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Research Article

Optimal Sensor Placement for Spatial Structure Based on Importance Coefficient and Randomness

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The current methods of optimal sensor placement are majorly presented based on modal analysis theory, lacking the consideration of damage process of the structure. The effect of different minor damage cases acting on the total spatial structure is studied based on vulnerability theory in structural analysis. The concept of generalized equivalent stiffness is introduced and the importance coefficient of component is defined. For numerical simulation, the random characteristics for both structural parameters and loads are considered, and the random samples are established. The damage path of each sample is calculated and all the important members on the damage failure path are listed; therefore the sensor placement scheme is determined according to the statistical data. This method is extended to dynamic analysis. For every dynamic time-history analysis, time-varying responses of the structure are calculated by selecting appropriate calculating interval and considering the randomness of structural parameters and load. The time-varying response is analyzed and the importance coefficient of members is sorted; finally the dynamic sensor placement scheme is determined. The effectiveness of the method in this paper is certified by example.

1. Introduction

Spatial structures, such as latticed shell and space truss, are three-dimensional representations of equilibrium equations and symbols of structural design, simulation, and construction in civil engineering. Spatial structures are widely used for many advantages such as large stiffness, light mass, and low cost, and structural styles are also novel and diversified.

Spatial structure has these mechanical characteristics as follows. It has three-dimensional mechanical characteristics and presents a space working state under loads [1]. It belongs to high-order statically indeterminate structure, but it is generally flexible and can be easily damaged and even destroyed under special loads and sudden disasters. Most of the spatial structures are symmetrical, the natural frequencies are usually close, and the dynamic characteristics are complex. The major loads have strong randomness, and the overall structure has a high sensitivity to defects. The initial defect and the damage of local elements can significantly influence the bearing capacity and the mechanical characteristics.

The operating environment of the spatial structure is complex, and the potential risk of damage and destruction is larger. In addition, spatial structures are inevitable to subject to environmental corrosion, long-term fatigue effects, or natural disasters, and then the damage accumulates during long service period [2, 3]. Furthermore, many large spatial structures are constructed as important building such as gymnasium, exhibition hall, and station hall. Therefore, it is necessary to detect the potential damage accurately and promptly in order to ensure structural capacity and safety. Under this background, intelligent health monitoring, sensors selection, and optimal placement and damage detection in construction and operation have an important theoretical value and practical significance [4, 5].

Over the past twenty years, various problems in health monitoring of spatial structure have been studied and many research achievements have been obtained, and some theoretical results and techniques have been applied to practical engineering. However, there are still some problems in the health monitoring of spatial structure, which need to be further studied and then get improved in the aspects such as optimal sensor placement, model updating, damage detection, and safety assessment.

According to the structural style and mechanical characteristics of spatial structures, placing the main sensors on the key joints of the spatial structure so as to detect the damage is reasonable and effective. However, there are a large number of joints, elements, and more degrees of freedom for large spatial structures; it is inconceivable and irrational to place the sensors on all the joints since the economy and the feasibility cannot be fulfilled. Hence, the locations and the number of the joints which will be placed on sensors should be optimized to achieve effective data acquisition and improve the accuracy of damage detection and maintenance efficiency, according to the specific type and performance of sensors. The methods on the optimal sensor placement have been developed from various aspects and many methodologies [6–8]. The most important methods are introduced and discussed in the following content from the dynamic mechanism and the applicability.

The most widely used method is modal assurance criterion (MAC) method or MinMAC method, which is carried out based on minimizing the value of the nondiagonal elements on the modal assurance criterion matrix when new sensors are added on the structure [9–11]. Minimizing the maximal off-diagonal term indicates less correlation exists in the corresponding mode shape vectors and renders the mode shapes discriminable from each other. This method, which belongs to the category of modal analysis, is to maximize the angles formed by unit mode shape vectors, which is equivalent to minimizing the dot product between them.

Although the modal information is sufficient and accurate for modal identification by MinMAC method, the damage detection cannot be achieved only by modal results. The energy is generally assumed as another parameter related to damage. For the energy concept, the most important sensor placement method is the modal kinetic energy (MKE) method [12]. It is established on the traditional heuristic visual inspection, which is to visually inspect the structural response, to examine the interested mode shapes, and to select locations with high amplitudes of responses. The MKE method gives a measure of the dynamic contribution of each degree of freedom for each of the target mode shapes and provides a rough idea where the maximum responses could be measured. Although MKE is suitable for simple structures, it encounters difficulty in large complicated structures; thus, the bottleneck problem occurs. The MKE method starts first from the selection of target modes by the modal kinetic energy as follows.

As another famous method, the effective independence (EI) method aims to select measurement positions that make the mode shapes of interest as linearly independent as possible while containing sufficient information about the target modal responses in the measurements [13, 14]. This method roots in the estimation theory by sensitivity analysis of the parameters to be estimated, and then it arrives at the maximization of the Fisher information matrix. There are many variants of the EI method. The so-called energy optimization technique is derived from EI by optimizing the kinetic energy matrix measured by candidate sensor locations.

The QR decomposition method proposed by Schedlinski and Link aims to locate the effective subset of the modal matrix [15]. The original modal matrix is decomposed into one unitary matrix and one trigonometric matrix by QR decomposition. The fundamental idea is to find the most linear independent rows of the modal matrix to minimize the off-diagonal terms of the MAC matrix.

The Guyan reduction method selects the master degrees as the locations of sensors during the process of Guyan reduction [16]. It is based on the assumption that low ratios of leading diagonal stiffness to mass terms indicate good degree of freedom to be retained in terms of describing the kinetic energy and that the inertia forces at slave coordinates are negligible compared with the elastic forces. A major disadvantage of the reduction methods is that they strongly depend on the meshing size of the FEM and are interested only in the lower modes, which are not always the case. It is useful to note that other sophisticated reduction techniques can also be employed. A relative promising method is static flexibility approach [17]. This method optimizes the static transformation matrix with the assumption that the best master degrees of freedom are those for which the FEM mode shapes can be represented as a linear combination of static flexibility shapes.

In conclusion, the deficiency in the current study on the optimal sensors placement method can be summarized as follows. Different optimal placement schemes can be obtained according to different placement criteria, and the guideline of comprehensive evaluation for various placement criteria and selecting the appropriate placement scheme according to the structural form and the engineering requirements needs intensive study. The current optimal sensor placement methods are mainly established based on dynamic analysis and modal analysis, which are only suitable for the dynamic sensors such as the accelerometer and the dynamic displacement meter. However, the optimal placement strategy on the static parameters such as stress, strain, crack, and long-term deformation is not comprehensive enough, and the placement scheme is usually determined by experience or the simple simulation. On the other hand, the existing optimal placement scheme is usually determined by combining the algorithm and the finite element model, but it is not enough to pay attention to the difference of the loading pattern, the randomness of materials and structures, and the model error and updating, which affects the feasibility of the theoretical optimization placement scheme. In addition, the minor damage and the damage evolution process are not adequately considered, which leads to the flexibility and the adaptability of the placement scheme being unsatisfactory.

In recent years, the concept, philosophy, and application of structural vulnerability continue to be valued [18–23]. Structural vulnerability, also viewed as the antonym of robustness, is usually defined as the characteristics and circumstances of a structural system that makes it susceptible to the damaging effects of a hazard. For example, the seismic vulnerability is the degree of damage to the built environment for a given intensity of earthquake motion [24]. Structural vulnerability can be expressed on a scale of 0 to 1, where 0 is no damage and 1 defines complete destruction. It is apparent that

the structural health monitoring and the sensor placement method can be studied within the system framework of structural vulnerability, and new methods and ideas maybe occur, which will be beneficial to the development of health monitoring.

According to the deficiency of the traditional method of optimal sensor placement, a new static-dynamic sensor placement method based on structural vulnerability and importance coefficient is presented in this paper, and the random characteristics are also involved, which can provide new ideas and methods for optimal sensor placement and application for complex spatial structures.

2. Structural Analysis considering Random Characteristics

The practical engineering structures inevitably have all kinds of initial random defects due to the material characteristics and the construction error; thus, the parameters such as structural stiffness, mass, and geometric size are not equal to the design value but fluctuate within a certain range and the design value is probably the mean value, so these parameters are stochastic. In addition, the actual value of the load on the structure is also random [25]. This randomness generates the difference of geometrical properties, bearing capacity, and damage evolution mechanism between the actual structure and the ideal design model.

Hence, the modal characteristics, damage performance, and the optimal sensor placement scheme based on the ideal model cannot be fully consistent with the actual condition, and there is a considerable deviation in many cases. Based on previous discussion, it is apparent that the structural performance will be recognized more intensively if the load pattern and the randomness of structure materials and dimensions are involved in structural analysis, and the statistical parameters and characterization of the structural static-dynamic performance can be calculated by probability statistical method [26–28]. On this basis, the optimal sensor placement will be more realistic and comprehensive, which can improve the accuracy of damage identification and health monitoring.

The Monte Carlo method is mostly used in the random analysis; the load pattern and the randomness of structural parameters can be considered. First, the probability distribution function (PDF) of load and parameters should be determined, and then a certain number of samples are selected randomly from the total sample library which are in conformity with the corresponding PDFs, in order to generate the operating cases with different load patterns. After that, the mechanical properties and the optimal scheme of each operating case are studied and the statistical results are achieved; then the final analysis result or the optimal scheme is determined based on the occurrence probability.

The advantages of the structure random analysis based on Monte Carlo method include the high accuracy and the precision which can be controlled, but the disadvantage is the low computational efficiency and being time consuming.

On the other hand, a function or a surface which is easy to be established substitutes for the implicit or complex real function or limit state surface in response surface method; the accuracy and the convenience are achieved simultaneously. For complex structures, the basic structural features can be represented by response surface function. On this basis, Monte Carlo sampling is carried out. Because only the value of special response surface function is calculated for each sample rather than restarting structural analysis, this method will obviously improve the computational efficiency than Monte Carlo method.

It is worth noting that the probabilistic design system (PDS) can be provided by some finite element software, such as ANSYS, and both Monte Carlo method and response surface method can be used by users to realize the structural random parameters analysis or reliability analysis. However, there are still many problems in this module, such as the limitation on the number of random input variables and the total amount of input and output, which lead to the bottleneck problem of the random analysis for large and complex structures.

Thus, the mathematical software such as MATLAB is suggested to be used for the preprocessing of parameters and the random variables generation especially for complex structures and then importing the data into the finite element command and carry out structural analysis in finite element software. Finally, the result data are saved and converted into the recognizable format; then the probability statistics and numerical analysis are performed in MATLAB.

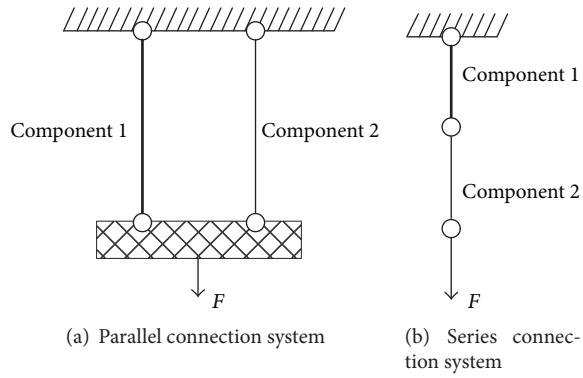
3. Vulnerability Evaluation Based on the Importance Coefficient

Spatial structures will inevitably damage due to various natural hazards and environmental effect. The damage includes the change of the materials property and the geometrical feature of the total structure or local members, besides the deterioration on the stiffness, the strength, the boundary, and the connection conditions.

The incident natural disasters such as earthquake and hurricane will cause serious structural damage in a very short period of time, and these damage types belong to the sudden damage. In addition, a certain degree of damage will occur in the structure due to the factors like environmental change, component degradation, fatigue, corrosion, and others, and these cases belong to the cumulative damage. Both the sudden damage and cumulative damage can be understood and analyzed through the concept of structural vulnerability.

The general definition of structural vulnerability indicates the conditional probability of the actual damage exceeds the designated damage degree when structure is subjected to a given load [22, 23]; this concept represents the relationship between the initial disturbance caused by an unexpected event or overload and the final consequence, and it also stands for the structural fragility and the bearing capacity for accidental damage.

As a derived concept, the importance coefficient of component based on vulnerability represents the influence on the performance of whole structure when the individual members damage or become invalid if the structure is subjected to unexpected events under normal loads, which



(a) Parallel connection system (b) Series connection system

Figure 1: Illustration of importance coefficient in different structures.

also indicates the constraint capacity of the component in a certain extent [29].

The current research on engineering vulnerability analysis mainly focuses on the seismic vulnerability of buildings and bridges, and the application of structural vulnerability in the field of structural health monitoring is not popular. In this paper, it is assumed that structural vulnerability can fully represent the characteristics of the structural damage and be used to evaluate the safety grade. In the theoretical framework of the structural vulnerability analysis, the importance coefficient and damage demand of structural component are analyzed in order to search for the structural vulnerable path and then install sensors according to important path. Hence, the needs of the cumulative damage and the sudden damage are both considered and the large scale of damage caused by local damage can be prevented. Furthermore, the robustness and the reliability of the structure are effectively ensured. Thus, the optimal sensor placement based on vulnerability analysis and important coefficient is studied in this paper.

In the traditional structural vulnerability analysis, it is usually necessary to simultaneously calculate the importance coefficient of the components and the energy which can destroy the components so as to find the circumstance that the components are most important or prone to damage; then the vulnerable path is determined according to the possible damage sequence. However, the components are fully removed in the process of damage analysis when the traditional method is used; that is, the components are completely destroyed. However, the actual possibility of a certain critical component completely destroyed is very small in normal state. On the contrary, the damage is often cumulative. Therefore, the traditional method should be improved for the application of optimal sensor placement.

In this study, the minor damage states, that is, the percent decrease of stiffness is less than 20%, which often occurs in the early stage of cumulative damage, are mainly discussed, and the target of the optimal sensor placement is to detect the minor damage rapidly and accurately. Since the required destruction energy of each component is small, all the components are assumed to be prone to damage, so there are no special damage demands for designated components in vulnerability analysis. Moreover, it should be pointed out that the initial random imperfections are not equivalent to the minor damage. The initial random imperfections are the inevitable errors in the process of material production and construction, and the minor damage is the damage that occurs in the operation process of structural members. In view of damage degree, the minor damage is obviously larger than the value of random imperfections.

From the above analysis, it is obvious that the definition and calculation of importance coefficient are the critical part to realize the vulnerability analysis and to search vulnerable path. There are two types of methods for evaluating the importance coefficient of structural component. The first type of methods mainly focuses on the intrinsic properties of the structural system, which is independent of the load patterns. The other type of methods belongs to the evaluation method related to load, which pays attention to both the properties of the structural system and the effects of the load on the structure.

In fact, the effective vulnerable path of the structure is not merely related to the structure geometry and the stiffness distribution but also intrinsically related to the load pattern. The stiffness involved in the evaluation method independent of load is not the effective stiffness of the structure resisting the external loads. For example, the practical importance of a redundant component with larger stiffness is very small because it has no contribution to load transfer, but the corresponding important coefficient calculated by above method may be larger than the value of the component with the great contribution to the load transfer but the stiffness is less.

In order to deeply illustrate the significance of importance coefficient and previous discussion, two simple systems are shown in Figure 1. In the parallel connection system, if the stiffness of component 1 is larger, the corresponding importance coefficient is larger because the sharing force of this component is large and the consequence is more serious if it is removed. In contrast, if either component is removed in the series connection system, the structure will be invalid; thus, the importance coefficients of these two components are the same though the stiffness is different. Hence, both

the structural characteristics and the load pattern should be considered in the importance evaluation for structural health monitoring, especially for the optimal sensor placement.

The applications of vulnerability and importance coefficient in health monitoring and sensor placement are necessary. The damage capacity of current damage detection methods is preferable for the structure with medium damage, but the results of minor damage detection are not satisfactory. For spatial structure, the minor damage of the components not only generates the potential safety hazard and reduces the structural service life, but also is directly harmful to the whole structure and weakens the normal operational performance. With the development of the damage, it will finally bring about serious damage on the structure and causes heavy casualties and property losses. Therefore, it is very necessary to study the consequence and the transfer path of minor damage by combining the random properties of the structure and the load pattern based on the structural vulnerability analysis. The study on searching possible damage path and installing various sensors on the important components becomes the first task to be solved.

4. Calculation of Component Importance Coefficient

To carry out vulnerability analysis and importance coefficient calculation, the quantifiable index must be presented. In view of the previous conclusion about the importance coefficient, the concept of the generalized equivalent stiffness is introduced into the calculation method of the importance coefficient of component. The relationship of the joint deformation Δ in a specified direction and the force F which is applied on the joint in the corresponding direction is given by

$$F = k\Delta, \tag{1}$$

where k is the equivalent stiffness of the component in the specified direction. If the structure is merely subjected to the load F, the components that participate in transferring the force, that is, contribute to the equivalent stiffness k, have importance, whereas the component, which has no contribution to the equivalent stiffness k, does not influence the state of the structural bearing capacity even if it is removed, so it is not important.

The above conclusion can be generalized to the larger and complex structure which is usually subjected to multiple loads. When the load distribution on the structure is determined, the joint deformation distribution can be also determined.

Assuming the resultant force acting on the joint i is F_i, the component force of F_i in the direction l is F_{li}, and the deformation of the joint i in the direction l is d_{li}. Then the structural equivalent stiffness of the joint i in the direction l is defined as

$$K_{li} = \frac{F_{li}}{d_{li}}. \tag{2}$$

On this basis, the generalized equivalent stiffness of the structure corresponding to the load pattern in the direction l is given by

$$K_{lg} = \sum_{i=1}^{n} K_{li} = \sum_{i=1}^{n} \frac{F_{li}}{d_{li}}, \tag{3}$$

where n is the number of joints.

The resultant forces acting on each joint can be generally decomposed into three principal axis directions including x, y, and z, and the generalized equivalent stiffness of the structure with respect to a given load pattern in three principal axis is K_{zg}, K_{yg}, and K_{xg}, respectively. The actual structure generalized equivalent stiffness is the vector sum of the generalized equivalent stiffness in all directions and is expressed as follows:

$$K_g = \sqrt{K_{xg}^2 + K_{yg}^2 + K_{zg}^2}. \tag{4}$$

The generalized equivalent stiffness of the structure with respect to a specified load pattern is a whole physical quantity which indicates the deformation capacity of the overall structure resisting a given load, which is related to the load distribution pattern on the structure and the structural stiffness. Under the action of a given load, the contribution of one component to the actual structural equivalent generalized stiffness evidently represents the significance in the force system; it also represents the importance of the components of the whole structure.

Therefore, the loss rate of the actual structural equivalent generalized stiffness caused by the component damage is used as an importance evaluation index of the structure system. The index is defined as the importance coefficient I, and the expression is

$$I = \frac{\left(K_{gp} - K_{gd}\right)}{K_{gp}}, \tag{5}$$

where K_{gp} is the initial generalized equivalent stiffness of the intact structure; K_{gd} is the generalized equivalent stiffness of the damaged structure. The component importance coefficient I is a constant range between 0 and 1. When I is 0, it indicates that the designated component has no effect on the actual generalized equivalent stiffness of the structure and does not contribute to the transmission path in the structural system; that is, this component is not important in the structure. If I is 1, it is evident that the component is very important, and once the failure occurs, the structure will not be able to sustain the given load.

In order to further explain the method and calculation process of the component importance coefficient based on the above formulae, a simple truss with 7 members is selected for analysis. The structural composition, the number of the joints, and the members are shown in Figure 2. All the members in the structure are the same, and the length is 3.0 m, the outer diameter of the member is 102 mm, and the wall thickness is 4 mm. Both the Compressive stiffness and the tensile stiffness are 460 kN/m.

Table 1: Importance coefficients of each member for different load patterns.

Importance coefficients	Member number						
	1	2	3	4	5	6	7
I_{A2}	0.018	0.018	0.079	0.000	0.000	0.079	0.023
I_{A5}	0.079	0.079	0.251	0.000	0.000	0.251	0.095
I_{B2}	0.023	0.023	0.072	0.011	0.011	0.072	0.044
I_{B5}	0.069	0.069	0.222	0.014	0.014	0.222	0.1417

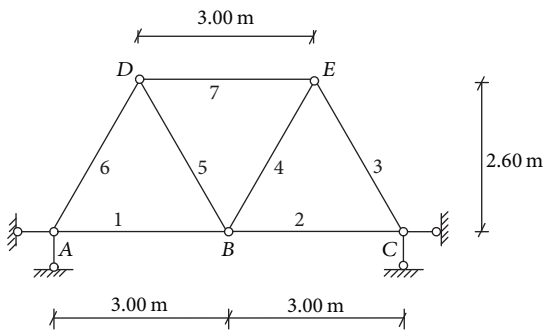


Figure 2: Illustration of importance coefficient in different structures.

The importance coefficient of each member under different static loading modes is calculated. In this study, two load patterns are considered; that is, load pattern A and load pattern B. For load pattern A, 10 KN vertical force is applied on joints D and E, respectively. For load pattern B, 20 KN vertical force is applied on joint B. It is assumed that each member has two types of degree of damage, so that the importance coefficient of each member is calculated according to (2), (3), and (5), and the calculation results are shown in Table 1.

Specifically, I_{A2} is the importance coefficient of the designated element when the load pattern A is applied and the damage degree of the corresponding element is 20% and other elements are intact. I_{A5} is the importance coefficient of the designated element when the load pattern A is applied and the damage degree of the corresponding element is 50% and other elements are intact. I_{B2} is the importance coefficient of the designated element when the load pattern B is applied and the damage degree of the corresponding element is 20% and other elements are intact. I_{B5} is the importance coefficient of the designated element when the load pattern B is applied and the damage degree of the corresponding element is 50% and other elements are intact.

It can be seen from the above results that the importance coefficient of certain member is different for different loading modes; a variety of possible load modes should be considered in order to calculate the importance of the members in a comprehensive way. In addition, for any damage degree and loading mode, the sequence of the importance of the member is given as members 3 and 6, member 7, members 1 and 2, and members 4 and 5, although the values are varied. Priority should be given to the strengthening and monitoring of the most important components.

5. Sensor Placement considering Importance Coefficient and Randomness

For spatial structures subjected to static loads, once the most important component damages, the stress distribution and the transfer path of the whole structure will change due to the change of the stiffness distribution. Therefore, the importance coefficient of each component will vary, and the maximum importance coefficient of all the remaining components can be obtained by recalculating the importance coefficient of each component. Following this set pattern, the multiple components with maximum importance coefficient can be gradually identified.

If the structure damages in the sequence of the permutation of the multiple components with maximum importance coefficient, the hazard of the whole structure is most severe. The damage of the multiple most important components will lead to much more loss of the generalized equivalent stiffness under given load condition. Thus, it causes a large deformation on the structure and deterioration of the normal performance.

From the perspective of the transfer path, the emergency capacity of the structure also greatly decreases, which will lead to structure destruction at top speed with the damage evolution and even causes the collapse. Hence, if the top important components are obtained by vulnerability analysis and the sensors are installed on corresponding joints to collect static response signals, the weak parts of the structure can be effectively monitored and the structural safety and reliability can be ensured.

For the minor damage detection and normal health monitoring, it is adequate to calculate the top three important members of the structure in the vulnerability analysis.

In addition, when the structure is subjected to ground motion or wind load, the applied forces on the structure involve the inertia force, gravity, and other dynamic actions at any moment in the dynamic process. Thus, the multiple most important members at any time step can be obtained according to the above method. Once the important components damage, the distribution of dynamic action will rearrange and the dynamic response will change obviously. Therefore, in order to evaluate the structural capacity to resist dynamic action, an appropriate time interval for vulnerability calculation should be determined and the time-varying responses of the structure in a dynamic process are extracted so as to calculate the importance coefficients of component. At last, the important members of the structure in a dynamic hazard are obtained and the static-dynamic parameters of these

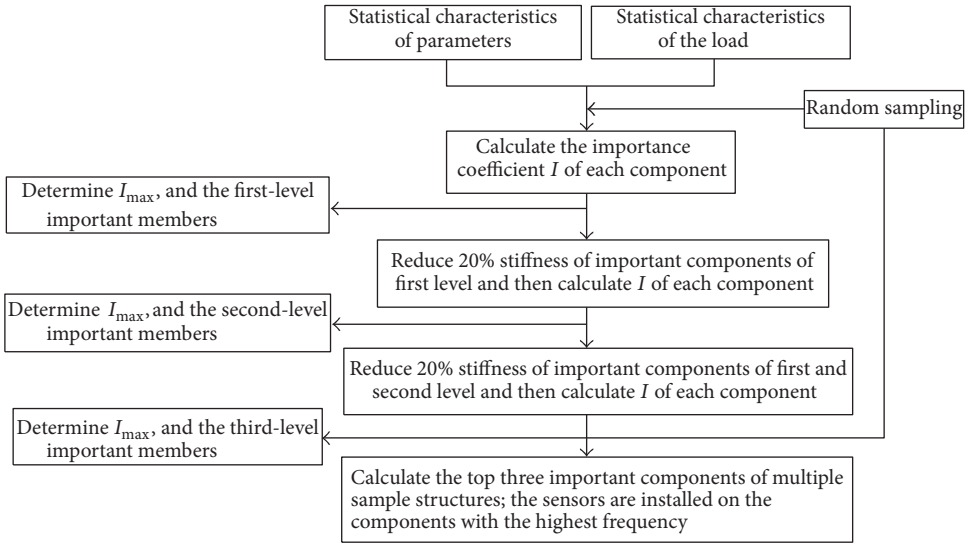


Figure 3: Flow chart of sensor placement considering static load.

members are monitored by statistical time-history analysis results.

The important coefficient calculation method for the deterministic structure and the specified load pattern is described above. For random cases, it is necessary to study a large number of sample conditions by considering the random characteristics of load and structure parameters. The most important components of each sample are obtained, respectively, and then the important members of all samples are counted. The components with the highest frequency are determined as the important components; then the sensors are installed on the components or the adjacent joint and the static-dynamic parameters are monitored.

In summary, the calculation flow diagram of the optimal sensor placement considering the importance coefficient and the randomness of structural parameters and static or dynamic load is shown in Figures 3 and 4, respectively.

6. Example Analysis

To verify the proposed method of optimal sensor placement considering importance coefficient and randomness, a single-layer spherical rib-circle latticed dome structure is selected as an analysis example, and the structural constitution is shown in Figures 5 and 6. The span of the structure is 40 m, the rise-span ratio is 1/4, and the material is normal carbon constructional steel with yield strength 2.35×10^8 N/m^2. For all the steel components, the external diameter is 200 mm and the internal diameter is 20 mm. The solid sphere with 160 mm diameter is used as the joint of the structure and welded with corresponding components. The assigned dead load to each joint on the surface is calculated as 15 kN/m^2.

The importance coefficient of the components when the structure is subjected to the static load is calculated. The normal uniform load and the snow load are involved, and the possible load patterns include the following four types: uniform load (load pattern I), combination of the uniform load and half edge uniform load (load pattern II), combination of the whole uniform load and 1/4 area uniform load (load pattern III), and the combination of the whole uniform load and the apex load (load pattern IV). According to the study [30], when randomness is considered, the statistical characteristics of the structure and load parameters are shown in Table 2, where L is the span.

The size of the random sample is determined according to the statistical characteristics and the amount of calculation. 100 random samples are simulated for each load pattern, and these loads are applied to 100 structures with random geometric parameters, respectively. Thus, 100 random models considering both the load and structural parameters are established. The first three important components of each model are calculated by finite element software ANSYS and numerical calculation software MATLAB. In the calculation of the importance coefficient of each component, the stiffness decreased by 20% to achieve the minor damage.

The initial displacements of joint 12 subjected to each type of load patterns in different random structures are shown in Figure 7; it is evident that the value has obvious randomness and discreteness. The importance coefficients I of the first level of element 51 near joint 12 in random structures are shown in Figure 8, and the results show the importance coefficient also has obvious randomness.

The importance coefficients of the structure under static load are calculated, and the vulnerability is evaluated based on different load patterns and statistical characteristics.

When the structure is subjected to a uniform load, as shown in Figure 9, the importance coefficients of different levels are calculated and analyzed. The frequency numbers of importance coefficients in the first three levels are shown in Figures 10(a)–10(c), respectively. The total frequency number of importance coefficients is shown in Figure 10(d). The numberings of the components whose frequency number is small are not listed. It is found that the probability that the rib

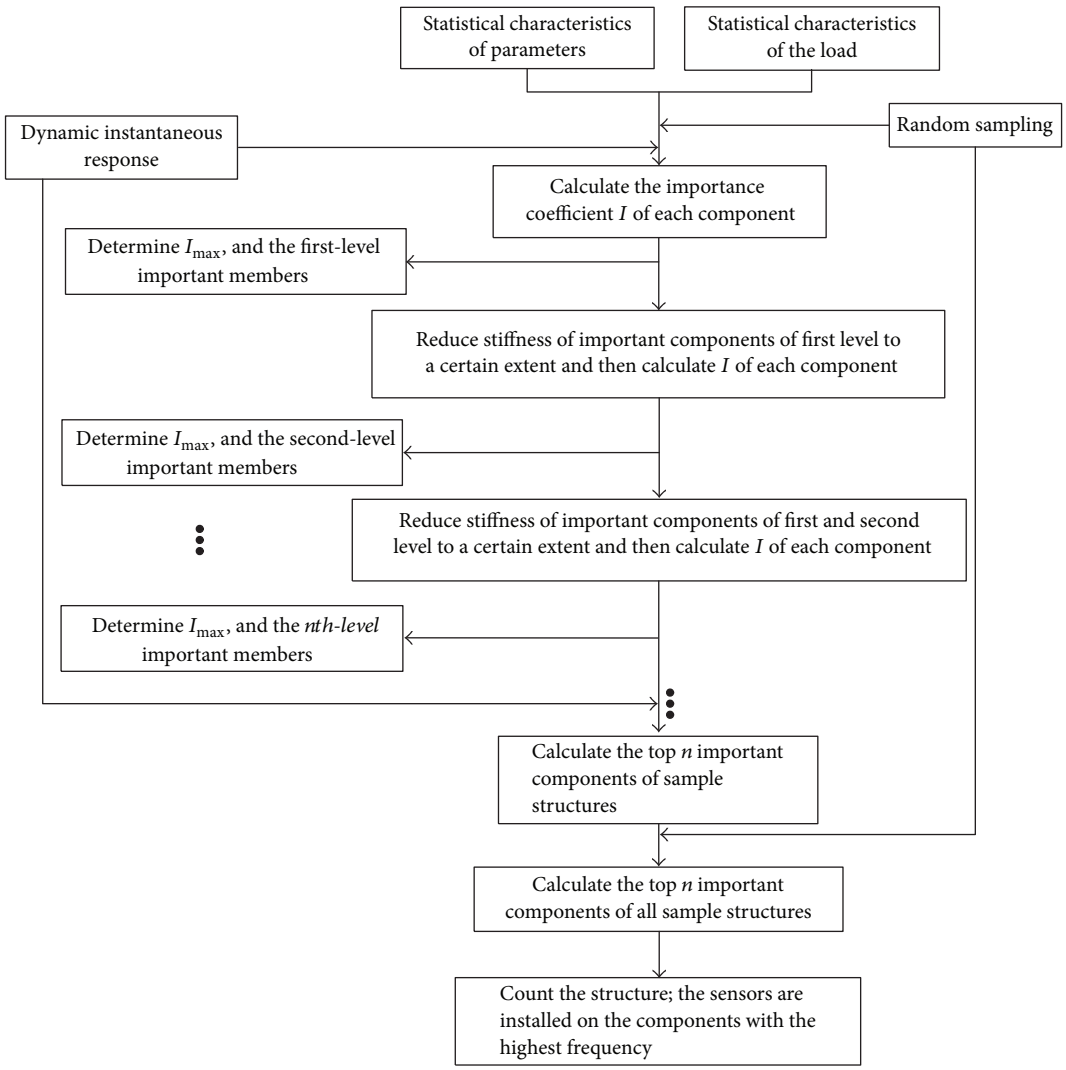


Figure 4: Flow chart of sensor placement considering dynamic load.

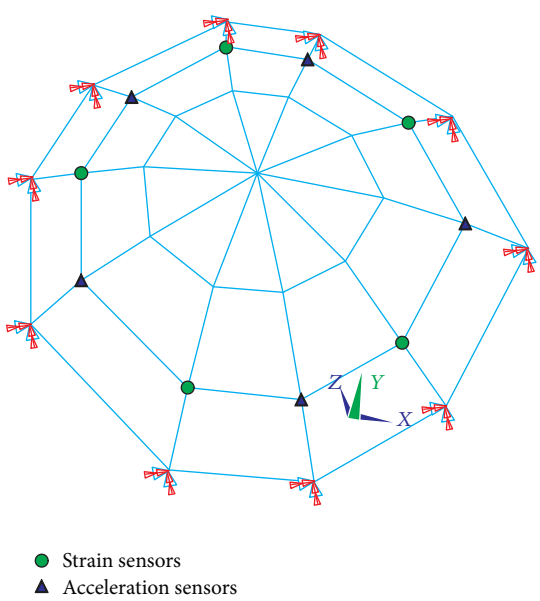
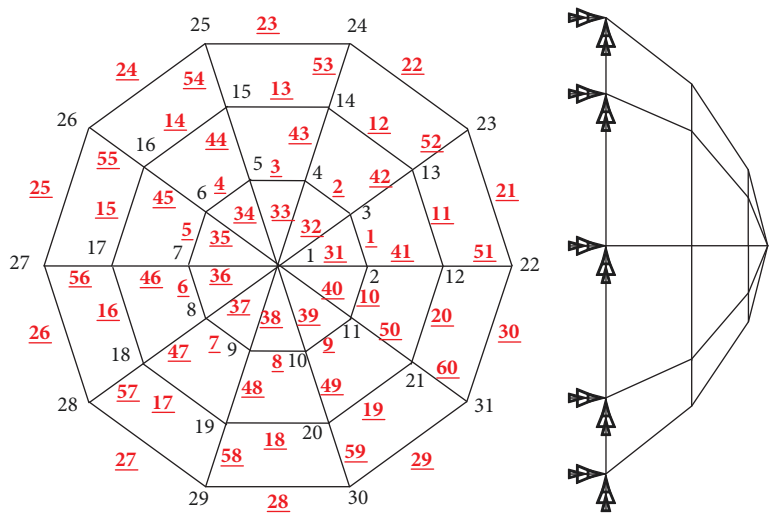
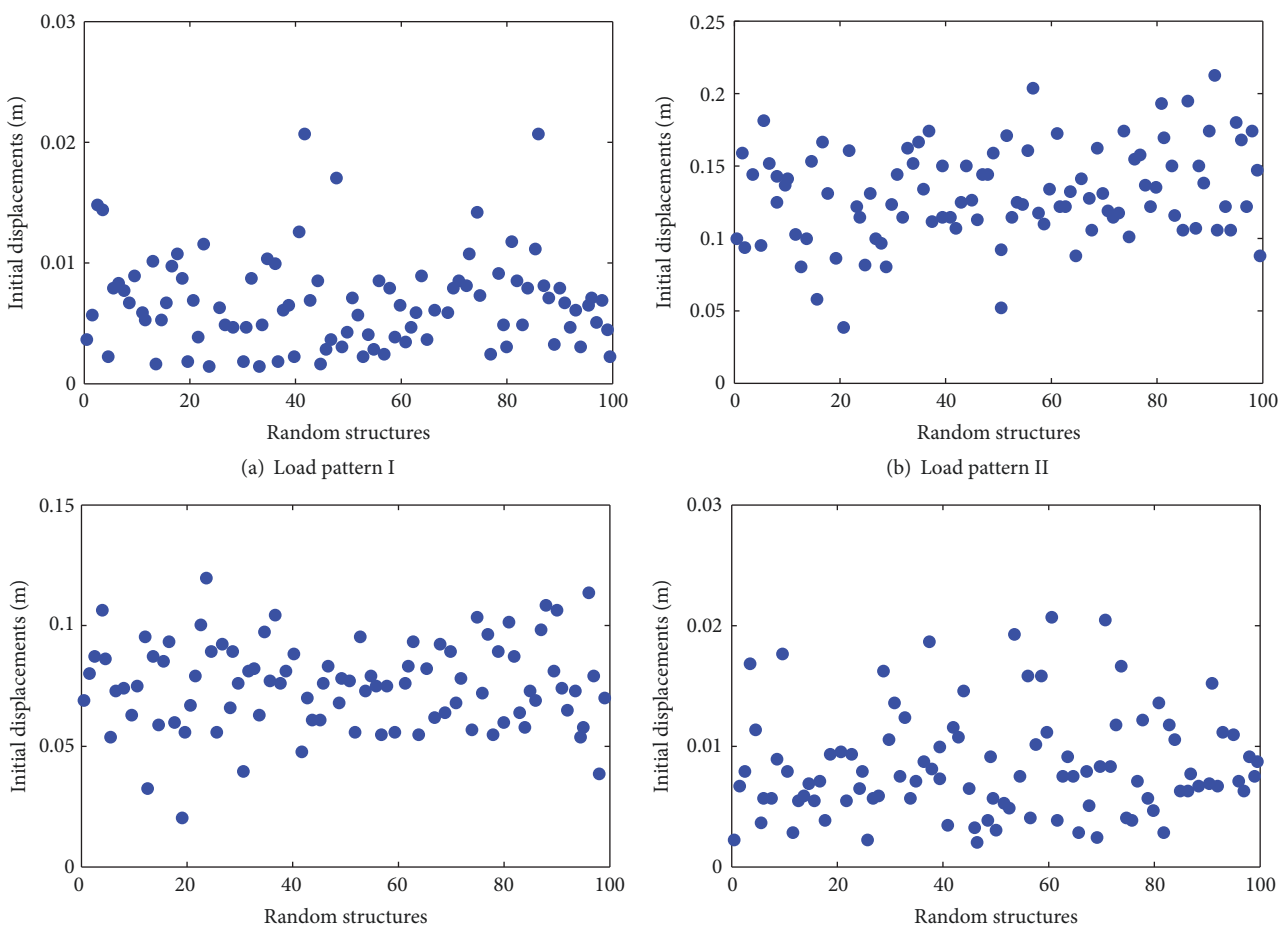


Figure 5: Reticulated dome structure and the sensors layout.



(a) (b)

Figure 6: Number of joints and components in dome.



(a) Load pattern I (b) Load pattern II

(c) Load pattern III (d) Load pattern IV

Figure 7: Random initial displacement of joint 12.

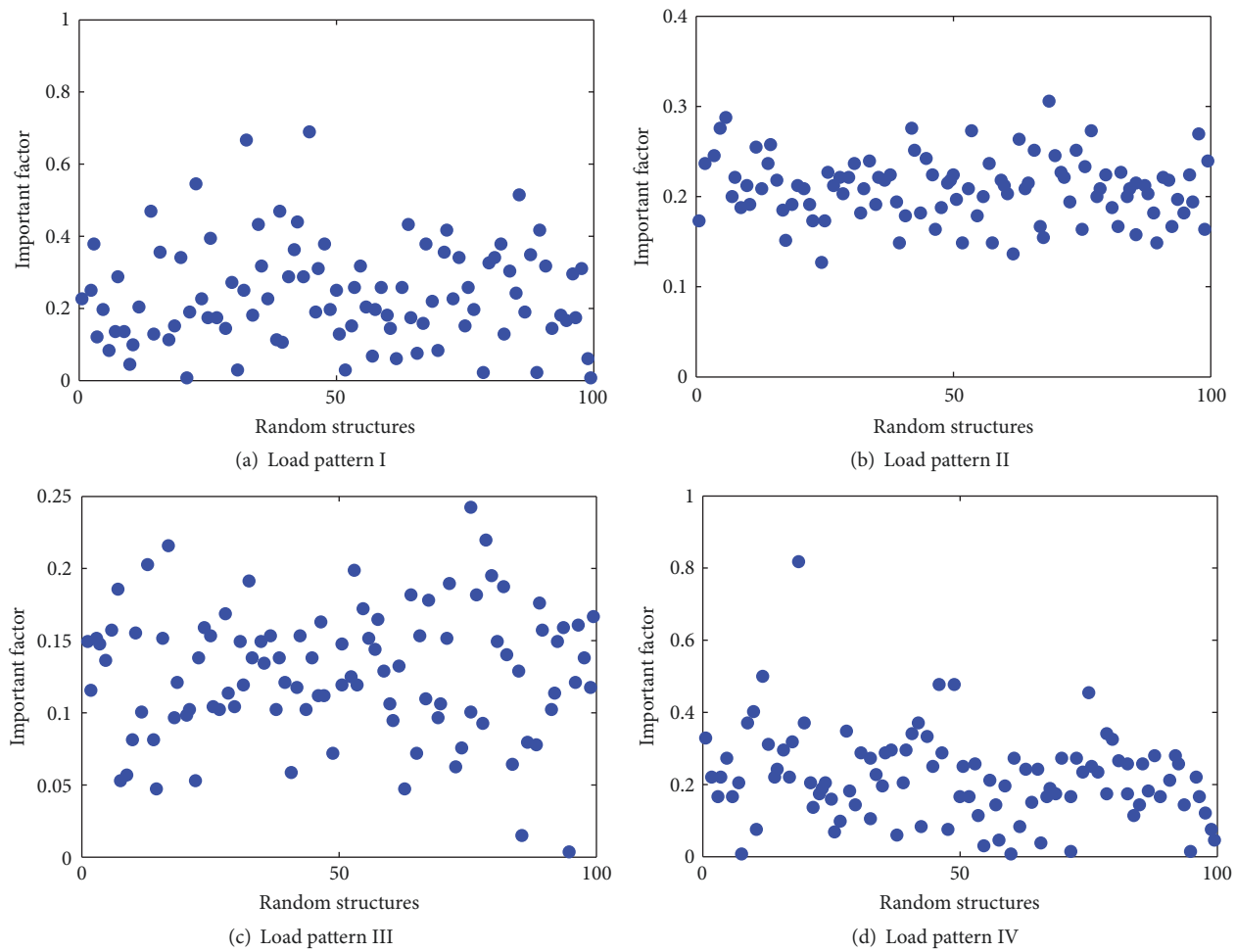


Figure 8: The first important factor of component 51.

elements in the base circle become important components is largest when the structure is subjected to uniform load.

Similar analysis is carried out according to the above method, the cumulated frequency where the main components become important components under various loads is shown in Figure 11, and the numberings of the components whose frequency number is less than 2 are not listed.

When the structure is subjected to the uniform load or the combination of the whole uniform load and the top load, the probability that the rib elements in the base circle become the important components is largest. When the structure is subjected to the combination of the uniform load and half or 1/4 uniform load, the rib elements in the base circle and the adjacent vertical elements especially at the junction of two kinds of uniform loads are the most important components.

Hence, the most important components range in element 51 to element 60 in most cases, that is, the rib elements at the base circle. Therefore, static strain sensors should be placed on these elements or corresponding joints. The definitive optimal placement scheme of static sensors is shown in Figure 3.

In what follows, the importance coefficients of the structure under dynamic action are analyzed and discussed. In

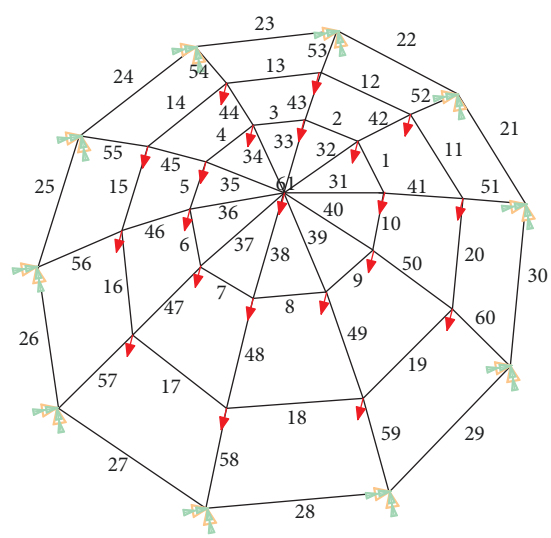
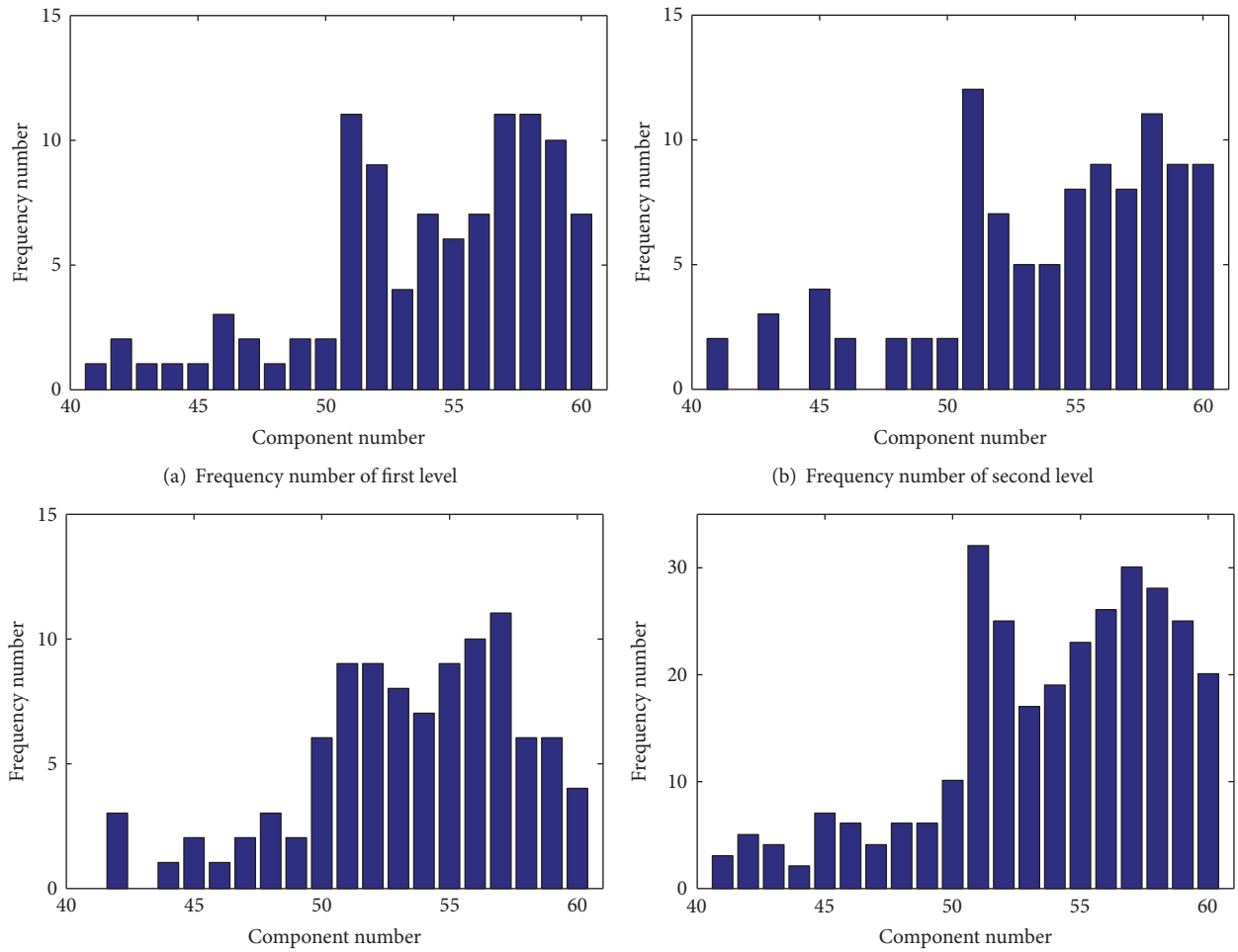


Figure 9: Uniform load pattern.

earthquake, the apparent deformation will occur in the shell structure subjected to the inertia force, and the importance coefficient of the components may vary in a short time. Thus,

Table 2: Statistical parameter for sample structure.

Parameter	Probability distribution	Mean	Standard deviation
Elastic modulus	Gaussian distribution	2.10×10^{11} N/m^2	6.18×10^9 N/m^2
Pipe diameter	Gaussian distribution	0.2 m	0.0002 m
Concentrated force on top	Gaussian distribution	50 KN	0.5 KN
Uniform load on joint	Gaussian distribution	10 KN	0.1 KN
Initial imperfections of position	Gaussian distribution	$(L/1000 + L/300)/2 = 0.087$ m	0.047 m



(a) Frequency number of first level

(b) Frequency number of second level

(c) Frequency number of third level

(d) Frequency number of first three levels

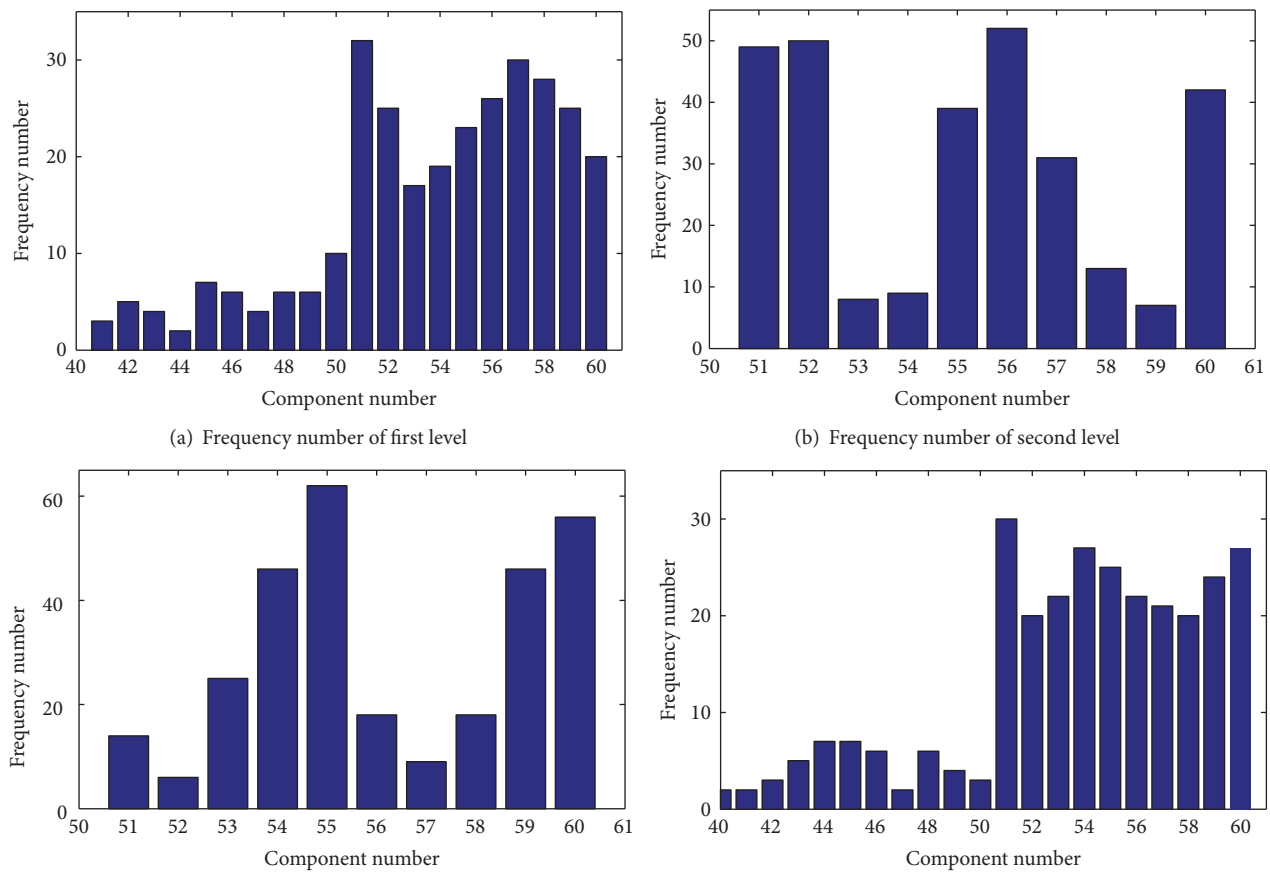
Figure 10: Frequency number under uniform load pattern.

the dynamic time-history analysis and adequate research are needed.

According to the site type, El Centro waves, Taft waves, Tianjin waves, and Kobe waves are used in dynamic analysis. Each earthquake wave comprises two horizontal seismic waves and one vertical seismic wave, the time interval is 0.02 s, the selected duration is 4.8 s, and the waveforms contain the maximum amplitudes.

In order to simplify the calculation, the dynamic displacement and the generalized equivalent stiffness of the structure are calculated by the time interval 0.2 s, which is equivalent to select 25 instantaneous states of the structure and calculate the statistical results from each earthquake wave.

The random parameters and their statistical distributions are in Table 1. For each group of earthquake waves, 30 random structure models are established. Therefore, the importance coefficients of 750 elements are calculated. According to the process in Figure 4, the probability values of the important coefficient of all elements are obtained by statistical calculation. Taking component 55 as an example, the importance coefficient results in different dynamic actions which are shown in Figure 12.



(a) Frequency number of first level

(b) Frequency number of second level

(c) Frequency number of third level

(d) Frequency number of first three levels

Figure 11: Accumulated frequency of members under various loads.

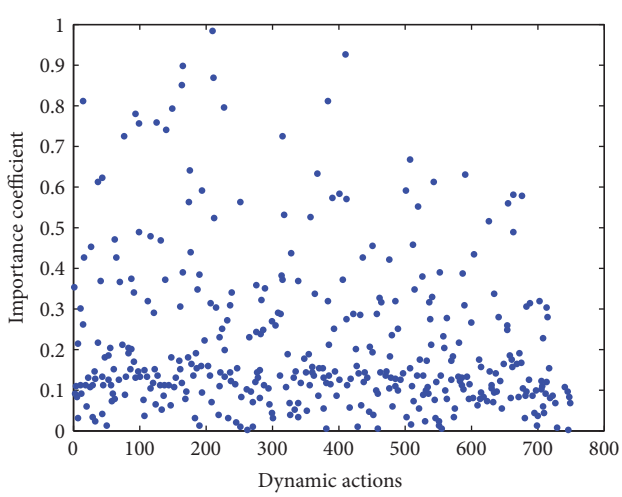


Figure 12: Importance coefficient of member 51 in various dynamic cases.

As the earthquake has great uncertainty, the elements of the similar location can be treated equally for the spatial structure with symmetry in evaluating the importance coefficient of each member.

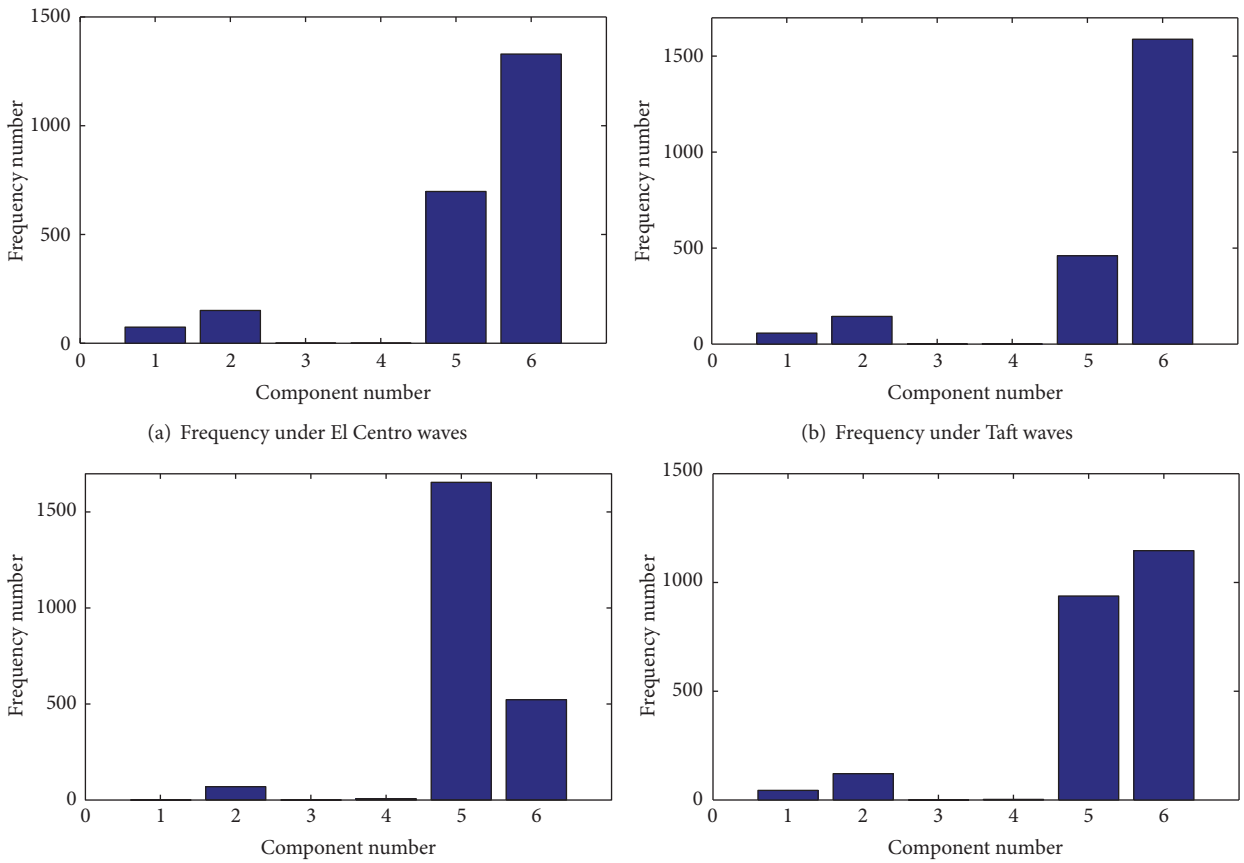
In this case, according to the locations, the elements on the first circle (elements 1–10) are classified as type 1, the elements on the second circle (elements 11–20) are classified as type 2, and the elements on the third circle (elements 21–30) are recorded as type 3.

The rib elements of the first circle (elements 31–40) are classified as type 4, the rib elements of the second circle (elements 41–50) are classified as type 5, and the rib elements of the third circle (elements 51–60) are classified as type 6. The accumulated frequency and the mean value of these six types of elements becoming the important members in three different earthquakes are shown in Figure 13.

From the above results, it can be seen that although the earthquake effect is random and the distribution of the importance coefficient of the components is slightly different in various earthquakes, the most important elements mainly belong to type 6 and type 5, that is, the rib elements of the second and third circle.

The final placement scheme of acceleration sensors is shown in Figure 4 according to the statistical calculation results and the experience of sensors placement.

It is worth noting that the optimal sensor locations according to modal assurance criteria method or energy method are generally at the top of the joints, which are



(a) Frequency under El Centro waves

(b) Frequency under Taft waves

(c) Frequency under Tianjin waves

(d) Frequency under Kobe waves

Figure 13: Frequency of importance coefficient of 6 types of components.

different from the results based on vulnerability method. Hence, the final placement scheme can be determined by sufficiently considering the results by various methods.

7. Conclusion

In the premise of considering randomness of the actual structure parameters and load patterns, the optimal sensor placement scheme based on the vulnerability is the effective application of the vulnerability theory in the field of health monitoring. The evaluation of the importance coefficient according to the change of the global generalized equivalent stiffness of the structure is a reasonable improvement and development for the vulnerability theory.

In addition, in order to meet the practical needs of health monitoring, the effect of components failure is replaced by the minor damage in the components in structural vulnerability analysis, and the most vulnerable path and important components involving the random characteristics of the structure and load pattern are intensely studied.

In general, the weak parts of the spatial structure in the actual operational environment can be effectively monitored by the sensors placement method proposed in this paper, and the identification capacity for initial damage stage is especially excellent. The corresponding method has significant application value in the field of health monitoring.

Different from the traditional method of optimal sensor placement based on modal theory or information theory, the optimal sensor placement based on vulnerability and importance coefficient is emphasized in searching the weak part firstly and to deduce the damage path, and it also can fully consider the random characteristics, which is different from the traditional methods. Although the results from different methods can vary slightly, they can absorb advantages from each other and ultimately achieve the goal of multiobjective optimization.

In the future, intensive studies can be carried out on reducing the number of simulated random samples effectively, exploring the mechanism of damage evolution, and fully combining with other sensors placement methods.

Conflicts of Interest

The authors declare that there are no conflicts of interest regarding the publication of this paper.

Acknowledgments

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