Research Article

Differential Movement Synchronous Tracking Control Strategy of Double-Shaking Table System Loading with Specimen

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Multisupport, multidimension, and nonuniform excitation seismic experiments have new requirements for shaking table array system in synchronous tracking control. Therefore, this article proposed a novel synchronous tracking strategy, differential movement synchronous tracking control (DMSTC) strategy, for double-shaking table system while taking the interaction between shaking tables and specimen into consideration. DMSTC Simulink model of the double-shaking table with specimen was established and simulations were conducted in various conditions. The results demonstrate the viability of the proposed DMSTC in that the frequency bandwidth of the double-shaking tables is expanded from 3.27 Hz to 64.57 Hz, the maximum value of differential movement synchronous error is decreased from 1.682 mm to 0.482 mm, and the maximum tracking errors of the two shaking tables decrease from 1.138 mm to 0.044 mm and from 1.030 mm to 0.497 mm, respectively.

1. Introduction

Electrohydraulic shaking table is a piece of the most important test equipment for replicating actual vibration situations in seismic tests and many other applications [1–6]. Multisupport, multidimension, and nonuniform excitation experiments have new requirements for shaking table array system in synchronous tracking control [2, 7–9]. Seismic tests of such large-span structures as bridges, dams, railways, pipelines, and so on should consider the traveling wave effect, partial coherent effect, and local site effect to accurately simulate their motion in earthquake. Therefore, synchronous tracking control strategy has become increasingly important and is in need of further development.

The common synchronous control strategies are the parallel control, the master/slave control, the virtual-shaft control, the cross-coupling control, the relative coupling control, adjacent cross-coupling control, ring coupling control, and so on [10–18]. Koren initially proposed the cross-coupling control [12], which, yet, is difficult to be applied to more than two systems. To overcome this limitation, Perez-Pinal et al. proposed the relative coupling control

which has stronger synchronous performance and better development [13, 14]. To reduce the complexity of multiple motor synchronization control systems, the adjacent cross-coupling control strategy and ring coupling control strategy were proposed by researchers [15, 16].

To improve synchronous control precision, the control algorithms are also further studied by researchers, such as traditional PID control, adaptive feed forward control, H_∞ control, iterative learning control, sliding mode control, fuzzy control, and neural network control [10, 11, 17–25]. Sun and Chiu developed a nonlinear control algorithm to address the motion synchronization of a dual-cylinder electrohydraulic lifting system [23]. Chen et al. proposed an integrated fuzzy controller design that consisted of a pair of cylinder controllers and a motion synchronization controller for synchronous motion and positioning in a hydraulic lifting system [24]. The proposed fuzzy control design effectively handled complex, ill-defined, and uncertain systems. In addition, Liu proposed an intelligent proportional-integral-derivative (PID) algorithm based on back propagation (BP) neural network arithmetic for a three-motor system and obtained satisfactory synchronous decoupling control [25].

A typical synchronization control scheme consists of synchronization control parameters, synchronization control strategy to calculate the parameters errors, and a control algorithm to improve synchronization control precision. C. Chen and L. Chen classified it into three types of control mode: position mode, velocity mode, and torque mode [20]. Tsuruta et al. considered motor torsion, displacement, and speed to improve the stability and synchronous control precision of the studied system [26, 27]. Moreover, Zhang et al. focused on the deviation of the array displacement, speed, and acceleration parameters of shaking tables to elevate the synchronous properties of the system [6]. Fang et al. utilized a synchronous control strategy that combined phase-shifting and amplitude-varying methods to achieve the synchronous motion of two parallel servos [28].

Many studies demonstrate that the feedforward compensator control is an effective method to improve a system's synchronous tracking accuracy. Zhang et al. applied the adaptive inverse control (AIC) algorithm to two shaking tables' synchronous tracking controls to improve the tracking precision [6]. In addition, a fuzzy tracking controller (FTC) was designed by Chen et al. to reduce the position-tracking error caused by the parameter uncertainties and nonlinearities of the hydraulic components [24].

However, to date, there are few synchronous tracking control researches in shaking table system with the consideration of the interaction between the specimen and the shaking table system. Particularly, the study of differential movement synchronous tracking control is rare. Therefore, a mathematical model of the double-shaking table system with specimen was built in this research; and the shaking table system of Beijing University of Technology (BJUT) was taken as the research object to verify the synchronous tracking strategy of the DMSTC for shaking table's motion, thereby establishing a Simulink model of the DMSTC. In this paper, the double-shaking table system was described in Section 1, and the hydraulic system was established in Section 2. Subsequently, the mathematical model of the double-shaking table system with specimen was established and the Simulink model of the DMSTC was presented in Section 3, and the simulation analysis was conducted in Section 4. Finally, in Section 5, a brief conclusion of the paper was given.

2. System Description

2.1. Introduction of the Double-Shaking Table System. The system of nine subarray shaking tables, composed of nine single shaking tables of $1\text{ m} \times 1\text{ m}$, can be used in shaking table's array experiments with diversified forms and in various positions. Figure 1 shows the structure of the double-shaking table system. Each single shaking table is composed of platform, supporting guide mechanism, servo valves, hydraulic cylinders, connecting rod, and base. As the core control unit, the servo valves use small control current to control the flow of high pressure oil. The hydraulic cylinders, as the force transform unit, can impose huge force on the platform through the supporting guide mechanism. Each system, with hydraulic cylinders and servo valves, has three

FIGURE 1: The structure diagram of the double-shaking table system with specimen.

exciters, one in X direction and two in Y horizontal direction, and three connecting rods in Z vertical direction.

2.2. The Establishment of the Hydraulic Cylinder Model. The hydraulic system is composed of the hydraulic cylinder, the servo valve, pressure sensor, energy accumulator, and so on. The hydraulic system can be presented in a series of difference equations in the establishment of shaking table's dynamic model. As the core component of shaking table, the servo valve's property is critical for the motion of shaking table.

2.2.1. The Servo Valve Model. The servo valve model can be established with the basis of the second-order oscillation link:

$$G_q(s) = \frac{K_v}{s^2/w_v^2 + (2\xi_v/w_v)s + 1}, \quad (1)$$

where w_v is the inherent frequency of the servo valve, ξ_v is the damping ratio of the servo valve, and K_v is the flow gain of the servo valve.

2.2.2. The Continuity Equations of Hydraulic Cylinder. Assuming that damping and spring effect are not involved in the hydraulic cylinder system [29], the continuity equations of hydraulic cylinder are

$$\begin{aligned} Q_L &= k_q G_q E - K_C p_L, \\ A_p p_L &= M \frac{d^2 x_p}{dt^2}, \\ Q_L &= A_p \frac{dx_p}{dt} + \frac{V}{4\beta} \frac{dp_L}{dt} + C_C p_L, \end{aligned} \quad (2)$$

where Q_L is represented as load flow, k_q is flow gain of slid valve near static operation point, G_q is servo value transfer function, E is control error signal, K_C is flow pressure parameter of slid valve near static operation point, p_L is load pressure, A_p is effective bearing area of valve, M is the gross mass of valve and its load deduction, x_p is displacement of valve, V is gross capacity of two hydraulic cylinder chamber, β is oil elastic modulus, and C_C is gross leakage coefficient of hydraulic cylinder.

Regarding the platform and specimen as an integrity, and load property as inertial load, the continuity equations of hydraulic cylinder can be obtained after the processing of Laplace transform. They are shown as follows:

$$\begin{aligned} Ms^2x &= A_p p_L, \\ Q_L &= A_p sx + \frac{V}{4\beta} sp_L + C_C p_L, \\ Q_L &= k_q G_q E - K_C p_L. \end{aligned} \quad (3)$$

3. The DMSTC Model of the Double-Shaking Table System with Specimen

3.1. The Deduction of the Interaction between the Double-Shaking Tables and Specimen. Figure 2 shows the simplified mechanical model of the double-shaking table system with specimen. M_1 and M_2 are the masses of the two platforms, respectively, and $M_1 = M_2 = M_T$; x_1 and x_2 are their displacements; F_1 and F_2 are exciting forces of their hydraulic systems; M_s is the mass of the specimen; x_s is the specimen's displacement; C is the specimen's damping coefficient; and K is the specimen's stiffness coefficient.

The force equilibrium equations are

$$\begin{aligned} M_s s^2 x_s + (Cs + k)(x_s - x_1) + (Cs + k)(x_s - x_2) &= 0, \\ M_1 s^2 x_1 + (Cs + k)(x_1 - x_s) &= F_1, \\ M_2 s^2 x_2 + (Cs + k)(x_2 - x_s) &= F_2. \end{aligned} \quad (4)$$

Then

$$\begin{aligned} [M_s s^2 + 2(Cs + k)]x_s - (Cs + k)x_1 - (Cs + k)x_2 &= 0, \\ (M_1 s^2 + Cs + k)x_1 - (Cs + k)x_s &= F_1, \\ (M_2 s^2 + Cs + k)x_2 - (Cs + k)x_s &= F_2. \end{aligned} \quad (5)$$

The force equilibrium equations are

$$\begin{aligned} A_p p_{L1} &= F_1, \\ A_p p_{L2} &= F_2. \end{aligned} \quad (6)$$

Presuming that the parameters of the two exciters are the same, the output flow of valve valves is

$$\begin{aligned} Q_{L1} &= k_q G_q E_1 - K_C p_{L1}, \\ Q_{L2} &= k_q G_q E_2 - K_C p_{L2}. \end{aligned} \quad (7)$$

The flow of exciters is

$$\begin{aligned} Q_{L1} &= A_p sx_1 + \frac{V}{4\beta} sp_{L1} + C_C p_{L1}, \\ Q_{L2} &= A_p sx_2 + \frac{V}{4\beta} sp_{L2} + C_C p_{L2}. \end{aligned} \quad (8)$$

FIGURE 2: Simplified mechanical model.

Equalizing (5) and (6), the result is

$$\begin{aligned} \left(\frac{V}{4\beta} s + K_C + C_C \right) p_{L1} &= k_q G_q E_1 - A_p sx_1, \\ \left(\frac{V}{4\beta} s + K_C + C_C \right) p_{L2} &= k_q G_q E_2 - A_p sx_2. \end{aligned} \quad (9)$$

Given

$$G_2 = \frac{V}{4\beta A_p^2} s + \frac{K_C + C_C}{A_p^2}, \quad (10)$$

then

$$\begin{aligned} A_p p_{L1} &= \frac{1}{G_2} \left(\frac{k_q G_q E_1}{A_p} - sx_1 \right), \\ A_p p_{L2} &= \frac{1}{G_2} \left(\frac{k_q G_q E_2}{A_p} - sx_2 \right). \end{aligned} \quad (11)$$

We substitute (9) into (4) to get

$$\begin{aligned} G_2 (M_1 s^2 + Cs + k)x_1 - G_2 (Cs + k)x_s &= \frac{k_q G_q E_1}{A_p} - sx_1, \\ G_2 (M_2 s^2 + Cs + k)x_2 - G_2 (Cs + k)x_s &= \frac{k_q G_q E_2}{A_p} - sx_2. \end{aligned} \quad (12)$$

We simplify (4) to get

$$x_s = \frac{Cs + k}{M_s s^2 + 2(Cs + k)} (x_1 + x_2). \quad (13)$$

The PID control transfer function is

$$G_{PID} = K_P \left(1 + \frac{1}{T_i s} + T_d s \right). \quad (14)$$

We combine (12) and (13) to get

$$\begin{aligned} \left[G_2 (M_1 s^2 + Cs + k) + s - \frac{G_2 (Cs + k)^2}{M_s s^2 + 2 (Cs + k)} \right] x_1 \\ - \frac{G_2 (Cs + k)^2}{M_s s^2 + 2 (Cs + k)} x_2 = \frac{G_q k_q E_1}{A_p}, \\ \left[G_2 (M_2 s^2 + Cs + k) + s - \frac{G_2 (Cs + k)^2}{M_s s^2 + 2 (Cs + k)} \right] x_2 \\ - \frac{G_2 (Cs + k)^2}{M_s s^2 + 2 (Cs + k)} x_1 = \frac{G_q k_q E_2}{A_p}. \end{aligned} \quad (15)$$

We take the displacement feedback gain, PID control transfer function, and the displacement meter transfer function into consideration to get

$$\begin{aligned} E_1 &= G_{PID} u_1 - G_d K d x_1, \\ E_2 &= G_{PID} u_2 - G_d K d x_2. \end{aligned} \quad (16)$$

We write (16) into (15) to get

$$\begin{aligned} \left[G_2 (M_1 s^2 + Cs + k) + s - \frac{G_2 (Cs + k)^2}{M_s s^2 + 2 (Cs + k)} \right. \\ \left. + \frac{G_q k_q}{A_p} G_d K d \right] x_1 - \frac{G_2 (Cs + k)^2}{M_s s^2 + 2 (Cs + k)} x_2 \\ = \frac{G_{PID} G_q k_q u_1}{A_p}, \\ \left[G_2 (M_2 s^2 + Cs + k) + s - \frac{G_2 (Cs + k)^2}{M_s s^2 + 2 (Cs + k)} \right. \\ \left. + \frac{G_q k_q}{A_p} G_d K d \right] x_2 - \frac{G_2 (Cs + k)^2}{M_s s^2 + 2 (Cs + k)} x_1 \\ = \frac{G_{PID} G_q k_q u_2}{A_p}. \end{aligned} \quad (17)$$

Given

$$\begin{aligned} G_3 &= G_2 (M_T s^2 + Cs + k) + s - \frac{G_2 (Cs + k)^2}{M_s s^2 + 2 (Cs + k)} \\ &\quad + \frac{G_q k_q}{A_p} G_d K d, \\ G_4 &= \frac{G_2 (Cs + k)^2}{M_s s^2 + 2 (Cs + k)}, \end{aligned} \quad (18)$$

then

$$\begin{aligned} G_3 x_1 - G_4 x_2 &= \frac{G_{PID} G_q k_q u_1}{A_p}, \\ G_3 x_2 - G_4 x_1 &= \frac{G_{PID} G_q k_q u_2}{A_p}. \end{aligned} \quad (19)$$

Finally, the system transfer functions can be obtained:

$$\begin{aligned} x_1 &= \frac{G_{PID} G_q k_q}{A_p} \frac{G_3 u_1 + G_4 u_2}{G_3^2 - G_4^2}, \\ x_2 &= \frac{G_{PID} G_q k_q}{A_p} \frac{G_3 u_2 + G_4 u_1}{G_3^2 - G_4^2}. \end{aligned} \quad (20)$$

3.2. The Proposition of the DMSC. Figure 3 shows the model of the double-shaking table system in different coordinates of the double-shaking table system, in which the input coordinate system and the output coordinate system were established. In the input coordinate system, u_1 and u_2 are input signals and ϕ is the desired rotation angle. In the output coordinate system, x_1 and x_2 are output signals, l is the distance between the shaking table and the central line of the double-shaking table system, and φ is the actual rotation angle of the double-shaking table system.

We substitute (13) into (12) to get

$$\begin{aligned} \left[G_2 (M_1 s^2 + Cs + k) + s \right] x_1 \\ - \frac{G_2 (Cs + k)^2}{M_s s^2 + 2 (Cs + k)} (x_1 + x_2) = \frac{G_q k_q E_1}{A_p}, \\ \left[G_2 (M_2 s^2 + Cs + k) + s \right] x_2 \\ - \frac{G_2 (Cs + k)^2}{M_s s^2 + 2 (Cs + k)} (x_1 + x_2) = \frac{G_q k_q E_2}{A_p}. \end{aligned} \quad (21)$$

Given

$$\begin{aligned} G_5 &= G_2 (M_T s^2 + Cs + k) + s, \\ G_4 &= \frac{G_2 (Cs + k)^2}{M_s s^2 + 2 (Cs + k)} \end{aligned} \quad (22)$$

and writing (22) into (21) to get

$$\begin{aligned} G_5 x_1 - G_4 (x_1 + x_2) &= \frac{G_q k_q E_1}{A_p}, \\ G_5 x_2 - G_4 (x_1 + x_2) &= \frac{G_q k_q E_2}{A_p}, \end{aligned} \quad (23)$$

then

$$\begin{aligned} (G_5 - 2G_4) (x_1 + x_2) &= \frac{G_q k_q}{A_p} (E_1 + E_2), \\ G_5 (x_1 - x_2) &= \frac{G_q k_q}{A_p} (E_1 - E_2). \end{aligned} \quad (24)$$

We deduct from the output signal coordinate system of Figure 3 with geometric method to acquire

$$\begin{aligned} x_1 &= x + l\varphi, \\ x_2 &= x - l\varphi. \end{aligned} \quad (25)$$

FIGURE 3: The model of the double-shaking table system in different coordinates.

FIGURE 4: Controlling block diagram.

Then

$$\begin{aligned} E_1 + E_2 &= 2e_d, \\ E_1 - E_2 &= 2e_a. \end{aligned} \quad (30)$$

We take the displacement feedback gain, PID control transfer function, and the displacement meter transfer function into consideration to get

$$\begin{aligned} e_d &= \frac{1}{2} [G_{\text{PID}}(u_1 + u_2) - G_d K d (x_1 + x_2)], \\ e_a &= \frac{1}{2} [G_{\text{PID}}(u_1 - u_2) - G_d K d (x_1 - x_2)]. \end{aligned} \quad (31)$$

We substitute (26) and (28) into (31) to get

$$\begin{aligned} e_d &= G_{\text{PID}} u - G_d K d x, \\ e_a &= G_{\text{PID}} l \phi - G_d K d l \phi. \end{aligned} \quad (32)$$

We write (32) into (30) to get

$$\begin{aligned} E_1 + E_2 &= 2(G_{\text{PID}} u - G_d K d x), \\ E_1 - E_2 &= 2(G_{\text{PID}} l \phi - G_d K d l \phi). \end{aligned} \quad (33)$$

We write (26) and (33) into (24) to get

$$\begin{aligned} \left(G_5 - 2G_4 + \frac{G_q k_q}{A_p} G_d K d \right) x &= \frac{G_{\text{PID}} G_q k_q}{A_p} u, \\ \left(G_5 + \frac{G_q k_q}{A_p} G_d K d \right) \phi &= \frac{G_{\text{PID}} G_q k_q}{A_p} \phi. \end{aligned} \quad (34)$$

Equation (34) can be transformed into

$$\begin{aligned} \frac{x}{u} &= \frac{G_{\text{PID}} G_q k_q}{A_p} \frac{1}{G_5 - 2G_4 + (G_q k_q / A_p) G_d K d}, \\ \frac{\phi}{\phi} &= \frac{G_{\text{PID}} G_q k_q}{A_p} \frac{1}{G_5 + (G_q k_q / A_p) G_d K d}. \end{aligned} \quad (35)$$

Then

$$\begin{aligned} x_1 + x_2 &= 2x, \\ x_1 - x_2 &= 2l\phi. \end{aligned} \quad (26)$$

We deduct from the input signal coordinate system in Figure 3 to acquire

$$\begin{aligned} u_1 &= u + l\phi, \\ u_2 &= u - l\phi. \end{aligned} \quad (27)$$

Then

$$\begin{aligned} u_1 + u_2 &= 2u, \\ u_1 - u_2 &= 2l\phi. \end{aligned} \quad (28)$$

In order to get the control error signal of the double-shaking table system, the controlling block diagram is designed as shown in Figure 4, in which e_d is the translation error and e_a is the rotation error.

We deduct from Figure 4 to get

$$\begin{aligned} E_1 &= e_d + e_a, \\ E_2 &= e_d - e_a. \end{aligned} \quad (29)$$

FIGURE 5: Simulink model of the DMSC.

We write (28) and (33) into (24) to get

$$\begin{aligned}
 & \left(G_5 - 2G_4 + \frac{G_q k_q}{A_p} G_a K d \right) (x_1 + x_2) \\
 &= \frac{G_{PID} G_q k_q}{A_p} (u_1 + u_2), \\
 & \left(G_5 + \frac{G_q k_q}{A_p} G_a K d \right) (x_1 - x_2) \\
 &= \frac{G_{PID} G_q k_q}{A_p} (u_1 - u_2).
 \end{aligned} \tag{36}$$

Given

$$\begin{aligned}
 G_6 &= G_5 - 2G_4 + \frac{G_q k_q}{A_p} G_a K d, \\
 G_7 &= G_5 + \frac{G_q k_q}{A_p} G_a K d,
 \end{aligned} \tag{37}$$

then

$$\begin{aligned}
 x_1 + x_2 &= \frac{G_{PID} G_q k_q}{A_p G_6} (u_1 + u_2), \\
 x_1 - x_2 &= \frac{G_{PID} G_q k_q}{A_p G_7} (u_1 - u_2).
 \end{aligned} \tag{38}$$

Finally, the system's transfer function can be acquired:

$$\begin{aligned}
 x_1 &= \frac{G_{PID} G_q k_q}{2A_p} \left[u_1 \left(\frac{1}{G_6} + \frac{1}{G_7} \right) + u_2 \left(\frac{1}{G_6} - \frac{1}{G_7} \right) \right], \\
 x_2 &= \frac{G_{PID} G_q k_q}{2A_p} \left[u_1 \left(\frac{1}{G_6} - \frac{1}{G_7} \right) + u_2 \left(\frac{1}{G_6} + \frac{1}{G_7} \right) \right].
 \end{aligned} \tag{39}$$

We write (20) and (39) into the form of transfer function matrix as follows:

$$\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} H_{11} & H_{12} \\ H_{21} & H_{22} \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \end{bmatrix}. \tag{40}$$

Then the system's transfer functions of double-shaking table system with specimen are

$$\begin{aligned}
 H_{11} = H_{11} = H_{22} &= \frac{G_{PID} G_q k_q}{A_p} \frac{G_3}{G_3^2 - G_4^2}, \\
 H_{12} = H_{12} = H_{21} &= \frac{G_{PID} G_q k_q}{A_p} \frac{G_4}{G_3^2 - G_4^2}.
 \end{aligned} \tag{41}$$

Then we apply the DMSC strategy to the double-shaking table system with specimen to get the transfer function:

$$\begin{aligned}
 H_{11} = G_{11} = G_{22} &= \frac{G_{PID} G_q k_q}{2A_p} \left(\frac{1}{G_6} + \frac{1}{G_7} \right), \\
 H_{12} = G_{12} = G_{21} &= \frac{G_{PID} G_q k_q}{2A_p} \left(\frac{1}{G_6} - \frac{1}{G_7} \right).
 \end{aligned} \tag{42}$$

Equation (35) shows that the displacement and the rotation are independent freedom degrees without coupling. Equation (41) shows the system's transfer functions of double-shaking table system with specimen. Equation (42) shows the system's transfer functions of double-shaking table system with specimen utilizing DMSC strategy.

The above-mentioned differential movement synchronous control is divided into two parts: error processing and error coupling. The Simulink model is shown in Figure 5.

3.3. The Design of the AIC. In Section 3.2, the DMSC is developed to improve the synchronization precision of the two shaking tables. In this section, this study combined the AIC based on Normalized Least Mean Square (NLMS) algorithm with the proposed synchronous strategy to realize

TABLE 1: The parameters of the double-shaking table model.

Parameters	Values
Mass of the platform	1t
Bulk modulus of oil	$7 \times 10^8 \text{ N/m}^2$
Flow coefficient of servo valve pressure	$1.0 \times 10^{-11} \text{ m}^5/\text{N/s}$
Spool flow gain	$6 \times 10^{-3} \text{ m}^3/\text{s}\cdot\text{V}$
Inherent frequency of servo valve	628 rad/s
Leakage coefficient of hydraulic cylinder	$7.5529 \times 10^{-15} \text{ m}^5/\text{N/s}$
Damping ratio of servo valve	0.7
Effective bearing area of valve	$4.73 \times 10^{-3} \text{ m}^2$
Displacement feedback gain	0.21 V/m
Equivalent volume of cylinder	$3.9023 \times 10^{-3} \text{ m}^3$
Damping ratio of sensor	0.7
Inherent frequency of sensor	942 rad/s

FIGURE 6: The adaptive inverse control system structure of the shaking table.

synchronous tracking control of the double-shaking table system.

As shown in Figure 6, the error signal calculated by the desired signal and the exciter output signal is utilized to adjust the weight coefficient of the controller, thereby realizing the waveform reproduction of the shaking table.

The weight updating formula of the NLMS algorithm is shown as follows:

$$\begin{aligned} \mathbf{w}(n+1) &= \mathbf{w}(n) + 2\mu e(n) \mathbf{x}(n), \\ \mu &= \frac{\alpha}{\beta + \|\mathbf{x}(n)\|^2}, \end{aligned} \quad (43)$$

where $\mathbf{w}(n)$ is the weight coefficient of the N -rank finite impulse response (FIR) digital filter, μ is the convergence factor constant, $e(n)$ is the error of the filter, $\mathbf{x}(n)$ is the input signal of the filter, and $\|\mathbf{x}(n)\|^2$ is the input signal's Euclidean norm square.

The designed adaptive inverse controller is shown in Figure 7. In this Simulink model, $\alpha = 0.25$ and $\beta = 0.5$.

3.4. The Model of the DMSTC

3.4.1. The Model of the Double-Shaking Table System with Specimen. The model of the double-shaking table system with specimen is shown in Figure 8. Table 1 shows the parameters of the model of the double-shaking table system

TABLE 2: The PID control parameters of the double-shaking table system.

PID parameters	K_p	K_i	K_d
Shaking table	0.6	0.2	0.025

and the PID control parameters are shown in Table 2. According to the hydraulic system model established in Section 2.1 and the deduction of the interaction between the shaking table and the specimen in Section 3.1, the model is established in Matlab/Simulink with the parameters in Table 1. In the simulations, the mass of the specimen is 2000 kg, the frequency is 3 Hz, and the damping ratio is 0.02.

3.4.2. The Model of the DMSTC. On the ground of the above deduction, analysis, and modeling, a synchronous tracking control system (including the double-shaking table system and specimen, the DMSC block, and the AIC blocks) is established in Matlab/Simulink, as shown in Figure 9.

4. Results and Discussions

4.1. The System's Frequency Response Performance. The frequency transform functions (FRFs) of the double-shaking table system without DMSTC and with DMSTC are shown in Figure 10. The frequency bandwidth of the system without DMSTC-H11 is only 3.27 Hz, which leads to different dynamic responses and synchronization error, while the frequency performance of the system with DMSTC-G11 is increased from 3.27 Hz to 64.57 Hz, which improves the accuracy of the signal representation and reduces the synchronization error simultaneously.

4.2. Random Wave Test. The random waves 1 and 2, which are input into shaking tables 1 and 2, respectively, are shown in Table 3. The results without DMSTC and with DMSTC are shown in Figure 11. For comparative study, the details of Figure 11(b) are enlarged.

Table 4 shows that, with DMSTC, the replication accuracy of each shaking table is improved. For shaking table 1, the

FIGURE 9: The DMSTC model.

FIGURE 10: The FRFs of the double-shaking table system.

For shaking table 2, the maximum tracking error, $\max(u_2 - x_2)$, decreases from 1.030 mm to 0.497 mm and the root mean square tracking error, $\text{RMS}(u_2 - x_2)$, decreases from 0.296 mm to 0.144 mm.

Figures 13 and 14 demonstrate that the differential movement synchronous performance is greatly improved. Namely, the error of the desired deviation and the accurate deviation decreases. The maximum value of the differential movement synchronous error, $\max(E)$, decreases from 1.682 mm to 0.482 mm, and the root mean square value of the differential movement synchronous error, $\text{RMS}(E)$, decreases from 0.541 mm to 0.145 mm.

5. Conclusion

In conclusion, this research proposed a novel synchronous tracking strategy, DMSTC strategy, for shaking table's motion to generate the control error signals while taking the interaction between shaking tables and specimen into consideration and to improve the system's tracking precision. DMSTC Simulink model of the double-shaking table with specimen was established and simulations were conducted in various conditions. The results of the study are as follows:

(1) The proposed DMSTC strategy extends the double-shaking table system's frequency bandwidth from 3.27 Hz to

FIGURE 11: Random wave test.

FIGURE 12: The tracking error of the double-shaking table.

FIGURE 13: The differential movement synchronous error.

FIGURE 14: Data analysis of the tracking errors and synchronous error.

64.57 Hz, which improves the frequency bandwidth of the double-shaking table system.

(2) The replication accuracy of each shaking table is improved. For shaking table 1, the correlative coefficient rises by 27.1%. For shaking table 2, the correlative coefficient rises from 58.6% to 82.7%.

(3) The tracking accuracy of each shaking table is improved. For shaking table 1, the maximum tracking error decreases from 1.138 mm to 0.044 mm. For shaking table 2, the maximum tracking error decreases by 51.74%.

(4) The differential movement synchronous error decreases, the maximum absolute value of that decreases by

71.34%, and the root mean square value of that decreases by 73.20%.

Conflicts of Interest

The authors declare that there are no conflicts of interest regarding the publication of this paper.

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