Research Article

Multiple-Frequency Controlled Synchronization of Two Homodromy Eccentric Rotors in a Vibratory System

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Multifrequency controlled synchronization of two homodromy eccentric rotors is investigated. The vibrating screen with two rotors is a typical underactuated system. So, the model of the vibrating screen is converted to a synchronization motion problem. Firstly, the mechanical-electromagnetic coupling dynamical model of the vibrating system is established. And then the fuzzy PID method is used for the two motors which are based on the master-slave control strategy. The slave motor uses the method of the phase ratio to trace the master motor and achieve the synchronous motion. Through the simulation, the two rotors cannot reach the state of multifrequency self-synchronization. And, then, it is realized by multifrequency controlled synchronization. The article presents the motion trails of the vibrating system between one and two times and realizes the zero phase difference after each period. Finally, some experiments are used to verify the accuracy and effectiveness of the conclusion in the simulation and analysis of the feature of the movement tracks.

1. Introduction

Synchronization is a movement which exists in the nature. The earliest detailed account on synchronization motion was made by Huygens, who observed that two clock pendulums suspended from stiff wooden beams could run in a steady state and move in opposition to each other at the same angular velocity [1]. Vibration machines are a series of machines by utilizing vibratory principle to achieve works and are widely used in various fields, like mining, metallurgy, coal, agriculture, mechanical engineering, etc. [2, 3]. The theory of synchronization is introduced by Blehman [4, 5] to experiment on the inductor motors which are driven by two eccentric rotors. Inoue and Araki [6, 7] presented their research results about 3 times frequency synchronization of a dual-motor driving plane motion vibrating machine. To apply the synchronization in the project, self-synchronization theory is introduced in various vibrating machines excited by two ERs to replace gears or chains in forced synchronization [1]. With the development of the self-synchronization, the method of two perturbation small parameters is introduced

to average angular velocity of two exciters and their phase difference [8]. Thus, the equation of frequency capture is deduced by averaging motion equations over their average period. Moreover, the experiment also has a rapid development [9, 10], which proves the stability and effectiveness of self-synchronization. However, self-synchronization has its own stable region which is a shortage. To solve this problem, controlled synchronization is introduced into the vibration machines.

For recent years, numbers of literatures have been finished to study the controlled synchronization. An adaptive feedforward control law is used by Tomizuka et al. on the motion synchronization [11, 12]. Li et al. use the adaptive sliding mode control algorithm to study the speed tracking and synchronization of multiple induction motors [13–15]. Kong xiangxi [16] et al. study the two nonidentical homodromy coupling exciters driven by inductor motors in a vibratory system with controlled synchronization. In their work, the master-slave control strategy is used to control the speed with the method of PI and adaptive sliding mode. Then the method of phase tracking is introduced into the

FIGURE 1: Dynamic model of the vibratory system.

phase synchronization. With the development of control method, intelligent control increases rapidly. Based on the PID method, the fuzzy PID method is widely used in the motor control [17–19].

The base frequency synchronization on the vibrating screen can only realize straight line and cycle motions which restrict the types of material and screening efficiency and the multifrequency self-synchronization has so many restrict factors that it is difficult to apply to engineering practice. However, the multifrequency controlled synchronization can present different kinds of motions so that the amplitude can be increased and finally the screening efficiency can be improved. So, in this article, multifrequency controlled synchronization of two homodromy eccentric rotors in a vibratory system is studied. In Section 2, the mechanical-electromagnetic coupling model of the vibration system is given and the responses of the three directions are shown. In Section 3, a modified master-slave strategy is used in the vibration system and fuzzy PID method is given to control the speed and phase of the motors. In Section 4, the numerical simulation and experimental verification are given to certify the effectiveness and stability. Finally, in Section 5, some conclusions are listed.

2. Dynamic Model of the Vibration System

2.1. The Model of the Mechanical System. As shown in Figure 1, the vibration system consists of one rigid body, two inductor motors with eccentric rotors, and four springs. o is the central point and mass center of the rigid body and the motions of the rigid body are along the x and y directions. Two inductor motors are symmetrically installed along the y axis on the rigid body which is established on an elastic foundation with four springs installed symmetrically. Using Lagrange's equations and choosing variables x , y , ψ , ϕ_1 , and ϕ_2 as the generalized coordinates, the differential equations of motion of the vibratory system can be derived as follows [16]:

$$\begin{aligned} M\ddot{x} + f_x\dot{x} + k_x x \\ = m_1 r (\dot{\phi}_1^2 \cos \phi_1 + \ddot{\phi}_1 \sin \phi_1) \end{aligned}$$

$$\begin{aligned} & + m_2 r (\dot{\phi}_2^2 \cos \phi_2 + \ddot{\phi}_2 \sin \phi_2) \\ M\ddot{y} + f_y\dot{y} + k_y y \\ & = m_1 r (\dot{\phi}_1^2 \sin \phi_1 - \ddot{\phi}_1 \cos \phi_1) \\ & + m_2 r (\dot{\phi}_2^2 \sin \phi_2 - \ddot{\phi}_2 \cos \phi_2) \\ J\ddot{\psi} + f_\psi\dot{\psi} + k_\psi \psi \\ & = m_1 r l_0 [\dot{\phi}_1^2 \sin(\phi_1 - \beta) - \ddot{\phi}_1 \cos(\phi_1 - \beta)] \\ & - m_2 r l_0 [\dot{\phi}_2^2 \sin(\phi_2 + \beta) - \ddot{\phi}_2 \cos(\phi_2 + \beta)] \\ J_1\ddot{\phi}_1 + f_1\dot{\phi}_1 = T_{e1} - T_{L1} \\ J_2\ddot{\phi}_2 + f_2\dot{\phi}_2 = T_{e2} - T_{L2} \end{aligned} \quad (1)$$

where M is the total mass of the vibratory system. $M = m_1 + m_2 + m_3$; m_1 and m_2 are the eccentric masses; m is the mass of the rigid body, and $m_2 = \eta m_1$; r is the eccentric radius; l_0 is the distance between the mass center and the rotary center of the eccentric rotor; J is the moment of inertia of the vibration system, and $J = M l_e^2 = J_p + (m_1 + m_2)(l_0^2 + r^2)$, in which J_p is the moment of inertia of the rigid body and l_e is the equivalent radius; J_1 and J_2 are the moment of inertia of the eccentric rotors, $J_1 = m_1 r^2$, and $J_2 = m_2 r^2$; f_x , f_y , and f_ψ are damping coefficients of the vibratory system in the x , y , and ψ directions, respectively; f_1 and f_2 are damping coefficients of the inductor motors. k_x , k_y , and k_ψ are the stiffness of the spring; T_{e1} and T_{e2} are the electromagnetic torques of the inductor motors. T_{L1} and T_{L2} are the time varying load torques which can be expressed as follows:

$$\begin{aligned} T_{L1} &= m_1 r [\ddot{y} \cos \phi_1 - \ddot{x} \sin \phi_1 + l_0 \dot{\psi}^2 \sin(\phi_1 - \beta) \\ & + l_0 \ddot{\psi} \cos(\phi_1 - \beta)] \\ T_{L2} &= m_2 r [\ddot{y} \cos \phi_2 - \ddot{x} \sin \phi_2 - l_0 \dot{\psi}^2 \sin(\phi_2 + \beta) \\ & - l_0 \ddot{\psi} \cos(\phi_2 + \beta)] \end{aligned} \quad (2)$$

2.2. *The Model of the Electromagnetic System.* Because the inductor motors need to be controlled, the model of the inductor motor is given as follows [20]:

$$\begin{aligned}\phi_{sd} &= L_s i_{sd} + L_m i_{rd} \\ \phi_{sq} &= L_s i_{sq} + L_m i_{rq} \\ \phi_{rd} &= L_m i_{sd} + L_r i_{rd} \\ \phi_{rq} &= L_m i_{sq} + L_r i_{rq}\end{aligned}\quad (3)$$

$$\begin{aligned}\dot{\phi}_{sd} &= -R_s i_{sd} + \omega_s \phi_{sq} - u_{sd} \\ \dot{\phi}_{sq} &= -R_s i_{sq} + \omega_s \phi_{sd} - u_{sq} \\ \dot{\phi}_{rd} &= -R_r i_{rd} + (\omega_s - \omega) \phi_{rq} \\ \dot{\phi}_{rq} &= -R_r i_{rq} - (\omega_s - \omega) \phi_{rd}\end{aligned}\quad (4)$$

Equations (3) and (4) are, respectively, flux-linkage equations and voltage equations of the inductor motor in the synchronous reference frame of the d and q axis. Because the model of the inductor motor is squirrel cage, the short circuit phenomenon appears in the rotor part. Hence, $u_{rd} = u_{rq}$. When the system is at steady state, $\dot{\phi}_{rd} = \text{constant}$ and $\dot{\phi}_{rq} = 0$. There are total nine variables in the synchronous reference frame of the d and q axis, in which five can be chosen to establish the state equation of an inductor motor in rotor field-oriented coordinate.

In this paper, ω - i_s - ϕ_r are chosen, where i_s represents i_{sd} and i_{sq} , ϕ_r represents ϕ_{rd} and ϕ_{rq} , and ω_s means rotate speed. Then the equation can be expressed as

$$\begin{aligned}\dot{\phi}_{rd} &= -\frac{\phi_{rd}}{T_r} + \frac{(\omega_s - n_p \omega)}{\phi_{rq}} + \frac{L_m i_{sd}}{T_r} \\ \dot{\phi}_{rq} &= -(\omega_s - n_p \omega) \phi_{rd} - \frac{\phi_{rq}}{T_r} + \frac{L_m i_{sq}}{T_r} \\ i_{sd} &= \frac{L_m \phi_{rd}}{(\sigma L_s L_r T_r)} + \frac{L_m n_p \omega \phi_{rq}}{(\sigma L_s L_r)} - \frac{(L_m^2 + R_s L_r T_r) i_{sd}}{(\sigma L_s L_r T_r)} \\ &\quad + \omega_s i_{sq} + \frac{u_{sd}}{(\sigma L_s)} \\ i_{sq} &= -\frac{L_m n_p \omega \phi_{rd}}{(\sigma L_s L_r)} + \frac{L_m \phi_{rq}}{(\sigma L_s L_r T_r)} - \omega_s i_{sd} \\ &\quad - \frac{(L_m^2 + R_s L_r T_r) i_{sq}}{(\sigma L_s L_r T_r)} + \frac{u_{sq}}{(\sigma L_s)}\end{aligned}\quad (5)$$

where the subscript s and r , respectively, represent stator and rotor; the subscript d and q , respectively, represent the d axis and q axis; i represents current; u represents voltage; R represents resistance; L_s and L_r are, respectively, self-inductance of the stator and rotor; L_m is mutual inductance of the stator and rotor. Tr is a rotor time constant, $Tr = L_r/R_r$; in a similar way, $T_s = L_s/R_s$; σ is leakage factor, $\sigma = 1 - L_m/(L_s L_r)$; n_p is the number of pole-pairs of the inductor

motor; ω is the mechanical speed; ω_s is the synchronous electric angular speed.

According to the model of the inductor motor, the following equation can be obtained. Because of $\dot{\phi}_{rd} = \text{constant}$ and $\dot{\phi}_{rq} = 0$, (6) can be expressed as

$$T_e = \frac{n_p L_m (i_{sq} \phi_{rd} - i_{sd} \phi_{rq})}{L_r} \quad (6)$$

$$T_e = \frac{n_p L_m i_{sq} \phi_{rd}}{L_r} \quad (7)$$

As shown above, n_p , L_m , ϕ_{rd} , and L_r are constants. Hence, (7) is an equation with the variable i_{sq} when the inductor motor is at the steady state.

2.3. *Responses of the Vibration System.* According to the document [8], in this article it is assumed that the average phase and phase difference are given as follows:

$$\begin{aligned}\varphi_1 &= \varphi + \alpha \\ \varphi_2 &= n(\varphi - \alpha)\end{aligned}\quad (8)$$

where n is the coefficient of the multifrequency synchronization of the vibrating system and it is a rational one. Then, taking the derivative of φ_1 and φ_2 , it can be derived as

$$\begin{aligned}\dot{\varphi}_1 &= \dot{\varphi} + \dot{\alpha} \\ \dot{\varphi}_2 &= n(\dot{\varphi} - \dot{\alpha})\end{aligned}\quad (9)$$

When the system is at the steady state, the mean value of the angular velocity is a constant [10, 11], where $\omega_0 = \int_0^T \dot{\varphi} dt / T$. Hence, (9) can be expressed as

$$\begin{aligned}\dot{\varphi}_1 &= (1 + \varepsilon_1 + \varepsilon_2) \omega_0 \\ \dot{\varphi}_2 &= n(1 + \varepsilon_1 - \varepsilon_2) \omega_0\end{aligned}\quad (10)$$

where $\dot{\varphi} = (1 + \varepsilon_1) \omega_0$ and $\dot{\alpha} = \varepsilon_2 \omega_0$. Taking (10) into (1), then the responses of the vibration system in the x , y , and ψ directions can be obtained as

$$\begin{aligned}x &= -\left(\frac{r_m r}{\mu_x}\right) [\cos(\varphi_1 + \gamma_x) + \eta n^2 \cos(\varphi_2 + \gamma_x)] \\ y &= -\left(\frac{r_m r}{\mu_y}\right) [\sin(\varphi_1 + \gamma_y) + \eta n^2 \sin(\varphi_2 + \gamma_y)] \\ \psi &= -\left(\frac{r_m r_l r}{\mu_\psi l_e}\right) \\ &\quad \cdot [\sin(\varphi_1 - \beta + \gamma_\psi) - \eta n^2 \sin(\varphi_2 + \beta + \gamma_\psi)]\end{aligned}\quad (11)$$

where $\omega_x^2 = k_x/M$, $\omega_y^2 = k_y/M$, $\omega_\psi^2 = k_\psi/J$, $\xi_x = f_x/(2\sqrt{k_x M})$, $\xi_y = f_y/(2\sqrt{k_y M})$, $\xi_\psi = f_\psi/(2\sqrt{k_\psi J})$, $r_m = m_1/M$, $r_l = l_0/l_e$, $\mu_x = 1 - \omega_x^2/\omega_0^2$, $\mu_y = 1 - \omega_y^2/\omega_0^2$, $\mu_\psi = 1 - \omega_\psi^2/\omega_0^2$, $\eta = m_2/m_1$, $\tan \gamma_x = 2\xi_x \omega_x/(\mu_x \omega_0)$, $\tan \gamma_y = 2\xi_y \omega_y/(\mu_y \omega_0)$, and $\tan \gamma_\psi = 2\xi_\psi \omega_\psi/(\mu_\psi \omega_0)$.

FIGURE 2: Flow diagram of the controlled system.

3. Design of the Controlled System

With the multifrequency self-synchronization theory, it is known that if the parameter n is different, the multifrequency synchronization has different stable regions. It is hard to give a uniform kinetic formula which is a shortage. Hence, the controlling method is used to satisfy the arbitrary positive real number n . In this section, the method consists of two parts. The controlling method is shown and how the controlled system works is explained.

3.1. Design of the Electromechanical Coupling System. As shown in Figure 2, the controlling law of the master-slave form is adopted. Inductor motor 1 is the master motor and inductor motor 2 is the slave motor. Inductor motor 2 traces inductor motor 1 with the phase angle. The rotor flux oriented control (RFOC) and fuzzy PID method are used on the electromagnetic system of the inductor motors. ω_t is the given speed which is transferred to the master inductor motor. With the fuzzy PID controlling method, the speed of motor 1 ω_1 is obtained. Then the feedback value ω_1 is used for two parts. One part is to control the master inductor motor with ω_t . The other is to obtain ϕ_1 through the integral method which is to trace the slave inductor motor. Finally, the speed of inductor motor 2 " ω_2 " and phase " ϕ_2 " which is fed back to the slave inductor motor are obtained which is fed back to the slave inductor motor. The RFOC in Figure 2 is shown as in Figure 3.

The given speed ω_t subtracts the feedback speed ω_1 , and then the fuzzy PID method is used to obtain the electromagnetic torque T_e^* which can calculate the stator current on the q axis i_{sq}^* . The stator current on the d axis i_{sd}^* can be obtained from the formulation $i_{sd}^* = \phi_{rd}^*/L_m$, where ϕ_{rd}^* is the given flux linkage. Hence, U_{sd}^* and U_{sq}^* can be obtained. The synchronization electric angle θ can be expressed as follows:

$$\theta = \int (\omega + \omega_s) dt \quad (12)$$

where ω_s is slip angular velocity, $\omega_s = L_m i_{sq}^* / \phi_{rd} T_r$. Through the coordinate transformation, $U_{s\alpha}^*$ and $U_{s\beta}^*$ can be obtained. Finally the speed of the motor ω and i_{abc} with the SVPWM technology can be acquired.

3.2. Design of the Fuzzy PID Method. According to the literature [18, 19], the fuzzy system can be structured by the following steps.

Step 1. Define the fuzzy set $A_i^{l_i}$ ($l_i = 1, 2, \dots, p_i$) with the number p_i of the variations x_i ($i = 1, 2$).

Step 2. Use the fuzzy rules with the number of $\prod_{i=1}^2 p_i$ to structure the fuzzy system $\hat{f}(x | \theta_f)$.

$$R^{(j)}: \text{if } x_1 \text{ is } A_1^{l_1} \text{ and } x_2 \text{ is } A_2^{l_2} \text{ then } \hat{f} \text{ is } E^{l_1 l_2} \quad (13)$$

where, $l_i = 1, 2, i = 1, 2$.

With the product inference engine, singleton obfuscator, and center average obfuscator, the output of the fuzzy system can be expressed as

$$\hat{f}(x | \theta_f) = \frac{\sum_{l_1=1}^{p_1} \sum_{l_2=1}^{p_2} \bar{y}_f^{l_1 l_2} \left[\prod_{i=1}^2 \mu_{A_i^{l_i}}(x_i) \right]}{\sum_{l_1=1}^{p_1} \sum_{l_2=1}^{p_2} \left[\prod_{i=1}^2 \mu_{A_i^{l_i}}(x_i) \right]} \quad (14)$$

where $\mu_{A_i^{l_i}}(x_i)$ is member function of x_i . Consider $\bar{y}_f^{l_1 l_2}$ as a free parameter in the set $\theta_f \in R^{\prod_{i=1}^2 p_i}$ and then introduce a column vector $\xi(x)$; (14) can be derived as

$$\hat{f}(x | \theta_f) = \theta_f^T \xi(x) \quad (15)$$

where $\xi(x)$ is a column vector with the dimension number of $\prod_{i=1}^2 p_i$, in which the l_1 th and l_2 th elements are

$$\xi_{l_1 l_2}(x) = \frac{\prod_{i=1}^2 \mu_{A_i^{l_i}}(x_i)}{\sum_{l_1=1}^{p_1} \sum_{l_2=1}^{p_2} \left[\prod_{i=1}^2 \mu_{A_i^{l_i}}(x_i) \right]} \quad (16)$$

FIGURE 3: RFOC: rotor flux oriented control.

FIGURE 4: The subjection function of variables. (a) is the error (e); (b) is the error change (ec); (c) is the parameter k_p ; (d) is the parameter k_i ; and (e) is the parameter k_d .

Two input variables and three output variables are given in this part. The input variables are, respectively, the error (e) and error change (ec). The output variables are, respectively, k_p , k_i , and k_d . Each of the five variables is divided into seven situations which are, respectively, NB, NM, NS, Z, PS, PM, and PB. NB and PB are the member functions of the model Z (*zmf*). The others are the member functions of the triangle model (*trimf*). The variables are shown in Figure 4.

Figure 4 shows the subjection function of variables. (a) is the error (e); (b) is the error change (ec); (c) is the parameter k_p ; (d) is the parameter k_i ; and (e) is the parameter k_d .

According to (13), forty-nine rules are established with the fuzzy control regulation in this article and then the following equations of the PID parameters can be obtained:

$$K_P = \frac{\left[\sum_{j=1}^{49} \mu_{K_{Pj}}(E, EC) \times K_{Pj} \right]}{\sum_{j=1}^{49} \mu_{K_{Pj}}(E, EC)} \quad (17)$$

$$K_I = \frac{\left[\sum_{j=1}^{49} \mu_{K_{Ij}}(E, EC) \times K_{Ij} \right]}{\sum_{j=1}^{49} \mu_{K_{Ij}}(E, EC)} \quad (18)$$

$$K_D = \frac{\left[\sum_{j=1}^{49} \mu_{K_{Dj}}(E, EC) \times K_{Dj} \right]}{\sum_{j=1}^{49} \mu_{K_{Dj}}(E, EC)} \quad (19)$$

With (17), (18), and (19), the surfaces of k_p , k_i , and k_d can be obtained in Figure 5.

FIGURE 5: The surfaces of the parameters with k_p , k_i , and k_d . (a) is the surface of k_p , (b) is the surface of k_i , and (c) is the surface of k_d . The horizontal axis x is e and the vertical axis y is ec in all the three diagrams.

TABLE 1: Parameters of the two inductor motors.

Parameters	Motor 1	Motor 2
Rated power P/kW	1.1	1.1
Pole pairs n_p	3	3
Rated frequency f_0/Hz	50	50
Rated voltage U/V	220	220
Rated speed $\omega/\text{(r/min)}$	910	910
Stator resistance R_s/Ω	40.4	40.5
Rotor resistance referred R_r/Ω	3.92	3.92
Stator inductance L_s/H	12	12.813
Rotor inductance referred L_r/H	1.222	1.222
Mutual inductance L_m/H	1.116	1.116
Rated flux linkage λ_{dr}^*/Wb	0.98	0.98
Damping coefficients $f_{1,2}/(\text{Nms/rad})$	0.005	0.005

In Figure 5, (a) is the surface of k_p , (b) is the surface of k_i , and (c) is the surface of k_d .

4. Results and Discussion

For this part, Matlab/Simulink is used to prove the effectiveness of the method and the stability of the vibration system. The base frequency controlled synchronization can be deemed as the multifrequency controlled synchronization in the special situation of $n = 1$. Actually, when the two rotors reach the synchronous state $\omega_1 - \omega_2 = 0$ equals $\omega_2/\omega_1 = 1$ and $\varphi_1 - \varphi_2 = 0$ equals $\varphi_2/\varphi_1 = 1$. Firstly, the simulation of the multifrequency self-synchronization on the situation $n = 1.5$ is shown. And, then, the situations $n = 1.2$ and $n = 1.5$ are given in detail. The other situations of the multifrequency controlled synchronization from one to two times are shown with the movement tracks of the frame. Finally, some experiments are given to demonstrate the validity of the method. The parameters of the two motors and system are, respectively, shown in Tables 1 and 2.

4.1. Numerical Simulation of the Multifrequency Self-Synchronization. According to the equations in Section 2,

TABLE 2: Parameters of the vibration system.

Parameters	Value
m/kg	246
$J_p/(\text{kg.m}^2)$	43.3
$k_x/(\text{N/m})$	129332
$k_y/(\text{N/m})$	105334
$k_\psi/(\text{Nm})$	30715
$f_x/(\text{Ns/m})$	615.5
$f_y/(\text{Ns/m})$	618
$f_\psi/(\text{Nsm})$	180.2
l_0/m	0.5
$\alpha/^\circ$	0
$\beta/^\circ$	30
m_0/kg	4
r/m	0.05

when the speed ratio and phase ratio are both 1.5 the two motors can be considered to be synchronous. In Figure 6, the given speeds of motors 1 and 2 are, respectively, 60 rad/s

FIGURE 6: The multifrequency self-synchronization on the situation $n = 1.2$, where $\alpha_0 = 0$, with α_0 being the initial phase difference. (a) Speeds of the motors; (b) phase difference; (c) response of the x direction; (d) response of the y direction; (e) response of the ψ direction.

and 90 rad/s in (a). In (b), the phase difference does not have a stable value. It is a monotone function over time. So when the vibrating system is in a period variation the two motors cannot reach the synchronization state. Different from the situation of $n = 1$, the amplitude value of the multifrequency self-synchronization is not the same value because of the amplitude superposition. The values of the crest and trough vary with the time periodically. A special phenomenon appears that the peaks of the amplitudes represent a sinusoidal variation in (c), (d), and (e).

4.2. Numerical Simulation of the Multifrequency Controlled Synchronization. Although the multifrequency self-synchronization cannot be realized, it can be realized with the controlled synchronization. In this part, the given speeds of the master motor in Figures 7 and 8 are both 60 rad/s. Then the slave motor traces the master motor with the method of the phase ratio. With this method, the master and slave motors can both reach the speed and phase synchronization state. In Figure 7, when the system reaches the stable state as shown in (d), the phase ratio equals 1.2 which verifies the effectiveness of the method. (e) represents that it is zero phase difference when the phase ratio reaches 1.2 times, which guarantees the accuracy of the equations above ($\varphi_1 - \varphi_2 = 0$ equals $\varphi_2/\varphi_1 = 1$). In a similar way, $\varphi_2 - 1.2 \times \varphi_1 = 0$ equals $\varphi_2/\varphi_1 = 1.2$. According to (8) to (10) in Section 2.3, if the phase ratio equals 1.2, the speed ratio also equals 1.2. The simulation result in (c) is consistent

with the theory. (a) and (b) are, respectively, speeds and load torques. From (a), it can be known that the speed of the slave motor is 72 rad/s. So the speed and phase both achieve multifrequency synchronization. (f), (g), and (h) are the responses of the x , y , and ψ directions, respectively. Compared with the base frequency synchronization, the values of each crest or trough in the periods do not present a standard sinusoidal variation. Due to the different speeds of the two motors, the crest and trough appear with the phenomenon of modes of vibration superposition, which leads to the different values of the crest and trough in one period. It is because the speed of motor 1 is faster than motor 2 speed. When the two motors rotate in the same direction, the amplitude is in a superposed state. Otherwise, it is in an offset state.

To demonstrate the arbitrariness of the multifrequency controlled synchronization, n is changed from 1.2 to 1.5. From (c) to (e) in Figure 8, it can be known that when the number n is 1.5 the speed and phase of the system are also synchronous. Similarly, the other numbers of the multifrequency parameters n are also tenable. With the speed ratio increasing, the values of the load torque also increase as shown in (b). Compared (f), (g), and (h) in Figure 7 with (f), (g), and (h) in Figure 8, the responses have different phenomenon which are influenced by the least common period. So a different parameter n represents a different response. Figure 9 shows the movement tracks of the frame from $n = 1.1$ to $n = 2$. From Figure 9, some interesting

FIGURE 7: Multifrequency controlled synchronization on the situation $n = 1.2$, where $\alpha_0 = 0$, with α_0 being the initial phase difference. (a) Speed of the motors; (b) load torque; (c) speed ratio; (d) phase ratio; (e) phase difference; (f) response of the x direction; (g) response of the y direction; (h) response of the ψ direction.

conclusions can be summarized. Different from the situation of $n = 1$, the movement tracks are not only an ellipse as well as some other inner circles. A regulation can be known that if the fractional part of the ratio is an odd number, the numbers of the minimum circle equal the decimal itself, shown as (a), (c), (g), and (i). If the fractional part of the ratio is an even number, the numbers of the minimum circle equal half the decimal, shown as (b), (d), (f), and (h). However, a special example is $n = 1.5$. Because $n = 1.5$ is a half period, it presents the phenomenon of the period which can be obtained from (e) in Figure 8. When the situation is $n = 2$, shown as (j), the external cycle which only exists in the situation of the integer part being 1 disappears and the inner cycle is retained. So the nonintegral period presents different phenomenon compared with the integral period and half-integral period.

4.3. Experimental Verification. To get a further confirmation of the result, the experiment of the multifrequency self-synchronization is given firstly. And, then, the experiment of the multifrequency controlled synchronization is shown. The experiment parameters are the same as the parameters in the simulation. In the experiment of the self-synchronization, the frequencies of motors 1 and 2 are, respectively, 30 Hz and 45 Hz in Figure 10(a). When the vibrating system is in a steady state of period variation the phase difference is not stable. Contrast with (b) in Figure 6, the same conclusion can be obtained. The multifrequency self-synchronization cannot be realized in the vibrating system. (c), (d), and (e) are the responses of the vibrating system. They have the same tendency as the simulation, which certifies the consistency between the simulation and the experiment.

FIGURE 8: Multifrequency controlled synchronization for the situation $n = 1.5$, where $\alpha_0 = 0$. α_0 is the initial phase difference. (a) Speed of the motors; (b) load torque; (c) speed ratio; (d) phase ratio; (e) phase difference; (f) response of x direction; (g) response of y direction; (h) response of ψ direction.

In the experiment of the multifrequency controlled synchronization, the speeds of the two motors are, respectively, set to 27 Hz and 40.5 Hz with two inverters. Then use a PLC (Programmable Logic Controller) to realize the master-slave control strategy. The photoelectric coder and the Hall sensor are used to measure and calculate the pulses. There are three acceleration sensors on the frame. One is on the x direction. Another is on the y direction in the center of the frame. The other is on the y direction in the edge of the frame. From (a) in Figure 11, it can be known that the speeds of the two motors are stable. (b) and (c) are, respectively, speed ratio and phase ratio which both equal 1.5. So when the vibrating system reaches the steady state the two rotors reach the synchronization state. Due to the error of the experiment, the phase difference in (d) is not zero. It floats around 15 degrees. Contrasting (e) in Figure 11 with (f) in Figure 8, the modes of vibration in the two figures are the same as the

enlarged figure. Similarly, (f) and (g) in Figure 11 are the same as (g) and (h) in Figure 8, respectively.

5. Conclusions

From the work above, the multifrequency controlled synchronization of two homodromy eccentric rotors in a vibrating system is studied. The multifrequency self-synchronization with the dynamic model in Figure 1 cannot be realized. To realize the multifrequency synchronization, the fuzzy PID method based on the master-slave strategy is introduced and the result is very effective. For proving the effectiveness, the multifrequency controlled synchronization can be realized in arbitrary situations by changing the parameter n which has an influence on the trajectories of the system. And, then, the trajectories of the system are shown

FIGURE 9: The movement track of the frame with multifrequency controlled synchronization on the situations from $n = 1.1$ to $n = 2$. (a) is the situation of $n = 1.1$ and (j) is the situation of $n = 2$.

FIGURE 10: The experiment of the multifrequency self-synchronization, where $\eta = 50\%$ and $\alpha_0 = 0$. (a) The speed of the two motors. (b) The phase difference. (c) The response of the x direction. (d) The response of the y_1 direction. (e) The response of the y_2 direction.

FIGURE 11: The experiment of the multifrequency controlled synchronization, where $\eta = 30\%$ and $\alpha_0 = 0$. (a) The speed of the two motors. (b) The speed ratio. (c) The phase ratio. (d) The phase difference. (e) The response of the x direction. (f) The response of the y_1 direction. (g) The response of the y_2 direction.

from $n = 1$ to $n = 2$. Finally, the different feature of the trajectories with the system is illuminated. Multifrequency controlled synchronization can increase the maximum amplitude of the vibrating system. Thereby, the efficiency of the vibrating screen which is simplified as in Figure 1 in this article can be promoted in the engineering.

Data Availability

The data used to support the findings of this study are available from the corresponding author upon request.

Conflicts of Interest

The authors declared no potential conflicts of interest with respect to the research, authorship, and/or publication of this article.

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