Research Article

Bifurcation and Chaos Analysis of the Spur Gear Transmission System for One-Way Clutch, Two-Shaft Assembly

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A four-degree-of-freedom nonlinear transverse and torsional vibration model of spur gear transmission system for one-way clutch, two-shaft assembly was developed, in which the one-way clutch was modeled as a piecewise nonlinear spring with discontinuous stiffness, considering the factors such as the time-varying gear mesh stiffness, static transmission error, and nonlinearity backlash. With the help of bifurcation diagrams, time domain response diagrams, phase plane diagrams, and Poincaré maps, the effects of the excitation frequency and the torsional stiffness of one-way clutch on the dynamic behavior of gear transmission system for one-way clutch, two-shaft assembly are investigated in detail by using Runge-Kutta method. Numerical results reveal that the system response involves period-1 motion, multi-periodic motion, bifurcation, and chaotic motion. Large torsional stiffness of one-way clutch can increase the impact and lead to instability in the system. The results can present a useful source of reference for technicians and engineers for dynamic design and vibration control of such system.

1. Introduction

The one-way clutch, two-shaft assembly is the key component of the two-speed transmission, which is widely used in loaders, helicopters, tanks, military trucks, and other gear transmission systems. The two-speed transmission is generally composed of the outer ring gear, intermediate input shaft, inner star wheel, and a plurality of cylinder rollers or sprags. The one-way clutch is made up of the outer ring gear, sprags, and the inner star wheel. The outer ring gear and the inner star wheel can be engaged or disengaged according to their relative angular speed, which can lead to the two-speed transmission having two kinds of output speed.

It is well known that gear mechanisms are widely used owing to their rather large technical advantages. However, they are known as one of the most important sources of vibration and noise in mechanical transmission systems. The vibration, noise, and wear performances are mainly affected by the gear pair dynamic characteristics.

There is a growing literature on nonlinear dynamics of gear pairs in the past decades. And to analyze the

nonlinear dynamic behaviors of gear transmission system, the different excitation of static transmission error, dynamic meshing stiffness, bearing contact stiffness, prime mover, clearance nonlinearity, and load fluctuation are added [1–7]. Walha et al. [8] investigated the dynamics of a two-stage gear system with mesh stiffness and backlash. Parker et al. [9] used a finite element/contact mechanic model to investigate the dynamic response of a spur gear pair. And the dynamic characteristics of a gear pair are analyzed across a wide range of frequency and torques. Al-Shyyab and Kahraman [10] adopted lumped parameter method to establish a nonlinear time-varying dynamic model for multimesh gear, considering the nonlinear gear gap and gear meshing stiffness. The multiterm harmonic balance method (HBM) was used to gain the steady-state period-1 response, including the Root Mean Square (RMS) and the mean value of the vibration amplitude across a wide range of frequency, taking into account different gear meshing stiffness. He et al. [4] analyzed the dynamic characteristics of a spur gear pair, considering the dynamic meshing stiffness and sliding friction. The influence of friction on dynamic transmission

error is discussed. In recent years, many scholars have studied the bifurcation and chaos characteristics of gear transmission system [11]. Wang et al. [12] made use of the bifurcation diagrams, phase plane diagrams, Poincaré maps, time domain response diagrams, and amplitude-frequency spectrums to investigate the effects of the pinion speed and stiffness on the dynamic behavior of gear transmission system for locomotive. Xiang et al. [13] built a six-degree-of-freedom nonlinear dynamic model of a spur gear-bearing system with time-varying stiffness, gear backlash, and surface friction. In [13], bifurcation diagrams of the mesh stiffness and the rotational speed ratio as control parameters were used to observe the dynamic orbits of the system. The chaotic motion was identified from the largest Lyapunov exponents, Poincaré maps, and the phase diagrams of the system. Chang-Jian and Chang [14] studied the dynamic orbits of a single-degree-of-freedom spur gear system by using bifurcation diagrams with the dimensionless damping coefficient and the dimensionless rotational speed ratio as control parameters. Gou et al. [15] used the simple cell mapping method and escape time algorithm to examine the bifurcation dynamic characteristics of the multiparameter coupling in torsion-vibration gear system. Although most scholars study the steady-state response analysis of gear transmission system, Khabou et al. [16] studied a spur gear dynamic behavior in the transient regime.

The dynamic analysis of one-way clutch, two-shaft assembly is to study the coupling nonlinear characteristics of one-way clutch and gear transmission system. Though application of one-way clutch, two-shaft assembly is extensive, literature on its dynamics is limited. In the aspects of dynamic characteristics study of one-way clutch, Vernay et al. [17] pointed out and quantified the possible relative sliding effects between sprag-type races of one-way clutches by experiments. Gill-Jeong [18] verified the nonlinear behavior analysis of a paired spur gear system with a one-way clutch for the first time. He found that the dynamic transmission error of a gear system with a one-way clutch was reduced over the entire frequency range compared to a pure gear system with three-degree-of-freedom torsional vibration model. However, the effects of parameters such as bearing stiffness, bearing backlash, torsion stiffness, and damping of clutch on the nonlinear behavior of gear system coupled with a one-way clutch were not considered. In order to investigate the effect of the one-way clutch on the dynamic performance of the coupling transmission, a great deal of research effort has been devoted to the study of the nonlinear dynamics of one-way clutches in belt-pulley systems. In such system, the one-way clutch is usually modeled as a two-degree-of-freedom model, which is regarded as piecewise linear [19]. Zhu and Parker [20] used a two-degree-of-freedom model to examine the nonlinear dynamics of a two-pulley belt system with a one-way clutch through three methods of multiterm harmonic balance, numerical integration, and the bifurcation software AUTO separately. Zhu and Parker [21] also used the method of multiple scales to obtain the periodic solutions of a two-pulley belt system with clearance-type nonlinearity. The stable steady-state periodic responses of a belt-drive system with a one-way clutch were studied comprehensively [22–25].

Ding and Zu [22] studied the effects of the system parameters on the nonlinear steady-state response of a belt-drive system with a one-way clutch and had a conclusion that one-way clutch not only decreases the resonance amplitude of the driven pulley and shaft's rotational vibration but also reduces the resonance region of the belt's transverse vibration. Ding [23] also presented the stable steady-state periodic response of a two-pulley belt-drive system coupled with one-way clutch under double excitation for the first time. The Galerkin method was adapted to exam whether the one-way device can efficiently reduce the rotation vibration and transverse vibration of the pulley belt dynamic system either.

In view of the lack of research for dynamic analysis for one-way clutch, two-shaft assembly, the present paper concentrates on building and analyzing a mathematical model for such system. For the first time, a four-degree-of-freedom nonlinear transverse and torsional vibration model of spur gear transmission system for one-way clutch, two-shaft assembly was modeled, considering the gear masses m_1 and m_2 , a nonlinear mesh stiffness $k_h(t)$, and mesh damping c_h . The one-way clutch is modeled as a piecewise nonlinear spring with discontinuous stiffness that separates the driven gear into two degrees of freedom. The main purpose is to inspect the influences of the gear speed and the torsional stiffness of one-way clutch on the system dynamic behavior and find out some useful reference information for technicians and engineers to dynamic design and vibration control of such system.

2. Working Principle of One-Way Clutch, Two-Shaft Assembly

Figure 1 shows the structural schematic of one-way clutch, two-shaft assembly, which is composed primarily of bearing (1, 2), gear (1, 2), sprags, input shaft, output shaft, and inner ring. Gear 1 is fixed on the input shaft by the method of spline interference fit. Gear 2, sprags, and the inner ring, which is connected to the output shaft through key connection, are formed into a one-way clutch. Figure 2 is used as a reference to explain the work principle of the one-way clutch. When the one-way clutch is not working, the tension of the spring ensures that the sprags keep in touch with gear 2 and the inner ring. When gear 2 begins to rotate clockwise under the action of gear 1 or the rotation speed of gear 2 exceeds the rotation speed of the inner ring, the sprag will rotate around its center in clockwise direction with the spring tension and the tangential friction at the contact point. With the sprag rotating, the distance of the two contact points between the sprag and gear 2 and the inner ring would become longer, and the power can be transmitted from gear 2 to the inner ring. As gear 2 and the inner ring are fully wedged by the sprags, the rotation speed of gear 2 and the inner ring will be the same. Conversely, gear 2 will be overrunning and the power could not be transmitted to the inner ring when gear 2 rotates anticlockwise or the rotation speed of gear 2 is slower than the rotation speed of the inner ring. So it can be concluded that the rotation direction of gear 1 determines whether the load can be transferred for the one-way clutch, two-shaft assembly.

FIGURE 1: Structural diagram of one-way clutch, two-shaft assembly.

FIGURE 2: Structural schematic of one-way clutch.

3. Mathematical Model

Figure 3 shows a schematic illustration of the dynamic model of one-way clutch, two-shaft assembly considered in the present analysis. In this model, r_1 is the base circle radius of gear 1, r_2 is the base circle radius of gear 2, I_1 is the mass moments of inertia of gear 1, I_2 is the mass moments of inertia of gear 2, I_o is the mass moment of inertia of inner ring, m_1 is the mass of gear 1, m_2 is the mass of gear 2, c_h is the damping coefficient of the gear mesh, $k_h(t)$ is the time-varying stiffness coefficient of the gear mesh, which can be obtained by assuming a rectangular wave as shown in (2) [8, 18], where k_h is the average mesh stiffness value, k_{hr} and ϕ_r are the r th Fourier coefficient and phase angle of $k_h(t)$, respectively, and $k_h/k_{tp} = \varepsilon_r$, $k_{hr}/k_{tp} = \sqrt{2 - 2\cos(2\pi r(\varepsilon_r - 1))}/(\pi r)$, $\phi_r = a \tan((1 - \cos(2\pi r(\varepsilon_r - 1)))/(\sin(2\pi r(\varepsilon_r - 1))))$, ε_r is the involute

contact ratio, $R = 5$, f_h is the force-displacement on the mesh line function of gear mesh, c_1 is the damping coefficient of bearing 1, c_2 is the damping coefficient of bearing 2, k_i , f_i are the contact stiffness coefficient and the force-transverse displacement function of i th bearing ($i = 1, 2$), K_c is the torsional stiffness coefficient of one-way clutch, and C_c is the torsional damping coefficient of one-way clutch. From the lumped parameter mass method and Newton's laws of motion, the five-degree-of-freedom nonlinear model of the geared clutch bearing system dynamic model can show that

$$\begin{aligned}
 m_1 y_1'' + c_1 y_1' + c_h ((r_1 \theta_1 - r_2 \theta_2)' + y_1' - y_2' - e') \\
 + k_1 f_1(y_1) \\
 + k_h(t) f_h(r_1 \theta_1 - r_2 \theta_2 + y_1 - y_2 - e) = -F_1, \\
 m_2 y_2'' + c_2 y_2' - c_h ((r_1 \theta_1 - r_2 \theta_2)' + y_1' - y_2' - e')
 \end{aligned}$$

FIGURE 3: Schematic diagram of one-way clutch, two-shaft assemblies.

$$\begin{aligned}
 & + k_2 f_2(y_2) \\
 & - k_h(t) f_h(r_1 \theta_1 - r_2 \theta_2 + y_1 - y_2 - e) = F_2, \\
 & I_1 \theta_1'' + c_h((r_1 \theta_1 - r_2 \theta_2)' + y_1' - y_2' - e') \cdot r_1 \\
 & + k_h(t) f_h(r_1 \theta_1 - r_2 \theta_2 + y_1 - y_2 - e) \cdot r_1 = T_1(t), \\
 & I_2 \theta_2'' - c_h((r_1 \theta_1 - r_2 \theta_2)' + y_1' - y_2' - e') \cdot r_2 \\
 & - k_h(t) f_h(r_1 \theta_1 - r_2 \theta_2 + y_1 - y_2 - e) \cdot r_2 = -T_2(t), \\
 & I_o \theta_o'' = T_2(t) - T_o,
 \end{aligned} \tag{1}$$

where gear 1 and gear 2 are assumed perfect involute spur gears with no modifications and the torsional and transverse flexibility of the input shaft and output shaft are neglected. Here, y_i and θ_i are the transverse and torsional displacements of the i th gear ($i = 1, 2$), θ_o is the torsional displacement of the inner ring, and $(\cdot)'$ means derivative with respect to time t . The input torque $T_1(t)$ is assumed to be fluctuating as $T_1(t) = T_{1m} + T_{1aT}(t)$, where T_{1m} is the average torque and $T_{1aT}(t)$ is the fluctuating torque, considered as $T_{1aT}(t) = T_{1aT} \sin(\omega_a t + \phi_T)$. Forcing term $T_2(t)$, which is connected with the piecewise nonlinear force-displacement function $f_c(x)$ shown in (4), and force-velocity function $f'_c(x)$ [26], is the torque transferred from gear 2 to the inner ring, as shown in (3). And the output torque $T_o(t)$ applied on the inner ring or output shaft is assumed to be constant: that is, $T_o(t) = T_{om}$. The external radial forces F_1 and F_2 are preloads which are applied to both rolling element bearings. The static transmission error $e(t)$, connected with the manufacturing and assembling of the gears, is considered sinusoidal excitation as $e(t) = \tilde{e} \sin(\omega_e t + \phi_h)$

$$k_h(t) = k_h \left(t + \frac{2\pi}{\omega} \right) = k_h + \sum_{r=1}^R k_{hr} \cos(r\omega t - \phi_r), \tag{2}$$

$$\begin{aligned}
 T_2(t) &= K_c f_c(\theta_2 - \theta_o)^{3/2} \\
 &+ C_c f'_c(\theta_2 - \theta_o) f_c(\theta_2 - \theta_o)^{1/4},
 \end{aligned} \tag{3}$$

$$f_c(x) = \begin{cases} x & x > 0, \\ 0 & x \leq 0. \end{cases} \tag{4}$$

It is assumed that $r_1 \theta_1 = y_{g1}$, $r_2 \theta_2 = y_{g2}$, and $r_2 \theta_o = y_{co}$, where the parameters of y_{g1} , y_{g2} , and y_{co} are defined as the displacement on the direction of mesh line of gear 1, gear 2 and the inner ring, respectively. Through defining new variables $q(t)$ and $q_o(t)$ as $q(t) = y_{g1} - y_{g2} + y_1 - y_2 - e(t)$ and $q_o(t) = y_{g2} - y_{co}$, (1) can be simplified further as

$$\begin{aligned}
 m_1 y_1'' + c_1 y_1' + c_h q' + k_1 f_1(y_1) + k_h(t) f_h(q) &= -F_1, \\
 m_2 y_2'' + c_2 y_2' - c_h q' + k_2 f_2(y_2) - k_h(t) f_h(q) &= F_2, \\
 m_c q'' - m_c y_1'' + m_c y_2'' + c_h q' + k_h(t) f_h(q) \\
 &= m_c \left(\frac{F_1(t) r_1^2}{I_1} + \frac{F_2(t) r_2^2}{I_2} \right) - m_c e''(t), \\
 q_o'' - \frac{c_h r_2^2}{I_2} q' - \frac{k_h(t) r_2^2}{I_2} f_h(q) \\
 &= -F_2(t) r_2^2 \left(\frac{1}{I_2} + \frac{1}{I_o} \right) + \frac{F_o r_2^2}{I_o}.
 \end{aligned} \tag{5}$$

Here, m_c is the equivalent gear pair mass. The piecewise linear displacement functions $f_i(y_i)$ and $f_h(q)$ include the half of backlash of bearings b_i ($i = 1, 2$) and the half of

backlash of the gears b_h , which can be expressed as (7) and (8).

$$m_c = \frac{1}{(r_1^2/I_1 + r_2^2/I_2)}, \quad (6)$$

$$f_i(y_i) = \begin{cases} y_i - b_i & y_i > b_i \\ 0 & |y_i| \leq b_i \\ y_i + b_i & y_i < -b_i \end{cases} \quad (i = 1, 2), \quad (7)$$

$$f_h(q) = \begin{cases} q - b_h & q > b_h \\ 0 & |q| \leq b_h \\ q + b_h & q < -b_h \end{cases} \quad (8)$$

A dimensionless form of (5) can be obtained by letting

$$z_i(t) = \frac{y_i(t)}{b_c},$$

$$\gamma_1(t) = \frac{q(t)}{b_c},$$

$$\gamma_2(t) = \frac{q_o(t)}{b_c},$$

$$\omega_i = \sqrt{\frac{k_i}{m_i}} \quad (i = 1, 2),$$

$$\omega_n = \sqrt{\frac{k_h}{m_c}},$$

$$\tau = \omega_n t,$$

$$\omega_h = \frac{\omega_e}{\omega_n},$$

$$\omega_T = \frac{\omega_a}{\omega_n},$$

$$\xi_{ii} = \frac{c_i}{2m_i\omega_n},$$

$$\xi_{i3} = \frac{c_h}{2m_i\omega_n} \quad (i = 1, 2),$$

$$\xi_{33} = \frac{c_h}{2m_c\omega_n},$$

$$\xi_{43} = \frac{c_h r_2^2}{2I_2\omega_n^2},$$

$$k_{ii} = \frac{\omega_i^2}{\omega_n^2},$$

$$k_{i3} = \frac{m_c}{m_i} + \sum_{r=1}^5 \frac{k_{hr}}{m_i\omega_n^2} \cos(r\omega_h\tau - \phi_r) \quad (i = 1, 2),$$

TABLE 1: Properties of one-way clutch, two-shaft assembly.

Item	Notation	Value
<i>Gears</i>		
Involute contact ratio	ε_r	1.5
Radius of gears	r_1, r_2	0.04 m
Rotational inertia of gears	I_1, I_2	0.0008 kg·m ²
Mesh stiffness	k_h	2×10^8 N/m
Gear mesh damping	c_h	707
Bearing contact damping	c_1, c_2	566
Quality of gears	m_1, m_2	1 kg
<i>One-way clutch</i>		
Rotational inertia of clutch	I_o	0.0008 kg·m ²
Torsional stiffness	$K_c = K_{co}$	2467 N·m/rad
Torsional damping	C_c	0.217

$$k_{33} = 1 + \sum_{r=1}^5 \frac{k_{hr}}{m_c\omega_n^2} \cos(r\omega_h\tau - \phi_r),$$

$$k_{43} = \frac{k_h r_2^2}{I_2\omega_n^2} + \sum_{r=1}^5 \frac{k_{hr} r_2^2}{I_2\omega_n^2} \cos(r\omega_h\tau - \phi_r),$$

(9)

where b_c is the characteristic length and the terms ω_a and ω_e are the fundamental excitation frequencies of external torque fluctuations and the internal static transmission error, respectively. And the dimensionless governing equations of motion are shown as (10), where z_i is the dimensionless transverse displacement of the i th gear ($i = 1, 2$), γ_1 is dimensionless difference between the dynamic transmission error and the static transmission error of the two gears, and γ_2 is the dimensionless dynamic transmission error between gear 2 and the inner ring.

4. Parametric Studies on Characteristics of Bifurcation and Chaos

In this section, a case study is analyzed to demonstrate the effect of the rotation speed of gear 2 and the torsional stiffness of one-way clutch on its dynamic characteristics.

For the structure of one-way clutch, two-shaft assemblies, gear 2 and the inner ring can be coupled or decoupled as the one-way clutch is active.

The main parameters of the one-way clutch, two-shaft assemblies are listed in Table 1. Equation (10) describes a strongly nonlinear dynamic system with time-varying gear mesh stiffness, nonlinear backlash, and nonlinear force-displacement function of one-way clutch. And it is solved by using the fourth-order Runge-Kutta method. In order to obtain a better understanding of the dynamic behavior of the one-way clutch, two-shaft assembly, the time series data were used to generate the bifurcation diagram, phase diagram, and Poincaré maps. The periodic responses are shown by varying the excitation frequency and the torsional

stiffness of one-way clutch. On the other hand, since the new introduced coordinates z_1 and z_2 have the similar nonlinear behavior, the dynamic response of the transverse vibration z_2 of gear 2 is selected to analyze the bifurcation and chaos characteristics. In order to find out the dynamic response of the windup angle of one-way clutch, the character of dynamic transmission error between gear 2 and the inner ring is chosen to be analyzed. Furthermore, in order to avoid the risk of obtaining transient solutions and minimize computing, the initial conditions of z_2 , z_2' , γ_1 , γ_1' , γ_2 , and γ_2' are all set to be zero.

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ -1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} z_1''(\tau) \\ z_2''(\tau) \\ \gamma_1''(\tau) \\ \gamma_2''(\tau) \end{bmatrix} + 2 \begin{bmatrix} \xi_{11} & 0 & \xi_{13} & 0 \\ 0 & \xi_{22} & -\xi_{23} & 0 \\ 0 & 0 & \xi_{33} & 0 \\ 0 & 0 & -\xi_{43} & 0 \end{bmatrix} \begin{bmatrix} z_1'(\tau) \\ z_2'(\tau) \\ \gamma_1'(\tau) \\ \gamma_2'(\tau) \end{bmatrix} + \begin{bmatrix} k_{11} & 0 & k_{13} & 0 \\ 0 & k_{22} & -k_{23} & 0 \\ 0 & 0 & k_{33} & 0 \\ 0 & 0 & -k_{43} & 0 \end{bmatrix} \begin{bmatrix} f_1(z_1(\tau)) \\ f_2(z_2(\tau)) \\ f_h(\gamma_1(\tau)) \\ 0 \end{bmatrix} = \{F(\tau)\},$$

$$\{F(\tau)\} = \begin{bmatrix} -\hat{F}_1 \\ \hat{F}_2 \\ \frac{m_c r_1^2}{I_1} \hat{F}_{1m} + \frac{m_c r_2^2}{I_2} \hat{F}_{21} \\ -\hat{F}_{22} r_2^2 \left(\frac{1}{I_2} + \frac{1}{I_o} \right) + \frac{\hat{F}_o r_2^2}{I_o} \end{bmatrix}$$

$$+ \begin{bmatrix} 0 \\ 0 \\ F_{ah} \omega_h^2 \\ 0 \end{bmatrix} \sin(\omega_h \tau + \phi_h)$$

$$+ \begin{bmatrix} 0 \\ 0 \\ \frac{m_c r_1^2}{I_1} \hat{F}_{1aT} \\ 0 \end{bmatrix} \sin(\omega_T \tau + \phi_T),$$

$$\hat{F}_i = \frac{F_i}{(m_i b_c \omega_n^2)}, \quad i = 1, 2,$$

$$\hat{F}_{1m} = \frac{F_{1m}}{(m_c b_c \omega_n^2)},$$

$$F_{ah} = \frac{\tilde{e}}{b_c},$$

$$\hat{F}_{1aT} = \frac{F_{1aT}}{(m_c b_c \omega_n^2)},$$

$$\hat{F}_o = \frac{F_o}{(b_c \omega_n^2)},$$

$$\hat{F}_{21} = \frac{K_c f_c(\gamma_2)^{3/2} b_c^{3/2}}{r_2^{5/2} m_c b_c \omega_n^2} + \frac{C_c f_c'(\gamma_2) f_c(\gamma_2)^{1/4} b_c^{5/4}}{r_2^{9/4} m_c b_c \omega_n},$$

$$\hat{F}_{22} = \frac{K_c f_c(\gamma_2)^{3/2} b_c^{3/2}}{r_2^{5/2} b_c \omega_n^2} + \frac{C_c f_c'(\gamma_2) f_c(\gamma_2)^{1/4} b_c^{5/4}}{r_2^{9/4} b_c \omega_n}. \quad (10)$$

4.1. Effect of Excitation Frequency. The ratio of the bearing stiffness to average gear mesh stiffness $\hat{k}$ is introduced to represent the change of support stiffness. The frequency parameter ω_h is assigned as control parameters. The bifurcation diagram of the dimensionless transverse displacement of gear 2 with excitation frequency ω_h change is presented in Figure 4.

It is well known that the jump phenomenon usually exists in the dynamic response of nonlinear system. It could be seen clearly in Figure 4 that there is a phenomenon of jumping at value of $\omega_h = 0.61$. The one-way clutch, two-shaft assemblies system mainly exhibits period-one motion at low values or at high excitation frequency: that is, $\omega_h < 0.69$ or $\omega_h > 1.66$. Just as shown in Figure 5(a), the time domain response diagram is a sine wave, the phase plane diagram is only one closed circle, and the Poincaré map is a single point. It is observed that the system goes into chaotic region within the range of $0.7 \leq \omega_h \leq 0.87$, $0.94 \leq \omega_h \leq 1.13$, and $1.3 \leq \omega_h \leq 1.59$. It can be seen in Figure 5(b) that the time domain response diagram shows a nonperiodic motion, the phase plane diagram is disorder, and the Poincaré map shows many discrete points. As ω_h is increased from 0.88 to 0.93 and from 1.14 to 1.19, the system exhibits unstable period-one motion, and the period-2 motion will be observed over the range of $1.2 \leq \omega_h \leq 1.29$. And as it is shown in Figure 5(c), the time domain response diagram shows a period motion, the phase plane diagram shows two closed circles, and the Poincaré map shows two unique points. Figure 5 presents time series, phase plane diagram, and Poincaré maps of Hopf bifurcation points at $\omega_h = 0.4, 0.7, 1.2, 1.3, 1.6$, and 1.63 . It can be seen that the motions of the one-way clutch, two-shaft assemblies system undergo period-1 motion, chaotic motion, period-2 motion, quasi-periodic motion, and period-7 motion.

Figure 6 presents the bifurcation diagram of the system response of dimensionless dynamic transmission error between gear 2 and the inner ring, which is essentially used to study the dynamic windup angle of one-way clutch, using excitation frequency ω_h as control parameter. It can be

FIGURE 4: z_2 versus ω_h , plots for $F_{ah} = 0.5$, $F_{at} = 0$, $b_h = 0$, $b_1 = b_2 = b_c = 0.01$, $\hat{k} = 5$, $K_c = 5K_{co}$, and $F_m = 1$.

observed that the relative reverse motion between gear 2 and the inner ring exhibits rather complex phenomenon with the increase of the excitation frequency ω_h . There exists some “frequency hopping” at $\omega_h = 0.26, 0.29$, and 0.61 , where the system undergoes the transient transition between period-one motion. Only period-one motion can be observed at low values of the excitation frequency, that is, $\omega_h < 0.69$, or at high values of the excitation frequency, that is, $\omega_h > 1.68$. The system performs period-2 motion within the range of $1.19 \leq \omega_h \leq 1.29$. The system exhibits chaotic motions within the range of $0.7 \leq \omega_h \leq 0.88$, $0.93 \leq \omega_h \leq 1.14$, and $1.3 \leq \omega_h \leq 1.68$. Unlike the transverse motion of gear 2, the relative reverse motion between gear 2 and the inner ring exhibits stable period-1 motion within the range of $0.89 \leq \omega_h \leq 0.92$, $1.16 \leq \omega_h \leq 1.18$. The system goes into chaotic motions through the quasi-periodic route around excitation frequency points $\omega_h = 0.71, 0.931$, and 1.31 and goes into chaotic motions through the quasi-periodic route around excitation frequency points $\omega_h = 0.881, 1.141$, and 1.681 .

Figure 7 shows the time series, phase plane diagram, and Poincaré maps of Hopf bifurcation points at $\omega_h = 0.4, 0.7, 1.2, 1.3, 1.6$, and 1.63 . It can be founded that the system behavior undergoes period-1 motion, chaotic motion, period-2 motion, quasi-periodic motion, and period-7 motion with the increase of the excitation frequency ω_h .

4.2. Effect of Torsional Stiffness of One-Way Clutch. Figure 8 presents the bifurcation diagram of the system response of dimensionless dynamic transmission error between gear 1 and gear 2, using the torsional stiffness of one-way clutch K_c as control parameter. It can be observed from the figure that, when the torsional stiffness is changed, there exist kinds of motion forms, such as periodic-1 motion, periodic-2 motion, periodic-4 motion, periodic-8 motion, and chaotic motion. In order to accurately distinguish the motion form, the time domain response diagrams, phase plane diagrams, and Poincaré maps are listed to illustrate the dynamic characteristics of the system in Figure 9. When K_c is less than $276000 \text{ N}\cdot\text{m}/\text{rad}$, only period-one motion can be observed. The system motion state will be changed from periodic-1 motion to periodic-2 motion, when the torsional

stiffness of one-way clutch is increased to $276000 \text{ N}\cdot\text{m}/\text{rad}$, and the state is continued until $331000 \text{ N}\cdot\text{m}/\text{rad}$. The system performs the transient periodic-4 motion within the range of $331000 \text{ N}\cdot\text{m}/\text{rad} \leq K_c \leq 341000 \text{ N}\cdot\text{m}/\text{rad}$ and exhibits chaotic motions within the range of $341000 \text{ N}\cdot\text{m}/\text{rad} \leq K_c \leq 676000 \text{ N}\cdot\text{m}/\text{rad}$, just as shown in Figure 9(c). It can be seen that the time domain response diagram shows a nonperiodic motion, the Poincaré map shows many discrete points, and the phase plane diagram is disorder. With the increasing of the torsional stiffness, the system leaves chaos and goes into periodic-8 motion from $676000 \text{ N}\cdot\text{m}/\text{rad}$ to $696000 \text{ N}\cdot\text{m}/\text{rad}$. When the torsional stiffness is greater than $696000 \text{ N}\cdot\text{m}/\text{rad}$, the system goes into another region of chaotic motion again until $811000 \text{ N}\cdot\text{m}/\text{rad}$. After undergoing the chaotic region, the system leaves chaotic motion and goes into periodic-1 motion again within the range of $811000 \text{ N}\cdot\text{m}/\text{rad} \leq K_c \leq 951000 \text{ N}\cdot\text{m}/\text{rad}$. When K_c is higher than $951000 \text{ N}\cdot\text{m}/\text{rad}$, the system leaves periodic-1 motion and returns into chaotic motion again.

5. Summary and Conclusions

A general multi-degree-of-freedom nonlinear dynamic model of one-way clutch, two-shaft assembly based on lumped parameter method (LPM) is proposed in this paper. The gear backlash, bearing backlash, a nonlinear mesh stiffness $k_h(t)$, and mesh damping c_h are considered in the model. The one-way clutch is modeled as a piecewise nonlinear spring with discontinuous stiffness that separates gear 2 into two degrees of freedom. The dynamic response of one-way clutch, two-shaft assemblies is studied over a wide range of frequencies. Speed sweeping is used to check the effect of one-way clutch on the nonlinear behavior of gear transmission system, by applying variable step-size Runge-Kutta method.

As mentioned above, with the increasing of excitation frequency and torsional stiffness of one-way clutch, the system of one-way clutch, two-shaft assembly is prone to involve periodic-1 motion, multiperiodic motion, and chaotic motion. Large torsional stiffness of one-way clutch can increase the impact and lead to instability in the system. To reduce the impact and increase stability, the excitation

FIGURE 5: The time series of z_2 , phase plane diagram, and Poincaré map under $F_{ah} = 0.5$, $F_{aT} = 0$, $b_h = 0$, $b_1 = b_2 = b_c = 0.01$, $\hat{k} = 5$, $K_c = 5K_{co}$, and $F_m = 1$ and different excitation frequency with them; (a) $\omega_h = 0.4$, (b) $\omega_h = 0.7$, (c) $\omega_h = 1.2$, (d) $\omega_h = 1.3$, (e) $\omega_h = 1.6$, and (f) $\omega_h = 1.63$.

FIGURE 6: γ_2 versus ω_n , plots for $F_{ah} = 0.5$, $F_{aT} = 0$, $b_h = 0$, $b_1 = b_2 = b_c = 0.01$, $\hat{k} = 5$, $K_c = 5K_{co}$, and $F_m = 1$.

frequency of the engine and the torsional stiffness of the one-way clutch are taken into account to be based on matching and optimization principles for the system of one-way clutch, two-shaft assembly.

Nomenclature

r_1 :	Base circle radius of gear 1	y_{g2} :	Displacement on direction of mesh line of gear 2
r_2 :	Base circle radius of gear 2	y_{co} :	Displacement on direction of mesh line of inner ring
I_1 :	Mass moment of inertia of gear 1	θ_1 :	Torsional displacement of gear 1
I_2 :	Mass moment of inertia of gear 2	θ_2 :	Torsional displacement of gear 2
I_o :	Mass moment of inertia of inner ring	θ_o :	Torsional displacement of inner ring
m_1 :	Mass representing inertia of gear 1	e :	Static transmission error
m_2 :	Mass representing inertia of gear 2	b_1 :	The half of backlash of bearing 1
m_o :	Mass representing inertia of inner ring	b_2 :	The half of backlash of bearing 2
m_c :	Equivalent gear pair mass	b_h :	The half of backlash of the gear pair
c_1 :	Viscous damping coefficient of bearing 1	b_c :	The characteristic length
c_2 :	Viscous damping coefficient of bearing 2	ω_e :	Excitation frequency of static transmission error
c_h :	Viscous damping coefficient of gear pair	ω_a :	Excitation frequencies of external torque
C_c :	Torsional damping of one-way clutch	ω_n :	Natural frequency $\omega_n = \sqrt{k_h/m_c}$
k_1 :	Contact stiffness of bearing 1	ε_r :	Involute contact ratio
k_2 :	Contact stiffness of bearing 2	z_1 :	Dimensionless transverse displacement of gear 1
k_h :	Time-varying mesh stiffness of gear pair	z_2 :	Dimensionless transverse displacement of gear 2
K_c :	Torsional stiffness of one-way clutch	γ_1 :	The dimensionless difference between the dynamic transmission error $(y_{g1} - y_{g2} + y_1 - y_2)/b_c$ and the static transmission of the gear pair $(y_{g1} - y_{g2} + y_1 - y_2 - e)/b_c$
f_1 :	Force-displacement function of bearing 1	γ_2 :	The dimensionless difference of the displacement on direction of mesh line of gear 2 and the displacement on direction of mesh line of inner ring of one-way clutch
f_2 :	Force-displacement of bearing 2	q :	The difference between the dynamic transmission error $(y_{g1} - y_{g2} + y_1 - y_2)$ and the static transmission (e) of the gear pair $(y_{g1} - y_{g2} + y_1 - y_2 - e)$
f_h :	Nonlinear displacement function of gear pair	q_o :	The difference of the displacement on direction of mesh line of gear 2 and the displacement on direction of mesh line of inner ring of one-way clutch.
F_1 :	Preload applied on bearing 1		
F_2 :	Preload applied on bearing 2		
F_o :	Force transmitted through one-way clutch		
T_1 :	Torque acting on gear 1		
T_2 :	Torque acting on gear 2		
T_o :	Torque acting on inner ring		
T_{1m} :	Average torque acting on gear 1		
T_{1aT} :	Fluctuating torque acting on gear 1		
y_1 :	Transverse displacement of gear 1		
y_2 :	Transverse displacement of gear 2		
y_{g1} :	Displacement on direction of mesh line of gear 1		

FIGURE 7: The time series of γ_2 , phase plane diagram, and Poincaré map under $F_{ah} = 0.5$, $F_{aT} = 0$, $b_h = 0$, $b_1 = b_2 = b_c = 0.01$, $\hat{k} = 5$, $K_c = 5K_{co}$, and $F_m = 1$ and different excitation frequency with them; (a) $\omega_h = 0.4$, (b) $\omega_h = 0.7$, (c) $\omega_h = 1.2$, (d) $\omega_h = 1.3$, (e) $\omega_h = 1.6$, and (f) $\omega_h = 1.63$.

FIGURE 8: γ_1 versus K_c , plots for $F_{ah} = 0.5$, $F_{aT} = 0$, $b_h = 0$, $b_1 = b_2 = b_c = 0.01$, $\hat{k} = 5$, $\omega_h = 1.25$, and $F_m = 1$.

FIGURE 9: The time series of γ_1 , phase plane diagram, and Poincaré map under $F_{ah} = 0.5$, $F_{aT} = 0$, $b_h = 0$, $b_1 = b_2 = b_c = 0.01$, $\hat{k} = 5$, $\omega_h = 1.25$, and $F_m = 1$ and different torsional stiffness of one-way clutch with them; (a) $K_c = 300000$ N·m/rad, (b) $K_c = 341000$ N·m/rad, (c) $K_c = 600000$ N·m/rad, and (d) $K_c = 688000$ N·m/rad.

Conflicts of Interest

The authors declare that there are no conflicts of interest regarding the publication of this paper.

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