Research Article

Design and Implementation of H_∞ Miscellaneous Information Feedback Control for Vehicle Suspension System

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In this paper, we investigate an H_∞ miscellaneous information feedback controller (HMIFC) for active suspension system stabilization. An auxiliary feedback signal is introduced in the control channel via a wireless communication network. Under this scheme, the onboard measured signal and network transmission signal are simultaneously used to improve the control performance. To handle the impact of network-induced delays, a delay-dependent stability criterion for the closed-loop system is derived based on Lyapunov theory and the linear matrix inequality (LMI) method. By considering an augmented Lyapunov-Krasovskii functional (LKF) and a relaxed integral inequality, a new existence condition of the miscellaneous feedback controller can be obtained. Compared with the traditional H_∞ controller, the designed controller can acquire better ride comfort subject to the same constraints. Finally, comparative simulation and experimental results prove the effectiveness of the presented H_∞ miscellaneous information feedback controller.

1. Introduction

Even as people have developed increasingly high requirements for traffic safety and ride comfort, a large number of casualties still occur due to traffic accidents. Many studies have shown that the vibrations caused by irregular road surfaces have an energy-draining effect on drivers, affecting their physical and mental health. Vibration in ambulances harms patients and aggravates their condition. A vehicle's suspension is an important vibration-isolating part. It effectively isolates the body from vibrations caused by the road surface to improve the ride comfort [1, 2], and it is important for the stability of a vehicle [3, 4]. Therefore, advanced vehicle suspension systems constitute an important research subject. Vehicle suspension can be divided into three types, according to their operating mechanism: passive, semiactive, and active [5]. Since an active suspension can generate a preset control force, it can achieve an ideal control effect. An active controller directly determines the vibration-absorption performance of an active suspension system; thus, it has received much

attention [1–13]. The common control methods include robust H_∞ control [2, 3], sliding mode control [5–11], adaptive control [14, 15], and fuzzy control [16, 17]. Among these, the H_∞ control is regarded as the most powerful design alternative, and many significant research results have been obtained [18–23].

Networked control systems (NCSs) have attracted increasing attention in some vehicles and active-vibration control systems due to their high reliability, easy installation, and simple maintenance [24–29]. For instance, Wang et al. [24] developed an event-triggered H_∞ control for active suspension systems with delays. In [25], network-based modeling and active control were studied for full-car suspension systems, and the I/O signals are transmitted via network to a central controller. In general, the active suspension systems are typical application of NCSs. As an effect of limited network bandwidth, the network-induced delays must be considered in controller design. A robust H_∞ control method was proposed for vehicle lateral control considering tire force saturation [26]. Network-based control was also applied to vehicle platoon control, and

numerical results were provided to show the effectiveness of the proposed control scheme [30]. An event-triggered H_∞ control was proposed for vehicle suspension systems [22]. Overall, network-based control is helpful and has potential to improve the vibration control performance of active suspension systems.

In research, vehicle active suspension control should reduce the amplitude of the acceleration of the body and consider the suspension dynamic deflection, tire dynamic load, and control saturation nonlinearity [1, 2]. Therefore, the control of an active suspension system is a multiobjective control problem. H_∞ control is an effective multiobjective control method that can effectively solve the conflict between the suspension effect and the handling performance of a vehicle [20]. There have been many important achievements in the research of H_∞ control of a vehicle active suspension [16–23]. Most research on H_∞ control of vehicle active suspension has been simulation-based and lacking in effective experimental verification. Considering the importance of robust H_∞ control for a vehicle active suspension system, it is necessary to study it from an experimental perspective.

Combining the advantages of onboard and network-based signal feedback, in this paper, we introduce an H_∞ miscellaneous information feedback control to improve the performance of a two degree-of-freedom (DOF) active suspension system. Considering network-induced delays, Lyapunov stability theory and the linear matrix inequality (LMI) method are applied to design the controller. Finally, the feasibility and practicability of the proposed solution are verified by simulations and experiments. The key contributions of this paper are summarized as follows.

First, an H_∞ miscellaneous information feedback control approach for a vehicle active suspension is proposed, which can stabilize the considered system and improve the drive comfort level, while meet suspension constrained requirements.

Second, an LMI-based miscellaneous controller design condition is derived using a relaxed integral inequality. Moreover, in view of the complexity of network channel, the network-induced delays are taken into consideration.

Finally, the superiorities of our proposed control strategy are illustrated deeply via both a numerical example and a real active suspension setup.

The rest of the paper is organized as follows. The suspension model and control objectives are formulated in Section 2. The main results are given in Section 3. Applications and conclusions are described in Sections 4 and 5, respectively.

2. Vehicle Suspension Modeling and Control Problem Formulation

2.1. Two DOF Suspension System Modeling. In this study, a simplified two DOF suspension model with an active actuator is considered, as shown in Figure 1. The parameters of the model are given in Table 1.

The linear dynamics of the two DOF suspension model can be mathematically described as

$$\begin{cases} m_s \ddot{z}_s + c_s (\dot{z}_s - \dot{z}_u) + k_s (z_s - z_u) = \vartheta u, \\ m_u \ddot{z}_u - c_s (\dot{z}_s - \dot{z}_u) - k_s (z_s - z_u) + c_u (\dot{z}_u - \dot{z}_r) \\ \quad + k_u (z_u - z_r) = -\vartheta u, \end{cases} \quad (1)$$

where z_s and z_u denote the car-body vertical displacement and wheel vertical displacement, respectively; z_r is the vertical disturbance input generated by an uneven road surface; u is the active control law to be designed; and ϑ represents the maximum control magnitude.

Define

$$\mathbf{x}(t) = \text{col}\{x_1(t) \ x_2(t) \ x_3(t) \ x_4(t)\}, \quad (2)$$

where

$$\begin{aligned} x_1(t) &= z_s(t) - z_u(t), \\ x_2(t) &= \dot{z}_s, \\ x_3(t) &= z_u - z_r, \\ x_4(t) &= \dot{z}_u. \end{aligned} \quad (3)$$

Using (2), the state space equation of the model (1) can be written as

$$\dot{\mathbf{x}}(t) = \mathbf{A}\mathbf{x}(t) + \mathbf{B}u(t) + \mathbf{B}_1\mathbf{f}(t), \quad (4)$$

where

$$\begin{aligned} \mathbf{A} &= \begin{bmatrix} 0 & 1 & 0 & -1 \\ -\frac{k_s}{m_s} & -\frac{c_s}{m_s} & 0 & \frac{c_s}{m_s} \\ 0 & 0 & 0 & 1 \\ \frac{k_s}{m_u} & \frac{c_s}{m_u} & -\frac{k_u}{m_u} & -\frac{c_s + c_u}{m_u} \end{bmatrix}, \\ \mathbf{B} &= \begin{bmatrix} 0 \\ \frac{\vartheta}{m_s} \\ 0 \\ -\frac{\vartheta}{m_u} \end{bmatrix}, \\ \mathbf{B}_1 &= \begin{bmatrix} 0 \\ 0 \\ -1 \\ \frac{c_u}{m_u} \end{bmatrix}, \\ \mathbf{f}(t) &= \dot{z}_r(t). \end{aligned} \quad (5)$$

Suppose that $\mathbf{f}(t)$ belongs to $L_2[0, \infty)$ and satisfies the following condition:

FIGURE 1: Two DOF suspension model.

TABLE 1: Model parameters.

Symbol	Value [1]	Nominal value (experiment)	Unit	Meaning
m_s	973	2.45	kg	Sprung mass
m_u	114	1	kg	Unsprung mass
k_s	42,720	900	N/m	Suspension stiffness
k_u	101,115	1,250	N/m	Tire stiffness
c_s	1,095	7.5	Ns/m	Suspension damping
c_u	14.6	5	Ns/m	Tire damping

$$\int_0^{\infty} \mathbf{f}^T(t) \mathbf{f}(t) dt \leq \kappa < +\infty, \quad (6)$$

where κ denotes a bounded constant.

The ride comfort of the vehicle can be effectively reflected by the magnitude of car-body vertical acceleration. Therefore, the main aim of control is to suppress the car-body vertical acceleration $\ddot{z}_s(t)$ as much as possible.

To avoid overload and decrease input energy consumption, consider the following constraint:

$$|\mathbf{u}(t)| \leq \vartheta. \quad (7)$$

The maximum suspension deflection should be less than an allowable value $z_{\max}$ for structural protection, and the dynamic tire load should not exceed the static load due to the road-holding stability requirement. Hence, the remaining constraints are

$$\begin{aligned} |z_s - z_u| &\leq z_{\max}, \\ |k_u(z_u - z_r)| &\leq (m_s + m_u)g. \end{aligned} \quad (8)$$

Using (7) and (8), the output equations are given as

$$\begin{aligned} \mathbf{z}_1(t) &= \ddot{z}_s, \\ \mathbf{z}_2(t) &= \begin{bmatrix} \frac{z_s - z_u}{z_{\max}} & \frac{k_u(z_u - z_r)}{(m_s + m_u)g} \end{bmatrix}^T, \quad |\mathbf{u}(t)| \leq 1. \end{aligned} \quad (9)$$

Then, the overall performance outputs become

where

$$\begin{aligned} \mathbf{z}_1(t) &= \mathbf{C}_1 \mathbf{x}(t) + \mathbf{D}_1 \mathbf{u}(t), \\ \mathbf{z}_2(t) &= \mathbf{C}_2 \mathbf{x}(t), \\ \mathbf{z}_3(t) &= \mathbf{u}(t), \end{aligned} \quad (10)$$

$$\begin{aligned} \mathbf{C}_1 &= \begin{bmatrix} -\frac{k_s}{m_s} & -\frac{c_s}{m_s} & 0 & \frac{c_s}{m_s} \end{bmatrix}, \\ \mathbf{D}_1 &= \frac{\vartheta}{m_s}, \\ \mathbf{C}_2 &= \begin{bmatrix} \frac{1}{z_{\max}} & 0 & 0 & 0 \\ 0 & 0 & \frac{k_u}{(m_s g + m_u g)} & 0 \end{bmatrix}. \end{aligned} \quad (11)$$

2.2. Problem Formulation. The aim of this paper is to develop an H_{∞} miscellaneous information feedback control scheme to stabilize a two DOF flexible active suspension system. The control structure of the scheme is shown in Figure 2. Clearly, the measured sensor signals are divided into two parts: $\mathbf{x}(t)$ and $\mathbf{x}(t-d(t))$. The first part can be directly measured by onboard sensors, and the remaining signals can be transmitted by the communication network. Note that the network-induced delay $d(t)$ must be considered in the control design. Therefore, a miscellaneous information feedback controller is designed as

FIGURE 2: Miscellaneous information feedback control diagram.

$$\mathbf{u}(t) = \mathbf{K}_1 \mathbf{x}(t) + \mathbf{K}_2 \mathbf{x}(t - d(t)), \quad (12)$$

where $\mathbf{K}_1$ and $\mathbf{K}_2$ denote the controller gains to be determined.

The network-induced delay contains the sensor-to-controller delay and controller-to-actuator delay. A natural assumption on the network-induced delay can be given as

$$\begin{aligned} 0 &\leq d(t) \leq h, \\ \mu_1 &\leq \dot{d}(t) \leq \mu_2, \end{aligned} \quad (13)$$

where h is the maximum value of $d(t)$ and μ_1 and μ_2 are the respective lower and upper bounds of $\dot{d}(t)$.

Substituting (12) into (4) and (10), a miscellaneous feedback control system can be obtained as follows:

$$\begin{aligned} \dot{\mathbf{x}}(t) &= \mathbf{A}\mathbf{x}(t) + \mathbf{B}\mathbf{K}_1 \mathbf{x}(t) + \mathbf{B}\mathbf{K}_2 \mathbf{x}(t - d(t)) + \mathbf{B}_1 \mathbf{f}(t), \\ \mathbf{z}_1(t) &= \mathbf{C}_1 \mathbf{x}(t) + \mathbf{D}_1 \mathbf{K}_1 \mathbf{x}(t) + \mathbf{D}_1 \mathbf{K}_2 \mathbf{x}(t - d(t)), \\ \mathbf{z}_2(t) &= \mathbf{C}_2 \mathbf{x}(t), \\ \mathbf{z}_3(t) &= \mathbf{K}_1 \mathbf{x}(t) + \mathbf{K}_2 \mathbf{x}(t - d(t)), \\ \mathbf{x}(\theta) &= \mathbf{0}, \quad \theta \in [-h, 0]. \end{aligned} \quad (14)$$

In summary, the aim is to design an H_∞ miscellaneous information feedback controller in the form of (12), such that system (14) has the following properties:

- (1) The system (14) is asymptotically stable for $\mathbf{f}(t) = \mathbf{0}$.
- (2) Under zero initial condition, the suspension system has an H_∞ constraint as follows:

$$\|\mathbf{z}_1(t)\| \leq \gamma \|\mathbf{f}(t)\|, \quad (15)$$

for a given scalar, $\gamma > 0$.

- (3) Under the prescribed disturbance energy bound, the following conditions are met:

$$|\{\mathbf{z}_2(t)\}_q| \leq 1, |\mathbf{z}_3(t)| \leq 1, \quad q = 1, 2. \quad (16)$$

Before proceeding, some useful lemmas are presented.

Lemma 1 [31, 32]. For a given matrix $\mathbf{R} > 0$ with appropriate dimension, symmetric matrices $\mathbf{X}_1 \in \mathfrak{R}^{3n \times 3n}$ and $\mathbf{X}_2 \in \mathfrak{R}^{3n \times 3n}$, and general matrices $\mathbf{Y}_1 \in \mathfrak{R}^{3n \times 3n}$ and $\mathbf{Y}_2 \in \mathfrak{R}^{3n \times 3n}$, if the following expressions hold

$$\begin{aligned} \begin{bmatrix} \tilde{\mathbf{R}} - \mathbf{X}_1 & \mathbf{Y}_1 \\ \mathbf{Y}_1^T & \tilde{\mathbf{R}} \end{bmatrix} &\geq 0, \\ \begin{bmatrix} \tilde{\mathbf{R}} & \mathbf{Y}_2 \\ \mathbf{Y}_2^T & \tilde{\mathbf{R}} - \mathbf{X}_2 \end{bmatrix} &\geq 0, \end{aligned} \quad (17)$$

then

$$\aleph(t) \geq \xi^T(t) \Upsilon_0 \xi(t), \quad (18)$$

where

$$\begin{aligned} \aleph(t) &= h \int_{t-d(t)}^t \dot{\mathbf{x}}^T(s) \mathbf{R} \dot{\mathbf{x}}(s) ds + h \int_{t-h}^{t-d(t)} \dot{\mathbf{x}}^T(s) \mathbf{R} \dot{\mathbf{x}}(s) ds, \\ \xi(t) &= \text{col}\{\mathbf{x}(t), \mathbf{x}(t-d(t)), \mathbf{x}(t-h), \boldsymbol{\rho}_1(t), \\ &\quad \boldsymbol{\rho}_2(t), \boldsymbol{\rho}_3(t), \boldsymbol{\rho}_4(t), \mathbf{f}(t), \dot{\mathbf{x}}(t)\}, \\ \Upsilon_0 &= \Gamma_1^T \tilde{\mathbf{R}} \Gamma_1 + (1-a) \Gamma_1^T \mathbf{X}_1 \Gamma_1 + \Gamma_2^T \tilde{\mathbf{R}} \Gamma_2 + a \Gamma_2^T \mathbf{X}_2 \Gamma_2 \\ &\quad + \Gamma_1^T [a \mathbf{Y}_1 + (1-a) \mathbf{Y}_2] \Gamma_2, \\ \Gamma_1 &= \text{col}\{\mathbf{e}_2 - \mathbf{e}_3, \mathbf{e}_2 + \mathbf{e}_3 - 2\mathbf{e}_4, \mathbf{e}_2 - \mathbf{e}_3 - 6\mathbf{e}_4 + 12\mathbf{e}_5\}, \\ \Gamma_2 &= \text{col}\{\mathbf{e}_1 - \mathbf{e}_2, \mathbf{e}_1 + \mathbf{e}_2 - 2\mathbf{e}_6, \mathbf{e}_1 - \mathbf{e}_2 - 6\mathbf{e}_6 + 12\mathbf{e}_7\}, \\ \tilde{\mathbf{R}} &= \text{diag}\{\mathbf{R}, 3\mathbf{R}, 5\mathbf{R}\}, \\ a &= \frac{(h-d(t))}{h}, \end{aligned} \quad (19)$$

with

$$\begin{aligned} \boldsymbol{\rho}_1(t) &= \int_{t-h}^{t-d(t)} \frac{\mathbf{x}(s)}{h-d(t)} ds, \\ \boldsymbol{\rho}_2(t) &= \int_{t-h}^{t-d(t)} \frac{(t-d(t)-s)\mathbf{x}(s)}{(h-d(t))^2} ds, \\ \boldsymbol{\rho}_3(t) &= \int_{t-d(t)}^t \frac{\mathbf{x}(s)}{d(t)} ds, \\ \boldsymbol{\rho}_4(t) &= \int_{t-d(t)}^t \frac{(t-s)\mathbf{x}(s)}{d^2(t)} ds, \end{aligned} \quad (20)$$

$$\mathbf{e}_i = [\mathbf{0}_{n \times (i-1)n}, \mathbf{I}, \mathbf{0}_{n \times (9-i)n}], \quad i = 1, 2, \dots, 9.$$

Lemma 2 [33]. For a function $\varphi(d) = d^2 \mathbf{g}_1 + d \mathbf{g}_2 + \mathbf{g}_3$, where $\mathbf{g}_i \in \mathfrak{R}$ ($i = 1, 2, 3$), $\varphi(d) < 0$ holds for all of the delay functions satisfying $d \in [0, h]$ if and only if $\varphi(0) < 0$, $\varphi(h) < 0$, and $-h^2 \mathbf{g}_1 + \varphi(0) < 0$.

3. Main Results

3.1. H_∞ Miscellaneous Information Feedback Controller Design. We are now in the position to design an H_∞ miscellaneous information feedback controller for the suspension system in (14). A computable criterion is presented to ensure the required conditions, as follows.

Theorem 1. For some prescribed scalars $h, \mu, \rho, \gamma, \vartheta_1$, and ϑ_2 , assume that there exist symmetric matrices $\mathbf{P} \in \mathfrak{R}^{3n \times 3n}$ ($\mathbf{P} > 0$), $\mathbf{Q} \in \mathfrak{R}^{3n \times 3n}$ ($\mathbf{Q} > 0$), $\mathbf{R} > 0$, $\mathbf{X}_1, \mathbf{X}_2$, and general matrices $\mathbf{X}, \mathbf{K}_1, \mathbf{K}_2, \mathbf{Y}_1, \mathbf{Y}_2$, such that

$$\begin{bmatrix} \Xi_0|_{\alpha=0} & \varepsilon_2^T \\ * & -\mathbf{I} \end{bmatrix} < 0, \quad (21)$$

$$\begin{bmatrix} \Xi_0|_{\alpha=1} & \varepsilon_2^T \\ * & -\mathbf{I} \end{bmatrix} < 0, \quad (22)$$

$$\begin{bmatrix} -h^2 \mathbf{g}_1 + \Xi_0|_{\alpha=1} & \varepsilon_2^T \\ * & -\mathbf{I} \end{bmatrix} < 0, \quad (23)$$

$$\begin{bmatrix} h\mathbf{Q}_{22} & \{\mathbf{C}_2\}_i^T \\ * & \frac{1}{\rho} \end{bmatrix} > 0, \quad i = 1, 2, \quad (24)$$

$$\mathbf{P}_{22} > \mathbf{Q}_{22}, \quad (25)$$

$$\begin{bmatrix} \tilde{\mathbf{R}} - \mathbf{X}_1 & \mathbf{Y}_1 \\ * & \tilde{\mathbf{R}} \end{bmatrix} > 0, \quad (26)$$

$$\begin{bmatrix} \tilde{\mathbf{R}} & \mathbf{Y}_2 \\ * & \tilde{\mathbf{R}} - \mathbf{X}_2 \end{bmatrix} > 0,$$

$$\begin{bmatrix} -\frac{h\mathbf{Q}_{22}}{2\rho} & \mathbf{K}_1^T & \mathbf{K}_2^T \\ * & -\mathbf{I} & \mathbf{0} \\ * & * & -\mathbf{I} \end{bmatrix} < 0, \quad (27)$$

where

$$\begin{aligned} \Xi_0|_{\alpha=0} &= [\mathbf{L}_1 + h\mathbf{L}_2]^T \mathbf{P} [\mathbf{L}_1 + h\mathbf{L}_2] - \mathbf{L}_4^T \mathbf{Q} \mathbf{L}_4 - (1 - \dot{d}(t)) \mathbf{L}_3^T \mathbf{P} \mathbf{L}_3 \\ &\quad + (1 - \dot{d}(t)) \mathbf{L}_5^T \mathbf{Q} \mathbf{L}_5 + 2\mathbf{L}_9^T \mathbf{P} [h\mathbf{L}_7 + h^2 \mathbf{L}_8] \\ &\quad - \mathbf{e}_8^T \gamma^2 \mathbf{I} \mathbf{e}_8 + 2\tilde{\mathbf{X}}^T \tilde{\mathbf{E}} + h^2 \mathbf{e}_9^T \mathbf{R} \mathbf{e}_9 - \Upsilon_0|_{\alpha=0}, \end{aligned}$$

$$\begin{aligned} \Xi_0|_{\alpha=1} &= \mathbf{L}_1^T \mathbf{P} \mathbf{L}_1 - \mathbf{L}_4^T \mathbf{Q} \mathbf{L}_4 - (1 - \dot{d}(t)) \mathbf{L}_3^T \mathbf{P} \mathbf{L}_3 \\ &\quad + (1 - \dot{d}(t)) [\mathbf{L}_5 + h\mathbf{L}_6]^T \mathbf{Q} [\mathbf{L}_5 + h\mathbf{L}_6] \\ &\quad + 2\mathbf{L}_{12}^T \mathbf{Q} [h\mathbf{L}_{10} + h^2 \mathbf{L}_{11}] - \mathbf{e}_8^T \gamma^2 \mathbf{I} \mathbf{e}_8 \\ &\quad + 2\tilde{\mathbf{X}}^T \tilde{\mathbf{E}} + h^2 \mathbf{e}_9^T \mathbf{R} \mathbf{e}_9 - \Upsilon_0|_{\alpha=1}, \end{aligned}$$

$$\tilde{\mathbf{X}} = \mathbf{X} \mathbf{e}_9 + \vartheta_1 \mathbf{X} \mathbf{e}_1 + \vartheta_2 \mathbf{X} \mathbf{e}_2,$$

$$\tilde{\mathbf{E}} = -\mathbf{e}_9 + \mathbf{A} \mathbf{e}_1 + \mathbf{B} \mathbf{K}_1 \mathbf{e}_1 + \mathbf{B} \mathbf{K}_2 \mathbf{e}_2 + \mathbf{B}_1 \mathbf{e}_8,$$

$$\Upsilon_0|_{\alpha=0} = \Gamma_1^T \tilde{\mathbf{R}} \Gamma_1 + \Gamma_1^T \mathbf{X}_1 \Gamma_1 + \Gamma_2^T \tilde{\mathbf{R}} \Gamma_2 + 2\Gamma_1^T \mathbf{Y}_2 \Gamma_2,$$

$$\Upsilon_0|_{\alpha=1} = \Gamma_1^T \tilde{\mathbf{R}} \Gamma_1 + \Gamma_2^T \tilde{\mathbf{R}} \Gamma_2 + \Gamma_2^T \mathbf{X}_2 \Gamma_2 + 2\Gamma_1^T \mathbf{Y}_1 \Gamma_2,$$

$$\mathbf{g}_1 = \mathbf{L}_2^T \mathbf{P} \mathbf{L}_2 + (1 - \dot{d}(t)) \mathbf{L}_6^T \mathbf{Q} \mathbf{L}_6 + 2\mathbf{L}_9^T \mathbf{P} \mathbf{L}_8 + 2\mathbf{L}_{12}^T \mathbf{Q} \mathbf{L}_{11},$$

$$\mathbf{P} = \begin{bmatrix} \mathbf{P}_{11} & \mathbf{P}_{12} & \mathbf{P}_{13} \\ * & \mathbf{P}_{22} & \mathbf{P}_{23} \\ * & * & \mathbf{P}_{33} \end{bmatrix},$$

$$\mathbf{Q} = \begin{bmatrix} \mathbf{Q}_{11} & \mathbf{Q}_{12} & \mathbf{Q}_{13} \\ * & \mathbf{Q}_{22} & \mathbf{Q}_{23} \\ * & * & \mathbf{Q}_{33} \end{bmatrix},$$

$$\mathbf{e}_2 = (\mathbf{C}_1 \mathbf{e}_1 + \mathbf{D}_1 \mathbf{K}_1 \mathbf{e}_1 + \mathbf{D}_1 \mathbf{K}_2 \mathbf{e}_2),$$

$$\mathbf{L}_1 = \text{col}\{\mathbf{e}_1, \mathbf{e}_1, \mathbf{0}\},$$

$$\mathbf{L}_2 = \text{col}\{\mathbf{0}, \mathbf{0}, \mathbf{e}_6\},$$

$$\mathbf{L}_3 = \text{col}\{\mathbf{e}_2, \mathbf{e}_1, \mathbf{0}\},$$

$$\mathbf{L}_4 = \text{col}\{\mathbf{e}_3, \mathbf{e}_1, \mathbf{0}\},$$

$$\mathbf{L}_5 = \text{col}\{\mathbf{e}_2, \mathbf{e}_1, \mathbf{0}\},$$

$$\mathbf{L}_6 = \text{col}\{\mathbf{0}, \mathbf{0}, \mathbf{e}_4\},$$

$$\mathbf{L}_7 = \text{col}\{\mathbf{e}_6, \mathbf{e}_1, \mathbf{0}\},$$

$$\mathbf{L}_8 = \text{col}\{\mathbf{0}, \mathbf{0}, \mathbf{e}_7\},$$

$$\mathbf{L}_9 = \text{col}\{\mathbf{0}, \mathbf{e}_9, (\dot{d}(t) - 1)\mathbf{e}_2\},$$

$$\mathbf{L}_{10} = \text{col}\{\mathbf{e}_4, \mathbf{e}_1, \mathbf{0}\},$$

$$\mathbf{L}_{11} = \text{col}\{\mathbf{0}, \mathbf{0}, \mathbf{e}_5\},$$

$$\mathbf{L}_{12} = \text{col}\{\mathbf{0}, \mathbf{e}_9, -\mathbf{e}_3\}.$$

(28)

Then, the system (14) is asymptotically stable subject to an H_∞ constraint (15), and the condition (16) is satisfied under $\kappa = (\rho - V(0))/\gamma^2$.

Proof. Choose an LKF candidate as

$$V(t) = \int_{t-d(t)}^t \boldsymbol{\eta}_1^T(t, s) \mathbf{P} \boldsymbol{\eta}_1(t, s) ds + \int_{t-h}^{t-d(t)} \boldsymbol{\eta}_2^T(t, s) \mathbf{Q} \boldsymbol{\eta}_2(t, s) ds + h \int_{-h}^0 \int_{t+\theta}^t \dot{\mathbf{x}}^T(s) \mathbf{R} \dot{\mathbf{x}}(s) ds d\theta, \quad (29)$$

where

$$\begin{aligned} \boldsymbol{\eta}_1(t, s) &= \begin{bmatrix} \mathbf{x}(s) \\ \mathbf{x}(t) \\ \int_{t-d(t)}^s \mathbf{x}(\theta) d\theta \end{bmatrix}, \\ \boldsymbol{\eta}_2(t, s) &= \begin{bmatrix} \mathbf{x}(s) \\ \mathbf{x}(t) \\ \int_{t-h}^s \mathbf{x}(\theta) d\theta \end{bmatrix}, \end{aligned} \quad (30)$$

$$\begin{aligned} \mathbf{P} &> 0, \\ \mathbf{Q} &> 0, \\ \mathbf{R} &> 0. \end{aligned}$$

Along with the trajectory of the system (14), the derivative of the chosen LKF can be obtained as

$$\begin{aligned} \dot{V}(t) &= \boldsymbol{\eta}_1^T(t, t) \mathbf{P} \boldsymbol{\eta}_1(t, t) - \boldsymbol{\eta}_2^T(t, t-h) \mathbf{Q} \boldsymbol{\eta}_2(t, t-h) \\ &\quad - (1 - \dot{d}(t)) \boldsymbol{\eta}_1^T(t, t-d(t)) \mathbf{P} \boldsymbol{\eta}_1(t, t-d(t)) \\ &\quad + (1 - \dot{d}(t)) \boldsymbol{\eta}_2^T(t, t-d(t)) \mathbf{Q} \boldsymbol{\eta}_2(t, t-d(t)) \\ &\quad + \int_{t-d(t)}^t 2 \boldsymbol{\eta}_1^T(t, s) \mathbf{P} \frac{\partial \boldsymbol{\eta}_1(t, s)}{\partial t} ds \\ &\quad + \int_{t-h}^{t-d(t)} 2 \boldsymbol{\eta}_2^T(t, s) \mathbf{Q} \frac{\partial \boldsymbol{\eta}_2(t, s)}{\partial t} ds \\ &\quad + h^2 \dot{\mathbf{x}}^T(t) \mathbf{R} \dot{\mathbf{x}}(t) - h \int_{t-d(t)}^t \dot{\mathbf{x}}^T(s) \mathbf{R} \dot{\mathbf{x}}(s) ds \\ &\quad - h \int_{t-h}^{t-d(t)} \dot{\mathbf{x}}^T(s) \mathbf{R} \dot{\mathbf{x}}(s) ds. \end{aligned} \quad (31)$$

For simplicity, denote

$$\begin{aligned} \boldsymbol{\xi}(t) &= \text{col}\{\mathbf{x}(t), \mathbf{x}(t-d(t)), \mathbf{x}(t-h), \boldsymbol{\rho}_1(t), \boldsymbol{\rho}_2(t), \\ &\quad \boldsymbol{\rho}_3(t), \boldsymbol{\rho}_4(t), \mathbf{f}(t), \dot{\mathbf{x}}(t)\}. \end{aligned} \quad (32)$$

Note that

$$\begin{aligned} &2 \boldsymbol{\xi}^T(t) \tilde{\mathbf{X}}^T [-\dot{\mathbf{x}}(t) + \mathbf{A} \mathbf{x}(t) + \mathbf{B} \mathbf{K}_1 \mathbf{x}(t) + \mathbf{B} \mathbf{K}_2 \mathbf{x}(t-d(t)) + \mathbf{B}_1 \mathbf{f}(t)] \\ &= 2 \boldsymbol{\xi}^T(t) \tilde{\mathbf{X}}^T \tilde{\mathbf{E}} \boldsymbol{\xi}(t) = 0, \end{aligned} \quad (33)$$

where

$$\begin{aligned} \tilde{\mathbf{X}} &= \mathbf{X} \mathbf{e}_9 + \vartheta_1 \mathbf{X} \mathbf{e}_1 + \vartheta_2 \mathbf{X} \mathbf{e}_2, \\ \tilde{\mathbf{E}} &= -\mathbf{e}_9 + \mathbf{A} \mathbf{e}_1 + \mathbf{B} \mathbf{K}_1 \mathbf{e}_1 + \mathbf{B} \mathbf{K}_2 \mathbf{e}_2 + \mathbf{B}_1 \mathbf{e}_8. \end{aligned} \quad (34)$$

From (31)–(33) and Lemma 1, one obtains

$$\begin{aligned} \dot{V}(t) + \mathbf{z}_1^T(t) \mathbf{z}_1(t) - \gamma^2 \mathbf{f}^T(t) \mathbf{f}(t) \\ \leq \boldsymbol{\xi}^T(t) (\mathbf{\Gamma}_0 + \boldsymbol{\varepsilon}_2^T \boldsymbol{\varepsilon}_2) \boldsymbol{\xi}(t), \\ = \boldsymbol{\xi}^T(t) [d^2(t) \mathbf{g}_1 + d(t) \mathbf{g}_2 + \mathbf{g}_3] \boldsymbol{\xi}(t), \end{aligned} \quad (35)$$

where

$$\begin{aligned} \Xi_0 &= [\mathbf{L}_1 + d(t) \mathbf{L}_2]^T \mathbf{P} [\mathbf{L}_1 + d(t) \mathbf{L}_2] - \mathbf{L}_4^T \mathbf{Q} \mathbf{L}_4 \\ &\quad - (1 - \dot{d}(t)) \mathbf{L}_3^T \mathbf{P} \mathbf{L}_3 + (1 - \dot{d}(t)) [\mathbf{L}_5 + (h - d(t)) \mathbf{L}_6]^T \\ &\quad \cdot \mathbf{Q} [\mathbf{L}_5 + (h - d(t)) \mathbf{L}_6], \\ &\quad + 2 \mathbf{L}_9^T \mathbf{P} [d(t) \mathbf{L}_7 + d^2(t) \mathbf{L}_8] \\ &\quad + 2 \mathbf{L}_{12}^T \mathbf{Q} [(h - d(t)) \mathbf{L}_{10} + (h - d(t))^2 \mathbf{L}_{11}], \\ &\quad + 2 \tilde{\mathbf{X}}^T \tilde{\mathbf{E}} - \mathbf{Y}_0 - \mathbf{e}_8^T \gamma^2 \mathbf{I} \mathbf{e}_8 + h^2 \mathbf{e}_9^T \mathbf{R} \mathbf{e}_9, \\ \mathbf{Y}_0 &= \mathbf{\Gamma}_1^T \tilde{\mathbf{R}} \mathbf{\Gamma}_1 + (1 - a) \mathbf{\Gamma}_1^T \mathbf{X}_1 \mathbf{\Gamma}_1 + \mathbf{\Gamma}_2^T \tilde{\mathbf{R}} \mathbf{\Gamma}_2 + a \mathbf{\Gamma}_2^T \mathbf{X}_2 \mathbf{\Gamma}_2 \\ &\quad + \mathbf{\Gamma}_1^T [a \mathbf{Y}_1 + (1 - a) \mathbf{Y}_2] \mathbf{\Gamma}_2, \\ \tilde{\mathbf{R}} &= \text{diag}\{\mathbf{R}, 3\mathbf{R}, 5\mathbf{R}\}, \\ \boldsymbol{\varepsilon}_2 &= (\mathbf{C}_1 \mathbf{e}_1 + \mathbf{D}_1 \mathbf{K}_1 \mathbf{e}_1 + \mathbf{D}_1 \mathbf{K}_2 \mathbf{e}_2), \\ \mathbf{g}_1 &= \mathbf{L}_2^T \mathbf{P} \mathbf{L}_2 + (1 - \dot{d}(t)) \mathbf{L}_6^T \mathbf{Q} \mathbf{L}_6 + 2 \mathbf{L}_9^T \mathbf{P} \mathbf{L}_8 + 2 \mathbf{L}_{12}^T \mathbf{Q} \mathbf{L}_{11}. \end{aligned} \quad (36)$$

According to Lemma 2, if (21)–(23) are satisfied, then $\dot{V}(t) + \mathbf{z}_1^T(t) \mathbf{z}_1(t) - \gamma^2 \mathbf{f}^T(t) \mathbf{f}(t) < 0$ holds for all of the delay functions satisfying $d(t) \in [0, h]$. A zero initial state has $\|\mathbf{z}_1(t)\| < \gamma \|\mathbf{f}(t)\|$, which implies that an H_∞ constraint is obtained. When $\mathbf{f}(t) = 0$, then $\dot{V}(t) < 0$ can be easily obtained by following similar lines to those above. This indicates that the system is also asymptotically stable.

From (29) and the above, we obtain that $h \mathbf{x}^T(t) \mathbf{Q}_{22} \mathbf{x}(t) + d(t) \mathbf{x}^T(t) (\mathbf{P}_{22} - \mathbf{Q}_{22}) \mathbf{x}(t) < \rho$, where $\rho = \gamma^2 \kappa + V(0)$. Therefore, we can deduce that $h \mathbf{x}^T(t) \mathbf{Q}_{22} \mathbf{x}(t) < \rho$ if (47) holds.

From (10) and (16), we obtain the following inequality:

$$\max_{t>0} \|\{\mathbf{z}_2(t)\}_i\|^2 < \frac{\rho}{h} \cdot \theta_{\max} \left(\hat{\mathbf{Q}}_{22}^{-1/2} \{\mathbf{C}_2\}_i^T \{\mathbf{C}_2\}_i \hat{\mathbf{Q}}_{22}^{-1/2} \right) \leq \mathbf{I}, \quad i = 1, 2, \quad (37)$$

which leads to

$$\begin{cases} \frac{\rho}{h} \cdot \hat{\mathbf{Q}}_{22}^{-1/2} \{\mathbf{C}_2\}_i^T \{\mathbf{C}_2\}_i \hat{\mathbf{Q}}_{22}^{-1/2} < \mathbf{I}, & i = 1, 2, \\ \kappa = (\rho - V(0))/\gamma^2, \end{cases} \quad (38)$$

where $\theta_{\max}(\cdot)$ represents maximal eigenvalue. Using the Schur complement, (46) is equivalent to (38).

According to Young's inequality [18], we have

$$\begin{aligned} \max_{t \geq 0} |u(t)|^2 &< \frac{2\rho}{h} \theta_{\max}(\mathbf{Q}_{22}^{-1/2} \mathbf{K}_1^T \mathbf{K}_1 \mathbf{Q}_{22}^{-1/2}) \\ &+ \frac{2\rho}{h} \theta_{\max}(\mathbf{Q}_{22}^{-1/2} \mathbf{K}_2^T \mathbf{K}_2 \mathbf{Q}_{22}^{-1/2}) \leq \mathbf{I}, \end{aligned} \quad (39)$$

which leads to

$$\begin{cases} \mathbf{K}_1^T \mathbf{K}_1 + \mathbf{K}_2^T \mathbf{K}_2 < \frac{h}{2\rho} \mathbf{Q}_{22}, \\ \kappa = (\rho - V(0))/\gamma^2. \end{cases} \quad (40)$$

Moreover, the aforementioned inequality is equivalent to (49). Hence, the constraints in (16) hold. This completes the proof. $\square$

Remark 1. Note that conditions (43)–(49) are all affine, and consequently convex with respect to $\hat{d}(t)$, and thus can be described by a given polytope with the two vertices $-\mu$ and μ .

Since the theorem contains coupling terms $\mathbf{X}^T \mathbf{B} \mathbf{K}_1$ and $\mathbf{X}^T \mathbf{B} \mathbf{K}_2$, the control gains are difficult to obtain directly by LMI solvers. Congruence transformations are used to transform this nonlinear theorem to a linear one.

Denote

$$\begin{aligned} \Delta_j &= \text{diag} \left\{ \underbrace{\mathbf{X}^{-1} \cdots \mathbf{X}^{-1}}_7 \quad \mathbf{I} \quad \mathbf{X}^{-1} \quad \mathbf{I} \right\}, \quad j = 1, 2, 3, \\ \Delta_4 &= \text{diag} \{ \mathbf{X}^{-1} \quad \mathbf{I} \}, \\ \Delta_5 &= \mathbf{X}^{-1}, \\ \Delta_6 &= \text{diag} \left\{ \underbrace{\mathbf{X}^{-1} \cdots \mathbf{X}^{-1}}_6 \right\}, \\ \Delta_7 &= \text{diag} \{ \mathbf{X}^{-1} \quad \mathbf{I} \quad \mathbf{I} \}, \\ \Delta &= \text{diag} \{ \mathbf{X}^{-1} \quad \mathbf{X}^{-1} \quad \mathbf{X}^{-1} \}. \end{aligned} \quad (41)$$

Perform a congruence transformation with Δ_p ($p = 1, 2, \dots, 7$) into (43)–(49) and set

$$\begin{aligned} \mathbf{X} &= \hat{\mathbf{X}}^{-1}, \\ \hat{\mathbf{K}}_1 &= \mathbf{K}_1 \hat{\mathbf{X}}, \\ \hat{\mathbf{K}}_2 &= \mathbf{K}_2 \hat{\mathbf{X}}, \\ \hat{\mathbf{R}} &= \mathbf{X}^{-T} \mathbf{R} \mathbf{X}^{-1}, \\ \hat{\hat{\mathbf{R}}} &= \Delta^T \hat{\mathbf{R}} \Delta, \\ \hat{\mathbf{X}}_1 &= \Delta^T \mathbf{X}_1 \Delta, \\ \hat{\mathbf{X}}_2 &= \Delta^T \mathbf{X}_2 \Delta, \\ \hat{\mathbf{Y}}_1 &= \Delta^T \mathbf{Y}_1 \Delta, \\ \hat{\mathbf{Y}}_2 &= \Delta^T \mathbf{Y}_2 \Delta, \\ \hat{\mathbf{P}} &= \Delta^T \mathbf{P} \Delta, \\ \mathbf{Q} &= \Delta^T \hat{\mathbf{Q}} \Delta. \end{aligned} \quad (42)$$

Then, a linear minimization theorem can be presented, as follows.

Theorem 2. For some prescribed scalars $h, \mu, \rho, \gamma, \vartheta_1$, and ϑ_2 , assume that there exist symmetric matrices $\hat{\mathbf{P}} \in \mathfrak{R}^{3n \times 3n}$ ($\hat{\mathbf{P}} > 0$), $\hat{\mathbf{Q}} \in \mathfrak{R}^{3n \times 3n}$ ($\hat{\mathbf{Q}} > 0$), $\hat{\mathbf{R}} > 0$, $\hat{\mathbf{X}}_1, \hat{\mathbf{X}}_2$, and general matrices $\hat{\mathbf{X}}, \hat{\mathbf{K}}_1, \hat{\mathbf{K}}_2, \hat{\mathbf{Y}}_1, \hat{\mathbf{Y}}_2$, such that

$$\begin{bmatrix} \hat{\mathbf{E}}_0|_{a=0} & \hat{\mathbf{E}}_2^T \\ * & -\mathbf{I} \end{bmatrix} < 0, \quad (43)$$

$$\begin{bmatrix} \hat{\mathbf{E}}_0|_{a=1} & \hat{\mathbf{E}}_2^T \\ * & -\mathbf{I} \end{bmatrix} < 0, \quad (44)$$

$$\begin{bmatrix} -h^2 \hat{\mathbf{g}}_1 + \hat{\mathbf{E}}_0|_{a=1} & \hat{\mathbf{E}}_2^T \\ * & -\mathbf{I} \end{bmatrix} < 0, \quad (45)$$

$$\begin{bmatrix} h \hat{\mathbf{Q}}_{22} & \hat{\mathbf{X}}^T \{\mathbf{C}_2\}_i^T \\ * & \frac{1}{\rho} \end{bmatrix} > 0, \quad i = 1, 2, \quad (46)$$

$$\hat{\mathbf{P}}_{22} > \hat{\mathbf{Q}}_{22}, \quad (47)$$

$$\begin{bmatrix} \hat{\hat{\mathbf{R}}} - \hat{\mathbf{X}}_1 & \hat{\mathbf{Y}}_1 \\ * & \hat{\hat{\mathbf{R}}} \end{bmatrix} > 0, \quad (48)$$

$$\begin{bmatrix} \hat{\hat{\mathbf{R}}} & \hat{\mathbf{Y}}_2 \\ * & \hat{\hat{\mathbf{R}}} - \hat{\mathbf{X}}_2 \end{bmatrix} > 0,$$

$$\begin{bmatrix} -\frac{h \hat{\mathbf{Q}}_{22}}{2\rho} & \hat{\mathbf{K}}_1^T & \hat{\mathbf{K}}_2^T \\ * & -\mathbf{I} & \mathbf{0} \\ * & * & -\mathbf{I} \end{bmatrix} < 0, \quad (49)$$

where

$$\begin{aligned}
\widehat{\Xi}_0|_{\alpha=0} &= [\mathbf{L}_1 + h\mathbf{L}_2]^T \widehat{\mathbf{P}} [\mathbf{L}_1 + h\mathbf{L}_2] - \mathbf{L}_4^T \widehat{\mathbf{Q}} \mathbf{L}_4 - (1 - \dot{d}(t)) \mathbf{L}_3^T \widehat{\mathbf{P}} \mathbf{L}_3 \\
&\quad + (1 - \dot{d}(t)) \mathbf{L}_5^T \widehat{\mathbf{Q}} \mathbf{L}_5 + 2\mathbf{L}_9^T \widehat{\mathbf{P}} [h\mathbf{L}_7 + h^2\mathbf{L}_8] - \mathbf{e}_8^T \gamma^2 \mathbf{I} \mathbf{e}_8 \\
&\quad + 2\Pi + h^2 \mathbf{e}_9^T \widehat{\mathbf{R}} \mathbf{e}_9 - \widehat{\mathbf{Y}}_0|_{\alpha=0}, \\
\widehat{\Xi}_0|_{\alpha=1} &= \mathbf{L}_1^T \widehat{\mathbf{P}} \mathbf{L}_1 - \mathbf{L}_4^T \widehat{\mathbf{Q}} \mathbf{L}_4 - (1 - \dot{d}(t)) \mathbf{L}_3^T \widehat{\mathbf{P}} \mathbf{L}_3 \\
&\quad + (1 - \dot{d}(t)) [\mathbf{L}_5 + h\mathbf{L}_6]^T \widehat{\mathbf{Q}} [\mathbf{L}_5 + h\mathbf{L}_6] \\
&\quad + 2\mathbf{L}_{12}^T \widehat{\mathbf{Q}} [h\mathbf{L}_{10} + h^2\mathbf{L}_{11}] - \mathbf{e}_8^T \gamma^2 \mathbf{I} \mathbf{e}_8 + 2\Pi \\
&\quad + h^2 \mathbf{e}_9^T \widehat{\mathbf{R}} \mathbf{e}_9 - \widehat{\mathbf{Y}}_0|_{\alpha=1}, \\
\Pi &= \mathbf{e}_1^T (\vartheta_1 \mathbf{A} \widehat{\mathbf{X}} + \vartheta_1 \mathbf{B} \widehat{\mathbf{K}}_1) \mathbf{e}_1 \\
&\quad + \mathbf{e}_1^T (\vartheta_1 \mathbf{B} \widehat{\mathbf{K}}_2 + \vartheta_2 \widehat{\mathbf{X}}^T \mathbf{A}^T + \vartheta_2 \widehat{\mathbf{K}}_1^T \mathbf{B}^T) \mathbf{e}_1 \\
&\quad + \vartheta_2 \mathbf{e}_2^T \mathbf{B} \widehat{\mathbf{K}}_2 \mathbf{e}_2 + \vartheta_1 \mathbf{e}_1^T \mathbf{B}_1 \mathbf{e}_8 + \vartheta_2 \mathbf{e}_2^T \mathbf{B}_1 \mathbf{e}_8 \\
&\quad + \mathbf{e}_1^T (\widehat{\mathbf{X}}^T \mathbf{A}^T + \widehat{\mathbf{K}}_1^T \mathbf{B}^T - \vartheta_1 \widehat{\mathbf{X}}) \mathbf{e}_9 \\
&\quad + \mathbf{e}_2^T (\widehat{\mathbf{K}}_2^T \mathbf{B}^T - \vartheta_2 \widehat{\mathbf{X}}) \mathbf{e}_9 + \mathbf{e}_8^T \mathbf{B}_1 \mathbf{e}_9 - \mathbf{e}_9^T \widehat{\mathbf{X}} \mathbf{e}_9, \\
\widehat{\mathbf{Y}}_0|_{\alpha=0} &= \Gamma_1^T \widehat{\mathbf{R}} \Gamma_1 + \Gamma_1^T \widehat{\mathbf{X}}_1 \Gamma_1 + \Gamma_2^T \widehat{\mathbf{R}} \Gamma_2 + 2\Gamma_1^T \widehat{\mathbf{Y}}_2 \Gamma_2, \\
\widehat{\mathbf{Y}}_0|_{\alpha=1} &= \Gamma_1^T \widehat{\mathbf{R}} \Gamma_1 + \Gamma_2^T \widehat{\mathbf{R}} \Gamma_2 + \Gamma_2^T \widehat{\mathbf{X}}_2 \Gamma_2 + 2\Gamma_1^T \widehat{\mathbf{Y}}_1 \Gamma_2, \\
\widehat{\mathbf{g}}_1 &= \mathbf{L}_2^T \widehat{\mathbf{P}} \mathbf{L}_2 + (1 - \dot{d}(t)) \mathbf{L}_6^T \widehat{\mathbf{Q}} \mathbf{L}_6 + 2\mathbf{L}_9^T \widehat{\mathbf{P}} \mathbf{L}_8 + 2\mathbf{L}_{12}^T \widehat{\mathbf{Q}} \mathbf{L}_{11}, \\
\widehat{\mathbf{P}} &= \begin{bmatrix} \widehat{\mathbf{P}}_{11} & \widehat{\mathbf{P}}_{12} & \widehat{\mathbf{P}}_{13} \\ * & \widehat{\mathbf{P}}_{22} & \widehat{\mathbf{P}}_{23} \\ * & * & \widehat{\mathbf{P}}_{33} \end{bmatrix}, \\
\widehat{\mathbf{Q}} &= \begin{bmatrix} \widehat{\mathbf{Q}}_{11} & \widehat{\mathbf{Q}}_{12} & \widehat{\mathbf{Q}}_{13} \\ * & \widehat{\mathbf{Q}}_{22} & \widehat{\mathbf{Q}}_{23} \\ * & * & \widehat{\mathbf{Q}}_{33} \end{bmatrix}, \\
\widehat{\mathbf{e}}_2 &= \mathbf{C}_1 \widehat{\mathbf{X}} \mathbf{e}_1 + \mathbf{D}_1 \widehat{\mathbf{K}}_1 \mathbf{e}_1 + \mathbf{D}_1 \widehat{\mathbf{K}}_2 \mathbf{e}_2, \\
\mathbf{L}_1 &= \text{col}\{\mathbf{e}_1, \mathbf{e}_1, \mathbf{0}\}, \\
\mathbf{L}_2 &= \text{col}\{\mathbf{0}, \mathbf{0}, \mathbf{e}_6\}, \\
\mathbf{L}_3 &= \text{col}\{\mathbf{e}_2, \mathbf{e}_1, \mathbf{0}\}, \\
\mathbf{L}_4 &= \text{col}\{\mathbf{e}_3, \mathbf{e}_1, \mathbf{0}\}, \\
\mathbf{L}_5 &= \text{col}\{\mathbf{e}_2, \mathbf{e}_1, \mathbf{0}\}, \\
\mathbf{L}_6 &= \text{col}\{\mathbf{0}, \mathbf{0}, \mathbf{e}_4\}, \\
\mathbf{L}_7 &= \text{col}\{\mathbf{e}_6, \mathbf{e}_1, \mathbf{0}\}, \\
\mathbf{L}_8 &= \text{col}\{\mathbf{0}, \mathbf{0}, \mathbf{e}_7\}, \\
\mathbf{L}_9 &= \text{col}\{\mathbf{0}, \mathbf{e}_9, (\dot{d}(t) - 1)\mathbf{e}_2\}, \\
\mathbf{L}_{10} &= \text{col}\{\mathbf{e}_4, \mathbf{e}_1, \mathbf{0}\}, \\
\mathbf{L}_{11} &= \text{col}\{\mathbf{0}, \mathbf{0}, \mathbf{e}_5\}, \\
\mathbf{L}_{12} &= \text{col}\{\mathbf{0}, \mathbf{e}_9, -\mathbf{e}_3\}.
\end{aligned} \tag{50}$$

Then, system (14) is asymptotically stable subject to an H_∞ constraint (15), and condition (16) is satisfied under $\kappa = (\rho - V(0))/\gamma^2$.

Hence, if the above theorem has a solution, then the miscellaneous feedback controller gains can be easily achieved by $\mathbf{K}_1 = \widehat{\mathbf{K}}_1 \widehat{\mathbf{X}}^{-1}$ and $\mathbf{K}_2 = \widehat{\mathbf{K}}_2 \widehat{\mathbf{X}}^{-1}$.

3.2. H_∞ Controller Design. In order to reveal the merits of the H_∞ miscellaneous information feedback controller, a traditional H_∞ controller is also designed as follows:

$$\mathbf{u}(t) = \mathbf{K}\mathbf{x}(t), \tag{51}$$

where $\mathbf{K}$ is an H_∞ control gain to be designed.

Then, the controlled suspension system can be expressed as

$$\begin{aligned}
\dot{\mathbf{x}}(t) &= \mathbf{A}\mathbf{x}(t) + \mathbf{B}\mathbf{K}\mathbf{x}(t) + \mathbf{B}_1\mathbf{f}(t), \\
\mathbf{z}_1(t) &= \mathbf{C}_1\mathbf{x}(t) + \mathbf{D}_1\mathbf{K}\mathbf{x}(t), \\
\mathbf{z}_2(t) &= \mathbf{C}_2\mathbf{x}(t), \\
\mathbf{z}_3(t) &= \mathbf{K}\mathbf{x}(t).
\end{aligned} \tag{52}$$

Corollary 1. Let scalars ρ and γ be given. If there exists a symmetric matrix $\mathbf{Q} > 0$ and general matrix $\mathbf{Y}$ satisfying

$$\begin{bmatrix} \mathbf{A}\mathbf{Q} + \mathbf{Q}\mathbf{A}^T + \mathbf{B}\mathbf{Y} + \mathbf{Y}^T\mathbf{B}^T & * & * \\ \mathbf{B}_1^T & -\gamma\mathbf{I} & * \\ \mathbf{C}_1\mathbf{Q} + \mathbf{D}_1\mathbf{Y} & \mathbf{D}_1 & -\gamma\mathbf{I} \end{bmatrix} < 0, \tag{53}$$

$$\begin{bmatrix} \frac{1}{\rho} \{\mathbf{C}_2\}_i \mathbf{Q} \\ * & \mathbf{Q} \end{bmatrix} > 0, \quad i = 1, 2, \tag{54}$$

$$\begin{bmatrix} \frac{1}{\rho} & \mathbf{Y} \\ * & \mathbf{Q} \end{bmatrix} > 0, \tag{55}$$

then, system (52) is asymptotically stable subject to an H_∞ constraint (15), and condition (16) is satisfied under $\kappa = (\rho - V(0))/\gamma^2$.

The proof of Corollary 1 is given in [2], and the H_∞ control gain can be computed by $\mathbf{K} = \mathbf{Y}\mathbf{Q}^{-1}$.

4. Application to Vibration Control of Active Suspension Systems

4.1. Simulation Results. In this section, simulation cases, including road shock and random road disturbance, validate the performance of the H_∞ miscellaneous control scheme. The suspension model parameters are given in Table 1.

The controller design parameters are set as $\rho = 1$, $z_{\max} = 0.1$ m, $\mu = 20$, $\vartheta = 2500$ N, $\vartheta_1 = 1.5$, and $\vartheta_2 = 1$. From [2], we know that the constraints in (16) can be guaranteed by choosing $\rho = 1$. The design parameters of the constrained

FIGURE 3: Frequency responses of car-body acceleration with different controllers.

H_∞ controller gain are given by $\rho = 1$, $z_{\max} = 0.1\text{m}$, and $\theta = 2500\text{N}$. Then, from Corollary 1 and Theorem 2, one can obtain the following control gains.

(i) Constrained H_∞ control gain:

$$\mathbf{K} = \begin{bmatrix} -3.3879 & -1.7614 & -0.8407 & 0.0397 \end{bmatrix}. \quad (56)$$

(ii) Proposed control gain ($h = 1\text{ms}$):

$$\begin{aligned} \mathbf{K}_1 &= \begin{bmatrix} 3.5105 & -5.1793 & -0.3099 & -0.1425 \end{bmatrix}, \\ \mathbf{K}_2 &= \begin{bmatrix} 13.4542 & 5.2620 & 2.0418 & -0.0091 \end{bmatrix}. \end{aligned} \quad (57)$$

(iii) Proposed control gain ($h = 10\text{ms}$):

$$\begin{aligned} \mathbf{K}_1 &= \begin{bmatrix} -7.8701 & -2.4436 & -15.4874 & -0.2807 \end{bmatrix}, \\ \mathbf{K}_2 &= \begin{bmatrix} 24.4565 & 2.2014 & 19.3895 & 0.2111 \end{bmatrix}. \end{aligned} \quad (58)$$

(iv) Proposed control gain ($h = 20\text{ms}$):

$$\begin{aligned} \mathbf{K}_1 &= \begin{bmatrix} 0.0226 & -0.0459 & -10.7519 & -0.2262 \end{bmatrix}, \\ \mathbf{K}_2 &= \begin{bmatrix} 16.5605 & -0.1164 & 15.5322 & 0.2079 \end{bmatrix}. \end{aligned} \quad (59)$$

(v) Proposed control gain ($h = 30\text{ms}$):

$$\begin{aligned} \mathbf{K}_1 &= \begin{bmatrix} 1.9540 & 0.0658 & -9.4600 & -0.2195 \end{bmatrix}, \\ \mathbf{K}_2 &= \begin{bmatrix} 14.4036 & -0.3316 & 14.5858 & 0.2247 \end{bmatrix}. \end{aligned} \quad (60)$$

FIGURE 4: Shock road disturbance.

FIGURE 5: Car-body acceleration results under the shock road disturbance.

In the following, the proposed controller and the constrained H_∞ controller are compared in the frequency and the time domains. Numerical results of car-body acceleration in the frequency domain are plotted in Figure 3. Then, from this figure, it is observed that the acceleration magnitudes of the proposed control are less than those of the constrained H_∞ control, especially in the 0.1 Hz–8 Hz range, which means that the vibration-damping performance of the proposed control is better for different delays. We can also note that the acceleration magnitude of the proposed control becomes large as the upper bound of delay increases.

Assume that the delay is less than 30 ms in the network; a shock road disturbance [19] plotted in Figure 4 is used to test the designed controllers. Time-domain simulation results are shown in Figures 5 and 6. In Figure 5, the comparative results of car-body acceleration indicate that the vibration

FIGURE 6: Constraint outputs of the proposed control under the shock road disturbance.

FIGURE 7: Random road disturbance.

magnitude suppressed by the proposed control is less than that suppressed by the constrained H_∞ control. Hence, the ride comfort is improved. Moreover, we can see from Figure 6 that the constraints (16) are also satisfied by the proposed H_∞ miscellaneous control.

To further illustrate the vibration-damping performance of the proposed controller, a random road disturbance, depicted in Figure 7 is employed. In Figures 8 and 9, the simulation results under this disturbance are provided. The results indicate that our method has a lower magnitude of car-body acceleration, while the constraint outputs (16) are all reasonable.

FIGURE 8: Car-body acceleration results under the random road disturbance.

FIGURE 9: Constraint outputs of the proposed control under the random road disturbance.

To quantitatively estimate the performance of the proposed controller, the following indicators are defined:

$$\|\ddot{\mathbf{z}}_1(t)\|_{\text{rms}} = \sqrt{\frac{1}{T} \int_0^T \ddot{\mathbf{z}}_1^2(t) dt}, \quad (61)$$

$$|\ddot{\mathbf{z}}_1(t)|_{\text{max}} = \max\{|\ddot{\mathbf{z}}_1(t)|, t \in [0, T]\}, \quad (62)$$

$$|\{\mathbf{z}_2(t)\}_1|_{\text{max}} = \max\{|\{\mathbf{z}_2(t)\}_1|, t \in [0, T]\}, \quad (63)$$

$$|\{\mathbf{z}_2(t)\}_2|_{\text{max}} = \max\{|\{\mathbf{z}_2(t)\}_2|, t \in [0, T]\}, \quad (64)$$

TABLE 2: Indicators of suspension outputs under random road disturbance.

Outputs	$\ \ddot{z}_1\ _{\text{rms}} (\text{m/s}^2)$	$ \ddot{z}_1 _{\text{max}} (\text{m/s}^2)$	$ \{z_2\}_1 _{\text{max}}$	$ \{z_2\}_2 _{\text{max}}$	$ \mathbf{u} _{\text{max}}$
Passive control	0.4300	1.2400	0.2564	0.1568	0
H_∞ control	0.2779	0.8900	0.1822	0.1045	0.17
HMIFC ($h = 1$ ms)	0.0948	0.3089	0.2477	0.1299	0.45
HMIFC ($h = 10$ ms)	0.1279	0.4145	0.2183	0.1225	0.43
HMIFC ($h = 20$ ms)	0.1427	0.4504	0.2139	0.1180	0.42
HMIFC ($h = 30$ ms)	0.1535	0.5150	0.2031	0.1155	0.41

(a)

(b)

FIGURE 10: Active suspension experiment. (a) Two DOF active suspension workstation. (b) Block diagram of control implementation.

$$|\mathbf{u}(t)|_{\text{max}} = \max\{|\mathbf{u}(t)|, t \in [0, T]\}. \quad (65)$$

The time period T is set to 20 ms, and a simulation is performed under the given random road disturbance. The computed results of indicators (61)–(65) are tabulated in Table 2. The results further support the merits of the proposed controller. Note that the constraints of the proposed control are slightly increased, but these are all far less than one. The theoretical analysis and simulation results show that the designed controller is useful for the vibration control of active suspension systems. Next, the H_∞ miscellaneous control scheme is implemented in a real two DOF flexible active suspension system.

4.2. Experimental Results. The proposed control is implemented in a real two DOF flexible active suspension setup. The experimental configuration and desired control diagram are shown in Figure 10. The established active suspension workstation produced by Quanser includes a set of real-time rapid-control prototyping software, a data acquisition card, a power amplifier, an emergency switch (EMS), and an active suspension setup. The setup can be used to simulate a real quarter-car active suspension, whose nominal parameters and functions can be found in Table 1 [34].

The proposed control and constrained H_∞ control are implemented under three different road excitations. Setting $\rho = 1$, $z_{\text{max}} = 0.02$ m, $\vartheta = 20$ m, $\vartheta_1 = 1.5$, $\vartheta_2 = 1$, $\mu = 20$, and

FIGURE 11: Hardware results of suspension displacement under square wave signal.

FIGURE 12: Hardware results of car-body acceleration under square wave signal.

$h = 30$ ms, by Theorem 2 and Corollary 1, the controllers are designed as follows:

(i) Constrained H_∞ control gain:

$$\mathbf{K} = [-35.5710 \quad -2.5701 \quad -6.7989 \quad 0.5388]. \quad (66)$$

(ii) Proposed control gain:

$$\begin{aligned} \mathbf{K}_1 &= [25.8727 \quad -0.2666 \quad -11.5398 \quad -0.2486], \\ \mathbf{K}_2 &= [17.7822 \quad 0.0341 \quad 18.3476 \quad 0.2273]. \end{aligned} \quad (67)$$

In the control implementation, the passive control is switched to active control at a time of about 7 s. The

FIGURE 13: Hardware results of active control force under square wave signal.

FIGURE 14: Hardware results of suspension displacement under sinusoidal wave signal.

network-induced delay is simulated by a time-varying delay satisfying (13). The controllers are first tested by a square-wave signal with amplitude 0.02 m. The hardware results are depicted in Figures 11–13. Figure 11 illustrates the sprung-mass displacements and vertical road input. The proposed control produces a smaller overshoot than the passive control and constrained H_∞ control. We can see from Figure 12 that the car-body acceleration is the least under the proposed active control method. The active control forces of these two controllers are described in Figure 13.

As shown in Figures 14–16, the experiment is performed for a sine-wave signal with amplitude 0.002 m and angular frequency 4π rad/s. It can be clearly observed from Figures 14 and 15 that the solid-red line (the proposed control) is lower than the dashed-blue lines (constrained H_∞

FIGURE 15: Hardware results of car-body acceleration under sinusoidal wave signal.

FIGURE 16: Hardware results of active control force under sinusoidal wave signal.

control), further verifying that the proposed control achieves better vibration-damping performance in the vertical direction. The active control forces plotted in Figure 16 are all less than the maximum limit of 20 N.

Finally, a random road signal generated by the shaping filter method [9] is employed to evaluate the performance of the designed controllers. The hardware results are shown in Figures 17–19. Note that the passive control results are not shown in this case, so as to exhibit the performances of the proposed controller more clearly. Based on the responses shown in Figures 17 and 18, we can say that the designed controller has the best vibration-damping performance with the help of an auxiliary network feedback. From Figure 19, it can be seen that the control input of the proposed control is large enough to stabilize the suspension system. Based on the previous hardware results, the computed results of indicators (61)–(65) are given in Table 3, which again shows

FIGURE 17: Hardware results of suspension displacement under random road signal.

FIGURE 18: Hardware results of car-body acceleration under random road signal.

the merits of the proposed control. Figure 20 gives the frequency response results of the car-body acceleration. From this figure, we can see that the proposed control has a small car-body acceleration gain in the 0.1 Hz–15 Hz range. Therefore, the control effect is improved in the low-frequency domain.

4.3. Discussion. It can be seen from the hardware and simulation results that both the constrained H_∞ control and proposed control can successfully attenuate the vibration of a car body from different road disturbances, but the proposed control is more outstanding for a given constraint parameter. In particular, the proposed control and the constrained H_∞ control are well verified by a real two DOF active suspension setup in this paper, which means that the vibration control performances of the two controllers are

FIGURE 19: Hardware results of active control force under random road signal.

TABLE 3: Indicators of suspension outputs in the hardware test.

Hardware test outputs	Random road excitation		
	Passive control	H_∞ control	HMIFC
$\ \ddot{z}_1\ _{\text{rms}} (\text{m/s}^2)$	0.0512	0.0352	0.0153
$ \ddot{z}_1 _{\text{max}} (\text{m/s}^2)$	0.3353	0.2515	0.1389
$ z_s - z_u _{\text{max}} (\text{mm})$	0.35	0.44	0.54
$ z_u - z_r _{\text{max}} (\text{mm})$	1.38	1.10	0.91
$ u _{\text{max}} (\text{N})$	0	0.4542	1.651

FIGURE 20: Frequency responses of car-body acceleration with different controllers.

also robust in the presence of some sensor noise and model error.

In summary, the designed H_∞ miscellaneous information feedback controller has a better vibration control characteristic and is superior to the H_∞ control without an auxiliary network feedback. It has great potential for implementation in a physical vehicle active suspension.

5. Conclusions

This paper has studied an H_∞ miscellaneous information feedback control scheme for a two DOF active suspension system. To improve the vibration control performance, an auxiliary feedback signal has been introduced in the control channel via a wireless communication network. A new existence condition based on the LMI method is given for the controller design. Finally, the proposed control has been successfully implemented in a real active suspension setup. The experiments have shown that the proposed control has better results, the car-body acceleration can be more significantly attenuated, and the constraints can be satisfied. Moreover, the designed controllers are robust to inherent sensor noise and model error, as confirmed by the experimental results. It should be pointed out that all states are measured in this scheme, and thus, observer-based robust control problem will be considered in the future work.

Data Availability

The simulation and experimental results are from MATLAB and the active suspension setup. Readers can request the results of this article by emailing the corresponding author, and the results will be provided in research gate.

Conflicts of Interest

The authors of this paper declare that there are no conflicts of interests regarding the publication of this paper.

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