Research Article

Experiments and Electromechanical Properties of a Broadband Piezoelectric Vibration Energy Harvester

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A broadband piezoelectric energy harvester (BPEH), consisting of a conventional linear piezoelectric energy harvester (CPEH) and an elastic magnifier, was presented in this paper. The improved two-degree-of-freedom lumped-parameter electromechanical model of the BPEH was established and the optimal external resistances under short-circuit and open-circuit resonance conditions were investigated to maximize the output power of the BPEH. The output voltage and output power of the BPEH obtained from the theoretical model were verified and found to be in reasonable agreement with the experimental results. The obtained results have shown that the maximal output powers under short-circuit and open-circuit resonance conditions are both 24 times that generated by the CPEH without elastic magnifier. The frequency space between the two peaks of the frequency-response curve of the BPEH is 14 Hz which is 7 times that of CPEH.

1. Introduction

The power requirements of small electronic devices have reduced drastically. This has motivated research into the possibilities of powering these devices by harvesting the vibrational energy available in their environments and using it to generate electricity. The ultimate objective is to remove the requirement of an external power source or battery replacement for small electronic devices, particularly when they are used in remote locations [1–3]. The basic transduction mechanisms that can be used to convert ambient vibrational energy to electrical energy are piezoelectric, electrostatic, and electromagnetic transductions [4–6]. Among these transduction mechanisms, piezoelectric transduction has attracted immense attention because of its low cost and high power density [7], and piezoelectric transducers such as conventional piezoelectric energy harvester (CPEH) are the simplest transducers to fabricate and are well suited to small-scale systems [8, 9].

Typically, a CPEH is a cantilevered unimorph/bimorph beam with or without a tip mass. The cantilevered beam is attached to a vibrating host structure and is therefore

subjected to a base excitation. Because the base excitation is always assumed to be harmonic, a well-known single-degree-of-freedom (SDOF) harmonic base excitation relationship has been widely used for modeling the dynamics [10] and parameter optimization [11]. However, an important issue in CPEH is that the maximum output power of the device is limited to a narrow bandwidth around the base resonant frequency. If the excitation frequency shifts slightly from the resonant condition, the output power is drastically decreased. To overcome this bandwidth issue of the CPEH, two innovative methods have been considered by researchers. One method is to tune the resonant frequency of the CPEH to match the ambient vibration frequency at all times [12–14]. The other method is to broaden the bandwidth with various approaches, including nonlinear oscillation techniques [15, 16], a multimodal energy harvester method [17], and coupled elastic structures [18, 19]. Other methods such as optimizing the parameters of the energy harvester [20] and impedance-matching technology [21] have also been proposed to improve the output power of the CPEH.

A dynamic magnifier has been extensively integrated into CPEH to improve the output power in recent studies.

FIGURE 1: Configuration of proposed BPEH.

Wang et al. [22] proposed an energy harvester combining a piezoelectric cantilever with a SDOF elastic magnifier to improve the output power of the CPEH. The distributed parameter model of the energy harvester was developed based on Hamilton's principle and Rayleigh-Ritz method. But the validity of the developed mathematical model should be verified by some experiments. Ma et al. [23] incorporated a dynamic magnifier, which works through the interaction of coupled elastic structures, into a CPEH to amplify the output power, and a two-degree-of-freedom (TDOF) lumped parameter model was derived. However, the developed mathematical model does not take into account in detail the electromechanical coupling between the mechanical and electrical fields and does not consider the effect of the design parameters of the dynamic magnifier or the external resistance on the output power of the energy harvester. Li et al. [24] investigated a miniature electric generator consisting of piezoelectric benders and elastic magnifier and derived the lumped parameter analytical model. The developed model, however, ignores the effect of the dynamic mode shape and the strain distribution of the CPEH and the electrical loads on the output power. Aldraihem and Baz [25] established the electromechanical coupling model of piezorods with a dynamic magnifier and studied the effect of the design parameters of the dynamic magnifier on the output power. Arafa et al. [26] implemented experiments to demonstrate the feasibility of a CPEH with a dynamic magnifier. Tang and Zuo [27] established a TDOF lumped parameter model to investigate the optimal output power of CPEH with a dynamic magnifier. These researches mainly focused on the maximum output power that could be achieved by ignoring the damping of the dynamic magnifier. Moreover, the TDOF lumped parameter models also ignored the strain distribution and the dynamic mode shape of the CPEH, which is critical to affect the electromechanical properties.

Despite the documented potential and advantages of the CPEH with dynamic magnifier, it has been shown that there is room for improvement and advancement for this device. One principle challenge is the accurate mathematical model

required to evaluate quickly the effect of parameter variation on the device and to calculate the maximum power harvesting performance. As abovementioned studies that present the TDOF lumped parameter models ignored the strain distribution and the dynamic mode shape of the CPEH and did not consider the effect of the dynamic magnifier damping on the output power, this research presents an improved TDOF lumped parameter electromechanical model of the BPEH. The developed mathematical model takes into account the strain distribution and the dynamic mode shape of the CPEH as well as the effect of the dynamic magnifier damping and the external resistance on the output power. The effects of some important parameters, such as the mass, stiffness, and damping ratios between the CPEH and the elastic magnifier, on the output properties of the BPEH were numerically studied. The optimal external resistances under short-circuit and open-circuit resonance conditions were investigated to maximize the output power of the BPEH. Finally, the output voltage and output power of the BPEH obtained from the theoretical model were verified and found to be in reasonable agreement with the experimental results. The obtained results verify the validity of the improved lumped parameter model of the BPEH.

2. Structure of the Proposed BPEH

The configuration of the proposed BPEH is shown in Figure 1. It consists of a CPEH and an elastic magnifier. The CPEH consists of a bimorph piezoelectric cantilever with a tip mass (M_t). Two piezoelectric plates (PZT_1 and PZT_2) oppositely polarized in the thickness direction are perfectly bonded to the top and bottom, respectively, of the substrate beam. The piezoelectric bimorph plates are connected in series. The electrode pairs covering the top and bottom faces of the piezoelectric plates are assumed to be thin so that their contribution to the thickness dimension is negligible. A simple electrical circuit consisting of a resistive load (R_L) is directly connected to the output terminal of the harvester. The elastic magnifier consists of a platform and a spring element,

and it is positioned between the CPEH and the base. The base is fixed on the vibration source using fixing screws. When an ambient vibration is generated in the base, the vibration amplitude of the CPEH will be amplified through the elastic magnifier.

3. Improved Lumped Parameter Electromechanical Model of BPEH

The improved electromechanical coupling model of BPEH is shown in Figure 2. The equivalent mass, equivalent damping, and equivalent stiffness of the CPEH are denoted by M_{eq} , C_{eq} , and K_{eq} , respectively; α and C_p represent the electromechanical coupling factor and the electrical capacitance of the PZTs, respectively; M_b , C_b , and K_b represent the platform mass, the friction damping of the contact surface, and the stiffness of the spring element of the elastic magnifier, respectively; u_b , y_b , and y_s denote the vibration displacements of the base, platform mass M_b , and equivalent mass M_{eq} , respectively; and V is the voltage of the load resistance R_L .

3.1. Definitions of Improved Model Parameters. Unlike previous studies [23–27], the improved model takes into account the important information on the dynamic mode shape and strain distribution of the CPEH. This important information is involved in the key parameters of equivalent mass M_{eq} and stiffness K_{eq} of the CPEH. Assuming the equivalent mass M_{eq} and stiffness K_{eq} of the CPEH takes the following forms [28]:

$$\begin{aligned} M_{eq} &= \beta_M \rho A L + M_t, \\ K_{eq} &= \frac{\beta_K EI}{L^3}, \end{aligned} \quad (1)$$

where ρA and EI are the average mass and bending stiffness of the CPEH, respectively, and L is the length of cantilevered beam of the CPEH. β_M and β_K are coefficients, which relate to the dynamic mode shape and strain distribution of the CPEH, and they can be determined based on the energy conservation principle. Let $w(x)$ be the dynamic bending deflection of the CPEH shown in Figure 1, and the relationship between the deflection and the bending mode shape $\phi(x)$ takes the following form:

$$w(x) = \mu(x) \phi(x), \quad (2)$$

where $\mu(x)$ is the proportional factor and $\phi(x)$ is the bending mode shape of the CPEH. Based on the elastic mechanics theories, the factor $\mu(x)$ can be calculated as

$$\mu(x) = \frac{w(x)}{\phi(x)} = \frac{M_t g x^2 (3L - x)}{6EI \phi(x)}, \quad (3)$$

where g is gravitational acceleration.

From (2) and (3), the equivalent mass and stiffness of the CPEH without the tip mass M_t can be obtained as follows according to the energy conservation principle:

$$\begin{aligned} m_{eq} &= \int_0^L \rho A \left[\frac{\bar{\mu}(x) \phi(x)}{\bar{\mu}(L) \phi(L)} \right]^2 dx, \\ k_{eq} &= \frac{L^3 M_t g}{EI \mu(L) \phi(L)} \cdot \frac{EI}{L^3}. \end{aligned} \quad (4)$$

Comparing (4) with the first term of the right-hand side of (1), respectively, the coefficients β_M and β_K can be determined:

$$\begin{aligned} \beta_M &= \frac{\int_0^L [\mu(x) \phi(x)]^2 dx}{[\mu(L) \phi(L)]^2 L}, \\ \beta_K &= \frac{L^3 M_t g}{EI \mu(L) \phi(L)}. \end{aligned} \quad (5)$$

The electromechanical coupling factor α and electrical capacitance C_p of the PZTs can be defined according to the well-known piezoelectric constitutive equation and the relationships between the stress and the strain of the CPEH, respectively [29, 30]:

$$\begin{aligned} \alpha &= \frac{\beta_\alpha e_{31} b (h_s + h_p)}{(2L)}, \\ C_p &= \frac{b L \epsilon_{33}^S}{(2h_p)}, \end{aligned} \quad (6)$$

where $\beta_\alpha = -L d\phi(x)/dx|_{x=L}$; e_{31} and ϵ_{33}^S are the piezoelectric and clamped dielectric constants, respectively; h_s and h_p are the thickness of the metal substrate and the PZTs, respectively; and b is the width of the cantilevered beam.

The damping factor ξ_{eq} of CPEH is related to the logarithmic decrement δ for underdamped vibrations via the following relationship [31]:

$$\xi_{eq} = \frac{\delta}{\sqrt{(2\pi)^2 + (\delta)^2}} \quad \text{where } \delta = \ln \left(\frac{A_k}{A_{k+1}} \right), \quad (7)$$

where A_k and A_{k+1} are any two successive amplitudes when the cantilever beam exhibits oscillation. Figure 3 shows the system used to test the damping factor of the CPEH. The CPEH vibrated in damped oscillation when it was excited by the hammer's impulse force. The tip displacement of the CPEH was detected by a laser displacement sensor (LDS) and was transferred to a data acquisition unit (DAU). The displacement waveform was displayed by personal computer (PC).

The stiffness K_b of the spring element of elastic magnifier was determined through experiments. Figure 4 shows the setup used to test the spring element stiffness. One end of the spring element was fixed on the pothook. Some standard mass pieces (1 kg/piece) were loaded onto the other end of the spring element, which extended the spring element.

FIGURE 2: Improved lumped parameter electromechanical model of BPEH.

FIGURE 3: Test of damping factor of CPEH.

FIGURE 4: Test setup for the spring element stiffness.

The elongation (ΔL) of the spring element was then observed using the indicator on the scale through the microscope. The force (F) acting on the spring element can be defined through calculating the number (N) of the standard mass pieces, by using $F = N \times g$, where g is the gravity acceleration. Thus, the stiffness of the spring element can be calculated as

$$K_b = \frac{F}{((1/n) \sum_{i=1}^n \Delta L_i)} = \frac{Ng}{((1/n) \sum_{i=1}^n \Delta L_i)}, \quad (8)$$

where n is the testing number of the elongation of the spring element.

3.2. Governing Equations of the Improved Electromechanical Model. The governing equations for the improved model shown in Figure 2 are then obtained:

$$\begin{aligned} & \begin{bmatrix} M_{eq} & 0 & 0 \\ 0 & M_b & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{Bmatrix} \ddot{y}_s \\ \ddot{y}_b \\ \dot{V} \end{Bmatrix} \\ & + \begin{bmatrix} C_{eq} & -C_{eq} & 0 \\ -C_{eq} & C_{eq} + C_b & 0 \\ \alpha & -\alpha & -C_p \end{bmatrix} \begin{Bmatrix} \dot{y}_s \\ \dot{y}_b \\ \dot{V} \end{Bmatrix} \\ & + \begin{bmatrix} K_{eq} & -K_{eq} & \alpha \\ -K_{eq} & K_{eq} + K_b & 0 \\ 0 & 0 & -\frac{1}{R_L} \end{bmatrix} \begin{Bmatrix} y_s \\ y_b \\ V \end{Bmatrix} \\ & = \begin{bmatrix} 0 \\ C_b \\ 0 \end{bmatrix} \dot{u}_b + \begin{bmatrix} 0 \\ K_b \\ 0 \end{bmatrix} u_b. \end{aligned} \quad (9)$$

Assuming that the base is vibrating in a harmonic manner, $u_b(t)$, $y_b(t)$, and $y_s(t)$ have the following forms:

$$\begin{aligned} u_b(t) &= U_b e^{i\omega t}, \\ y_b(t) &= Y_b e^{i\omega t}, \\ y_s(t) &= Y_s e^{i\omega t}, \end{aligned} \quad (10)$$

where U_b is the amplitude of the harmonic vibration at frequency ω . Y_b and Y_s are the vibration amplitudes of M_b and M_{eq} , respectively. The output voltage of the load resistance R_L can be expressed as

$$V(t) = V_0 e^{i\omega t}. \quad (11)$$

Substituting (10) and (11) into (9), we can obtain the relative displacement $\bar{y}_b$ (the displacement of mass M_b with respect to the base displacement); the relative tip velocity $\bar{v}_s$ (the velocity of mass M_{eq} with respect to the base acceleration); the relative output voltage $\bar{V}_0$ (the output voltage of load resistance R_L with respect to the base acceleration); and the relative output power $\bar{P}_0$ (the output power of load resistance R_L with respect to the base acceleration square) as follows:

$$\bar{y}_b = \frac{y_b(t)}{u_b(t)} = \frac{A_0 + r}{1 + r - a\Omega^2 + 2A_0(1 + d) - (1 + A_0)(1 + A_0 + B_0)/(1 - \Omega^2 + A_0 + B_0)}, \quad (12)$$

$$\bar{v}_s = \frac{\dot{y}_s(t)}{\dot{u}_b(t)} = -\frac{j}{\omega} \cdot \frac{A_0 + B_0}{1 - \Omega^2 + A_0 + B_0} \bar{y}_b, \quad (13)$$

$$\bar{V}_0 = \frac{V(t)}{\ddot{u}_b(t)} = \frac{C_0(\Omega^2 - 1)}{-\omega^2(1 - \Omega^2 + A_0 + B_0)} \cdot \bar{y}_b, \quad (14)$$

$$\bar{P}_0 = \frac{P(t)}{\ddot{u}_b^2(t)} = \frac{\bar{V}_0^2}{R_L}, \quad (15)$$

where $A_0 = 2j\xi_{eq}\Omega$, $B_0 = \theta\Omega k^2 j/(1 + \theta\Omega j)$, $C_0 = \alpha\omega j/(C_p\omega j + 1/R_L)$, $a = M_b/M_{eq}$, $r = K_b/K_{eq}$, $d = C_b/C_{eq}$, $\Omega = \omega/\omega_{eq}$, $\xi_{eq} = C_{eq}/(2M_{eq}\omega_{eq})$, $\theta = \omega_{eq}R_L C_p$, $k^2 = \alpha^2/(K_{eq}C_p)$, $g = \omega_{eq}/\omega_b$, $\omega_b = \sqrt{K_b/M_b}$, and $\omega_{eq} = \sqrt{K_{eq}/M_{eq}}$.

From (9), we can obtain the resonance frequency and antiresonance frequency of the BPEH:

$$\begin{aligned} f_r &= f_{eq}; \\ f_{anti} &= \sqrt{1 + k^2} f_{eq}, \end{aligned} \quad (16)$$

where $f_{eq} = \omega_{eq}/(2\pi)$ is the resonance frequency of the CPEH.

To guarantee that the BPEH has more output power over a frequency bandwidth ranging from f_r to f_{anti} than a constituent CPEH, the amplitude of the minimum displacement $|y_s(t)|$ occurring at $f = f_{eq}$ within the frequency range must satisfy the following relationship:

$$\min(|y_s(t)|) > |y_s^{CPEH}(t)|, \quad (17)$$

where $y_s^{CPEH}(t)$ is the tip displacement of the CPEH at $f = f_{eq}$, and it can be expressed as

$$y_s^{CPEH}(t) = \frac{j2\xi_{eq}\omega_{eq}\omega + \omega_{eq}^2 + C_0/M_{eq}}{j2\xi_{eq}\omega_{eq}\omega + \omega_{eq}^2 - \omega^2 + C_0/M_{eq}} u_b(t). \quad (18)$$

The tip displacement $y_s(t)$ of the BPEH can be obtained from (13) as follows:

$$y_s(t) = \frac{A_0 + B_0}{1 - \Omega^2 + A_0 + B_0} \cdot \frac{A_0 + r}{1 + r - a\Omega^2 + 2A_0(1 + d) - (1 + A_0)(1 + A_0 + B_0)/(1 - \Omega^2 + A_0 + B_0)} u_b(t). \quad (19)$$

From (17) the critical mass ratio a_c can be determined. When $a > a_c$, the electrical output of the BPEH is larger than that of the CPEH over a frequency range between f_r and f_{anti} .

4. Numerical Analysis

Numerical calculations were carried out to analyze the performances of the BPEH using MATLAB software. The material and geometric parameters of the CPEH for numerical calculations are listed in Table 1. The set of parameters of the cantilever beam is chosen here only for testing the validity of the BPEH; further details of how the parameters are chosen can be found in [32].

Figure 5 shows the relative motion, $\bar{y}_b$, of mass M_b of the BPEH with various mass ratios a when $g = 1$ and $R_L = 1 \text{ k}\Omega$. It indicates that the BPEH has two peaks that are sensitive to the mass ratio. As the mass ratio a increases,

the left peak decreases and the right peak increases, and the distance between the two peaks narrows, forming a wideband frequency window. Moreover, for a given mass M_{eq} of the CPEH, when the mass ratio $a = 1e - 10$ and the stiffness ratio $r = 1e + 10$, resp., the elastic magnifier becomes rigid and the BPEH shown in Figure 1 reduces to a CPEH. The vibration amplifying role of the elastic magnifier disappears, so that the relative motion curve of mass M_b is a line and is identical to that of the CPEH shown in Figure 5.

Figure 6 shows the tip relative velocity, $\bar{v}_s$, of the piezoelectric cantilever of the BPEH with different mass ratios a when $g = 1$ and $R_L = 1 \text{ k}\Omega$. The tip relative velocity, $\bar{v}_s$, of the piezoelectric cantilever is also sensitive to the mass ratio. As the mass ratio a increases, the left peak increases and the right peak decreases. This indicates that some mechanical energy of the elastic magnifier is transferred to the piezoelectric cantilever. Particularly, when the mass ratio $a = 20$, there is a minimum tip velocity between the two peaks, which is equal to the maximum tip velocity of the CPEH. Thus, we

TABLE 1: Material and geometric parameters of the CPEH.

Item	Value
Piezoelectric plate density ρ_p (kg/m ³)	7450
Substrate plate density ρ_s (kg/m ³)	8920
Piezoelectric plate stiffness E_p (GPa)	28.45
Substrate plate stiffness E_s (GPa)	113
Strain constant d_{31} (C/m)	-190×10^{-12}
Stress constant e_{31} (C/m)	-4.08
Vacuum permittivity ϵ_0 (F/m)	8.854×10^{-12}
Absolute permittivity ϵ_{33}^s (F/m)	$1200\epsilon_0$
Beam length L (mm)	50.8
Beam width b (mm)	30.8
Piezoelectric plate thickness h_p (mm)	0.26
Substrate plate thickness h_s (mm)	0.15
Damping ratio of elastic magnifier ξ_b	0.002
Damping ratio of piezoelectric beam ξ_{eq}	0.02
Tip mass M_t (kg)	0.011
Load resistance R_L (Ω)	1000

can determine the critical mass ratio $a_c = 20$ when $g = 1$ and $R_L = 1 \text{ k}\Omega$.

Figures 7 and 8 show the output voltage $\bar{V}_0$ and the output power $\bar{P}_0$ of the BPEH with different mass ratios a when $g = 1$ and $R_L = 1 \text{ k}\Omega$. A semilogarithmic coordinate is used for clarifying the difference of each output power curve in Figure 7. The output voltage and the output power behave similarly to the tip relative velocity $\bar{v}_s$ of the piezoelectric cantilever (in Figure 5). As the mass ratio a increases, both the output voltage and the output power increase, which indicates that a larger mass ratio a results in higher voltage and output power. Moreover, when the mass ratio $a = 1e - 10$ and the stiffness ratio $r = 1e + 10$, the voltage and the power curves of the BPEH are identical to those of the CPEH.

Figure 9 shows the effect of the damping ratio $q = \xi_b/\xi_{eq}$ of the BPEH on the output power when $g = 1$, $a = r = 20$, and $R_L = 1 \text{ k}\Omega$. As the damping ratio q increases, the peak power of the BPEH decreases. When the damping coefficient ξ_b of the elastic magnifier is larger than that of the CPEH, the dynamic magnified function of the elastic magnifier becomes

FIGURE 5: Relative displacement $\bar{y}_b$ with various a when $g = 1$ and $R_L = 1 \text{ k}\Omega$.

too weak to increase the peak power of the BPEH. This means that a small ξ_b or $\xi_b = 0$ of the elastic magnifier is suitable for enhancing the output power of the BPEH. So the damping coefficient ξ_b of the elastic magnifier is zero in the following research.

5. Optimal Resistance Load and Output Power

Suppose g , a , r , Ω , and q of the BPEH are fixed except for the load resistance R_L ; then, we can obtain the design criterion for reaching maximum power flow under steady-state operation by tuning the load resistance R_L according to the following expression:

$$\left. \frac{\partial}{\partial R_L} \bar{P}_0(g, a, r, \Omega, q, R_L) \right|_{g, a, r, \Omega, q} = 0. \quad (20)$$

The notation R_L^{opt} is used to represent the solution of (20), where the superscript “opt” denotes functions evaluated at the optimal load resistance R_L^{opt} . The optimization problem of the BPEH is formulated as follows:

$$\left\langle \text{Find the optimal load resistance } R_L^{\text{opt}} \text{ that maximizes } \bar{P}_0(g, a, r, \Omega, q, R_L^{\text{opt}}) \right\rangle. \quad (21)$$

Figure 10 shows the output power $\bar{P}_0$ of the BPEH with various load resistances at the resonant frequency ($f_r' = 31.9 \text{ Hz}$) and antiresonant frequency ($f_{ar}' = 32.3 \text{ Hz}$) of CPEH. Both power curves show peak values that correspond to the optimal load resistance. According to (20), the two optimal load resistances calculated under resonant and antiresonant states are $136 \text{ k}\Omega$ and $177 \text{ k}\Omega$, which are identified in Figure 10 as R_{L-r}^{opt} and R_{L-ar}^{opt} , respectively.

Figure 11 shows the optimal output power $\bar{P}_0^{\text{opt}}$ of the BPEH with changing excitation frequencies at the two optimal load resistances when $g = 1$ and $a = r = 20$. As can be seen in Figure 11, the peak output power generated at a load resistance of $136 \text{ k}\Omega$ is close to that generated at a load resistance of $177 \text{ k}\Omega$. Thus, the use of matching load resistances under resonance and antiresonance frequency conditions produces almost the same

FIGURE 6: Tip relative velocity $\bar{v}_s$ with various a when $g = 1$ and $R_L = 1 \text{ k}\Omega$.

FIGURE 7: Output voltage $\bar{V}_0$ with various a when $g = 1$ and $R_L = 1 \text{ k}\Omega$.

amount of maximum power although they have different voltage and current outputs. Table 2 lists the resonance frequency and the output power of the BPEH with two different load resistances of $136 \text{ k}\Omega$ and $177 \text{ k}\Omega$, respectively. From Table 2, we can obtain that the frequency bandwidth of BPEH is 7 Hz, which is about 16 times that of CPEH, and the peak output power of the BPEH under resonance and antiresonance frequency conditions is almost 40 times that generated by the CPEH. The analytical results indicate that the BPEH cannot only enhance the output performances but also broaden the frequency bandwidth.

FIGURE 8: Output power $\bar{P}_0$ with various a when $g = 1$ and $R_L = 1 \text{ k}\Omega$.

FIGURE 9: Output power $\bar{P}_0$ with various q when $g = 1$, $a = r = 20$, and $R_L = 1 \text{ k}\Omega$.

6. Experimental Verification of the Numerical Results

6.1. Prototype Model Arrangement. The value of the mass ratio should be as large as possible for the design of a high-performance BPEH. Here, we present a simple prototype model to explore the main aspects of the BPEH.

The CPEH and BPEH prototypes were developed as shown in Figure 12, respectively. The CPEH was made of a phosphor bronze substrate (mass density: 8920 kg/m^3 , Young's modulus: 113 GPa) and two PZT (PZT-5A, mass density: 7450 kg/m^3 , Young's modulus: 28.45 GPa) layers. The equivalent mass (M_{eq}) and stiffness (K_{eq}) of the CPEH shown

FIGURE 10: Output power $\bar{P}_0$ with various R_L at resonant and antiresonant frequencies of CPEH.

in Figure 12(a) were evaluated to be 12.9 g and 5.2×10^2 N/m, respectively; the mass (M_b) and the stiffness (K_b) of the elastic magnifier were measured to be 123 g and 7.9×10^3 N/m, so the mass ratio a and stiffness ratio r of the BPEH are 10 and 15, respectively.

6.2. Verification of the Numerical Results. The experimental setup used for measuring the voltage-to-base acceleration of BPEH and CPEH is shown in Figure 13. The BPEH/CPEH was fixed onto the base platform of the vibrator. An arbitrary waveform generator generated a voltage signal to cause the electromagnetic vibrator (JZ-1) to oscillate and excite the BPEH/CPEH to vibrate harmonically. A low mass accelerometer (YJ9A) was used to detect the vibration acceleration of the base platform. A laser beam from a laser displacement sensor (LDS) projects on the tip end of the BPEH/CPEH and is detected also by LDS to become the displacement signal at the tip end of the BPEH/CPEH. Both the acceleration signal $\ddot{u}_b(t)$ of the base platform and the tip displacement signal $y_s(t)$ of the BPEH/CPEH were acquired by a dynamic signal analyzer (COINV INV1601B) and transformed into the frequency domain. The dynamic response and the resonant frequencies were thus obtained and shown by a personal computer. The output voltage signal $V(t)$ of the BPEH/CPEH was acquired and shown by an oscilloscope. Thus, the output voltage $\bar{V}_0$ and the output power $\bar{P}_0$ of the load resistance R_L with respect to the base acceleration could be calculated with the following relationships:

$$\begin{aligned}\bar{V}_0 &= \frac{V(t)}{\ddot{u}_b} \text{ [V/g]}, \\ \bar{P}_0 &= \frac{\bar{V}_0^2}{R_L} \text{ [mW/g}^2\text{]}.\end{aligned}\quad (22)$$

FIGURE 11: Optimal output power of BPEH versus vibration frequency for two optimal load resistances when $g = 1$ and $a = r = 20$.

Figure 14 shows comparisons of the experimental and theoretical results of the output voltage (Figure 14(a)) and output power (Figure 14(b)) of the CPEH when the load resistance R_L is 3 kΩ and 470 kΩ, respectively. The maximum output voltage and output power measured are 2.4 [V/g] and 1.93 [mW/g²] when R_L is 3 kΩ, and the maximum voltage and power measured are 104 [V/g] and 23 [mW/g²] when R_L is 470 kΩ. Figure 15 shows the comparisons of the experimental and theoretical results of the output voltage and output power of the CPEH under resonance state (Figure 15(a)) and antiresonance state (Figure 15(b)), respectively. The comparisons indicate that the theoretical results trace the experimental results reasonably accurately.

TABLE 2: Analytical results of BPEH with two different load resistances when $a = 20$.

Node	$R_L = 136 \text{ k}\Omega$		$R_L = 177 \text{ k}\Omega$	
	Frequency [Hz]	Power [mW/g ²]	Frequency [Hz]	Power [mW/g ²]
BPEH				
Left peak	28.75	935.5	28.77	1003.81
Middle well	31.91	27.8	32.35	27.9
Right peak	35.75	274.8	35.77	268.68
CPEH	31.91	27.8	32.35	27.9

FIGURE 12: Prototypes of BPEH and CPEH.

FIGURE 13: Experiment setup for the BPEH and CPEH.

Figure 16 shows comparisons of the experimental and theoretical results of the output voltage and output power of the BPEH when $a = 10$, $r = 15$ and the load resistances, R_L , are 136 k Ω and 177 k Ω , respectively. The corresponding comparisons when $a = 10$, $r = 7$ at the load resistances, R_L , which are 136 k Ω and 177 k Ω , respectively, are shown in Figure 17. All the comparisons shown in Figures 16 and 17

indicate that the theoretical results trace the experimental results reasonably accurately, which clearly shows the consistency of the coupling lumped parameter model proposed here. The corresponding measured data are listed in Table 3. By changing the stiffness ratio (or the stiffness of the elastic magnifier), different peak voltage and power are recorded. It should be noted that long spring element with small stiffness

FIGURE 14: Frequency responses of output voltage and power of the CPEH under different load resistances.

TABLE 3: Measured results of BPEH with different a , r and load resistances R_L .

	$a = 10$			
	$r = 15$	$r = 15$	$r = 7$	$r = 7$
	$R_L = 136 \text{ k}\Omega$	$R_L = 177 \text{ k}\Omega$	$R_L = 136 \text{ k}\Omega$	$R_L = 177 \text{ k}\Omega$
Left peak				
Frequency [Hz]	33	31	24	24
Voltage [V/g]	110.8280	102.5641	271.6418	310.8108
Power [mW/g ²]	90.3151	59.4316	542.5681	545.7817
Middle well				
Frequency [Hz]	38	37	32	32
Voltage [V/g]	26.3309	25.1998	15.1858	17.3844
Power [mW/g ²]	5.0979	3.5877	1.6956	1.7074
Right peak				
Frequency [Hz]	42	42	38	38
Voltage [V/g]	226.6667	197.5940	82.1918	95.6522
Power [mW/g ²]	377.7778	220.5760	49.6727	51.6912

FIGURE 15: Output voltage and power of CPEH at resonance and antiresonance with the varying resistance.

may be weak to maintain compression and elongation in the longitudinal direction of the spring, such that transverse movement of the spring element may happen and buckling occurs.

From Figure 16 and Table 3, in the case of $a = 10$ and $r = 15$, the measured maximal output powers of the BPEH at the two optimal impedances are $377.7778 \text{ (mW/g}^2\text{)}$ and $220.5760 \text{ (mW/g}^2\text{)}$, respectively, which are 16 and 9 times that generated by the CPEH at load resistance $R_L = 470 \text{ k}\Omega$, respectively. And the frequency spaces between the two peaks at the two optimal impedances are 9 and 11 Hz, respectively.

From Figure 17 and Table 3, in the case of $a = 10$ and $r = 7$, the measured maximal output powers of the BPEH at the two optimal impedances are $545.7817 \text{ (mW/g}^2\text{)}$

and $542.5681 \text{ (mW/g}^2\text{)}$, respectively, which are both about 24 times that generated by the CPEH at load resistance $R_L = 470 \text{ k}\Omega$, respectively. And the frequency spaces between the two peaks at the two optimal impedances are both 14 Hz.

Based on Table 3, as the stiffness ratio increases, the left peak power decreases and the right peak power increases under the same mass ratio. It is also found during the experiments that when the stiffness ratio is close to 100, the left peak and the right peak of the power curve become one peak, whose frequency is close to the resonance frequency of the CPEH.

The above results verify the validity of the improved lumped parameter model presented above, and the obtained results also indicate that the elastic magnifier added to

FIGURE 16: Frequency responses of output voltage and power of the BPEH under different load resistances when $a = 10$ and $r = 15$.

the CPEH cannot only enhance the output performances but also broaden the frequency bandwidth.

7. Conclusions and Future Work

A BPEH combining a CPEH with an elastic magnifier is presented to enhance the electromechanical properties. An improved lumped parameter electromechanical model is developed and testified by some experiments, and the design optimization problem of the BPEH is studied. Both the theoretical and the experimental results indicate that

choosing an appropriate mass ratio and stiffness ratio of the BPEH can greatly improve the output voltage and output power. In the case of $a = 10$ and $r = 7$, the measured maximal output powers of the BPEH at the two optimal impedances are $545.7817 \text{ (mW/g}^2\text{)}$ and $542.5681 \text{ (mW/g}^2\text{)}$, respectively, which are both 24 times that generated by the CPEH at load resistance $R_L = 470 \text{ k}\Omega$. And the frequency spaces between the two peaks at the two optimal impedances are both 14 Hz. Therefore, the inclusion of the elastic magnifier is beneficial to miniature generators in harvesting vibrational energy from the environment.

FIGURE 17: Frequency responses of output voltage and power of the BPEH under different load resistances when $a = 10$ and $r = 7$.

Concerning the vision of the proposed BPEH, when the mass ratio a is larger than the critical mass ratio a_c , the electrical output of the BPEH is larger than that of the CPEH over a frequency range between f_r and f_{ar} . However, from a practical viewpoint, the realization of a large mass ratio within the limited space of a BPEH will be challenging.

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

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