Research Article

Damage Degree Detection of Cracks in a Locomotive Gear Transmission System

XinChang Liu,1 Qi Sun,2 and ChunJun Chen2

1Southwest Jiaotong University, State Key Laboratory of Traction Power, Chengdu, China
2Mechanical Engineering College, Southwest Jiao Tong University, Chengdu, China

Correspondence should be addressed to XinChang Liu; kuanxin07@163.com

Received 24 June 2018; Revised 21 October 2018; Accepted 29 October 2018; Published 18 November 2018

1. Introduction

The gear transmission system transmits the torque of the motor and is a key element of the locomotive system. During locomotive operation, the damage to the gear transmission system spreads rapidly and affects the locomotive’s operational safety. In this paper, a method is proposed to detect the degree of tooth root crack damage. First, a dynamic locomotive model with a gear transmission is built, and the vertical acceleration of the locomotive subsystem (car body, bogie frame, wheelset, and motor) vibrations is obtained under various degrees of tooth root crack damage on the gear transmission system. By comparing the characteristics of those signals, the subsystem that is more sensitive to the effect of the tooth root crack is found. The characteristic parameters of the sensitive subsystem are calculated, and a multidimensional characteristic parameter matrix is established. The multidimensional characteristic parameter matrix is optimized and reduced by principal component analysis (PCA). Using the Grey relational analysis method, the degree of tooth root crack damage is detected. The proposed method demonstrates the ability to recognize the degree of tooth root crack damage.

1.1. Literature Review

The gear transmission system is a vital component of the locomotive bogie. During locomotive operation, the damage to the gear transmission system spreads rapidly and affects the locomotive’s operational safety [1]. Therefore, the detection method of the gear transmission system is the key to ensure locomotive safety.

In recent years, scholars worldwide have carried out much research on gear dynamics. Wu et al. [2] established a six-degree-of-freedom dynamic model and analyzed gear dynamic behavior with a change in the gear root crack. Through the dynamic analysis of the gear transmission system, Endo et al. [3] noted that the change in the meshing stiffness and the geometric error of the tooth shape are important causes of tooth root crack failure. The dynamic behavior of gear wear failure is analyzed by Ding and Kahraman [4]. Cui et al. [5] established the dynamic model of the gear rotor coupling system and studied the influence of the rotational speed on the dynamic response. In 2011, Ma and Chen [6] studied the nonlinear dynamics of a gear system with a tooth root crack. To research the spalling mechanism, Ma et al. established a dynamic model of a gear pair with local spalling [7]. Bahk and Parker developed an analytical model used to study the influence of tooth profile modification on the planetary gear dynamics [8]. In 2015, Li and Peng developed a multi-degree-of-freedom nonlinear gear model that considers the meshing force and tribological force of the gear pair [9]. To analyze the influence of helical gear wear on the dynamic response of the gear transmission, Brethee et al. developed a comprehensive dynamic model that consists of 18 degrees of freedom [10].

At present, scholars have studied the fault diagnosis of the gear transmission system and put forward a series of effective diagnosis methods. To monitor the mechanical condition of a railway traction system, Kia et al. used time-frequency signal processing techniques to address the machine current signal [1]. Shao et al. introduced the support
vector machine (SVM) method for damage detection of the mechanical transmission system [11]. In 2011 [12], Henao et al. put forward an electromagnetic-torque estimation method to detect tooth damage in a high-speed railway traction system. Cao et al. proposed a method for detecting gear damage based on empirical mode decomposition (EMD) and high-order cumulant (HOC) analysis [13]. This method is used to identify the damage to the gear transmission system under various operating conditions. In 2013, [14], Yang and Cheng proposed a damage classification method based on local mean decomposition and principle component analysis methods. In 2015, [15], Kia et al. presented a noninvasive technique using space vector analysis of the stator current to detect gear tooth surface damage faults. In 2016, [16], Heydarzadeh et al. used the discrete wavelet transform and deep neural networks to realize the automatic detection of gear faults. In 2017, [17], Chai et al. introduced a method using resonance-based sparse signal decomposition combined with envelope spectrum analysis to extract the fault features of motor current. Peng et al. realized the fault detection of wind turbine drivetrain gearboxes using the recurrent neural network [18]. Ding et al. proposed a new method to detect the fault of planetary gearboxes based on local mean decomposition and permutation entropy [19].

However, the main object of those methods is to detect the existence of damage. The detection of the degree of damage of rotating machinery is the focus of this field. In this paper, the locomotive subsystem which can be used to reflect the gear tooth root crack is found. The degree of tooth root crack damage is detected based on the PCA and Grey relational analysis methods.

2. Locomotive-Track Vertical Coupled Dynamics Model with a Gear Transmission and Gear Mesh Stiffness Model with Tooth Crack Damage

2.1. Vertical Dynamic Model of Locomotive-Track Coupled with Gear Transmission. A two-dimensional vertical dynamic model of the locomotive-track coupled with the gear transmission system is developed. The dynamic model is composed of two parts: the locomotive subsystem and the track subsystem. The locomotive is modeled as a multirigid body system running on the track. It consists of one car body, two bogie frames, four wheelsets, four motors, and a gear transmission system. These components are connected by spring-damper elements. The car body connects with the bogie frame by a secondary suspension. The bogie frame is supported on the wheelset via the primary suspension. One end of the motor is fixed on the bogie frame while the other end of the motor is fixed on the wheelset. Each component of this mathematical model has two vibrational degrees of freedom, which are torsional and vertical.

The track subsystem contains the rail, the sleepers, the ballasts, and the subgrade. These components are connected by spring-damper elements. The rail is regarded as a continuous Bernoulli–Euler beam. It is mounted on the sleepers by the rail pads. Zhai et al. [20] provide a method of modeling railway ballasts. In the new model, the shear stiffness and shear damping are introduced between the adjacent ballasts. Each component of this mathematical model has one vibrational degree of freedom, which is vertical.

The notation used in this study is shown in Tables 1–3. The schematic of the total dynamic system is depicted in Figure 1(a). The schematic of the gear transmission is depicted in Figure 1(b). Figure 1(c) is a detailed illustration of the gear transmission system. The gear transmission consists of motor, pinion, and gear. The dashed circle represents the base circle of the gear. The rotor of the motor is linked with the pinion by the torsional spring-damper element. The pinion drives the gear to rotate through the engaging force. The gear is mounted on the wheelset. In this system, only the rotational motion of the components is considered.

As the locomotive is modeled as a multirigid body, the differential equations of the locomotive with the gear transmission can be established by D’Alembert’s principle.

Vertical vibration of car body:

\[ M_c \ddot{z}_c + F_{ct1} + F_{ct2} = M_c g. \]  

Pitch motion of car body:

\[ J_c \ddot{\beta}_c + F_{ct1} \dot{\beta}_c - F_{ct2} \dot{\gamma}_c = 0. \]  

Vertical vibration of bogie frame:

\[ M_u \ddot{Z}_u + F_{tw(2i-1)} + F_{tw(2i)} + F_{tmo(2i-1)} + F_{tmo(2i)} - F_{ct1} = M_u g, \]  

\[(i = 1, 2). \]

Pitch motion of bogie frame:

\[ J_{tu} \ddot{\alpha}_u + F_{tw(2i-1)} + F_{tw(2i)} + F_{tmo(2i-1)} + F_{tmo(2i)} - F_{ct1} = 0, \]  

\[(i = 1, 2). \]

Vertical vibration of motor:

\[ M_{motor} \ddot{Z}_{motor} + F_{motor} + F_{motor} + (-1)^i F_{motor} \cos (\alpha_0) = M_{motor} g, \]  

\[(i = 1, 2, 3, 4). \]

Pitch motion of motor:

\[ J_{motor} \ddot{\alpha}_{motor} - (-1)^i F_{motor} \dot{\alpha}_{motor} + (-1)^i F_{motor} \dot{\alpha}_{motor} + T_{motor} = 0, \]  

\[(i = 1, 2, 3, 4). \]

Vertical vibration of wheelset:

\[ M_{wheel} \ddot{Z}_{wheel} + F_{wheel} + F_{wheel} + (-1)^i F_{wheel} \cos (\alpha_0) + 2P_l = M_{wheel} g, \]  

\[(i = 1, 2, 3, 4). \]

Rotational motion of motor rotor:

\[ J_\alpha \dot{\theta}_{motor} = T_{motor} - T_{rpi}, \]  

\[(i = 1, 2, 3, 4). \]

Rotational motion of the pinion:
Formulas:

- Mass moment of inertia of wheelset: \( J_c \)
- Mass of bogie frame: \( M_b \)
- Mass moment of inertia of bogie frame: \( I_b \)
- Mass moment of inertia of wheelset: \( J_w \)
- Mass of motor: \( M_m \)
- Mass moment of inertia of motor: \( J_m \)
- Angular displacement of car body/bogie frame: \( \beta_{C} \)
- Angular displacement of wheelset/motor: \( \beta_{m} \)
- Linear displacement of wheelset/motor in vertical: \( Z_{v} \)
- Linear displacement of wheelset/motor in horizontal: \( Z_{r} \)
- Linear displacement of rail/sleeper/ballast in vertical: \( Z_{v} \)
- Linear displacement of rail/sleeper/ballast in horizontal: \( Z_{r} \)
- Stiffness/damping of secondary suspension: \( K_{p}/C_{p} \)
- Stiffness/damping of primary suspension: \( K_{p}/C_{p} \)
- Stiffness/damping of axle-hung bearing: \( K_{p}/C_{p} \)
- Stiffness/damping of discrete ballast: \( K_{p}/C_{p} \)
- Stiffness/damping of subgrade: \( K_{p}/C_{p} \)

**Table 1: Notation used in the locomotive subsystem.**

<table>
<thead>
<tr>
<th>Notation</th>
<th>Physical meaning</th>
</tr>
</thead>
<tbody>
<tr>
<td>( M_c )</td>
<td>Mass of car body</td>
</tr>
<tr>
<td>( I_c )</td>
<td>Mass moment of inertia of car body</td>
</tr>
<tr>
<td>( M_b )</td>
<td>Mass of bogie frame</td>
</tr>
<tr>
<td>( I_b )</td>
<td>Mass moment of inertia of bogie frame</td>
</tr>
<tr>
<td>( J_w )</td>
<td>Mass moment of inertia of wheelset</td>
</tr>
<tr>
<td>( M_m )</td>
<td>Mass of motor</td>
</tr>
<tr>
<td>( J_m )</td>
<td>Mass moment of inertia of motor</td>
</tr>
<tr>
<td>( \beta_{C}/\beta_{m} )</td>
<td>Angular displacement of car body/bogie frame</td>
</tr>
<tr>
<td>( \beta_{C}/\beta_{m} )</td>
<td>Angular displacement of wheelset/motor</td>
</tr>
<tr>
<td>( Z_{v}/Z_{r} )</td>
<td>Linear displacement of wheelset/motor in vertical</td>
</tr>
<tr>
<td>( K_{p}/C_{p} )</td>
<td>Stiffness/damping of secondary suspension</td>
</tr>
<tr>
<td>( K_{p}/C_{p} )</td>
<td>Stiffness/damping of primary suspension</td>
</tr>
<tr>
<td>( K_{p}/C_{p} )</td>
<td>Stiffness/damping of axle-hung bearing</td>
</tr>
<tr>
<td>( K_{p}/C_{p} )</td>
<td>Stiffness/damping of discrete ballast</td>
</tr>
<tr>
<td>( K_{p}/C_{p} )</td>
<td>Stiffness/damping of subgrade</td>
</tr>
</tbody>
</table>

**Table 2: Notation used in the track subsystem.**

<table>
<thead>
<tr>
<th>Notation</th>
<th>Physical meaning</th>
</tr>
</thead>
<tbody>
<tr>
<td>( M_r )</td>
<td>Rail quality of unit length</td>
</tr>
<tr>
<td>( M_{b} )</td>
<td>Mass moment of inertia of car body</td>
</tr>
<tr>
<td>( M_{b} )</td>
<td>Quality of discrete ballast</td>
</tr>
<tr>
<td>( E_{b} )</td>
<td>Resistance to bending stiffness of rail</td>
</tr>
<tr>
<td>( Z_{v}/Z_{r} )</td>
<td>Linear displacement of rail/sleeper/ballast in vertical</td>
</tr>
<tr>
<td>( K_{p}/C_{p} )</td>
<td>Stiffness/damping of rail pad</td>
</tr>
<tr>
<td>( K_{p}/C_{p} )</td>
<td>Stiffness/damping of discrete ballast</td>
</tr>
<tr>
<td>( K_{p}/C_{p} )</td>
<td>Stiffness/damping of subgrade</td>
</tr>
</tbody>
</table>

**Table 3: Notation used in the gear transmission system.**

<table>
<thead>
<tr>
<th>Notation</th>
<th>Physical meaning</th>
</tr>
</thead>
<tbody>
<tr>
<td>( K_{p}/C_{p} )</td>
<td>Torsional stiffness/damping between rotor and pinion</td>
</tr>
<tr>
<td>( K_{p}/C_{p} )</td>
<td>Torsional stiffness/damping between gear and wheelset</td>
</tr>
<tr>
<td>( K_{m}/C_{m} )</td>
<td>Stiffness/damping of gear meshing</td>
</tr>
<tr>
<td>( T_{m} )</td>
<td>Torque that actuates the rotor</td>
</tr>
</tbody>
</table>

\[
J_{\pi}\ddot{\theta}_{\pi} = T_{\pi} + F_{\text{me}}R_{\pi}, \quad (i = 1, 2, 3, 4). \quad (9)
\]

Rotational motion of the gear:
\[
J_{\pi}\ddot{\theta}_{\pi} = F_{\text{me}}R_{\pi} - T_{\text{me}}, \quad (i = 1, 2, 3, 4). \quad (10)
\]

Rotational motion of the wheelset:
\[
J_{w}\ddot{\theta}_{w} = T_{\text{me}} - 2F_{\text{creep}}R_{w}. \quad (11)
\]

The forces and moments are given by the following formulas:

**Force between the car body and bogie:**
\[
F_{\text{ct}} = K_{x}(Z_{c} - Z_{i}) - (-1)^{i}J_{f} \dot{\xi}, \quad (i = 1, 2, 3). \quad (12)
\]

**Force between the bogie and wheelset:**
\[
F_{\text{me}} = K_{x}(Z_{\text{me}} - Z_{\text{wi}}) - (-1)^{i}J_{f} \dot{\xi}, \quad (i = 1, 2, 3, 4). \quad (13)
\]

Force between the bogie and motor:
\[
F_{\text{mot}} = K_{m}[Z_{\text{me}} - Z_{\text{wi}} - (-1)^{i}(J_{f} \dot{\xi})], \quad (i = 1, 2, 3, 4). \quad (14)
\]

Force between the motor and wheelset:
\[
F_{\text{mwi}} = K_{m}[Z_{\text{me}} - Z_{\text{wi}} - (-1)^{i}(J_{f} \dot{\xi})], \quad (i = 1, 2, 3, 4). \quad (15)
\]

In equations (12)–(15), the symbol “round” refers to the function for rounding. \( L_{d} \) denotes the half distance between the centers of the two bogie frames. \( L_{s} \) is the half distance between the centers of two wheelsets. \( L_{g} \) represents the longitudinal distance from the motor center of gravity to the center of the frame. \( L_{s} \) represents the longitudinal distance from the pinion shaft to the motor center of gravity. \( L_{g} \) represents the longitudinal distance from the pinion shaft to the gear shaft.

The symbols \( P, F_{\text{m}}, \) and \( F_{\text{creep}} \) respectively, represent the wheel-rail interaction force, the gear transmission mesh force, and the wheel-rail interaction creep force. The gear transmission mesh force, \( P \) and \( F_{\text{creep}} \) are shown as follows:
\[
P = \left( \frac{1}{G} \right)^{1/2} \delta Z(t). \quad (16)
\]
\[
F_{\text{creep}} = \mu Q, \quad (17)
\]
\[
F_{\text{m}} = F_{\text{m}} + \delta + C_{m} \times \delta, \quad (18)
\]

where the symbol \( \delta \) represents the transmission error. The dots above the variables \( \delta \) indicate a first-order differential operation. The transmission error \( \delta \) can be calculated by the following formula:
\[
\delta_{i} = -R_{p} \dot{\theta}_{p} - R_{g} \dot{\theta}_{g} - (-1)^{i}(Z_{\text{mot}} - Z_{\text{wi}})X_{q, i} \cos \alpha_{0}, \quad (19)
\]

where \( R_{p} \) and \( R_{g} \) denote the radius of the pinion and gear base circles. Where \( \dot{\theta}_{p} \) and \( \dot{\theta}_{g} \) denote the angular displacements of the pinion and gear. The meaning of symbol \( Z_{\text{mot}} \) and \( Z_{\text{wi}} \) can be found in Table 1. In this paper, the manufacturing and the assembly error are ignored.

The symbol \( \delta Z(t) \) in equation (16) represents the wheel-rail contact constant. The symbol \( \delta Z(t) \) represents the wheel-rail elastic compressive deformation. Zhai and Cai [21] notes that the wheel-rail elastic compressive deformation includes the static compressive deformation, and it is calculated as follows:
\[
\delta Z(t) = Z_{wi}(t) - Z_{w}(x_{wi}, t), \quad (20)
\]

where the symbols \( Z_{wi} \) and \( Z_{w} \) are the vertical distances of the wheel and the rail on the contact point, respectively.
Figure 1: Vertical dynamics model of locomotive-track coupled with gear transmission: (a) total dynamic system, (b) schematic of the gear transmission, and (c) detail illustration of the gear transmission system.
The symbol $Q$ in equation (17) represents the normal load of the wheel-rail contact. The symbol $\mu$ in equation (17) represents the adhesion coefficient of the wheel-rail interface. Ishikawa gives the formula as follows:

$$\mu = c \cdot \exp(-a \cdot v_{\text{slip}}) - d \cdot \exp(-b \cdot v_{\text{slip}}),$$

(21)

where $a$, $b$, $c$, and $d$ are calculated constants and $v_{\text{slip}}$ is the difference between the wheel rotational velocity and the locomotive velocity.

2.2. Gear Mesh Stiffness Model with Tooth Crack Damage. During gear transmission system operation, the tooth root is subjected to repeated bending stress. After some time, cracks will appear at the tooth root site. When the teeth with tooth root cracks enter the engagement, the mesh stiffness changes.

In 1987, Yang and Lin [22] used the potential energy method to calculate the gear mesh stiffness. They assumed that the gear meshing energy consists of three parts: $U_b$ (hertz energy), $U_a$ (bending potential energy), and $U_s$ (axial compressive energy). Tian [23] improved the mesh stiffness model in 2004. In this method, the $U_s$ (shear energy) was considered.

In 2013, Chen and Shao [24] propose a much accurate method for calculating gear mesh stiffness. In this method, they consider the effect of the tooth root fillet-foundation deflection and the range of the cantilever beam is between the base circle to the dedendum circle. In order to calculate the mesh stiffness of tooth profile modifications and profile shifted gear, Ma et al. [25] revised the model put forward by Chen. In 2016, Ma and Zeng [26] put forward a method to calculate the mesh stiffness of gear pairs with tip relief.

During gear transmission system operation, the tooth root is subjected to repeated bending stress. After some time, cracks will appear at the tooth root site. When the teeth with tooth root cracks enter the engagement, the mesh stiffness changes. References [27, 28] consider that a crack develops at the root of a single tooth. In most cases, the crack is a straight line. Based on the above study, the crack propagation path is assumed to be a smooth and continuous line. Reference [29] indicates that when the gear has a tooth root crack, parameters $I_s$ and $A_s$ will change. In 2014, Ma et al. [30, 31] put forward a model to calculate the mesh stiffness of gear with tooth root crack considering the accurate transition curve of tooth root.

In 2013, Mohammed et al. [32] assumed that the crack of the gear tooth is parabolic, and the accuracy of gear stiffness calculation is improved. In order to calculate the mesh stiffness of gear with tooth crack in planetary gearbox, Liang developed an improved beam model to analyze the effect of the crack evolution on gear mesh stiffness.

In this paper, the mesh stiffness of the healthy gear can be calculated by equation (22), where $k_i$, $k_0$, $k_r$, $k_s$ represent the gear mesh stiffness. Those indicators calculated by the formula equation (23), where $F$ represents the acting force at the contact point. $F$ can be divided into $F_s$ and $F_b$, where $F_s$ is the radial force and $F_b$ is the tangential force. The schematic of gear force is shown in Figure 2. $G$ and $E$ denote the shear modulus and Young’s modulus, respectively. $I_s$ and $A_s$ are the area moment of inertia and section area, respectively, where the distance from the tooth root is $x$. The total mesh gear potential energy can be calculated by equation (24). Chaari et al. [33] noted that the deformation of the base circle has an effect on the gear mesh stiffness. The mesh stiffness can be calculated by equation (25). The coefficients $L^*$, $M^*$, $P^*$, and $Q^*$ can be calculated by equation (26), and $X_i^*$ represents these coefficients $L^*$, $M^*$, $P^*$, and $Q^*$. The coefficients $A_i$, $B_i$, $C_i$, and $D_i$ can be obtained by Table 4. $\theta_i$ is an angle that refers to half the base circle thickness. $r_i$ and $r_{\text{int}}$ represent the basic circle radius and gear bore, respectively. $h_{ti}$ is calculated by equation (27).

$$k = \sum_{i=1}^{2} \left( \frac{1}{k_{bi}} + \frac{1}{k_{s1,i}} + \frac{1}{k_{s2,i}} + \frac{1}{k_{f1,i}} + \frac{1}{k_{f2,i}} \right).$$

(22)

$$\frac{1}{k_i} = \cos^2 \alpha \frac{EL}{B_i h_i^2} \left\{ L^* \left( \frac{h_{ti}}{S_i} \right)^2 + M^* \left( \frac{h_{ti}}{S_i} \right) + P^* \left( 1 + Q^* \tan^2 \alpha \right) \right\},$$

(25)

$$X_i^* = A_i + B_i h_i^2 + C_i h_i^2 \frac{1}{\theta_i} + D_i + E_i h_i^2 + F_i,$$  

(26)

$$h_{ti} = r_i, r_{\text{int}},$$

(27)

$$I_s = \begin{cases} \frac{1}{12} (h_{c1} + h_s)^3 L, & x \leq g_c, \\ \frac{1}{12} (2h_s)^3 L, & x > g_c, \end{cases}$$

(28)
Principal component analysis (PCA) is a statistical analysis method that maps multiple features into a few comprehensive features. To systematically analyze a practical problem, indicators that are extracted using a time-frequency domain analysis need to be considered. However, each indicator reflects some information about the problem, and there is a certain correlation between the indicators. Principal component analysis takes effective data dimensionality reduction technology to extract several comprehensive and irrelevant features that reflect the comprehensive test information as much as possible [34, 35]. The principal component analysis process is shown as follows:

1. The standardization of data \( X = (X_1, X_2, \ldots, X_p)^T \) is a \( p \)-dimensional random vector, and \( X_i = (X_{i1}, X_{i2}, \ldots, X_{ip})^T \) is the sample data, where \( i = 1, 2, \ldots, n \). \( Z \) is the standardization matrix that is calculated by equation (30):
\[
Z_{ij} = \frac{x_{ij} - \bar{x}_j}{s_j}, \quad i = 1, 2, \ldots, n, \quad j = 1, 2, \ldots, p, \quad (30)
\]

where
\[
\bar{x}_j = \frac{\sum_{i=1}^{n} x_{ij}}{n},
\]
\[
s_j^2 = \frac{\sum_{i=1}^{n} (x_{ij} - \bar{x}_j)^2}{n-1}.
\]

2. The calculation of the correlation coefficient matrix \( R \) of the standardization matrix \( Z \) is done in equation (32).
\[
R = Z^T Z \frac{n-1}{n-1} \quad (32)
\]

3. Calculate the principal component \( U \). The characteristic root \( \lambda \) of the correlation coefficient matrix \( R \) can be obtained by equation (33). The eigenvector of the characteristic root \( \lambda_i \) is \( b_i \). \( I \) is the characteristic vector matrix.
\[
|R - \lambda I| = 0. \quad (33)
\]

The contributions ratio of the \( i \)th principal component can be calculated by equation (33). The cumulative contribution rate of the former \( m \) components can be calculated by equation (35). The number of the principal components can be determined by the value of \( \tau \). Finally, we can obtain the principal component \( U \) by equation (36):
\[
t_j = \frac{\lambda_j}{\sum_{k=1}^{m} \lambda_k}, \quad (34)
\]
\[
\tau = \frac{\sum_{j=1}^{m} \lambda_j}{\sum_{k=1}^{m} \lambda_k}, \quad (35)
\]
\[
U_j = Z^T b_j, \quad j = 1, 2, \ldots, m. \quad (36)
\]

3.2. Grey Relational Analysis. The Grey relational analysis is a method of quantitative description of the development and change of a system [36]. It determines the state of the system by calculating the correlation degree between the standard mode and the unknown mode. The process of detecting the degree of damage using the Grey relational analysis is shown as follows:

1. Structure standard damage mode matrix. In this paper, the standard mode matrix is established using the simulation data of the locomotive car
body. The standard mode matrix \([X_0(k)]\) is shown as follows:

\[
[X_0(k)] = \begin{bmatrix}
X_{01} \\
X_{02} \\
\vdots \\
X_{0n}
\end{bmatrix} = \begin{bmatrix}
X_{01}(1) & X_{01}(2) & \cdots & X_{01}(k) \\
X_{02}(1) & X_{02}(2) & \cdots & X_{02}(k) \\
\vdots & \vdots & \ddots & \vdots \\
X_{0n}(1) & X_{0n}(2) & \cdots & X_{0n}(k)
\end{bmatrix},
\]

(37)

where \(k\) is the number of damage states and \(n\) is the number of indicators of each standard state.

In this paper, we choose a series of statistical indicators (such as RMS, kurtosis, \(S_a\), \(S_r\), and FM4) to analysis the degree of gearbox gear crack damage. RMS, kurtosis, \(S_a\), \(S_r\), and FM4 can be calculated by

\[
\text{RMS} = \sqrt{\frac{1}{N} \sum_{n=1}^{N} (x(n) - \bar{x})^2},
\]

\[
\bar{x} = \frac{1}{N} \sum_{n=1}^{N} x(n),
\]

\[
\text{kurtosis} = \frac{(1/N)\sum_{n=1}^{n} (x(n) - \bar{x})^4}{\left[(1/N)\sum_{n=1}^{n} (x(n) - \bar{x})^2\right]^2},
\]

\[
S_a = \frac{(1/N)\sum_{n=1}^{n} |x(n) - \bar{x}|}{\left[(1/N)\sum_{n=1}^{n} |x(n) - \bar{x}|\right]},
\]

\[
S_r = \frac{(1/N)\sum_{n=1}^{n} |d(n) - \overline{d}|^{3/2}}{\left[(1/N)\sum_{n=1}^{n} |d(n) - \overline{d}|\right]^{3/2}},
\]

\[
\text{FM4} = \left(\frac{1}{(1/N)\sum_{n=1}^{n} (d(n) - \overline{d})^4}\right)^{1/4},
\]

where \(x(n)\) is the amplitude of the signal and \(N\) is the number of data. \(\bar{x}\) is the mean value of the data and \(d\) is the difference signal of \(x(n)\). \(\overline{d}\) is the mean value of \(d\). The detailed flowchart for correlation degree is shown in Figure 3.

(2) Calculation of the correlation coefficient. Assume \(X_i(j) = (X_1(j), X_2(j), \ldots, X_n(j))^T\) is the characteristic vector calculated from the data to be checked. \(X_i(j)\) is the \(i\)th characteristic parameter. The correlation coefficient can be calculated by

\[
\eta_i(k) = \frac{\min, \min[|X_0(k) - X_i(0)| + \rho \max, \max[|X_0(k) - X_i(0)|]}{[|X_0(k) - X_i(0)| + \rho \max, \max[|X_0(k) - X_i(0)|]},
\]

(39)

where \(\rho\) is the resolution coefficient. In this paper, the value of \(\rho\) is 0.5.

(3) Detection of damage degree. The vector of the correlation degree \(\gamma\) is shown as follows:

\[
\gamma = \left[\frac{1}{n} \sum_{j=1}^{n} \eta_j(1), \frac{1}{n} \sum_{j=1}^{n} \eta_j(2), \ldots, \frac{1}{n} \sum_{j=1}^{n} \eta_j(k)\right].
\]

(40)

The damage mode corresponding to the maximum element of \(\gamma\) is the state of the data to be checked.

3.3. Detection of Degree of Locomotive Gearbox Gear Damage.

The process of detecting the degree of gearbox gear damage is shown as follows:

(1) Decompose the fault range that needs to be detected into \(m\) parts.

(2) Calculate the \(n\) indicators of the vertical acceleration of car body vibration under each part that was decomposed in Section 1. These indicators make up the standard damage mode matrix. The matrix and the calculated indicators that are to be verified make up a matrix with the size of \((m + 1) \times n\).

(3) Use the method of PCA to reduce the dimension of matrix.

(4) Solve the correlation degree vector between the signal to be verified and the standard mode using the Grey relational analysis method.

(5) Determine the maximum value of the correlation degree vector corresponding to the degree of gear damage. The damage degree detection chart is shown in Figure 4.

4. Simulation and Result Discussion

With the development of the vertical dynamics model of the locomotive-track coupled with the gear transmission, the dynamic responses of the locomotive subsystem can be obtained. We adopt the locomotive model reported earlier. The running speed of the locomotive is 100 km/h. The radius of the wheelset is 0.625 m. During locomotive operation, the mesh force of the gear transmission system is the primary excitation that causes locomotive vibrations. The notation for the main parameters of the gearbox is in Table 5. Using the above parameters, we can obtain the gear mesh frequency \(f_{mn}\) and the rotation frequency of the pinion \(f_p\): 884.8 Hz and 36.9 Hz, respectively.

The total mesh stiffness of gear pair can be calculated by equations (22)–(29). When there is a crack in the gear tooth root, the mesh stiffness of the gear pair will change. The mesh stiffness of gear pair under different crack sizes is calculated and shown in Figure 5. The angle \(\theta\) between the crack direction and the tooth center is 45°. As shown, when there is a crack on the pinion, the mesh stiffness of the gear pair is decreased.

4.1. Acceleration Comparison of the Car Body, Frame, and Wheelset. The dynamic model of the locomotive is
a large-scale system that contains the car body, bogie, and wheelset. To determine the component with an acceleration that shows more sensitivity to a tooth root crack, we compare the acceleration data of the car body, bogie frame, and wheelset.

Figure 6(a) is the time domain diagram of vertical vibration acceleration of car body when there is no damage on the gear. Figure 6(b) is the time domain diagram of vertical vibration acceleration of car body when the crack length of the pinion is 4 mm. By comparing these diagrams, we find that when there is a crack on the root of the pinion, the amplitude of the acceleration of the car body is bigger.

Figure 7(a) is the frequency spectrum diagram (in logarithm form) of vertical vibration acceleration of car body when there is no damage on the gear. Figure 7(b) is the frequency spectrum diagram of vertical vibration acceleration of car body when the crack length of the pinion is 4 mm. And the frequency range of these two diagrams is 0–10000 Hz. As we can see, the mesh frequency ($f_m = 848.8$ Hz) and the harmonics of the gear pair is apparent. In order to better observe the low frequency part of Figure 7, we restrict the abscissa range of Figure 7 between 0 and 1200 Hz and obtain Figure 8. As we can see, when there is a crack on the pinion, the rotation frequency of the pinion ($f_p = 36.9$ Hz) and the harmonics of the rotation frequency of the pinion appear.

Figure 9(a) is the time domain diagram of vertical vibration acceleration of car body when there is no damage on the gear. Figure 9(b) is the time domain diagram of vertical vibration acceleration of car body when the crack length of the pinion is 4 mm. By comparing these diagrams, we find that when there is a crack on the root of the pinion, the amplitude of the acceleration of the car body is bigger.

Figure 10(a) is the frequency spectrum diagram (in logarithm form) of vertical vibration acceleration of car body when there is no damage on the gear. Figure 10(b) is the frequency spectrum diagram of vertical vibration acceleration of car body when the crack length of the pinion is 4 mm. And the frequency range of these two diagrams is 0–10000 Hz. As we can see, the mesh frequency ($f_m = 848.8$ Hz) and the harmonics of the gear pair are apparent. In order to better observe the low frequency part of Figure 10, we restrict the abscissa range of Figure 10 between 0 and 1200 Hz and obtain Figure 11. As we can see, when there is a crack on the pinion, the rotation frequency of the pinion ($f_p = 36.9$ Hz) and the harmonics of the rotation frequency of the pinion appear.

Figure 12(a) is the time domain diagram of vertical vibration acceleration of car body when there is no damage on the gear. Figure 12(b) is the time domain diagram of vertical vibration acceleration of car body when the crack

---

**Table 5: Main parameters of the gearbox.**

<table>
<thead>
<tr>
<th>Notation</th>
<th>Physical meaning</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$m$</td>
<td>Module (mm)</td>
<td>Pinion</td>
</tr>
<tr>
<td>$\alpha_0$</td>
<td>Pressure angle (°)</td>
<td>20</td>
</tr>
<tr>
<td>$h^*$</td>
<td>Addendum coefficient</td>
<td>1</td>
</tr>
<tr>
<td>$c^*$</td>
<td>Tip clearance coefficient</td>
<td>0.25</td>
</tr>
<tr>
<td>$W$</td>
<td>Face width (mm)</td>
<td>175</td>
</tr>
<tr>
<td>$Z$</td>
<td>Number of teeth</td>
<td>23</td>
</tr>
<tr>
<td>$x^*$</td>
<td>Tooth profile shift coefficient</td>
<td>0.362</td>
</tr>
</tbody>
</table>

**Figure 5: Mesh stiffness.**

---

Shock and Vibration
Figure 6: Time domain diagram of vertical vibration acceleration of car body: (a) health gear (b) the crack length of the pinion is 4 mm.

Figure 7: Frequency spectrum diagram of vertical vibration acceleration of car body (frequency range: 0–10000): (a) health gear (b) the crack length of the pinion is 4 mm.

Figure 8: Frequency spectrum diagram of vertical vibration acceleration of car body (frequency range: 0–1200): (a) health gear (b) the crack length of the pinion is 4 mm.
length of the pinion is 4 mm. By comparing these diagrams, we find that the crack of the pinion has little effect on the vertical vibration acceleration of the wheelset.

Figure 9: Time domain diagram of vertical vibration acceleration of bogie: (a) health gear (b) the crack length of the pinion is 4 mm.

Figure 10: Frequency spectrum diagram of vertical vibration acceleration of bogie (frequency range: 0–10000): (a) health gear (b) the crack length of the pinion is 4 mm.

Figure 11: Frequency spectrum diagram of vertical vibration acceleration of bogie (frequency range: 0–1200): (a) health gear (b) the crack length of the pinion is 4 mm.

Figure 13(a) is the frequency spectrum diagram (in logarithm form) of vertical vibration acceleration of car body when there is no damage on the gear. Figure 13(b) is
frequency spectrum diagram of vertical vibration acceleration of car body when the crack length of the pinion is 4 mm. And the frequency range of these two diagrams is 0–10000 Hz. As we can see, the mesh frequency \( f_m \) (equals 848.8 Hz) and the harmonics of the gear pair is apparent. In order to better observe the low frequency part of Figure 13, we restrict the abscissa range of Figure 13 between 0 and 1200 Hz and obtain Figure 14. As we can see, when there is a crack on the pinion, the rotation frequency of the pinion (\( f_p = 36.9 \) Hz) and the harmonics of the rotation frequency of the pinion appear. In addition, the difference in the gear pair mesh frequency and the pinion rotation frequency is appear.

By comparing the vertical acceleration of the car body, bogie, and wheelset vibrations, we find that when there is a crack on the pinion, the amplitude of the acceleration of these vibrations are larger in the time history and the frequency spectrum. The rotation frequency of the pinion and the harmonics of the rotation frequency of the pinion appear. Due to the influence of the spring-damper system, the vertical acceleration of the car body and frame vibrations attenuates much more than in the wheelset. The vertical acceleration of the car body is more sensitive to the effect of the tooth root crack.

### 4.2 Detection of Degree of the Gear Box Gear Crack Damage by the Vertical Acceleration of the Car Body Vibration

Through the above analysis, we find that the vertical acceleration of the car body vibration is more sensitive to the effect of the tooth root crack. Therefore, we choose the vertical acceleration of the car body vibration to detect the degree of gear box gear crack damage. In this paper, the crack length of the pinion from 0 to 5 mm is simulated and analyzed. The length of the crack grows in 0.5 mm increments. The gear pair mesh stiffness under different crack sizes is calculated and shown in Figure 3. As shown, with an increase in tooth root crack length, the gear meshing stiffness decreases.

The process of detecting the degree of gearbox gear damage is shown as follows:

1. Decompose the fault range that needs to be detected into \( m \) parts. In this paper, the crack length of the pinion is increased from 0 to 5 mm in 0.5 mm increments. Therefore, the damage states that it is 11.
(2) Calculate the 5 indicators (RMS, kurtosis, $S_\alpha$, $S_r$, and FM4) of the vertical acceleration of car body vibration under each damage states. And those indicators of matrix are shown in Table 6. These indicators make up the standard damage mode matrix. The acceleration of the car body vibration with a crack length of 3.25 mm was verified. The 5 indicators of the signal to be verified are shown in Table 7. The standard damage mode matrix and the calculated indicators that are to be verified make up a matrix with the size of $12 \times 5$.

(3) Use the method of PCA to reduce the dimension of matrix. The principal component analysis method proposed in the Section 3.1 is applied to deal with the standard damage mode matrix.

(4) Use the Grey relational analysis method proposed in the Section 3.2 to solve the correlation degree vector between the signal to be verified and the standard

---

### Table 6: Indicators of the vertical acceleration of car body vibration under each damage states.

<table>
<thead>
<tr>
<th></th>
<th>0 mm</th>
<th>0.5 mm</th>
<th>1 mm</th>
<th>1.5 mm</th>
<th>2 mm</th>
<th>2.5 mm</th>
<th>3 mm</th>
<th>3.5 mm</th>
<th>4 mm</th>
<th>4.5 mm</th>
<th>5 mm</th>
</tr>
</thead>
<tbody>
<tr>
<td>Kurtosis</td>
<td>1.5097</td>
<td>1.5102</td>
<td>1.5131</td>
<td>1.5204</td>
<td>1.5345</td>
<td>1.5569</td>
<td>1.5898</td>
<td>1.6333</td>
<td>1.6893</td>
<td>1.7553</td>
<td>1.8268</td>
</tr>
<tr>
<td>FM4</td>
<td>1.5002</td>
<td>1.5002</td>
<td>1.5003</td>
<td>1.5006</td>
<td>1.5012</td>
<td>1.5021</td>
<td>1.5036</td>
<td>1.5056</td>
<td>1.5084</td>
<td>1.5120</td>
<td>1.5165</td>
</tr>
<tr>
<td>rms</td>
<td>$6.72e^{-4}$</td>
<td>$6.72e^{-4}$</td>
<td>$6.73e^{-4}$</td>
<td>$6.74e^{-4}$</td>
<td>$6.76e^{-4}$</td>
<td>$6.80e^{-4}$</td>
<td>$6.85e^{-4}$</td>
<td>$6.92e^{-4}$</td>
<td>$7.01e^{-4}$</td>
<td>$7.12e^{-4}$</td>
<td></td>
</tr>
<tr>
<td>$S_\alpha$</td>
<td>1.6521</td>
<td>1.6525</td>
<td>1.6549</td>
<td>1.6614</td>
<td>1.6737</td>
<td>1.6931</td>
<td>1.7217</td>
<td>1.7606</td>
<td>1.8130</td>
<td>1.8792</td>
<td>1.9558</td>
</tr>
<tr>
<td>$S_r$</td>
<td>1.2033</td>
<td>1.2035</td>
<td>1.2043</td>
<td>1.2065</td>
<td>1.2108</td>
<td>1.2175</td>
<td>1.2275</td>
<td>1.2407</td>
<td>1.2578</td>
<td>1.2781</td>
<td>1.3003</td>
</tr>
</tbody>
</table>

---

### Table 7: Indicators of the vertical acceleration of car body vibration when crack length is 3.25.

<p>| | | | | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Kurtosis</td>
<td>FM4</td>
<td>rms</td>
<td>$S_\alpha$</td>
<td>$S_r$</td>
<td></td>
</tr>
<tr>
<td>3.25 mm</td>
<td>1.6023</td>
<td>1.5042</td>
<td>$6.82e^{-4}$</td>
<td>1.7404</td>
<td>1.2345</td>
</tr>
</tbody>
</table>

---

**Figure 14:** Frequency spectrum diagram of vertical vibration acceleration of wheelset (frequency range: 0–1200): (a) health gear (b) the crack length of the pinion is 4 mm.

**Figure 15:** Normalization curve of the correlation degree vector.
mode. So, we can obtain the normalization curve of the correlation degree vector, as shown in Figure 12. The maximum correlation degree vector is 0.9765. The crack length corresponding to the maximum correlation degree is 3 mm. Therefore, the crack length of the detected data is between 2.5 and 3.5 mm. It can be seen from Figure 12 that the correlation curve is consistent. The value of the correlation degree becomes larger when the value of the standard damage mode approaches the data to be verified.

5. Conclusions

In this paper, a vertical dynamic model of the locomotive-track coupled with the gear transmission is established. Using a comprehensive model, the dynamic response of the locomotive subsystem is obtained under various degrees of tooth root crack damage in the gear transmission system. By comparing the vertical acceleration for vibrations in an undamaged locomotive and in one with a gear tooth root crack, we find that when there is a gear tooth root crack on the pinion of the gear transmission system, the rotation frequency and the harmonics of the rotation frequency of the pinion appear. The vertical acceleration of the car body vibration is more sensitive to the effect of the tooth root crack. The degree of tooth root crack damage is detected using PCA and the Grey relational analysis methods.

The track excitation is inevitable when a locomotive operating at track. However, in this paper, track excitation is not considered. The dynamic response of a locomotive subsystem to track excitation will be investigated in the future.

Data Availability

The data used to support the findings of this study are available from the corresponding author upon request.

Conflicts of Interest

The authors declare that there are no conflicts of interest regarding the publication of this paper.

Acknowledgments

This project is supported by National Key Research and Development Program of China (grant no. 2017YFB1201103-07).

References

[19] C. Ding, B. Z. Zhang, F. Z. Feng et al., "Application of local mean decomposition and permutation entropy in fault


