustrates band structure of a two-dimensional PPC-C obtained by PWE-PPC-C, where structural, circuit, and other relevant parameters are all the same as those of [22]. Piezoelectric patches are arrayed as tetragonal lattice with lattice constants $a_1 = a_2 = 80$ mm. Band structure with all propagation directions is obtained conveniently by PWE-PPC-C as shown in Figure 4. Band structures of two particular orientations, $\theta = 0$ and $\pi/4$, are computed by Finite Element method [22]. It can be noted that location and width of band gaps of these two orientations obtained by PWE-PPC-C and Finite Element method are in good agreement. Comparisons on band gaps between PWE-PPC-C and Finite Element method are further listed in Table 2. Therefore, PWE-PPC-C can correctly solve two-dimensional PPC-C and obtain its band structure with all propagation directions.

The proposed PWE-PPC-C is further used to study a two-dimensional PPC-C with orthogonal lattice. Both unit cell and piezoelectric patches are rectangular as shown in Figure 1. Piezoelectric patches are arrayed as orthogonal lattice as shown in Figure 2(a). Geometrical parameters of unit cell of the PPC are listed in Table 1. Circuit parameters $R_L = 50 \Omega$ and $L = 0.25$ H. Here $n_1 = n_2 = 15$; thus the number of reciprocal vectors is 961. Figure 5 illustrates the band structure of the PPC-C that a complete LR gap exists in all directions and spans from 573 Hz to 584 Hz; two incomplete Bragg gaps appear between 756 and 960 Hz in $\Gamma \rightarrow X$ direction and between 1336.2 and 1656.8 Hz in $Y \rightarrow \Gamma$ direction, respectively. Directions $\Gamma \rightarrow X$ and $Y \rightarrow \Gamma$ of reciprocal space correspond to horizontal and vertical directions of primitive space, respectively. It can be found from Figure 5 that the Bragg gap frequency of vertical direction is higher than that of horizontal direction. This is because lattice constant of vertical direction, a_2 , is smaller than that of horizontal direction, a_1 . It can be inferred that the Bragg gap frequency decreases with increasing of lattice constant in a certain direction. Therefore, the PPC-C with orthorhombic as well as tetragonal lattice can be calculated by the proposed PWE-PPC-C.

We further investigate the effect of circuit parameters on band gap by the proposed PWE-PPC-C. The LR band gap is often used to isolate vibration because of the low frequency. Therefore, the following discussions are focused on the first LR band gap. Firstly, the inductance is fixed at $L = 0.25$ H. Band structures of the two-dimensional PPC-C of orthogonal

FIGURE 6: LR band gaps of the two-dimensional PPC-C with different circuit parameters: (a) $R_L = 200 \Omega$, $L = 0.25$ H; (b) $R_L = 400 \Omega$, $L = 0.25$ H; (c) $R_L = 50 \Omega$, $L = 0.20$ H; (d) $R_L = 50 \Omega$, $L = 0.30$ H.

lattice with different resistances are shown in Figures 5, 6(a), and 6(b), where resistance $R_L = 50$, 200, and 400 Ω , respectively. The center frequency and bandwidth of the LR band gap both decrease with increasing of the resistance. Then the resistance is fixed at $R_L = 50 \Omega$. Band structures of the two-dimensional PPC-C of orthogonal lattice with different inductances are shown in Figures 6(c), 5, and 6(d), where the inductance $L = 0.2$, 0.25, and 0.30 H, respectively. These figures illustrate that the larger the inductance, the lower the center frequency and the narrower the bandwidth of the LR band gap.

PWE-PPC-C is further applied to calculate dependence of the LR band gap distribution on circuit parameters of the two-dimensional PPC. The LR band gap distribution and estimated oscillation frequency versus the inductance for different resistance are shown in Figure 7. The center frequency and bandwidth of the LR band gap both increase

FIGURE 7: Distribution of the LR band gap of the two-dimensional PPC-C versus the inductance for different resistance.

FIGURE 8: Distribution of the LR band gap of the two-dimensional PPC-C versus the resistance for different inductance.

FIGURE 9: Band structure of the two-dimensional PPC-C of triangular lattice.

with decreasing of the inductance. The tendency is same as that reported in related references. Figure 8 shows that distribution of the LR band gap and estimated oscillation frequency versus the resistance for different inductance. The center frequency and bandwidth of the LR band gap both decrease with increasing of the resistance. Transmission factor is often used to measure the wave propagation properties of the PPC with finite period. Nevertheless, curves of transmission factor versus the frequency show that the larger the resistance, the wider the band gap. The reason is that damping effect is enhanced by increasing of the resistance; thus band gap of near field wave is broadened. One also can find from Figures 7 and 8 that the frequency of LR band gaps is lower than the estimated oscillation frequency of the resonant shunting circuits. This is because the oscillation frequency depends on the dynamic capacitance but not on the static capacitance [32].

Finally, the advantage of PWE-PPC-C in effectively calculating the PPC-C of various lattices is demonstrated. Figure 9 displays the band structure of the PPC with piezoelectric

FIGURE 10: Band structure of the two-dimensional PPC-C of hexagonal lattice.

patches arranged as triangular lattice (see Figure 2(b)), where the lattice constant $a = 80$ mm and the radius of the piezoelectric patches $R = 30$ mm. It can be found that a completely tiny LR band gap exists between 375 and 384 Hz and two Bragg band gaps locate between 1685.9 and 1686.6 Hz in $\Gamma \rightarrow K$ direction and between 1131.3 and 1541.5 Hz in $\Gamma \rightarrow M$ direction, respectively. Moreover, Figure 10 displays band structure of a PPC-C of hexagonal lattice. The hexagonal lattice ($a = 80$ mm) consists of elastic matrix and piezoelectric patches ($R = 30$ mm) as shown in Figure 2(c). A Bragg band gap appears between 1019 and 1142 Hz along $P \rightarrow Q$ direction. Therefore, from band structures of four kinds of lattices as shown in Figures 4, 5, 9, and 10, it can be found that LR band gap is a complete band gap in all directions if it exists. But Bragg band gaps only exist in some directions. Moreover, Bragg band gap appears in different direction for different lattice type of the PPC-C.

6. Conclusions

An enhanced plane wave expansion method is proposed to solve the PPC connected with resonant shunting circuits (PWE-PPC-C). A two-dimensional PPC-C model has been presented to demonstrate in detail solving process of PWE-PPC-C. Piezoelectric patches of the two-dimensional PPC are arrayed as orthorhombic lattice with two different lattice constants in horizontal and vertical directions. The PWE-PPC-C method has been validated by two ways. On one hand, PWE-PPC-C has consistent result with Transfer Matrix method in solving one-dimensional PPC-C. On the other hand, PWE-PPC-C agrees well with Finite Element method in solving a two-dimensional PPC-C of tetragonal lattice.

The PPC-C of orthorhombic lattice is studied by PWE-PPC-C. Both LR band gaps and Bragg band gaps can be calculated by PWE-PPC-C effectively. Dependence of the first LR band gap on the resistance and the inductance is further investigated by PWE-PPC-C in detail.

The band structures of the PPC-C of triangular and hexagonal lattices are obtained by PWE-PPC-C, which

demonstrate the advantage of PWE-PPC-C in solving PPC-C of various arrangements. Some redundant lines appear near the LR band gap in the figure of band structure, whose generation mechanism will be further clarified in the future work.

Competing Interests

The authors declare that they have no competing interests.

Acknowledgments

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