Research Article

Study of Baffle Boundary and System Parameters on Liquid-Solid Coupling Vibration of Rectangular Liquid-Storage Structure

Wei Jing,¹ Xuansheng Cheng,^{1,2} Wei Shi,¹ Jin Fan,¹ and Huan Feng¹

¹School of Civil Engineering, Lanzhou University of Technology, Lanzhou, Gansu 730050, China

²Western Center of Disaster Mitigation in Civil Engineering of the Ministry of Education, Lanzhou, Gansu 730050, China

Correspondence should be addressed to Xuansheng Cheng; cxs702@126.com

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In order to study the vibration problem of liquid-solid coupling of rectangular liquid-storage structure with horizontal elastic baffle, ignoring the influence of surface gravity wave, two different velocity potential functions corresponding to the liquid above and below the elastic baffle are assumed; based on the theory of mathematical equation and energy method, the formulas of basic frequency of liquid-solid coupling vibration system are derived, the baffle joined to the tank wall with 3 kinds of boundary conditions, namely, four edges simply supported, two opposite edges clamped and two opposite edges simply supported, and four edges clamped; the influence rules of baffle length-width ratio, the ratio of baffle height to liquid level, baffle elastic modulus, baffle density, baffle thickness, and liquid density on the coupling vibration performance are studied. The results show that the frequency of the clamped boundary is minimum; the influences of baffle length-width ratio and relative height on the basic frequency are much greater than that of the other system parameters; the relation between baffle length-width ratio and the frequency is exponential, while baffle relative height has a parabola relation with the frequency; the larger the baffle length-width ratio, the closer the baffle to the liquid level; the coupling frequency will be reduced more obviously.

1. Introduction

Rectangular liquid-storage structure is widely used in water supply and drainage system, sewage treatment, petrochemical industry, TLD of high rise structure, and so on. With the rapid development of our economy, the liquid-storage structures have already started to develop in the direction of large amount and large scale. The special performance of this kind of structure is that it will bear extra dynamic liquid pressure under external excitations.

As a measure to reduce the liquid sloshing, the baffle has been widely used in the field of aerospace, automotive storage tanks, structural engineering, and so on. But setting baffles in the liquid-storage structure, not only does the complexity of the structure itself increase, but also the analysis difficulty is further increased due to the coupling vibration of elastic baffle and liquid [1]; a large amount of researches on the

structure with baffle has been carried out all over the world. Wang et al. [2] studied the liquid sloshing characteristics of the spherical tank with baffle, finding that the basic frequency of the spherical tank is decreased after adding baffle. Yue et al. [3] used two potential functions to solve the liquid sloshing problem of the tank with elastic baffle; the results showed that the baffle position in liquid has a great influence on the reducing sloshing effect. Yang et al. [4] carried out finite element simulation calculation and experimental determination of liquid sloshing in a cylindrical liquid-storage tank with an elastic baffle, finding that when the baffle was closer to the liquid surface, its influence on the coupling frequency is greater. Hao et al. [5] used the single Lagrangian method to study stable equilibrium problems of storage tank with different boundary elastic baffle. Biswal and Bhattacharyya [6] considered the dynamic interaction of liquid-structure-baffle system by using the

finite element method and studied the influences of baffle size and position on the liquid sloshing and structural response. Shahrokhi et al. [7] studied the influences of baffle position on the liquid flow pattern of liquid-storage structure by using computational fluid dynamics (CFD). Wang et al. [8] got the potential functions of each subdomain liquid by using the separation variable method and superposition principle and studied the fluid-solid coupling characteristics of the liquid-storage structure with elastic baffle. Song et al. [9] established a mathematical model of water free surface by using the boundary element method in order to solve the problem of liquid sloshing in a rectangular tank with baffle and pointed out that the reducing sloshing function of baffle is mainly achieved by changing the frequency distribution of original liquid-storage structure. Xue et al. [10] simulated a cube liquid-storage structure with multibaffles and got the conclusion that baffle can effectively reduce the amplitude and dynamic liquid pressure of liquid sloshing. Wang et al. [11] divided the fluid domain into several simple subdomains, and the influences of location, inner radius, and the number of circular baffles on the coupling vibration of the liquid-storage structure were studied. Hasheminejad et al. [12] studied the transient horizontal response of the tank with baffle and the effective baffle shape that can suppress the lateral force. Shekari [13] divided the fluid domain into two parts with baffle, using boundary element method to study the basic mode of liquid-storage structure with baffle, summarizing the maximum seismic responses of the structure with baffle; then the baffle effect on the vibration control of liquid-storage structure is proved. Ebrahimian et al. [14] applied the feature analysis method to study the basic frequency and formation of liquid-storage structure with baffle. Goudarzi and Farshadmanesh [15] studied the free vibration of liquid-storage structure with different size and different position baffle; the results showed that the liquid sloshing height was reduced by 50% owing to adding baffle.

In summary, although the baffle can exert an inhibitory effect on the liquid sloshing of liquid-storage structure, its effect on the fluid-solid coupling vibration is influenced by many factors. So the effect of elastic horizontal baffle on the fluid-solid coupling vibration of rectangular liquid-storage structure will be studied further in this paper, supposing the liquid is of no rotation, of no viscosity, and incompressible, because the occurrence probability of gravity wave in the structure of civil engineering is small [16]; therefore, the surface gravity wave is ignored. Assuming that the wall of liquid-storage structure is rigid and its baffle is elastic, two different velocity potential functions corresponding to the liquid above and below the elastic baffle are defined; based on the theory of mathematical equation and energy method, the formulas of basic frequency of liquid-solid coupling vibration of rectangular storage structure with flexible baffle are derived, the baffle joined to the tank wall with 3 kinds of boundary conditions, namely, four edges simply supported, two opposite edges clamped and two opposite edges simply supported, and four edges clamped; the conclusions can provide theoretical basis for the reducing sloshing design of reinforced concrete rectangular liquid-storage structure.

2. Fluid-Solid Coupling Model and Basic Theory

Assuming that the liquid is in an ideal state, the length, width, and height of the rectangular liquid-storage structure are a , b , and c , respectively, the distance between elastic baffle and the tank bottom is H , liquid level height is h , and h is greater than H ; supposing the wall is rigid, liquid sloshing is of small amplitude. Ignoring hollows in the elastic baffle, although in practical engineering application, in order to make the liquid flow through the baffle, some holes need to be set in the baffle; if the holes are small and arranged in centralized manner, their effect on the coupling vibration can be ignored; when the holes layout does not conform to the above provisions, the calculation results should be modified by considering the liquid flow effect caused by baffle holes [17]; the analytical model is shown in Figure 1.

When the liquid sloshing is of small amplitude, linear potential flow theory can be used to solve the coupling vibration problem. Assuming that the baffle is located below the stationary liquid level, in this case, the velocity potentials for both sides of liquid of the baffle can be expressed as $\phi_1(x, y, z, t)$ and $\phi_2(x, y, z, t)$; ϕ_1 and ϕ_2 all satisfy the Laplace equation [18]:

$$\nabla^2 \phi_j = 0, \quad j = 1, 2. \quad (1)$$

According to the existing literature [19], for common liquids, because σ/ρ_w is small, for simplicity, liquid surface tension σ can be ignored. After neglecting the surface tension of liquid sloshing, the kinematic and dynamic boundary conditions for the first-order liquid sloshing problem can be expressed as follows:

$$\frac{\partial^2 \phi_1}{\partial t^2} + g \frac{\partial \phi_1}{\partial z} \Big|_{z=h} = 0, \quad (2)$$

where g is the gravity acceleration.

On the contact surface of rigid wall and liquid, the liquid velocity potential functions satisfy the following boundary conditions:

$$\begin{aligned} \frac{\partial \phi_j}{\partial x} \Big|_{x=0,a} &= 0, \\ \frac{\partial \phi_j}{\partial y} \Big|_{y=0,b} &= 0, \end{aligned} \quad (3)$$

$$j = 1, 2,$$

$$\frac{\partial \phi_2}{\partial z} \Big|_{z=0} = 0. \quad (4)$$

According to Bernoulli equation, the relation between liquid sloshing pressure and velocity potential is $P = -\rho_w(\partial\phi/\partial t)$; thus, the total sloshing pressure of the baffle under the action of upper and lower liquid is as follows:

$$P^H = \left(-\rho_w \frac{\partial \phi_1}{\partial t} \right) - \left(-\rho_w \frac{\partial \phi_2}{\partial t} \right), \quad (5)$$

FIGURE 1: Analytical model of liquid-storage structure.

where ρ_w is liquid density and P^H is total liquid sloshing pressure.

Supposing the deflection in the point (x, y, H) is $W(x, y, t)$, then the differential equation of baffle vibration is

$$\nabla^4 W(x, y, t) + \frac{\rho}{D} \frac{\partial^2 W(x, y, t)}{\partial t^2} = -\frac{P^H}{D}, \quad (6)$$

where D is the baffle flexural rigidity, $D = Ed^3/[12(1-\nu)]$, ρ is baffle density, d is baffle thickness, and ν is Poisson's ratio.

On the contact surface of liquid and baffle, the velocities in the z direction should be equal to each other; namely,

$$\frac{\partial \phi_1}{\partial z} = \frac{\partial \phi_2}{\partial z} = \frac{\partial W}{\partial t} \Big|_{z=H}. \quad (7)$$

3. Coupling Vibration Solution of Liquid-Storage Structure with Baffle

3.1. Coupled Frequency. Using the method of separation of variables, the velocity potential functions that satisfy (1) and (3) can be expressed as [3, 20, 21]

$$\begin{aligned} \phi_1 &= A_{mn} \cos\left(\frac{m\pi}{a}x\right) \cos\left(\frac{n\pi}{b}y\right) \left(e^{-K_{mn}z} + B_{mn}e^{K_{mn}z}\right) \\ &\quad \cdot e^{i\omega_{mn}t}, \\ \phi_2 &= C_{mn} \cos\left(\frac{m\pi}{a}x\right) \cos\left(\frac{n\pi}{b}y\right) \left(e^{-K_{mn}z} + D_{mn}e^{K_{mn}z}\right) \\ &\quad \cdot e^{i\omega_{mn}t}, \end{aligned} \quad (8)$$

where $K_{mn} = \sqrt{(m\pi/a)^2 + (n\pi/b)^2}$, $(m, n = 1, 2, \dots)$; ω_{mn} is the basic frequency of the coupling system.

Considering that the liquid sloshing is of small amplitude, in the free liquid surface, z is equal to h approximately; taking (8) into (2) and (4) into (7), we can get

$$B_{mn} = \frac{(gK_{mn} + \omega_{mn}^2)e^{-K_{mn}h}}{(gK_{mn} - \omega_{mn}^2)e^{K_{mn}h}}, \quad (9)$$

$$D_{mn} = 1, \quad (10)$$

$$C_{mn} = \frac{A_{mn}(B_{mn}e^{K_{mn}H} - e^{-K_{mn}H})}{e^{K_{mn}H} - e^{-K_{mn}H}}. \quad (11)$$

Taking (8), (9), (10), and (11) into (5), the liquid dynamic pressure acting on the baffle can be obtained:

$$\begin{aligned} P^H &= -\rho i \omega_{mn} \cos\left(\frac{m\pi}{a}x\right) \cos\left(\frac{n\pi}{b}y\right) \\ &\quad \cdot A_{mn} (e^{-K_{mn}H} + B_{mn}e^{K_{mn}H}) \\ &\quad \cdot \left[1 - \text{cth}(K_{mn}H) \frac{B_{mn}e^{K_{mn}H} - e^{-K_{mn}H}}{B_{mn}e^{K_{mn}H} + e^{-K_{mn}H}} \right] e^{i\omega_{mn}t}. \end{aligned} \quad (12)$$

Assume that the deflection of elastic baffle is [20, 21]

$$W(x, y, t) = W_{mn}(x, y) e^{i\omega_{mn}t}. \quad (13)$$

By (6), (12), and (13), the following equation can be obtained:

$$D\nabla^4 W_{mn} - \rho_{mn}\omega_{mn}^2 W_{mn} = 0, \quad (14)$$

where ρ_{mn} is a function of ω_{mn} ; it can be called dynamic density [21], and

$$\begin{aligned} \rho_{mn} &= \rho \\ &- \frac{\rho_w}{K_{mn}} \left[\frac{B_{mn} e^{K_{mn}H} + e^{-K_{mn}H}}{B_{mn} e^{K_{mn}H} - e^{-K_{mn}H}} - \text{cth}(K_{mn}H) \right]. \end{aligned} \quad (15)$$

By (14), the maximum kinetic energy $T_{\max}$ and the potential energy $V_{\max}$ of the elastic baffle can be obtained, respectively:

$$T_{\max} = \frac{1}{2} \bar{\rho}_{mn} \omega_{mn}^2 \int_0^a \int_0^b W_{mn}^2(x, y) dx dy,$$

$$\begin{aligned} V_{\max} &= \frac{1}{2} D \int_0^a \int_0^b \left\{ \left(\frac{\partial^2 W_{mn}}{\partial x^2} + \frac{\partial^2 W_{mn}}{\partial y^2} \right)^2 - 2(1-\nu) \right. \\ &\quad \cdot \left. \left[\frac{\partial^2 W_{mn}}{\partial x^2} \cdot \frac{\partial^2 W_{mn}}{\partial y^2} - \left(\frac{\partial^2 W}{\partial x \partial y} \right)^2 \right] \right\} dx dy. \end{aligned} \quad (16)$$

According to the law of energy conservation,

$$V_{\max} = T_{\max}. \quad (17)$$

Taking (16) into (17), then

$$\omega_{mn}^2 \frac{\rho_{mn}}{D} = d_{mn}, \quad (18)$$

where

$$d_{mn} = \frac{\int_0^a \int_0^b \left\{ \left(\partial^2 W_{mn} / \partial x^2 + \partial^2 W_{mn} / \partial y^2 \right)^2 - 2(1-\nu) \cdot \left[\left(\partial^2 W_{mn} / \partial x^2 \right) \cdot \left(\partial^2 W_{mn} / \partial y^2 \right) - \left(\partial^2 W_{mn} / \partial x \partial y \right) \right] \right\} dx dy}{\int_0^a \int_0^b W_{mn}^2 dx dy}. \quad (19)$$

Taking (9) and ρ_{mn} of (14) into (18),

$$a_{mn} \omega_{mn}^4 - b_{mn} \omega_{mn}^2 + c_{mn} = 0, \quad (20)$$

where

$$\begin{aligned} a_{mn} &= [\rho K_{mn} + \rho_w \text{cth}(K_{mn}H)] \text{ch}[K_{mn}(h-H)] \\ &\quad + \rho_w \text{sh}[K_{mn}(h-H)]; \\ b_{mn} &= g K_{mn} \{ \rho K_{mn} \text{sh}[K_{mn}(h-H)] + \rho_w \text{cth}(K_{mn}H) \\ &\quad \cdot \text{sh}[K_{mn}(h-H)] + \rho_w \text{cth}[K_{mn}(h-H)] \} \\ &\quad + D d_{mn} K_{mn} \text{ch}[K_{mn}(h-H)]; \\ c_{mn} &= D d_{mn} K_{mn}^2 g \text{sh}[K_{mn}(h-H)]. \end{aligned} \quad (21)$$

The basic frequency of the coupling system can be obtained by solving (20):

$$\omega_{mn}^2 = \frac{b_{mn} - \sqrt{b_{mn}^2 - 4a_{mn}c_{mn}}}{2a_{mn}}. \quad (22)$$

As can be seen from (18), in order to obtain the basic frequency of the coupled vibration, the dynamic density ρ_{mn} and d_{mn} must be known firstly; through the above analysis, the liquid dynamic density ρ_{mn} has been obtained; it can be seen that the parameters such as liquid height, horizontal baffle height, and baffle size have a great influence on ρ_{mn} . The main factor affecting the expression of d_{mn} of elastic baffle is the boundary conditions. Based on the existing research, the paper derives the expression of d_{mn} of liquid-storage structure with baffle, and the baffle joined to the tank wall with 3 kinds of boundary conditions, namely, four

edges simply supported, two opposite edges clamped and two opposite edges simply supported, and four edges clamped; by doing this, the basic frequency of the coupling vibration of the rectangular liquid-storage structure with elastic baffle can be solved.

(1) *Four Edges of Baffle Simply Supported.* For baffle with four edges simply supported, its boundary conditions are as follows:

$$\begin{aligned} W|_{x=0,a} &= 0, \\ W|_{y=0,b} &= 0. \end{aligned} \quad (23)$$

Considering the boundary conditions that (23) satisfies, we can set

$$W_{mn}(x, y) = \sin \frac{m\pi x}{a} \sin \frac{n\pi y}{b}. \quad (24)$$

Taking (24) into the expression of d_{mn} of (18) gets the following equation:

$$d_{mn} = \pi^4 \left(\frac{m^2}{a^2} + \frac{n^2}{b^2} \right)^2. \quad (25)$$

(2) *Two Opposite Edges of Baffle Clamped and Two Opposite Edges of Baffle Simply Supported.* For two opposite edges of baffle clamped and two opposite edges of baffle simply supported, the boundary conditions are as follows:

$$\begin{aligned} W|_{x=0,a} &= 0, \\ \frac{\partial W}{\partial x} \Big|_{x=0,a} &= 0, \\ W|_{y=0,b} &= 0. \end{aligned} \quad (26)$$

For two opposite edges of baffle clamped and two opposite edges of baffle simply supported, the deflection equation should satisfy (26); we can suppose

$$W_{mn}(x, y) = \left(1 - \cos \frac{2m\pi x}{a}\right) \sin \frac{n\pi y}{b}. \quad (27)$$

Taking (27) into the expression of d_{mn} of (18), we can get

$$d_{mn} = \frac{1}{3} \left(\frac{2m\pi}{a}\right)^4 + \left(\frac{n\pi}{b}\right)^4 + \frac{2}{3} \left(\frac{2m\pi}{a}\right)^2 \left(\frac{n\pi}{b}\right)^2. \quad (28)$$

(3) *Four Edges of Baffle Clamped.* For baffle with four clamped edges, it has the following boundary conditions:

$$\begin{aligned} W|_{x=0,a} &= 0, \\ \frac{\partial W}{\partial x}|_{x=0,a} &= 0, \\ W|_{y=0,b} &= 0, \\ \frac{\partial W}{\partial y}|_{y=0,b} &= 0. \end{aligned} \quad (29)$$

Considering boundary conditions equations (29) for a rectangular baffle with four clamped edges, the baffle deflection equation can be expressed as

$$W_{mn}(x, y) = \left(1 - \cos \frac{2m\pi x}{a}\right) \left(1 - \cos \frac{2n\pi y}{b}\right). \quad (30)$$

Taking (30) into the expression of d_{mn} of (18), then

$$\begin{aligned} d_{mn} &= \frac{1}{3} \left[\left(\frac{2m\pi}{a}\right)^4 + \left(\frac{2n\pi}{b}\right)^4 + \frac{2}{3} \left(\frac{2m\pi}{a}\right)^2 \left(\frac{2n\pi}{b}\right)^2 \right]. \end{aligned} \quad (31)$$

Through the above derivation, as can be seen from (20) and (22), the main factors affecting the basic frequency of the coupling vibration of rectangular liquid-storage structure with baffle include liquid density, baffle density, baffle thickness, the ratio of baffle height to liquid level, the ratio of baffle length to width, and baffle boundary conditions. For convenience of engineering application, supposing the ratio of baffle length to width is $\lambda = b/a$ and the ratio of baffle height to liquid level is $\gamma = H/h$ and taking $m = n = 1$, then the fundamental frequency of the coupling system can be gotten by (22):

$$\omega_{11}^2 = \frac{b_{11} - \sqrt{b_{11}^2 - 4a_{11}c_{11}}}{2a_{11}}, \quad (32)$$

where

$$\begin{aligned} b_{11} &= gK_{11} \{ \rho K_{11} \text{sh} [K_{11} (h - H)] + \rho_w \text{cth} [K_{11} H] \\ &\quad \cdot \text{sh} [K_{11} (h - H)] + \rho_w \text{cth} [K_{11} (h - H)] \} \\ &\quad + Dd_{11} K_{11} \text{ch} [K_{11} (h - H)]; \\ c_{11} &= Dd_{11} K_{11}^2 g \text{sh} [K_{11} (h - H)]; \\ K_{11} &= \frac{\pi}{a} \sqrt{1 + \frac{1}{\lambda^2}}. \end{aligned} \quad (33)$$

3.2. *Theory of Coupled Modes.* In order to solve the coupled modes corresponding to the basic coupled frequency of liquid-storage structure with horizontal elastic baffle, we can assume (34) based on (8):

$$\phi_j = \Phi_j(x, y, z) e^{i\omega_{mn}t}; \quad (j = 1, 2). \quad (34)$$

Taking (34) into continuity equation (1) of liquid sloshing in the fluid domain V , (2) of kinematic and dynamic boundary conditions of liquid free surface ∂S_f , boundary condition equations (3) and (4) in the liquid-solid interaction wall ∂S_w , and same speed condition equation (7) in the baffle ($z = H$), respectively, then the differential boundary value equation of characteristic mode function can be obtained:

$$\begin{aligned} \nabla^2 \Phi_j &= 0, \quad (j = 1, 2); \quad V, \\ \frac{\partial \Phi_1}{\partial z} &= \frac{\omega_{mn}^2}{g} \Phi_1; \quad \partial S_f, \\ \frac{\partial \Phi_j}{\partial x} &= \frac{\partial \Phi_j}{\partial y} = \frac{\partial \Phi_2}{\partial z} = 0; \quad \partial S_w, \\ \frac{\partial \Phi_1}{\partial z} &= \frac{\partial \Phi_2}{\partial z} = \frac{\partial W}{\partial t} = 0; \quad z = H. \end{aligned} \quad (35)$$

Equation (35) can be solved by FEM method, and the equation should be firstly transformed into functional extremes problem:

$$\delta L(\Phi) = 0, \quad (36)$$

where L is the functional; its expression is [22]

$$\begin{aligned} L &= \int_V \left[\left(\frac{\partial \Phi}{\partial x}\right)^2 + \left(\frac{\partial \Phi}{\partial y}\right)^2 + \left(\frac{\partial \Phi}{\partial z}\right)^2 \right] dV \\ &\quad - \frac{\omega_{mn}^2}{g} \int_{\partial S_f} \Phi_j^2 dS. \end{aligned} \quad (37)$$

After the finite element discretization of the liquid, Φ of each element V_e can be obtained by interpolation method [23]:

$$\Phi(x, y, z) = \sum_{k=1} N_k \Phi_k^e = \mathbf{N}^T \Phi_e, \quad (38)$$

where $\mathbf{N} = (N_1, N_2, \dots, N_j)^T$ is shape function array in the fluid domain and $\Phi_e = (\Phi_{e1}, \Phi_{e2}, \dots, \Phi_{ek})$ is the corresponding node array.

Similarly, Φ of each element S_{fe} in the free surface can be expressed as

$$\Phi(x, y, z) = \sum_{l=1} \tilde{N}_l \Phi_l^e = \tilde{\mathbf{N}}^T \Phi_e, \quad (39)$$

where $\tilde{\mathbf{N}} = (\tilde{N}_1, \tilde{N}_2, \dots, \tilde{N}_l)$ is shape function array in the free surface and $\Phi_e = (\Phi_{e1}, \Phi_{e2}, \dots, \Phi_{el})$ is the corresponding node array.

Taking (38) and (39) into L ,

$$\begin{aligned} L &= \sum \Phi_e^T \mathbf{C}_e \Phi_e - \frac{\omega_{mn}^2}{g} \sum \Phi_e^T \mathbf{D}_e \Phi_e \\ &= \Phi^T \mathbf{C} \Phi - \frac{\omega_{mn}^2}{g} \Phi^T \mathbf{D} \Phi, \end{aligned} \quad (40)$$

where $\mathbf{C}$ and $\mathbf{D}$ are formed by element matrixes $\mathbf{C}_e$ and $\mathbf{D}_e$, Φ is integrated node variable array, and

$$\mathbf{C}_e = \sum_{V_e} \left(\frac{\partial \mathbf{N}}{\partial x} \frac{\partial \mathbf{N}^T}{\partial x} + \frac{\partial \mathbf{N}}{\partial y} \frac{\partial \mathbf{N}^T}{\partial y} + \frac{\partial \mathbf{N}}{\partial z} \frac{\partial \mathbf{N}^T}{\partial z} \right) dV \quad (41)$$

$$\mathbf{D}_e = \int_{S_{fe}} \tilde{\mathbf{N}} \tilde{\mathbf{N}}^T dS_f.$$

Taking (40) into (36),

$$\mathbf{C}\Phi - \frac{\omega_{mn}^2}{g} \mathbf{D}\Phi = 0. \quad (42)$$

4. Numerical Examples and Discussions

According to (32), the basic frequency of the liquid-solid coupling vibration of rectangular liquid-storage structure with horizontal baffle can be solved, in order to study the coupling vibration characteristics of solid-liquid coupling system more comprehensively, assuming the baffle joined to the tank wall with 3 kinds of boundary conditions, taking a variety of values for the main system parameters of this kind of structure; by doing this, statistical results can be obtained. The liquid height h is 3 m, the baffle width is 4 m, and the other parameters are as follows.

4.1. Verification of Proposed Method. Equation (22) is the basic frequency of three-dimensional coupling system considering the elasticity of horizontal baffle, when the baffle is assumed to be rigid; namely, baffle bending stiffness D tends to infinity, the fluid-solid coupling vibration in the upper baffle can be approximately equivalent to liquid-storage structure without baffle, and the liquid coupling frequency above the rigid baffle with four clamped edges can be obtained by (22):

$$\omega_{mn} = \sqrt{g \left[\left(\frac{m\pi}{a} \right)^2 + \left(\frac{n\pi}{b} \right)^2 \right] \cdot \text{th} \left[\sqrt{\left(\frac{m\pi}{a} \right)^2 + \left(\frac{n\pi}{b} \right)^2} \cdot (h - H) \right]}. \quad (43)$$

For the two-dimensional rectangular liquid-storage structure, the coupling frequency can be expressed as [23]

$$\omega_n = \sqrt{\frac{gn\pi}{b} \text{th} \left(\frac{n\pi(h-H)}{b} \right)}. \quad (44)$$

By comparing (43) and (44) of basic coupling frequency in the three- and two-dimensional coupling vibration problem, the calculation method rationality of the coupling frequency is explained to a certain extent.

On the other hand, in order to further verify the method rationality of the coupling vibration of rectangular liquid-storage structure with different boundary baffle, the corresponding calculation model is established by ADINA. Baffle length to width ratio λ is 1.0, baffle height-liquid level ratio γ is 0.1, 0.3, 0.5, 0.7, and 0.9, respectively, baffle thickness d

FIGURE 2: Results comparison.

is 6 cm, baffle elastic modulus E is 30 GPa, baffle density ρ is 2500 kg/m³, and ρ_w is 1000 kg/m³. Comparison of calculation results of ADINA and (32) is shown in Figure 2.

As seen from Figure 2, the difference of coupling frequency results calculated by the proposed method and ADINA is small; besides, with the change of γ , change trends of frequency corresponding to the two methods are consistent; then the validity of the present method is verified.

4.2. Analysis of Boundary and Parameter Influence

4.2.1. Effect of Baffle Length-Width Ratio on the Basic Frequency of Liquid-Solid Coupling Vibration. The ratios of baffle length to width λ ($\lambda = b/a$) are, respectively, 1.0, 1.5, 2.0, 2.5, and 3.0, baffle thickness d is 6 cm, the ratio of baffle height to liquid level γ ($\gamma = H/h$) is 0.5, liquid density ρ_w is 1000 kg/m³, baffle density ρ is 2500 kg/m³, baffle elastic modulus E is 30 GPa, and the influence of λ on the basic frequency of the coupling vibration is shown in Figure 3. As can be seen from Figure 3, the natural frequencies of the coupling system under the three boundary conditions are decreased with the increase of baffle length-width ratio, and the relationship between length-width ratio and the natural frequency is exponential. Under the same length-width ratio, natural frequency corresponding to four edges of baffle simply supported is maximum, natural frequency corresponding to two opposite edges simply supported and two opposite edges clamped is middle, and natural frequency corresponding to four edges clamped is minimum; namely, the basic frequency of the coupling system decreases with the strengthening of the baffle constraint. Therefore, horizontal baffle should be connected to tank wall with four clamped edges when conducting sloshing reduction design; it not only

FIGURE 3: Effect of baffle length-width ratio λ on the basic frequency of liquid-solid coupling vibration.

is convenient for construction, but also can reduce the basic frequency of fluid-solid coupling vibration.

4.2.2. Effect of Baffle Height-Liquid Level Ratio on the Basic Frequency of Liquid-Solid Coupling Vibration. The ratios of baffle height to liquid level γ ($\gamma = H/h$) are, respectively, 0.1, 0.2, 0.3, 0.4, 0.5, 0.7, 0.8, and 0.9, baffle length-width ratio λ is 1.0, baffle thickness d is 6 cm, liquid density ρ_w is 1000 kg/m³, baffle density ρ is 2500 kg/m³, baffle elastic modulus E is 30 GPa, and the influence of γ on the basic frequency of the coupling vibration is shown in Figure 4. Figure 4 shows that the basic frequency of the coupling vibration under the three boundary conditions decreases with the increase of baffle height to liquid level ratio; when the ratio γ changes in the range of 0.1~0.5, the basic frequency reduces relatively slow, but when γ changes from 0.5 to 0.9, that is, the baffle is closer to the liquid surface, the trend that frequency decreases is much faster; therefore, in the actual project, baffle should be designed as far as possible near to the liquid surface. When the ratio γ is smaller, the basic frequency corresponding to the three boundary conditions has a certain difference; namely, basic frequency corresponding to four edges simply supported is maximum, basic frequency corresponding to two opposite edges simply supported and two opposite edges clamped is middle, and basic frequency corresponding to four edges clamped is minimum, but with the increase of γ , the influence of baffle boundary conditions on the basic frequency of the coupling vibration is gradually weakened; when the baffle position is close to the liquid level, the frequency corresponding to three kinds of boundary conditions is basically equivalent. On the whole, the relationship between coupling vibration frequency and the ratio γ is parabola.

FIGURE 4: Effect of baffle height to liquid level ratio γ on the basic frequency of liquid-solid coupling vibration.

FIGURE 5: Effect of baffle thickness on the basic frequency of liquid-solid coupling vibration.

4.2.3. Effect of Baffle Thickness on the Basic Frequency of Liquid-Solid Coupling Vibration. The baffle thickness d is, respectively, 6 cm, 8 cm, 10 cm, and 12 cm; in these cases, the relative baffle thickness d/a is in the scope of elastic thin plate, the ratios λ and γ are 1.0 and 0.5, ρ is 2500 kg/m³, ρ_w is 1000 kg/m³, E is 30 GPa, and the influence of d on the basic frequency of the coupling vibration is shown in Figure 5. As can be seen from Figure 5, the basic frequencies corresponding to the three boundary conditions all decrease with the increase of baffle thickness d , but with the increase of d , the trend that frequencies decrease becomes much gentler. When the baffle thickness is much thinner, the difference of the frequency corresponding to the three boundary conditions is

FIGURE 6: Effect of baffle modulus of elasticity on the basic frequency of liquid-solid coupling vibration.

relatively larger, but with the increase of baffle thickness, the influence of boundary conditions on the basic frequency of the coupling vibration is gradually weakened.

4.2.4. Effect of Baffle Elastic Modulus on the Basic Frequency of Liquid-Solid Coupling Vibration. The baffle elastic modulus E is, respectively, 22 GPa, 25.5 GPa, 28 GPa, 30 GPa, 31.5 GPa, and 32.5 GPa, ρ is 2500 kg/m³, d is 6 cm, the ratios λ and γ are 1.0 and 0.5, ρ_w is 1000 kg/m³, and the influence of E on the basic frequency of the coupling vibration is shown in Figure 6. The results show that when the elastic modulus E changes in the process of small to large value, the frequency of simply supported edges is obviously larger than that of the other two kinds of boundary conditions, and the basic vibration frequency decreases with the increase of elastic modulus, so in the design of reinforced concrete rectangular liquid-storage structure, in order to reduce the coupling vibration frequency, higher grade concrete can be used for baffle.

4.2.5. Effect of Baffle Density on the Basic Frequency of Liquid-Solid Coupling Vibration. The baffle density ρ is, respectively, 2100 kg/m³, 2200 kg/m³, 2300 kg/m³, 2400 kg/m³, 2500 kg/m³, and 2600 kg/m³, d is 6 cm, the ratios λ and γ are 1.0 and 0.5, ρ_w is 1000 kg/m³, E is 30 GPa, and the influence of ρ on the basic frequency of the coupling vibration is shown in Figure 7. As can be seen from Figure 7, the effect of baffle density on basic frequency of the coupling system is very small; therefore, the influence of baffle density on the basic frequency cannot be considered in the design.

4.2.6. Effect of Liquid Density on the Basic Frequency of Liquid-Solid Coupling Vibration. The liquid density ρ_w is, respectively, 800 kg/m³, 900 kg/m³, 1000 kg/m³, and 1100 kg/m³, d is 6 cm, the ratios λ and γ are 1.0 and 0.5, ρ is 2500 kg/m³, E is 30 GPa, and the influence of ρ_w on the basic frequency of the coupling vibration is shown in Figure 8. As can be

FIGURE 7: Effect of baffle density on the basic frequency of liquid-solid coupling vibration.

FIGURE 8: Effect of liquid density on the basic frequency of liquid-solid coupling vibration.

seen from Figure 8, the basic frequencies of the coupling vibration corresponding to the three boundary conditions all increase slowly with the increase of liquid density. Under various liquid densities, the frequency of simply supported edges is larger than that of the other two kinds of boundary conditions; compared with other parameters of the system, liquid density is positively correlated with basic frequency, but in the actual engineering, the difference of liquid density is small in general, so the effect of liquid density on the basic frequency also cannot be considered.

4.3. Solution of Coupled Modes. The results gotten from Section 4.2 show that the coupling vibration frequencies are affected by the parameters λ and γ significantly more than the other system parameters; due to limited space, the modes

FIGURE 9: Coupled modes corresponding to λ . FIGURE 10: Coupled modes corresponding to γ .

corresponding to λ (1.2, 1.6, 2.0, 2.4) and γ (0.5, 0.5, 0.7, 0.9) are listed in Figures 9 and 10.

5. Conclusions

(1) The influence of baffle boundary conditions on the basic frequency of the coupling vibration of rectangular liquid-storage structure cannot be ignored. In general, under the same conditions, the basic frequency of the coupling vibration corresponding to four edges of baffle simply supported is maximum, frequency corresponding to two opposite edges

simply supported and two opposite edges clamped is middle, and frequency corresponding to four edges clamped is minimum. Therefore, horizontal baffle should be connected to tank wall with four clamped edges when conducting sloshing reduction design; it not only is convenient for construction, but also can reduce the basic frequency of fluid-solid coupling vibration.

(2) Many of system parameters have a negative relation with the basic frequency, such as baffle elastic modulus, baffle thickness, baffle length-width ratio, and the ratio of baffle height to liquid level, while the liquid density is positively correlated with the basic frequency of the coupling vibration, but in the actual engineering, the difference of liquid density is small in general, so the effect of liquid density on the basic frequency of the coupling vibration cannot be considered. Besides, the effect of baffle density on the coupling frequency is very small and its effect can also be neglected.

(3) In contrast, the influences of length-width ratio and baffle height relative to liquid level on the basic frequency of the coupling vibration are much greater than that of the other system parameters; the larger the baffle length-width ratio, the closer the position of horizontal baffle to the liquid level; the coupling frequency will be reduced more obviously, and the relation between baffle length-width ratio and the coupling vibration frequency is exponential, while the ratio of baffle height to liquid lever has a parabola relation with the coupling vibration frequency.

Competing Interests

The authors declare that there are no competing interests regarding the publication of this paper.

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