Research Article

Research on Automatic Balance Control of Active Magnetic Bearing-Rigid Rotor System

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In this paper, in order to solve the problem of unbalance vibration of rigid rotor system supported by the active magnetic bearing (AMB), automatic balancing method is applied to suppress the unbalance vibration of the rotor system. Firstly, considering the dynamic and static imbalance of the rotor, the detailed dynamic equations of the AMB-rigid rotor system are established according to Newton’s second law. Then, in order to rotate the rotor around the inertia axis, the notch filter with phase compensation is used to eliminate the synchronous control current. Finally, the variable-step fourth-order Runge–Kutta iteration method is used to solve the unbalanced vibration response of the rotor system in MATLAB simulation. The effects of the rotational speed and phase compensation angle on the unbalanced vibration control are analysed in detail. It is found that the synchronous control currents would increase rapidly with the increase of rotational speed if the unbalance vibration cannot be controlled. When the notch filter with phase shift is used to balance the rotor system automatically, the control current is reduced significantly. It avoids the saturation of the power amplifier and reduces the vibration response of the rotor system. The rotor system can be stabilized over the entire operating speed range by adjusting the compensation phase of the notch filter. The method in the paper is easy to implement, and the research result can provide theoretical support for the unbalance vibration control of AMB-rotor systems.

1. Introduction

Compared with mechanical bearings, active magnetic bearings have the advantages of no wear, no lubrication, low noise, and long life and can be controlled actively. Active magnetic bearings are widely used in aerospace, machine tools, turbomachinery, vacuum and cleanroom systems, and other fields [1]. The rotor inertia axis does not usually coincide with the geometric axis due to the processing error of the rotor and the nonuniformity of the raw materials, which can result in imbalance of the rotor system. Although the imbalance of the rotor system can be corrected by the method of static and dynamic balance, there will always be residual imbalance. Unbalance vibration is the main excitation source of the rotor system during operation. The unbalance excitation force is proportional to the square of the rotation speed, and this excitation force increases sharply as the rotation speed increases. If unbalance vibration cannot be effectively controlled, the bearing reaction force caused by unbalanced centrifugal force will be transmitted to the casing, and large noise will be generated. Moreover, the synchronous control current caused by unbalance vibration is approximately proportional to the square of the rotational speed. It may cause the saturation of the power amplifier or the saturation of the control force of AMBs, which affects the stability of the AMB-rotor system at high speed seriously. Therefore, it is very necessary to study the suppression of unbalance vibration of AMB-rotor systems.

For the study of suppression of unbalance vibration, scholars have proposed many methods. In principle, there are mainly two kinds of methods: the first one is to make the rotor rotate around the inertia axis by cancelling the synchronous electromagnetic force, that is, the automatic balancing method; the second one is to compensate unbalance vibration force and rotate the rotor around the geometrical axis, that is, unbalance compensation method [2]. The
former is widely used in occasions where the requirement for rotational accuracy is not high, and the air gap of AMBs is large enough. It has the advantage of reducing the housing vibrations and avoiding of dynamic power amplifier saturation. The latter is more suitable for high precision applications, but it needs high bearing force and high amplifier power in the presence of large residual rotor unbalance.

At present, many scholars have done some important research on the suppression of unbalance vibration. Regarding automatic balancing, Herzog et al. [3] solved the problem of stability of the closed-loop system by inserting the T-matrix into the adaptive notch filter to remove the synchronous control current. Shafai et al. [4, 5] and Vahedforough et al. [6] introduced an automatic balancing method that can obtain the Fourier coefficients of unbalance vibration by an iterative algorithm. Lum et al. [7] realized the automatic adjustment of the control system by identifying the rotor unbalance characteristics online. The proposed method does not depend on the change of rotation speed, but relies on the accuracy of the model. Li et al. [8] proposed an automatic balancing method using double-loop feed-forward control to offset the unbalance effect. Cui et al. [9] used the imbalance identification method to identify static mass imbalance and realized the zero-displacement control of the magnetic suspension rotor. Tang and Chen [10] adopted adaptive noise filter using the least mean square (LMS) algorithm and achieved the automatic balance of AMB rotor systems. On the imbalance compensation, Kejian et al. [11] proposed a method of generating control signals based on the real-time position of rotor imbalance mass. Nonami and Liu [12] realized feed-forward unbalance compensation through the iterative algorithm, but the stability of the algorithm was only explained by simulation. Schuhmann et al. [13] and Taguchi et al. [14] used the Kalman filter to compensate the unbalanced vibrations and improved the position accuracy of the rotor. Based on the Takagi–Sugeno fuzzy model of the nonlinear electromagnetic bearing system, Huang and Lin [15] proposed a dynamic output feedback control method. Setiawan et al. [16, 17] used the Lyapunov method to develop an algorithm that compensates for sensor output and mass imbalance in AMB. In addition, some scholars also made comparative analysis of two methods of controlling unbalance vibration. Kejian et al. [18] analysed the inherent relationship between imbalance compensation and automatic balancing and proposed a unified control method. Shi et al. [19] compared two kinds of adaptive feed-forward compensation algorithms and achieved the control purpose by adjusting the synchronization signal injected at the summing node of the feedback control loop. In the field of unbalance vibration control, some important progresses have been made in the previous literatures. But the models of AMB-rotor systems were usually simplified, and some practical situations were often overlooked for ease of analysis.

Firstly, a radial magnetic bearing model is established, and the bearing force is linearized in this paper. The displacement stiffness and the current stiffness are calculated according to the specific structural parameters of magnetic bearings. Then, the geometrical relationship between the coordinates of AMB, sensor, and the mass center of the rotor is considered, and the fact that the magnetic bearing actuator and the neighboring sensors are usually not collocated in a standard AMB system is described. According to Newton’s second law, a detailed dynamical equation of the AMB-rigid rotor system is established. Afterwards, the optimization method was used to optimize the control parameters of PID controllers in order to reduce the unbalance vibration response of the rotor system. On this basis, the notch filters with phase compensation is used to eliminate the synchronous electromagnetic force, so that the rigid rotor rotates about the inertia axis to achieve balance automatically. Finally, the effects of the rotational speed and phase compensation angle on the unbalanced vibration control are analysed in detail.

2. AMB-Rotor System Dynamic Model

2.1. Model of Radial Active Magnetic Bearing. Figure 1 shows the configuration of an 8-pole radial AMB. The radial electromagnetic bearing is driven in a differential mode. In X- and Y-directions, a pair of electromagnets is driven by the sum of the bias current \(i_0\) and the control currents \(i_x\) or \(i_y\), and the other pairs of electromagnets are driven by the difference between the bias current \(i_0\) and the control currents \(i_x\) or \(i_y\).

It is assumed that the leakage magnetic flux, eddy current loss, and edge effect of the magnetic flux are not considered. According to the literature [1, 2], the expression of the electromagnetic force is as follows:

\[
f_x = f_x^+ - f_x^- = \frac{\mu_0 N^2 A}{4} \left[ \left( i_0 + i_x \right)^2 - \left( \delta_0 - x \right)^2 \right] \cos \alpha_x,
\]

(1)

where \(\mu_0\) is the permeability in vacuum, \(N\) is the number of turns of the coil, \(A\) is the projected area of the pole surface, \(i_0\) and \(i_x\) represent the bias current and control current in the X-direction, respectively, and \(\delta_0\) and \(x\) denote the initial clearance and the displacement of the rotor in the X-direction, respectively.

In general, the control current is much smaller than the bias current, and the rotor displacement does not exceed 1/10 of the initial gap of the electromagnetic bearing. By Taylor’s series expansion of equation (1) at the balance point \((i_0, \delta_0)\), linearized expression of electromagnetic force in the X-direction can be obtained as follows:

\[
f_x = k_1 i_x + k_2 x,
\]

(2)

where \(k = \mu_0 N^2 A \cos \alpha_x/4\), current stiffness coefficient \(k_1 = 4k_i_0/\delta_0^2\), and displacement stiffness coefficient \(k_2 = 4k_i_0/\delta_0^2\). Similarly, the electromagnetic force in the Y-direction can be obtained:

\[
f_y = k_1 i_y + k_3 y.
\]

(3)

The structural parameters of the radial active magnetic bearings used in this study are shown in Table 1. According to equation (2), the displacement stiffness coefficient \(k_2\) and current stiffness coefficient \(k_1\) of the radial electromagnetic bearing can be calculated:
degrees of freedom of rigid rotor systems have only weak couplings, axial motion is often considered as a single degree of freedom in the magnetic bearing-rotor system. So, we also consider the axial and radial motions of the rotor separately. In this paper, only the dynamic characteristics of the radial freedom of the rotor are studied. Table 2 shows the parameters of the AMB-rotor system in simulation.

According to Newton’s second law, dynamic equations of the rotor can be expressed as follows:

\[
\begin{align*}
\mathbf{M} \ddot{\mathbf{q}}_r + \mathbf{G} \Omega = & \mathbf{B} \mathbf{f}_m, \\
\mathbf{M} &= \begin{bmatrix} m & 0 & 0 \\ 0 & J_y & 0 \\ 0 & 0 & J_x \end{bmatrix}, \\
\mathbf{G} &= \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & J_z \Omega \\ 0 & -J_z \Omega & 0 \end{bmatrix}, \\
\mathbf{B} &= \begin{bmatrix} I_{\text{ma}} & -I_{\text{mb}} & 0 \\ 0 & 0 & 1 \\ 0 & 0 & I_{\text{ma}} - I_{\text{mb}} \end{bmatrix}, \\
\mathbf{f}_m &= \begin{bmatrix} f_{x\text{a}} \\ f_{x\text{b}} \\ f_{y\text{a}} \\ f_{y\text{b}} \end{bmatrix}.
\end{align*}
\]

where

\[
\begin{align*}
\mathbf{q}_r &= (x_1, y_1, -\alpha_1)^T, \\
\mathbf{f}_m &= \begin{bmatrix} f_{x\text{a}} \\ f_{x\text{b}} \\ f_{y\text{a}} \\ f_{y\text{b}} \end{bmatrix}, \\
\mathbf{M} &= \begin{bmatrix} m & 0 & 0 \\ 0 & J_y & 0 \\ 0 & 0 & J_x \end{bmatrix}, \\
\mathbf{G} &= \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & J_z \Omega \\ 0 & -J_z \Omega & 0 \end{bmatrix}, \\
\mathbf{B} &= \begin{bmatrix} I_{\text{ma}} & -I_{\text{mb}} & 0 \\ 0 & 0 & 1 \\ 0 & 0 & I_{\text{ma}} - I_{\text{mb}} \end{bmatrix}, \\
\mathbf{f}_m &= \begin{bmatrix} f_{x\text{a}} \\ f_{x\text{b}} \\ f_{y\text{a}} \\ f_{y\text{b}} \end{bmatrix}.
\end{align*}
\]

According to equations (2) and (3), the magnetic bearing force can be expressed as follows:

\[
\mathbf{f}_m = \begin{bmatrix} f_{x\text{a}} \\ f_{x\text{b}} \\ f_{y\text{a}} \\ f_{y\text{b}} \end{bmatrix} = \begin{bmatrix} k_x & k_x & k_x & k_x \\ k_y & k_y & k_y & k_y \\ i_{x\text{a}} & i_{x\text{b}} & i_{y\text{a}} & i_{y\text{b}} \end{bmatrix} \begin{bmatrix} x_{\text{ma}} \\ x_{\text{mb}} \\ y_{\text{ma}} \\ y_{\text{mb}} \end{bmatrix}.
\]

that is, \( \mathbf{f}_m = K_y \mathbf{y}_m + K_i \mathbf{i} \).
The rotor studied in this paper is rigid, and the displacement of the rotor is very small. According to the geometric relationship, the conversion between the rotor displacement at magnetic bearings and the neighboring sensors are usually not collocated in a standard AMB system. It may lead to instability of the closed-loop AMB system at certain rotor speeds for some specific plant configurations. Therefore, it is unreasonable to use the rotor displacements at the magnetic bearing actuators as the rotor displacements measured by the sensors directly. According to the geometric relationship, the conversion between the rotor displacement at sensors \(y_s\) and the rotor geometric center \(q_g\), can be expressed as follows:

\[
y_s = \left[\begin{array}{c}
x_{sa} \\
x_{sb} \\
y_{sa} \\
y_{sb}
\end{array}\right] = \left[\begin{array}{cccc}
1 & l_{ma} & 0 & 0 \\
1 & -l_{mb} & 0 & 0 \\
0 & 0 & 1 & l_{ma} \\
0 & 0 & 1 & -l_{mb}
\end{array}\right] \left[\begin{array}{c}
x_g \\
\beta_g \\
y_g \\
-\alpha_g
\end{array}\right] = T_m q_g. \quad (9)
\]

In AMB-rotor systems, eddy current sensors are commonly used to measure rotor displacement. The eddy current sensors usually cannot be integrated into the electromagnetic bearings in industrial applications due to the limitations of the manufacturing technology. So, the magnetic bearing actuators and the neighboring sensors are usually not collocated in a standard AMB system. It may lead to instability of the closed-loop AMB system at certain rotor speeds for some specific plant configurations in conjunction with local PID control [1]. Therefore, it is unreasonable to use the rotor displacements at the magnetic bearing actuators as the rotor displacements measured by the sensors directly. According to the geometric relationship, the conversion between the rotor displacement at sensors \(y_s\) and the rotor geometric center \(q_g\), can be expressed as follows:

\[
y_s = \left[\begin{array}{c}
x_{sa} \\
x_{sb} \\
y_{sa} \\
y_{sb}
\end{array}\right] = \left[\begin{array}{cccc}
1 & l_{sa} & 0 & 0 \\
1 & -l_{sb} & 0 & 0 \\
0 & 0 & 1 & l_{sa} \\
0 & 0 & 1 & -l_{sb}
\end{array}\right] \left[\begin{array}{c}
x_g \\
\beta_g \\
y_g \\
-\alpha_g
\end{array}\right] = T_s q_g. \quad (10)
\]

Rotor imbalances include static imbalance and dynamic imbalance. Static unbalance is caused by the distance between the inertial axis of the rotor and the geometric axis. The dynamic imbalance is due to the deflection angle between the inertial axis of the rotor and the geometric axis. According to references [20, 21], the imbalance vector of the rotor can be expressed as follows:

\[
u_b = q_l - q_g = \left[\begin{array}{c}
\epsilon \cos (\Omega t + \theta) \\
\sigma \sin (\Omega t + \varphi) \\
\epsilon \cos (\Omega t + \varphi)
\end{array}\right], \quad (11)
\]

where \(\epsilon\) and \(\sigma\) are the amplitudes of static and dynamic unbalance, respectively. \(\theta\) and \(\varphi\) denote the initial phases of static and dynamic unbalance, respectively.

Substituting equations (9)–(11) into equation (6), the following equation can be obtained:

\[
M_s \ddot{y}_s + C_s \dot{y}_s + K_s y_s = f_u + f_i, \quad (12)
\]

where \(M_s = MT_s^{-1}\), \(C_s = GT_s^{-1}\), \(K_s = BK_s T_m T_s^{-1}\), \(f_u = -M \dot{u}_b - G \dot{u}_b\), and \(f_i = BK_i\).

By simplifying equation (12), we can get the following equations:
\[
\ddot{x}_a + \frac{J_{la}}{J_{ls}} \left( \dot{y}_a - \dot{y}_b \right) = \left( \frac{k_x}{m} + \frac{k_{la}}{J_{ls}} \right) x_a - \left( \frac{b}{m} + \frac{k_{la}}{J_{ls}} \right) \left( \dot{x}_a - \dot{x}_b \right)
\]

\[
- \left( \frac{k_x}{m} b + \frac{k_{la}}{J_{ls}} d \right) x_{ab} = \frac{k_i}{m} \left( i_{xa} + i_{xb} \right)
\]

\[
+ \frac{k_{la}}{J_{y}} \left( l_{ma} x_{ab} - l_{mb} x_{ab} \right) + \epsilon \Omega^2 \cos(\Omega t + \theta)
\]

\[
+ \left( \frac{J_y - J_{la}}{J_{y}} \right) \sigma \Omega^2 \sin(\Omega t + \phi),
\]

\[
\ddot{x}_b - \frac{J_{lb}}{J_{ls}} \left( \dot{y}_a - \dot{y}_b \right) = \left( \frac{k_x}{m} + \frac{k_{lb}}{J_{ls}} c \right) x_a - \left( \frac{b}{m} + \frac{k_{lb}}{J_{ls}} d \right) \left( \dot{x}_a - \dot{x}_b \right)
\]

\[
- \left( \frac{k_x}{m} b - \frac{k_{lb}}{J_{ls}} d \right) x_{ab} = \frac{k_i}{m} \left( i_{xa} + i_{xb} \right)
\]

\[
- \frac{k_{lb}}{J_{y}} \left( l_{ma} y_{ab} - l_{mb} y_{ab} \right) + \epsilon \Omega^2 \cos(\Omega t + \theta)
\]

\[
+ \left( \frac{J_y - J_{lb}}{J_{y}} \right) \sigma \Omega^2 \sin(\Omega t + \phi),
\]

\[
\ddot{y}_a - \frac{J_{la}}{J_{ls}} \left( \dot{x}_a - \dot{x}_b \right) = \left( \frac{k_x}{m} + \frac{k_{la}}{J_{ls}} c \right) y_a - \left( \frac{b}{m} + \frac{k_{la}}{J_{ls}} d \right) \left( \dot{x}_a - \dot{x}_b \right)
\]

\[
- \left( \frac{k_x}{m} b + \frac{k_{la}}{J_{ls}} d \right) y_{ab} = \frac{k_i}{m} \left( i_{ya} + i_{yb} \right)
\]

\[
- \frac{k_{la}}{J_{x}} \left( l_{ma} y_{ab} - l_{mb} y_{ab} \right) + \epsilon \Omega^2 \sin(\Omega t + \theta)
\]

\[
+ \left( \frac{J_x - J_{la}}{J_{x}} \right) \sigma \Omega^2 \cos(\Omega t + \phi),
\]

\[
\ddot{y}_b + \frac{J_{lb}}{J_{ls}} \left( \dot{x}_a - \dot{x}_b \right) = \left( \frac{k_x}{m} + \frac{k_{lb}}{J_{ls}} c \right) y_a - \left( \frac{b}{m} + \frac{k_{lb}}{J_{ls}} d \right) \left( \dot{x}_a - \dot{x}_b \right)
\]

\[
- \left( \frac{k_x}{m} b - \frac{k_{lb}}{J_{ls}} d \right) y_{ab} = \frac{k_i}{m} \left( i_{ya} + i_{yb} \right)
\]

\[
+ \frac{k_{lb}}{J_{x}} \left( l_{ma} y_{ab} - l_{mb} y_{ab} \right) + \epsilon \Omega^2 \sin(\Omega t + \theta)
\]

\[
+ \left( \frac{J_x - J_{lb}}{J_{x}} \right) \sigma \Omega^2 \cos(\Omega t + \phi),
\]

where \( I_s = I_{sa} + I_{sb}, \ a = 2I_{sa} + I_{mb} - I_{ma}, \ b = 2I_{sa} - I_{ma} + I_{mb}, \ c = I_{ma} + I_{mb} + I_{sa} + I_{sb}, \ \) and \( d = I_{ma} - I_{mb} + I_{sa} - I_{sb} \).

Considering that the transverse moment of inertia \( I_x \) and \( J_y \) studied in this paper is much larger than the polar moment of inertia \( I_z \) and \( J_{lb} \) or \( J_{la} \), and \( |ak_x/m + c\sigma J_{la}/I_{ja}| \gg |bk_x/m + d\sigma J_{la}/I_{ja}|, \ |ak_x/m - c\sigma J_{lb}/I_{ja}| \ll |bk_x/m - d\sigma J_{lb}/I_{ja}| \), gyro effect and the coupling of two magnetic bearings can be completely ignored. Therefore, equation (13) can be simplified as follows:

2.3. PID Decentralized Control System. The control system of magnetic bearings generally consists of sensors, controllers, and power amplifiers. The block diagram of one degree of freedom AMB control system is shown in Figure 3. \( G_{c}(s) \), \( G_{c}(s) \), and \( G_{c}(s) \) denote the transfer functions of the sensor, controller, and power amplifier, respectively. \( G_{p}(s) \) is the transfer function from the control current to the rotor displacement, and \( i_{ea} \) is the equivalent unbalance disturbance.

The PID controller is widely used in industrial AMB systems because of its high precision, convenient parameter adjustment, and good stability. The PID controller is also used to control rigid rotors with a decentralized control scheme in this paper. The transfer function of a standard PID controller can be expressed as follows:

\[
G_{c}(s) = \frac{k_p}{s} + k_{i} + k_{d} s.
\]

In practice, due to the dynamic characteristics of the system and high-frequency noise, the ideal PID controller generally cannot meet the needs of the actual system [22]. So, it is necessary to make a reasonable transformation of the ideal controller. In this paper, the PID controller with incomplete derivation is used. The transfer function of the controller is

\[
G_{c}(s) = \frac{k_p}{s} + \frac{k_{i}}{\tau_{d} s + 1} + \frac{k_{d} s}{\tau_{d} s + 1},
\]

where \( k_p \), \( k_d \), and \( k_t \) denote the proportional coefficient, differential coefficient, and integral coefficient of the PID controller, respectively. \( \tau_{d} \) is the differential time constant.
Moreover, the higher proportional feedback gain at the moment, the rotor may vibrate intermittently. If the impact interference occurs, the magnet flux becomes saturated when the rotor displacement is very small. If the impact interference occurs, the interference on the dynamic characteristics of rotor systems. On the one hand, if the stiffness is too high, it will cause the stiffness to become too high when the rotor displacement is very small. If the impact interference occurs at the moment, the rotor may vibrate intermittently. Moreover, the higher proportional feedback gain \( k_p \) will result in too high closed-loop characteristic frequency and high bandwidth requirements for controllers, sensors, and power amplifiers [1]. On the other hand, when equivalent stiffness is too low, the effect of the negative stiffness \( k_x \) on the system becomes significant. In this paper, the equivalent stiffness of AMB is chosen to be 3~5 times of the negative stiffness \( k_x \) of the magnetic bearing, and the corresponding proportional coefficient \( k_p \) of the PID controller is about 2.4~4.7. After the range of \( k_p \) is determined, the range of the differential coefficient \( k_d \) can be determined. Too small a damping ratio would make the relative stability of the system worse, and too much damping would amplify the feedback noise signal. From the vibration characteristics of the second-order system, when the damping ratio is 0.6~0.8, the better overall performance can be obtained. Therefore, the differential coefficient \( k_d \) is determined to be about 0.005~0.009. The simulation shows that the integral coefficient \( k_i \) has little influence on the unbalance vibration response of the system, and the integral gain \( k_i \) can be set as 0 when \( k_p \) and \( k_d \) are optimized.

In order to reduce the unbalance vibration displacement and the corresponding synchronous control current of the rotor system under different rotational speeds, \( k_p \) and \( k_d \) of the PID controller are optimized by the optimization method. The corresponding objective function \( W \) can be expressed as follows:

\[
W = P \cdot \sum_{i=1}^{n} (\log A_i + 5) + Q \cdot \sum_{i=1}^{n} (\log B_i + 1),
\]

where \( A_i \) and \( B_i \) are the displacement amplitude and the control current amplitude of the unbalance vibration of the rotor at the sensor \( A \) at a certain speed, respectively. \( n \) is the number of selected speed. \( P \) and \( Q \) are the weights of the vibration displacement amplitude and control current amplitude, which are set as 0.4 and 0.6, respectively. The order of \( A_i \) and \( B_i \) is quite different, being \( 10^{-5} \) and \( 10^{-1} \), respectively. The order of both can be the same through the corresponding mathematical operations in equation (20).

Within the range of the PID controller parameters, 25 groups of \( k_p \) and \( k_d \) were selected for simulation. Figures 4(a) and 4(b) show the displacement amplitude and the corresponding control current amplitude versus speed under three groups of \((k_p, k_d)\), respectively. Finally, the values of the objective function at different \( k_p \) and \( k_d \) are calculated, and the final \( k_p \) and \( k_d \) are determined to be 2.5 and 0.006, respectively. After determining the appropriate \( k_p \) and \( k_d \), the \( k_i \) can be adjusted in the stable region to eliminate the static error of the system. The simulation analysis shows that the variation of the differential coefficient \( k_i \) has no obvious effect on the vibration response of the system. The final determined \( k_i \) is 2.5.

### 3. Automatic Balance Control of AMB-Rotor System

In order to ensure the stable operation of the magnetic bearing-rotor system within the full operating speed range, the notch filter with phase shift is used in this paper. The notch filters with phase shift have less parameter and can achieve phase compensation; so, it is easy to design [23]. The...
The unbalance control block diagram of the AMB-rotor system is shown in Figure 5. The sine and cosine signals can be generated by the internal sine and cosine blocks, and the speed can be measured in real time by an external photoelectric encoder.

As shown in Figure 5, \( x_f(t) \) and \( c_f(t) \) denote the input and output signals of \( N_f \), respectively. The feedback part can be expressed by

\[
c_f(t) = \sin(\Omega t + \theta) \int \sin(\Omega t)x_f(t)dt + \cos(\Omega t + \theta) \int \cos(\Omega t)x_f(t)dt.
\]

(21)

Assuming that the rotational speed \( \Omega \) is a constant and does not vary with time, derivate equation (21) twice with respect to time is

\[
\dot{c}_f(t) = \Omega \cos(\Omega t + \theta) \int \sin(\Omega t)x_f(t)dt - \Omega \sin(\Omega t + \theta) \int \cos(\Omega t)x_f(t)dt + \cos \theta \dot{x}_f(t),
\]

(22)

\[
\ddot{c}_f(t) - \Omega^2 c_f(t) = \cos \theta \ddot{x}_f(t) - \Omega \sin \theta \dot{x}_f(t).
\]

(23)

As shown in equation (23), the equation is a linear differential equation and the Laplace transform can be expressed as follows:

\[
N_f(s) = \frac{c_f}{x_f} = \frac{s \cos \theta - \Omega \sin \theta}{s^2 + \Omega^2}.
\]

(24)

Then, the transfer function from \( v(t) \) to \( e(t) \) can be given by

\[
E(s) = \frac{e}{v} = \frac{1}{1 + \omega N_f(s)} = \frac{s^2 + \Omega^2}{s^2 + \epsilon \Omega \cos \theta s + (\Omega^2 - \epsilon \Omega^2 \sin \theta)}.
\]

(25)

The frequency response of the \( E(s) \) under different phase angles \( \theta \) is shown in Figure 6. The unbalance compensation methods with the generalized notch filters may reduce stability margin of the closed-loop system [24]. The notch filter with phase shift can provide phase lead to compensate the loss of phase introduced by the notch filter and improve the stability of the rotor system in the lower frequency range. The phase of the \( E(s) \) can be adjusted by tuning the parameter \( \theta \). The transfer function from the external unbalance disturbance \( u_i \) to the control current \( i \) can be given by

\[
G_{ui}(s) = -\frac{G_k(s)E(s)}{1 + G_k(s)G_p(s)E(s)},
\]

(26)

where \( G_k(s) = G_c(s)G_q(s)G_p(s) \).

Substituting equation (25) into equation (26) yields

\[
G_{ui} = -\frac{(s^2 + \Omega^2)G_k(s)}{\epsilon \Omega (\cos \theta \cdot s - \Omega \sin \theta) + (s^2 + \Omega^2)(1 + G_k(s)G_p(s))}.
\]

(27)

So, the following equation can be obtained:

\[
G_{ui}(s)\big|_{s=\omega j} = \left\{ \begin{array}{ll}
0, & \omega \in (\Omega - \delta, \Omega + \delta),
G_{ui}, & \omega \notin (\Omega - \delta, \Omega + \delta),
\end{array} \right.
\]

(28)

where \( \delta \) is a very small value.

As shown in equation (28), when the speed of the rotor is in the neighborhood of the set speed, the magnitude of the transfer function from the external unbalanced force \( u \) to the control current \( i \) is almost equal to zero. So, the synchronous component of control current induced by the residual unbalance of the rotor can be attenuated to 0 in theory.

The stability of the closed-loop system can be determined by solving the characteristic equation of the closed-loop system. The characteristic equation of the closed loop system is

\[
\epsilon \Omega (\cos \theta \cdot s - \Omega \sin \theta) + (s^2 + \Omega^2)(1 + G_k(s)G_p(s)) = 0.
\]

(29)

When the roots of equation (29) are all in the left half plane of the complex plane, the system is stable.
there are roots in the right half plane of the complex plane with the change of speed, the system will be unstable. It can be seen from equation (29) that the roots of the closed-loop system are related to speed $\Omega$ and phase angle $\theta$. (Xhe characteristic roots of the closed loop system can be all located in the left half plane of the complex plane by adjusting the phase angle $\theta$ at different speed ranges.

4. Simulation Results and Analysis

4.1. Comparison of Control Effects at Different Speeds. The unbalance force is directly proportional to the square of the speed and will increase dramatically with the speed increasing. Excessive unbalance force may cause big control current, which may make a great impact to the stability of the rotor system due to the saturation characteristics of the power amplifier. So, it is necessary to study the vibration responses of the rotor system at different speeds. The input speed of the notch filter can be measured by the photoelectric encoder. Under normal circumstances, the maximum measurement error of the optical encoder is less than 0.14%. The speed measurement error is set to 0.1% in the simulation. In addition, considering the sensor’s measurement noise in reality, the Gaussian noise block in the MATLAB/simulink is used to simulate the sensor’s measurement noise. The mean of the Gaussian noise block is set to 0, and the variance is set to $10^{-11}$. The compensation phase angle $\theta$ in the notch filter is set to $-60^\circ$.

Since the AMB-rotor system has similar characteristics in the $X$- and $Y$-directions, only the dynamic response of the rotor at the sensor $A$ in the $X$-direction will be analysed next. Figures 7–9 show the comparison of the rotor dynamic response before and after adding the unbalance control at the rotor speeds of 3000 rpm, 5000 rpm, and 8000 rpm, respectively. Among them, Figures 7(a) and 9(a) are the time-domain diagrams and FFT spectrums of the displacement and control current of the rotor without unbalance control added. Figures 7(b) and 9(b) are the time-domain diagrams and FFT spectrums of the displacement and control current of the rotor with unbalance control added.

Comparing Figures 7(a) and 9(a), it can be found that the vibration displacement of the rotor and the corresponding synchronous control current increase rapidly with the increase of the rotation speed when the rotor unbalance vibration is not controlled. (Xhe amplitude of synchronous vibration displacement increases from $3 \times 10^{-5}$ m to $5.5 \times 10^{-5}$ m, and the corresponding synchronous control current increases from 0.3 A to 0.95 A. Compared with the growth rate of synchronous vibration displacement, the synchronous control current increases more obviously. If the rotation speed is further increased, the saturation of the power amplifier or the saturation of the electromagnetic force may be caused. It is detrimental to the stability of the AMB-rotor system. Comparing Figures 7(b) and 9(b), it is not difficult to find that
the vibration displacement of the rotor and the corresponding synchronous control current do not increase obviously as the rotational speed increases when the imbalance control method is applied. Synchronous vibration displacement remains within $3 \times 10^{-5}$ m, and the corresponding synchronous control current is kept at a very low level, less than 0.2 A. It ensures that the power amplifier does not saturate due to excessive synchronous control current. The stability of the AMB-rotor system can be guaranteed.
In order to further study the control effect of unbalance vibration of the rotor system in the range of full operating speed (0–8000 rpm), the curves of rotor displacement amplitude and control current amplitude versus rotation speed are plotted. Since the AMB-rotor system has similar dynamics in the X- and Y-directions, only the dynamic response of the rotor at the sensor A in the X-direction is analysed. Figures 10(a) and 10(b) show the amplitude of the rotor displacement and the corresponding control current amplitude versus speed in X-direction, respectively. As shown in Figure 10, when the speed is higher than 3000 rpm, the amplitude of rotor vibration displacement and the amplitude of the corresponding synchronous control current are lower than that before adding the unbalance control, and the control effect is ideal. However, when the rotor speed is lower than 3000 rpm, the amplitude of the vibration displacement of the rotor increases with the unbalance control added. Moreover, the amplitudes of control current are larger at some speeds after unbalance control is added, and the control effect is not satisfactory.

4.2. Influence of Phase Angle of Notch Filter on Control Effect. The compensation phase of the notch filter affects the stability of the rotor system. From the previous analysis of Figure 10, it can be seen that the control effect of unbalance vibration is not satisfactory in a rotation speed range below 3000 rpm when the phase angle in the notch filter is set to −60°. When the phase angle of the notch filter is set to −140°, the amplitude of the vibration displacement and the magnitude of the control current vary with the rotational speed in the low speed region, as shown in Figure 11.

When the unbalance vibration suppression method is applied, it is obvious that the amplitudes of vibration displacement of the rotor decrease at the speeds below 5000 rpm, and the corresponding synchronous control current amplitudes stabilize at a lower level. In the low speed range, control effect of unbalance vibration is relatively good. However, the amplitude of the vibration displacement and the control current of the rotor rapidly increase after adding the notch filter when the speed is higher than 5000 rpm. This should be due to the fact that a closed-loop right pole appears and the system becomes unstable at high rotational speeds when the phase angle in the notch filter is set to −140°.

Comparing with Figures 10 and 11, it is not difficult to find that the control effect of the unbalance vibration is ideal in the low speed region when the phase angle of the notch filter is set to −140°; in the high speed region, the control effect of the unbalance vibration is better when the phase angle of the notch filter is set to −60°. In order to make the unbalance vibration control effect of the rotor system ideal over the entire operating speed range, the phase angle of the notch filter can be adjusted in different speed ranges. The phase angle of the notch filter is set to −140° at speeds below 5000 rpm, and the phase angle is set to −60° at speeds above 5000 rpm. The three-dimensional frequency spectrums of the vibration displacement of the rotor and the control current in the X-direction at the sensor A are shown in Figures 12 and 13, respectively.

As shown in Figures 12 and 13, the vibration displacement of the rotor and the corresponding control current are reduced after the imbalance control is added over the entire operating speed range. The unbalance vibration of
Figure 10: Comparison of vibration response at different speeds: (a) vibration displacement and (b) control current.

Figure 11: Comparison of vibration response at different speeds: (a) vibration displacement and (b) control current.

Figure 12: Three-dimensional frequency spectrums of the displacement (a) without unbalance control and (b) with unbalance control.
the rotor is effectively suppressed, which avoids the saturation of the power amplifier due to excessive synchronous control current.

5. Conclusions

In this paper, we consider the noncollocation between the magnetic bearings and the sensors along the axial direction and establish the detailed dynamic model of the AMB-rigid rotor system. In order to rotate the rotor around the inertia axis, the synchronous current caused by rotor imbalance is canceled, and the synchronous electromagnetic force of AMBs is suppressed by using the notch filter with phase shift. The effects of the rotational speed and phase compensation angle on the unbalanced vibration responses and the stability of the rotor system are analysed in detail. According to research on suppression of unbalance vibration of the AMB-rotor system, the following conclusions can be obtained:

(1) If the unbalance vibration of the rotor is not controlled, the synchronous control current will increase rapidly with the increase of rotational speed. When the notch filter with phase shift is used to balance the rotor system, the control current is significantly reduced. It avoids the saturation of the power amplifier and reduces the vibration response of the rotor system.

(2) The compensation phase of the notch filter will affect the stability of the rotor system, and the unbalanced vibration of the rotor system can be controlled in a piecewise way. By adjusting the compensation phase of the notch filter at different speed ranges, the rotor system can achieve stability over the entire operating speed range.

Regrettably, no experimental verification has been carried out in this paper because the experimental facilities are being built, and the experimental conditions are not yet available. The corresponding experimental verification will be carried out in subsequent studies.

Data Availability

The datasets supporting the conclusions of this article are included within the article.

Conflicts of Interest

The authors declare no conflicts of interest.

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References


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