Research Article

Instantaneous Modal Parameter Identification of Linear Time-Varying Systems Based on Chirplet Adaptive Decomposition

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Instantaneous modal parameter identification of time-varying dynamic systems is a useful but challenging task, especially in the identification of damping ratio. This paper presents a method for modal parameter identification of linear time-varying systems by combining adaptive time-frequency decomposition and signal energy analysis. In this framework, the adaptive linear chirplet transform is applied in time-frequency analysis of acceleration response for its higher energy concentration, and the response of each mode can be adaptively decomposed via an adaptive Kalman filter. Then, the damping ratio of the time-varying systems is identified based on energy analysis of component response signal. The proposed method can not only improve the accuracy of instantaneous frequency extraction but also ensure the antinoise ability in identifying the damping ratio. The efficiency of the method is first verified through a numerical simulation of a three-degree-of-freedom time-varying structure. Then, the method is demonstrated by comparing with the traditional wavelet and time-domain peak method. The identified results illustrate that the proposed method can obtain more accurate modal parameters in low signal-to-noise ratio (SNR) scenarios.

1. Introduction

System identification techniques have received significant attention in civil, aerospace, and mechanical engineering in the past few decades [1]. Most of them assume that the analyzed systems are linear time-invariant (LTI), i.e., the output for such systems does not change with a delay in the input. However, in reality, this assumption is not valid for many engineering structures which are damaged or subjected to strong excitation. Thus conventional modal analysis methods and experimental techniques are not appropriate for these linear time-varying (LTV) systems [2]. In such cases, for the purpose of health monitoring or damage identification, it is necessary to develop reliable approaches for the identification of time-varying characteristics.

In terms of LTV system identification, many methods have been developed for various time-varying cases. These methods generally belong to two categories: time-domain methods and time-frequency-domain methods [3].

In the time domain, identification methods based on the time-varying model such as state-space model and time series model have become the main research objects since the time-domain description of LTV systems is very convenient. Liu [4] proposed a subspace-based identification technique and used eigenvalues of the estimated transition matrices to characterize the dynamic properties. The subspace is constructed by the singular value decomposition (SVD) which costs too much computational time. Authors in [5–7] further presented improved algorithm to overcome such drawback. Pang et al. [7] utilized the projection approximation subspace tracking (PAST) technique instead of the SVD. Zhou et al. [8] presented a postprocess method to improve the quality of identification results in low signal-to-noise ratio scenarios. Meanwhile, parameter identification is established based on the autoregressive-moving-average...
(ARMA) model which can match the structures of real-world processes. In order to process nonstationary signals and identify the parameters of time-varying structures, many approaches were also proposed based on the time-dependent autoregressive moving average (TARMA) model [9]. The TARMA model can be utilized as an efficient tool to reveal the dynamic characteristics of nonstationary signals due to its simplicity and effectiveness. However, a continuing challenge is how to establish a precise TARMA model. Typically, the recursive least-squares (RLS) approach and the Kalman filter [10] were applied to solve the problem. Recently, Spiridonakos and Fassois [11] presented an adaptable functional series TARMA model for nonstationary signal modelling, which employs the B-spline function to model the TARMA coefficients adaptively and structurally. Yang et al. [12] developed a moving Kriging technique for identification accuracy of instantaneous modal parameters of time-varying structures. Dziedziech et al. [18] used the wavelet-based frequency response function (FRF) for modal identification of time-variant systems. An identification approach based on the empirical wavelet transform (EWT) was developed for LTV systems by Xin et al. [19].

As seen from the above analysis, conventional TFA methods are suitable for processing such nonstationary signals of LTV systems. However, they are insufficient to provide a TF representation with high resolution due to their inherent restrictions. HT can process multicomponent signals only when combined with EMD which always has the shortage of modal aliasing and end swing phenomena. For short-time TFA tools like WT [20, 21], they are established on the assumption of the considered signal being piecewise stationary in a short time. This means that they essentially use horizontal lines to approximate the IF feature. To improve the performance of the TFA methods, many effective methods have been proposed. One relatively effective strategy named synchrosqueezing transform (SST) [22] is to squeeze the TF coefficients into the instantaneous frequency trajectory. However, the strongly modulated frequency component and noise could destroy the original TF representation of the signal heavily. Another strategy named parameterized TFA was inspired by the fact that using adaptive window function (such as adding chirp rate in base function) in WT can achieve a much higher TF resolution. Thus the chirplet transform (CT) was designed by Deng et al. [23]. By using an extra parameter, chirp rate, the CT is able to create a concentrated TF representation for the linear modulated signal. In order to search the best chirp rate according to the TF signature of nonstationary signal, Yu and Zhou [24] developed an adaptive algorithm named GLCT. In terms of the TF characteristics of vibration response, the responses of LTV systems are usually characterized by multicomponent amplitude-modulation (AM) and frequency-modulation (FM) signals. Therefore, the CT is particularly suitable for identifying the instantaneous frequency of time-varying signals. In addition, for nonstationary signals with noise, the Vold–Kalman (VK) filter can more effectively decompose the multicomponent signals [25] compared with EMD.

This paper presents a time-frequency decomposition method to extract the individual modes from the free response signal of LTV systems. The IF is extracted based on GLCT as the complex carrier wave of the signal, and then, the VK filter is constructed to perform signal decomposition adaptively. Using this method, the IF of each time-varying component can be identified more accurately, and the amplitude envelope of each signal component can be also obtained. Thus many damping ratio identification methods based on amplitude envelope were proposed such as the time-domain method [26, 27]. The identification of damping ratio is of great value for many engineering applications such as flutter analysis of aircraft. But this task is of challenges since the amplitude envelope is easily disturbed by noise which leads to a much lower identification accuracy of damping ratio than frequency. Considering this shortcoming, the improvement based on energy analysis [28] is explored to make the identification method insensitive to noise. The damping ratio is identified by energy ratio of adjacent half-periodic signals, which can effectively eliminate the effect of noise on amplitude. The aim of this paper is to improve the identification accuracy of instantaneous modal parameters, i.e., instantaneous frequencies and damping ratios.

With this prime objective, this paper is organized as follows: the theory of time-frequency decomposition based on GLCT and VK filter is briefly introduced in Section 2. The energy analysis for damping ratio identification of LTV systems is presented in Section 3. Section 4 shows a series of comparative numerical simulation examples to illustrate the performance of identification. In these comparative examples, the Morlet wavelet and time-domain peak method are selected for comparison to identify instantaneous frequencies and damping ratios, respectively. Finally, Section 5 provides the discussion and conclusions on the identification results.

2. Theoretical Background

Using the adaptive TF method for vibration signal decomposition consists of two main steps: (1) performing TFA
of the target vibration signals by using GLCT to extract the TF ridge of each time-varying component and (2) constructing the filtering band based on the extracted TF ridge, then decomposing the target vibration signals into multicomponent AM and FM signals to obtain the amplitude envelope. To apply this TF method for LTV systems, we can identify the instantaneous frequencies based on the TF ridge and identify the damping ratio based on the amplitude envelope, respectively.

2.1. General Linear Chirplet Transform (GLCT). GLCT is an approach to obtain the best demodulated operator for signals with time-varying IF, which has better performance than any individual CT in generating TF representation. According to the theory of Ville, an analytic signal with time-varying IF can be written as

\[ s(t) = a(t)e^{j\int \varphi(t) dt}, \]  

where \( a(t) \) is the amplitude and \( \varphi(t) \) is the IF of the signal.

In a short time \( \tau \), \( \varphi(t) \) can be regarded as a linear equation approximately, which can be expanded by the Taylor expansion:

\[ \varphi(\tau) = \varphi(t) + \varphi'(t)(\tau - t), \]  

where \( \varphi(t) \) is the IF at time \( t \) and \( \varphi'(t) \) is the first-order derivative of IF at time \( t \).

The CT of signal \( s(t) \) which considers the time-variant demodulated operator can be written as

\[ CT(t, \omega, c(t)) = \int_{-\infty}^{\infty} s(\tau) \cdot \omega(\tau - t) \cdot e^{j\omega t} \cdot e^{-j\omega(\tau-t)/2} d\tau, \]  

where \( \omega(t) \) is a window function and \( e^{-j\omega(\tau-t)/2} \) is the time-variant demodulated operator.

The CT amplitude in IF \( \varphi(t) \) \( (\omega = \varphi(t)) \) can be calculated by

\[ |CT(t, \omega, c(t))| = \int_{-\infty}^{\infty} \omega(\tau - t) \cdot a(\tau) \cdot e^{j\varphi(t)} \cdot e^{-j\varphi(t)(\tau-t)/2} \cdot e^{-j\varphi(t)t/2} \cdot e^{-j\varphi(t)t/2} d\tau \]  

\[ \leq \int_{-\infty}^{\infty} \omega(\tau - t) a(\tau) d\tau, \]  

From equation (4), it can be seen that, when \( c(t) \) in CT is consistent with \( \varphi'(t) \) of the analyzed signal, the CT amplitude will achieve the maximum, and then the better TF representation with high energy concentration will be obtained. But for multicomponent signals, the IF feature cannot be known in advance, which leads to that it is hard to determine the demodulated operator accurately.

A feasible solution is to use a series of discrete demodulated operators to approximate the best demodulated operator [24]. Actually the demodulated operator is also a rotating operator of signals on the TF result, and the rotating degree is \( \arctan(-c) \). So a parameter \( \alpha \) is introduced to characterize the parameter \( c \) as

\[ \alpha = \arctan \left( \frac{2T_s}{F_s c} \right), \quad \alpha \in \left( -\pi, -\pi \right), \]  

\[ \text{where} \quad T_s \quad \text{is the sampling time and} \quad F_s \quad \text{is the sampling frequency.} \]

In a discrete way, the parameter \( \alpha \) is divided into \( N_d \) values on the TF plane from \( -\pi/2 \) to \( \pi/2 \) as

\[ \alpha = \frac{\pi}{2} + \frac{\pi}{2N_d} + \frac{2\pi}{2N_d + 1}, \quad \frac{\pi}{2N_d + 1}, \ldots, \frac{\pi}{N_d} + \frac{\pi}{N_d + 1}. \]  

Then, equation (3) can be rewritten as

\[ CT(t, \omega, c(t)) = \int_{-\infty}^{\infty} s(\tau) \cdot \omega(\tau - t) \cdot e^{j\omega t} \cdot e^{-j\pi(\tau-t)/2} \cdot e^{-j\pi(\tau-t)^2/4} d\tau. \]  

CT amplitude in IF \( \varphi(t) \) \( (\omega = \varphi(t)) \) can be calculated by

\[ \text{GSpec}(t, \omega) = |CT(t, \omega, c_m)|^2, \]  

where \( c_m \) is a discrete value when the CT amplitude achieves the maximum in TF point \( (t, \omega) \), which can be calculated by

\[ c_m = \arg \max |CT(t, \omega, c(t))|. \]  

In order to demonstrate the better TF representation of GLCT, WT and GLCT are used for TF analysis of a time-varying signal, respectively. The results are shown in Figure 1. It is shown that GLCT can adaptively track the instantaneous frequency trajectory at different TF points. Compared with WT, GLCT uses adaptive window function instead of rectangular TF window to achieve a much higher TF resolution. Thus, this approach can more adaptively and accurately estimate IF than traditional TFA tools especially for signals with time-varying IF.

2.2. Vold–Kalman Filter. In terms of nonstationary signal decomposition, various interesting approaches have been developed such as EMD [13], empirical wavelet transform
(EWT) [19], and Kalman filter [29]. But considering a multicomponent signal with time-varying IF in low SNR scenarios, the VK filter is more adapted for decomposing this signal [30]. Since the VK filter needs an estimation of the IF firstly, a novel time-frequency decomposition method is presented when GLCT is applied for IF extraction. In this section, the decomposition process is expounded in detail.

Consider a multicomponent analytic signal composed of component signal in equation (1) with random noise which can be written as

\[ s(t) = \sum_{k=1}^{K} a_k(t)\Phi_k(t) + \mu(t) \]

\[ = \sum_{k=1}^{K} a_k(t)e^{j\int \phi_k(t)dt} + \mu(t) \quad (10) \]

where \( K \) is the number of component signals and \( \mu(t) \) is a zero mean white noise.

In a discrete way, the signal is rewritten as

\[ s(n) = \sum_{k=1}^{K} a_k(n)\Phi_k(n) + \mu(n), \quad i = 1, 2, \ldots, K, \quad (11) \]

where \( n = 1, 2, \ldots, N \) and \( N \) is the total number of sampling points.

By using GLCT in TFA of this multicomponent signal, the IF of each component can be estimated. Thus, the complex-valued carrier wave \( \Phi_k(n) \) can be constructed.

2.2.1. Structural Equation. It is assumed that the amplitude envelope \( a_k(t) \) is the low-frequency modulation of the complex carrier wave \( \Phi_k(t) \). In other words, envelope is locally approximated by a low-order polynomial, and it is filtered in some specific way. This condition can be expressed by a structural equation with the nonhomogeneity term \( \varepsilon_k(n) \). The polynomial order designates the number of the filter poles. The equations for \( l \)-pole filter are given by

\[ \nabla^l a_k(n) = \varepsilon_k(n). \quad (12) \]

Taking \( l = 2 \) as an example, the corresponding equation is given by

\[ \nabla^2 a_k(n) = a_k(n-1) - 2a_k(n) + a_k(n+1) = \varepsilon_k(n). \quad (13) \]

The system of the structural equations containing all the samples takes the following form, which can be rewritten in the matrix form:

\[ \begin{bmatrix} -2 & 1 & 0 & \cdots & 0 & 0 \\ 1 & -2 & 1 & \cdots & 0 & 0 \\ \vdots & \vdots & \ddots & \ddots & \vdots & \vdots \\ 0 & 0 & 0 & 1 & -2 \end{bmatrix} \begin{bmatrix} a_k(1) \\ a_k(2) \\ \vdots \\ a_k(N) \end{bmatrix} = \begin{bmatrix} \varepsilon_k(1) \\ \varepsilon_k(2) \\ \vdots \\ \varepsilon_k(N) \end{bmatrix}, \quad (14) \]

\[ \begin{bmatrix} A_1 \\ A_2 \\ \vdots \\ A_K \end{bmatrix} = \mathbf{U}, \quad (16) \]

where \( S = [W_1 \ W_2 \ W_3 \ \cdots \ W_K] \), \( \mathbf{W}_k = \begin{bmatrix} \Phi_k(1) & 0 & \cdots & 0 \\ 0 & \Phi_k(2) & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & \Phi_k(N) \end{bmatrix} \), \( \mathbf{A}_k = \begin{bmatrix} a_k(1) \\ a_k(2) \\ \vdots \\ a_k(N) \end{bmatrix}, \mathbf{U} = \begin{bmatrix} \mu(1) \\ \mu(2) \\ \vdots \\ \mu(N) \end{bmatrix} \), and \( N \) is the number of sampling points.

2.2.2. Data Equation. Taking the measurement noise into account, the measurement equation is written in the matrix form:

\[ \nabla^2 a_k(n) = a_k(n-1) - 2a_k(n) + a_k(n+1) = \varepsilon_k(n). \quad (13) \]

The system of the structural equations containing all the samples takes the following form, which can be rewritten in the matrix form:

\[ \begin{bmatrix} -2 & 1 & 0 & \cdots & 0 & 0 \\ 1 & -2 & 1 & \cdots & 0 & 0 \\ \vdots & \vdots & \ddots & \ddots & \vdots & \vdots \\ 0 & 0 & 0 & 1 & -2 \end{bmatrix} \begin{bmatrix} a_k(1) \\ a_k(2) \\ \vdots \\ a_k(N) \end{bmatrix} = \begin{bmatrix} \varepsilon_k(1) \\ \varepsilon_k(2) \\ \vdots \\ \varepsilon_k(N) \end{bmatrix}, \quad (14) \]

\[ \begin{bmatrix} A_1 \\ A_2 \\ \vdots \\ A_K \end{bmatrix} = \mathbf{U}, \quad (16) \]

where \( S = [W_1 \ W_2 \ W_3 \ \cdots \ W_K] \), \( \mathbf{W}_k = \begin{bmatrix} \Phi_k(1) & 0 & \cdots & 0 \\ 0 & \Phi_k(2) & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & \Phi_k(N) \end{bmatrix} \), \( \mathbf{A}_k = \begin{bmatrix} a_k(1) \\ a_k(2) \\ \vdots \\ a_k(N) \end{bmatrix}, \mathbf{U} = \begin{bmatrix} \mu(1) \\ \mu(2) \\ \vdots \\ \mu(N) \end{bmatrix} \), and \( N \) is the number of sampling points.
envelope \( a_k(t) \). The additional condition for the equation solution is that the variances of the nonhomogeneity terms \( \varepsilon_k(n) \) and noise \( \mu(n) \) have to be minimal while maintaining the given relationship between them.

The weighted sum of equations (15) and (16) gives the loss function:

\[
J = \sum_{k=1}^{r} \epsilon^T \epsilon + U^T U
\]

\[
= \sum_{k=1}^{r} A_k^H G^T G A_k + \left( S^T - \sum_{k=1}^{r} A_k^H W_k^H \right) \left( S - \sum_{k=1}^{r} W_k A_k \right),
\]

where \( r \) is a weighting factor. The choice of a large value for the weighting factor \( r \) leads to the highly selective filtration in the frequency domain. In contrast, fast convergence with low-frequency resolution is achieved by choosing the small \( r \) value. The limit value of \( r \) [30] is shown in Table 1.

The first derivative of equation (17) with respect to the vector \( A_k \) gives a condition for the minimum of this function, which is called a normal equation:

\[
\frac{\partial J}{\partial A_k} = 0.
\]

Thus the unknown envelope \( A_k \) results from equation (18).

### 3. Instantaneous Modal Parameter Identification of Linear Time-Varying Systems

#### 3.1. Modal Response of Time-Varying Structures

The motion equation of a \( p \)-DOF LTV structure is given as

\[
M(t)\ddot{x}(t) + C(t)\dot{x}(t) + K(t)x(t) = f(t),
\]

where \( M(t) \), \( C(t) \), and \( K(t) \) are time-varying mass, damping, and stiffness matrices, respectively; \( x(t) \) is the displacement response vector, \( \dot{x}(t) \) and \( \ddot{x}(t) \) are the velocity and acceleration response vectors, respectively; and \( f(t) \) is the excitation force vector on the structure.

Structural acceleration responses can be obtained as the superposition of \( P \) vibration modal responses [31], and when an impulse force is applied on the \( p \)-th dof, the acceleration response of the \( k \)-th generalized modal coordinate is given as

\[
\ddot{x}_k(t) = \sum_{p=1}^{P} A_{pk} e^{-\xi_k(t)\omega_k(t)t} \cos(\omega_k(t)t + \varphi_{pk}),
\]

where \( \omega_k(t) = \omega_{pk}(t) \sqrt{1 - \xi_k(t)^2} \) and \( \xi_k(t) \), \( \omega_{pk}(t) \), and \( \omega_k(t) \) denote the \( k \)-th modal damping ratio, modal frequency, and natural frequency, respectively. It can be further written as the superposition of multicomponent AM-FM signals.

The acceleration impulse response signal of the time-varying structure is similar to multicomponent harmonic signals of equation (10). It can be adaptively decomposed by using the TF decomposition method proposed in this paper, thereby reconstructing each component signal, i.e., \( A_k(t) \cos(\varphi_k(t)) \).

Then, the instantaneous modal parameters can be identified by using the following estimator:

\[
\begin{align*}
\omega_k(t) & = \frac{d\varphi_k(t)}{dt}, \\
\eta_k(t) & = \frac{d \ln(\omega_k(t))}{dt} - \frac{1}{\omega_k(t)}, \\
\xi_k(t) & = \sqrt{\eta_k(t)^2 + 1}.
\end{align*}
\]

#### 3.2. Energy Analysis Method for the Identification of Damping Ratio

As already noted, the higher precision of the identification of the IF can be obtained from the well-established TF representation, but for damping ratio identification based on amplitude envelope in equation (21), it has much lower identification accuracy since the amplitude envelope is easily disturbed by noise. In order to enhance the identification accuracy of the damping ratio, the estimator based on signal energy analysis for LTV systems is presented.

It can be assumed that the signal is time-invariant in a short time (that is, the frequency of the signal is not changed in a sampling period). For most signals without fast time-varying IF, this assumption can really be valid. Consider a time-varying signal as shown in Figure 2; the zero position of this signal is denoted as \( t_q \), and the IF is considered to be constant within a sampling period \( T_q \) around \( t_q \). In other words, the IF of the signal from time \( t_q - T_q/2 \) to time \( t_q + T_q/2 \) is denoted as \( \omega_{dk}(t_q) \). The signal energy of the first and second half sampling periods is calculated by \( E_{ku} \) and \( E_{kd} \), respectively. They are given by

\[
E_{ku} = \int_{t_q - T_q/2}^{t_q + T_q/2} \mathbf{x}_k^2(t) dt = A_{ok}^2 \int_{t_q - T_q/2}^{t_q + T_q/2} e^{-2\xi_k(t_q)\omega_{ok}(t_q)t} \cdot \cos^2(\omega_{dk}(t_q)t + \varphi_{ok}) dt,
\]

\[
E_{kd} = \int_{t_q - T_q/2}^{t_q + T_q/2} \mathbf{x}_k^2(t) dt = A_{ok}^2 \int_{t_q - T_q/2}^{t_q + T_q/2} e^{-2\xi_k(t_q)\omega_{ok}(t_q)t} \cdot \cos^2(\omega_{dk}(t_q)t + \varphi_{ok}) dt.
\]

According to \( \cos^2(\omega_{dk}(t_q)t + \varphi_{ok}) = 1 - \sin^2(\omega_{dk}(t_q)t + \varphi_{ok}) \), equation (22) can be rewritten as

\[
E_{ku} = A_{ok}^2 \int_{t_q - T_q/2}^{t_q + T_q/2} e^{-2\xi_k(t_q)\omega_{ok}(t_q)t} \cdot (1 - \sin^2(\omega_{dk}(t_q)t + \varphi_{ok})) dt.
\]
According to the inherent characteristics of the vibration system, the following conditions can be satisfied:

\[ \omega_{dk}(t_q) T_q = 2\pi, \]
\[ \omega_{dk}(t_q) = \omega_{0k}(t_q) \sqrt{1 - \xi_k^2(t_q)}. \]  

(26)

Then, \( \Gamma_2 \) can be rewritten as

\[ \Gamma_2 = \frac{e^{j2\varphi_{0k} \cdot \left[ -j2\omega_{dk}(t_q) - 2\xi_k(t_q)\omega_{0k}(t_q) \right]}}{4\omega_{0k}^2(t_q)} \cdot e^{\left( j2\omega_{dk}(t_q) - 2\xi_k(t_q)\omega_{0k}(t_q) \right) (t_q - (T_q/2))} \cdot \left[ e^{\left( j2\omega_{dk}(t_q) - 2\xi_k(t_q)\omega_{0k}(t_q) \right) (T_q/2)} - 1 \right] \]
\[ = \frac{-j2\omega_{dk}(t_q) - 2\xi_k(t_q)\omega_{0k}(t_q)}{4\omega_{0k}^2(t_q)} \cdot e^{j2\omega_{0k}(t_q)(t_q - (T_q/2)) + \varphi_{0k}} \cdot e^{-2\xi_k(t_q)\omega_{0k}(t_q)(t_q - (T_q/2))} \cdot e^{-2\pi\xi_k(t_q)\sqrt{1 - \xi_k^2(t_q)} - 1}. \]  

(27) 

In the same way, equation (23) can be rewritten as
\[
E_{qd} = \frac{A_{nk}^2}{2} \int_{t_q}^{t_q} (T_{q/2}) e^{-2\xi_k(t_q)\omega_{nk}(t_q)} \, dt \\
+ \frac{A_{nk}^2}{2} \int_{t_q}^{t_q} (T_{q/2}) e^{-2\xi_k(t_q)\omega_{nk}(t_q)} \cdot \cos 2(\omega_{nk}(t_q)t + \varphi_{nk}) \, dt \\
= \frac{A_{nk}^2}{2} \times B_3 + \frac{A_{nk}^2}{2} \times B_4,
\]

where \( B_4 \) can be calculated by the real part of \( \Gamma_4 \):

\[
\Gamma_4 = -\frac{j2\omega_{nk}(t_q) - 2\xi_k(t_q)\omega_{nk}(t_q)}{4\omega_{nk}^2(t_q)} e^{j2[\omega_{nk}(t_q)(t_q-T_{q/2})]+\varphi_{nk}} \cdot e^{-2\xi_k(t_q)\omega_{nk}(t_q)} t_q \cdot \left[ e^{-2\xi_k(t_q)} \sqrt{1-\xi_k(t_q)} - 1 \right].
\]

(29)

From the above equations, the relationships can be obtained as follows:

\[
\begin{cases}
B_3 = e^{-\xi_k(t_q)\omega_{nk}(t_q)} T_{q/2} B_1, \\
B_4 = e^{-\xi_k(t_q)\omega_{nk}(t_q) T_{q/2}} B_2.
\end{cases}
\]

(30)

Therefore, when the IF has already been identified, the instantaneous damping ratio can be identified by the relationship between signal energy \( E_{qd} \) and signal energy \( E_{qd} \):

\[
\begin{cases}
E_{qd} = e^{-\xi_k(t_q)\omega_{nk}(t_q)} T_{q/2} E_{qu}, \\
\xi_k(t_q) = -\ln \left( \frac{E_{qd}}{E_{qu}} \right) \left( \omega_{nk}(t_q) T_{q/2} \right).
\end{cases}
\]

(31)

In such a way, the damping ratio \( \xi_k(t_q) \) at each point \( t_q \) can be identified using the same method. Compared with the identification method based on amplitude envelope, the method based on energy analysis can effectively improve the robustness to noise.

### 4. Simulation Example

In order to validate the proposed method, a 3-DOF mass-spring-dashpot system with time-varying stiffness and time-varying damping is simulated as a numerical example. The LTV system is shown in Figure 3.

The initial parameters are given by

\[
\begin{align*}
m_1 &= 0.5, \ m_2 = m_3 = 1 \text{ (kg)}, \\
k_1 &= 8000, \ k_2 = 10000, \ k_3 = 6000, \ k_4 = 4000 \text{ (N/m)}, \\
c_1 &= c_2 = 0.2, \ c_3 = c_4 = 0.3 \text{ (Ns/m)}.
\end{align*}
\]

(32)

The stiffness and damping variations for the case are given by

\[
\begin{align*}
\Delta k_1 &= 12000 \left[ \frac{\tan(10 \times (-2)^2)}{10(1 - 2)^2} \right], \\
\Delta k_3 &= 1200 \sin(1.5\pi t) \cdot t, \\
\Delta k_5 &= \Delta k_4 = 0, \\
\Delta c_1 &= \Delta c_2 = 0, \\
\Delta c_3 &= -0.3 \sin(0.8\pi t), \\
\Delta c_4 &= 0.3t.
\end{align*}
\]

(33)

Assuming that an instantaneous impulse excitation is applied on the mass \( m_3 \) at the initial time, the Newmark\( \beta \) algorithm is used to compute the structural free response. The sampling time is 3 s, and the acceleration response data of the mass \( m_2 \) are shown in Figure 4. Then the identification method can be performed on the data.

To further validate the adaptiveness and improvement for the TF representation based on GLCT, the Morlet wavelet is selected as a TFA tool to extract the IF as a comparison. The TFA results are shown in Figures 5 and 6, respectively. It can be seen that the response signal contains three components. The IF of the first component varies slowly with time, and the IF of the other two components varies rapidly. For the TF representation around point (2 s, 20 Hz), instantaneous frequency trajectory based on GLCT is more clear and obvious and the TF representation based on GLCT has a higher resolution. However, at TF intervals (1−2 s, 15−25 Hz) and (1−3 s, 35−45 Hz), the Morlet wavelet appears to be inadequate for TF representation of these signal components with fast time-varying IF.

Therefore, the IF of the time-varying system can be identified by TF ridge extraction. The IF of each mode extracted by using the Morlet and GLCT are shown in Figures 7–9. Black lines in these figures are the theoretical values, and red dotted lines and blue dotted lines are the identified results by using GLCT and Morlet, respectively. The results clearly indicate that GLCT conduces to better performance in tracking fast time-varying IF than Morlet.

Once the IF of each mode is identified, the three time-varying components of the acceleration response can be exactly separated by the VK filter. The decomposition results are shown in Figure 10.

Moreover, the amplitude envelope of the three components is also computed, which can be used for the identification of the time-varying damping ratio. To further validate the robustness of the method based on energy analysis, the estimator based on signal amplitude in equation (21) is used to estimate the time-varying damping ratio as a comparison. The identified results are presented in Figures 11–13, and significant fluctuations are observed by using the estimator in equation (21). By comparison, the energy method based on half-period integral can effectively resist noise and get higher identification accuracy. Since the identification of the damping ratio is full of challenge and most modal identification methods are invalid for LTV systems, important...
Figure 3: 3-DOF time-varying system.

Figure 4: Acceleration response signal.

Figure 5: TFA results of GLCT.

Figure 6: TFA results of the Morlet wavelet.
improvements have been made in instantaneous modal parameter identification by using energy analysis. In order to investigate the performance of the identification method, a root-mean-square error (RMSE) over the total sampling time is further computed to quantify the accuracy of identified results as

$$\text{RMSE} = \sqrt{\frac{1}{N_s} \sum_{i=1}^{N_s} (\vartheta_i - \bar{\vartheta})^2}, \quad (34)$$

where $N_s$ represents the number of sampling points and $\vartheta_i$ and $\bar{\vartheta}$ represent the theoretical and identified instantaneous modal parameters, i.e., instantaneous frequencies and damping ratios, respectively. Therefore, the calculated RMSE values of instantaneous frequencies and damping ratios are listed in Table 2.

To further investigate the performance and reliability of the proposed method, different signal-to-noise ratios (SNR) of noise are added to the response of this structure. The same procedure as above is followed to identify the instantaneous modal parameters, and a mean absolute percentage error (MAPE) over the total sampling time is further computed to quantify the accuracy of the identified results as

$$\text{MAPE} = \frac{1}{N_s} \sum_{i=1}^{N_s} \left| \frac{\vartheta_i - \bar{\vartheta}}{\bar{\vartheta}} \right| \times 100\%, \quad (35)$$

where $N_s$ represents the number of sampling points and $\vartheta_i$ and $\bar{\vartheta}$ represent the theoretical and identified instantaneous modal parameters.

![Figure 7: Theoretical results and identified results of the 1st mode.](image)

![Figure 8: Theoretical results and identified results of the 2nd mode.](image)

![Figure 9: Theoretical results and identified results of the 3rd mode.](image)
Therefore, the calculated MAPE values under the effects of measurement noise are listed in Tables 3 and 4.

By comparing the RMSE values and MAPE values using different identification methods, it can be found that the proposed method has better performance in tracking time-varying parameters. The results also show the good adaptive ability and high accuracy even in low SNR scenarios. Especially for the identification of damping ratio, the energy analysis algorithm outperforms the traditional time-domain method.

For practical application of the proposed method, instantaneous frequencies and damping ratios can be identified only using one set of data from single-output. However, for the estimation of mode shapes of LTV systems, there are two hard issues in the implementation of this algorithm. Firstly, multiple sets of data from multioutput are needed to establish the mode shape matrix. Secondly, it is necessary to identify the time-varying mode shapes at each time point, which will lead to a large amount of calculation. Therefore, further studies should be done to estimate mode shapes and improve algorithm efficiency.
Table 2: Errors of identification results of instantaneous frequency.

<table>
<thead>
<tr>
<th>RMSE</th>
<th>Mode</th>
<th>Morlet</th>
<th>GLCT</th>
</tr>
</thead>
<tbody>
<tr>
<td>Instantaneous frequency</td>
<td>1st</td>
<td>0.306</td>
<td>0.195</td>
</tr>
<tr>
<td></td>
<td>2nd</td>
<td>1.882</td>
<td>0.230</td>
</tr>
<tr>
<td></td>
<td>3rd</td>
<td>0.599</td>
<td>0.331</td>
</tr>
</tbody>
</table>

Table 3: Errors of identification results of instantaneous damping ratio.

<table>
<thead>
<tr>
<th>MAPE (%)</th>
<th>Mode</th>
<th>SNR (dB)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Noise-free</td>
<td>100</td>
<td>50</td>
</tr>
<tr>
<td>Morlet</td>
<td>1st</td>
<td>2.421</td>
</tr>
<tr>
<td></td>
<td>2nd</td>
<td>8.141</td>
</tr>
<tr>
<td></td>
<td>3rd</td>
<td>1.091</td>
</tr>
<tr>
<td>GLCT</td>
<td>1st</td>
<td>1.553</td>
</tr>
<tr>
<td></td>
<td>2nd</td>
<td>1.305</td>
</tr>
<tr>
<td></td>
<td>3rd</td>
<td>0.911</td>
</tr>
</tbody>
</table>

Table 4: Errors of identification results of the instantaneous damping ratio.

<table>
<thead>
<tr>
<th>MAPE (%)</th>
<th>Mode</th>
<th>SNR (dB)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Noise-free</td>
<td>100</td>
<td>50</td>
</tr>
<tr>
<td>Time-domain algorithm</td>
<td>1st</td>
<td>&gt;50</td>
</tr>
<tr>
<td></td>
<td>2nd</td>
<td>&gt;50</td>
</tr>
<tr>
<td></td>
<td>3rd</td>
<td>&gt;50</td>
</tr>
<tr>
<td></td>
<td>3rd</td>
<td>26.783</td>
</tr>
</tbody>
</table>
5. Conclusion

This paper proposes a method based on adaptive TF decomposition for instantaneous modal parameter identification of LTV systems. This method consists of two main steps. The first step is the IF extraction algorithm based on GLCT. By applying the algorithm to perform the VK filter, the free response of LTV systems can be decomposed into multiple mode responses. Then the other step is the energy analysis algorithm based on each mode response for the identification of the instantaneous damping ratio. Based on the results of numerical simulations, the corresponding conclusions can be concluded as following:

(1) GLCT is a more effective TFA tool than WT for the response signals of LTV systems. It can obtain the better TF representation with high energy concentration and therefore extract the IF with better accuracy.

(2) Since the wavelet transform is essentially a signal filter, application of a TFA tool with high energy concentration can enhance antinoise capability of parameter identification. By applying GLCT to perform the VK filter, it not only improves the adaptability of decomposition but also has good antinoise ability. It can be seen from the results of error analysis that our method has good immunity to noise.

(3) By adding different SNRs of noise, identification of the instantaneous damping ratio based on the time-domain algorithm and energy analysis algorithm is also discussed. The energy analysis algorithm can eliminate the influence of amplitude fluctuation in the time domain and improve the identification accuracy of the damping ratio. The results show that the energy analysis algorithm has better recognition accuracy, convergence, and noise immunity compared with the traditional method.

Data Availability

The raw data used to support the findings of this study have not been made available because the data also forms part of an ongoing study.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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