Research Article

New Analytical Method to Evaluate Casing Integrity during Hydraulic Fracturing of Shale Gas Wells

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An accurate analysis of casing stress distribution and its variation regularities present several challenges during hydraulic fracturing of shale gas wells. In this paper, a new analytical mechanical-thermal coupling method was provided to evaluate casing stress. For this new method, the casing, cement sheath, and formation (CCF) system was divided into three parts such as initial stress field, wellbore disturbance field, and thermal stress field to simulate the processes of drilling, casing, cementing, and fracturing. The analytical results reached a good agreement with a numerical approach and were in-line with the actual boundary condition of shale gas wells. Based on this new model, the parametric sensitivity analyses of casing stress such as mechanical and geometry properties, operation parameters, and geostress were conducted during multifracturing. Conclusions were drawn from the comparison between new and existing models. The results indicated that the existing model underestimated casing stress under the conditions of the geostress heterogeneity index at the range of 0.5–2.25, the fracturing pressure larger than 25 MPa, and a formation with large elastic modulus or small Poisson's ratio. The casing stress increased dramatically with the increase of in situ stress nonuniformity degree. The stress decreased first and then increased with the increase of fracturing pressure. Thicker casing, higher fluid temperature, and cement sheath with small modulus, large Poisson's ratio, and thinner wall were effective to decrease the casing stress. This new method was able to accurately predict casing stress, which can become an alternative approach of casing integrity evaluation for shale gas wells.

1. Introduction

During the multistage fracturing process, fracturing fluids are pressed into a borehole with a high pump rate and pressure. The complex downhole environments—high pressure and large temperature variation—increase the risk of casing deformation. The volume fracturing technique effectively reconstructs shale reservoirs; however, frequent and serious casing deformation failures occur [1]. There were over 36 wells with casing deformation (including 112 horizontal wells by 2017) during fracturing process in some shale gas plays. Drilling tools were blocked, and serious deformation sections were abandoned before fracturing operation completion [2–4]. Therefore, casing integrity has always been the main issue in designing shale gas wells under

harsh conditions, and accurately calculating casing stress becomes primary to guarantee the casing safety. A casing stress analysis presents several challenges regarding the drilling, completion, and fracturing phases of shale gas wells. Many scholars have predicted the casing stress by simplifying the actual situations.

Analytical solutions have been developed under different conditions for casing-cement sheath-formation system. In-plane and out-of-plane analyses for the stress field around an internally pressurized, cased, cemented, and remotely loaded circular hole were developed [5]. Taking the in situ stresses and well trajectory into account, the mechanical model of casing in the directional well under in situ stresses is established [6]. Other scholars gave a comprehensively analytical solution of the stress

distribution in a casing-cement-formation system under anisotropic in situ stresses. Fang et al. [7] developed a multilayer cemented casing system in the directional well under anisotropic formation to investigate the collapse resistance of casing under nonuniform in situ stress and anisotropic formation. Based on the stress function method, a three-dimensional model of the casing-cement sheath-formation (CCF) system was proposed subjected to linear crustal stress, and then an analytical solution of the model was obtained [8].

A finite element method (FEM) had been proposed to achieve a better understanding of the ultimate collapse strength of casing [9]. Using this method, casing stress was better analyzed, subjected to external and nonuniform loading caused by void and pressure [10]. The results of a series of finite element studies of the cemented casing under a variety of stress conditions for both burst and collapse were presented, demonstrating the inadequacy of accepted design equations under many cemented conditions [11]. A purpose-built finite element model was applied to simulate the radial displacement of a casing string constrained within an outer wellbore [12]. Casing stress on the inner wall under the condition of elliptic casing was calculated and analyzed to improve the designing and correction of casing strength [13]. The main controlling factor to the plastic limit load of defective casing was analyzed by the combination of high temperature and high steam injection pressure [14].

However, the above analytical or FEM solutions are obtained by setting the casing and cement together instantly at the beginning of the analysis. The strains induced by the initial stress are included in the model, which is not in-line with the actual situation. In fact, initial stress has already existed in the formation before wellbore excavation. The wellbore stress redistributed after removing the rocks that originally occupied the borehole volume and just affected the stress and displacement near wellbore zones. In view of this, more sophisticated solutions have been developed with additional parameters and appropriate assumptions regarding the drilling, completion, and fracturing phases.

Patillo and Kristiansen [15] developed a finite model starting with the virgin reservoir, which considered prewellbore depletion, drilling the wellbore, installation of casing and cement, and subsequent draw down. Behavior of various configurations during subsequent draw down permitted them to be ranked according to the life expectancy of the resulting completion. Gray et al. [16] presented a staged-finite element procedure during the well construction which considered sequentially the stress states and displacements at and near the wellbore. But, temperature, flow, and poroelasticity effects were not included. Mackay and Fontoura [17] carried a numerical analysis to determine the effect of salt creep and contacts amongst the materials before, during and after drilling the well, focusing on the drilling of the wellbore and on the hardening of the cement. Zhang et al. [18] used the staged-finite element modeling approach to simulate the well construction processes and injection cycle, using a “realistic” bottom-hole state of stress to simulate the microannuli generation by the tensile debonding of the cement-formation interface. Simone et al. [19] developed an analytical solution of single and double casing configurations to assess the

stresses during the drilling, construction, and production phases. But the nonuniform boundary stress was not considered in this model. Liu et al. [20] presented an analytical method for evaluating the stress field within a casing-cement-formation system of oil/gas wells under anisotropic in situ stresses in the rock formation and uniform pressure within the casing. However, the temperature was not considered in this model.

In this work, to evaluate the thermal and mechanical stresses of casing, an analytical model of casing-cement sheath-formation system was established considering wellbore construction. The boundary stresses in the wellbore coordinate system were obtained through three-dimensional rotations from principal in situ stresses. The casing stress was obtained by dividing the model into three parts such as initial stress field, disturbance stress field, and thermal stress field. The continuous homogeneous isotropy and linear elasticity were taken into consideration. Solutions were validated by a finite element method. Sensitivity analyses were conducted to estimate the influence of different factors on casing stress such as property of cement sheath and casing, fracturing pressure, fluid temperature, and initial geostress. Useful countermeasures were put forward to decrease casing stress during the fracturing operation.

2. Method and Basic Conditions Analysis

2.1. Method Comparison

2.1.1. Existing Method. For existing method, the casing, cement sheath, and formation were set together at the beginning of analysis. Then, temperature boundary and loads were added to the system to calculate wellbore stress distribution as shown in Figure 1. This method did not consider the process of wellbore construction and hydraulic fracturing. So, the wellbore stress distribution could not accurately be calculated under the condition of formation strain induced by the initial in situ stress.

2.1.2. New Method. To exactly predict the wellbore stress distribution, a casing-cement sheath-formation model using a new analysis method was provided. The analysis process was divided into four steps (Figure 2). Before drilling, the initial stresses such as normal stress and shear stress were loaded in the model and initial strains were produced. The constrained boundary conditions were assigned to the entire model which reached initial mechanical equilibrium. During drilling, the rock was excavated, causing stress concentration around the borehole. The drilling mud pressure P_m was applied to the internal face of the wellbore. During casing and cementing, casing and cement sheath elements were added simultaneously to the model and the cement hardening procedure was not considered. Initial stress and strain in casing and cement sheath were ignored. The cement slurry pressure P_c was applied to the internal casing wall. During fracturing, a low temperature T_n and the fracturing pressure P_f were assigned in the internal casing wall. The wellbore stress field was obtained under the condition of thermal-mechanical coupling.

FIGURE 1: Loading process of the existing method. Outer boundary temperature, T_f ; inner boundary temperature, T_n ; normal stress, σ_x and σ_y ; shear stress τ_{xy} .

FIGURE 2: Loading process of the new method. Outer boundary temperature, T_f ; inner boundary temperature, T_n ; normal stress, σ_x and σ_y ; shear stress, τ_{xy} ; drilling mud pressure, P_m ; cement slurry pressure, P_c ; fracturing pressure, P_f .

2.2. Stress Transformation. The stress state and coordinate transformation system are shown in Figure 3. The coordinate rotation processes from the principal in situ stress coordinate system to the wellbore coordinate system are shown as $X_H Y_H Z_Y \rightarrow X' Y' Z' \rightarrow X'' Y'' Z'' \rightarrow X''' Y''' Z''' \rightarrow XYZ$: first, rotating anticlockwise φ around the Z_v -axis and rotating clockwise β around the Z_v -axis; second, rotating anticlockwise α around the Y'' axis (after the second time rotation); and finally, rotating anticlockwise 90° around the Z''' axis (after the third time rotation).

In the principal stress coordinate system, the principal horizontal stress matrix is σ^0 , whose components are maximum principal stress, σ_H ; the minimum principal stress, σ_h ; and the overburden pressure, σ_v , shown in equation (2). The stress matrix in the wellbore coordinate system is σ . According to the right-hand rule, the direction cosine matrices rotating around X-axis, Y-axis, and Z-axis are shown in equation (1) [21]. The wellbore boundary stress is obtained by the three-dimensional rotations from the principal in situ stress coordinate system shown in equation (3):

$$C_{x\alpha_x} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos \alpha_x & \sin \alpha_x \\ 0 & -\sin \alpha_x & \cos \alpha_x \end{bmatrix}, \quad C_{y\alpha_y} = \begin{bmatrix} \cos \alpha_y & 0 & -\sin \alpha_y \\ 0 & 1 & 0 \\ \sin \alpha_y & 0 & \cos \alpha_y \end{bmatrix}, \quad (1)$$

$$C_{z\alpha_z} = \begin{bmatrix} \cos \alpha_z & \sin \alpha_z & 0 \\ -\sin \alpha_z & \cos \alpha_z & 0 \\ 0 & 0 & 1 \end{bmatrix}, \quad \sigma^0 = \begin{bmatrix} -\sigma_H & 0 & 0 \\ 0 & -\sigma_h & 0 \\ 0 & 0 & -\sigma_v \end{bmatrix}, \quad (2)$$

$$\sigma = C_{z90} C_{y\alpha} C_{z\beta} C_{zj} \sigma^0 C_{zj}^T C_{z\beta}^T C_{y\alpha}^T C_{z90}^T, \quad (3)$$

FIGURE 3: The coordinate rotation processes: (a) stress state of a horizontal well and (b) coordinate transformation system. Principal in situ stress coordinate system, $X_H Y_H Z_H$; geodetic coordinate system, $X'Y'Z'$; coordinate system, $X''Y''Z''$ after the second time rotation; coordinate system, $X'''Y'''Z'''$ after the third time rotation; local wellbore coordinate system, XYZ ; wellbore deviation angle, α ; wellbore azimuth angle, β ; the maximum horizontal stress azimuth angle, φ .

where C_{α_x} , C_{α_y} , and C_{α_z} are the coordinate rotation matrices of the x , y , and z directions; σ^0 and σ are the stress matrices in the principal stress coordinate and local wellbore coordinate systems; and α_x , α_y , and α_z are the rotation angles in a counterclockwise direction when looking towards the origin coordinate.

2.3. Basic Hypotheses. A thermo-pressure coupling model of casing-cement sheath-formation (CCF) system was established (Figure 4). The boundary stresses of σ_x , σ_y , and τ_{xy} were obtained by using equation (3). Compared to the longitude of the well, the radial dimension was very small and a long cylindrical model was loaded by forces that were perpendicular to the axial line and did not vary in length. Both the geometric form of the object and the external loads exerted on the object did not change along the longitudinal (z -axis) direction and that the length of the object might be treated as an infinite one. There were no additional restrictions on the external loads. The wellbore stress was obtained under this kind of stress state called the plane strain problem [22, 23].

For simplicity, some assumptions have been made [24]:

- (1) Geometry: the casing, cement, and borehole were concentric circles, which were assumed to be perfectly bonded to each other at each interface. The perfect bonding mathematically indicated that the continuity of radial stress and displacement was satisfied at each interface.
- (2) Thermal effect: the stress induced by wellbore temperature variation was assumed to be steady state and the time effect was ignored.
- (3) Material: to simplify the complex property of strong anisotropy and well-developed bedding planes of shale formation [25], the formation was assumed to be a linear elastic material with an infinite radius ($R_4 \rightarrow \infty$). The cement sheath was also a complex material. The 3D images revealed the evolution of a

large connected pore network with characteristic widths on the micrometer scale as hydration proceeded [26]. It was assumed to be an elastic material neglecting the complex microstructure. It is generally accepted that the casing was an elastic-plastic material, and the casing yielding had nothing to do with hydrostatic pressure. In the model, it was assumed that the deformation of the casing was within the elastic range.

2.4. Stress Superposition. The boundary compression normal stresses ($-\sigma_x, -\sigma_y$) were decomposed into uniform stress (p_0) and deviator stress (s) expressed as equation (4) in the Cartesian coordinate system. The other boundary stress was shear stress (τ_{xy}):

$$\begin{bmatrix} -\sigma_x & 0 \\ 0 & -\sigma_y \end{bmatrix} = \begin{bmatrix} -p_0 & 0 \\ 0 & -p_0 \end{bmatrix} + \begin{bmatrix} -s & 0 \\ 0 & s \end{bmatrix}, \quad (4)$$

where the uniform stress $p_0 = (\sigma_x + \sigma_y)/2$ and the deviator stress $s = (\sigma_x - \sigma_y)/2$.

According to the basic hypotheses in Section 2.2, the deformation history of all phases in the CCF system was independent with each other. The principal of the linear superposition for the stress was applied as shown in the following equation:

$$\sigma_{\text{stress}}^i = \sigma_{\text{Uniform-stress}}^i + \sigma_{\text{Deviator-stress}}^i + \sigma_{\text{Shear-stress}}^i + \sigma_{\text{Thermal-stress}}^{Ti} \quad (5)$$

The stress distribution of the thermal-pressure coupling model around a wellbore was decomposed into four parts as shown in Figure 5. The stress field of the first part was induced by the uniform stress and inner casing pressure. The second one was induced by the deviator stress, and the third one was induced by the shear stress. Thermal stress of the fourth part was induced by the temperature variation.

FIGURE 4: CCF composite assembly. Formation boundary temperature, T_f ; internal casing temperature, T_i ; internal casing pressure, P_i ; radius, R_i , $i = 1, 2, 3, 4$ present the radii of the internal casing wall, outer casing wall, internal wellbore, and formation boundary, respectively; the counterclockwise angle from the x -direction to the calculated point, θ .

FIGURE 5: Stress decompositions. Inner casing pressure, P_i ; thermal stress, σ_T .

It was convenient to convert the Cartesian coordinate system into the polar coordinate system to calculate wellbore stress. The normal boundary stresses in the polar coordinate system were expressed as follows under the conditions of the infinite outer boundary radius of R_4 :

$$\begin{cases} \sigma_r^{30}|_{r=\infty} = -p_0 - s \cos(2\theta), \\ \tau_{r\theta}^{30}|_{r=\infty} = s \sin(2\theta), \end{cases} \quad (6)$$

where σ_r^{30} and $\tau_{r\theta}^{30}$ are the initial normal and shear stresses in the formation, respectively.

Since temperature and stress were coupled, the stress distribution around a cased wellbore induced by temperature variation was hard to solve in the closed form. However, the steady-state condition made the temperature and stress decouple and the problem analytically solvable [27].

3. Stress Distribution around Wellbore

3.1. Stress Induced by Uniform Stress. Under the condition of the uniform internal pressure and external stress, the stress

and displacement in a thin wall cylinder were obtained by using the following equations shown in Figure 6:

$$u_r^i = \frac{1}{2G_i} \left[(1 - 2\mu_i) A_i' r + C_i' \frac{1}{r} \right] q_i + \frac{1}{2G_i} \left[(1 - 2\mu_i) B_i' r + C_i' \frac{1}{r} \right] q_{i+1} - r \epsilon_r^{i0}, \quad (7)$$

$$\begin{cases} \sigma_r^i = \left(A_i' - C_i' \frac{1}{r^2} \right) q_i + \left(B_i' - C_i' \frac{1}{r^2} \right) q_{i+1}, \\ \sigma_\theta^i = \left(A_i' + C_i' \frac{1}{r^2} \right) q_i + \left(B_i' + C_i' \frac{1}{r^2} \right) q_{i+1}, \end{cases} \quad (8)$$

where σ_r^i , σ_θ^i , and u_r^i are the radial stress, tangential stress, and radial displacement, respectively; σ_r^{i0} , σ_θ^{i0} , and u_r^{i0} are the initial radial stress, tangential stress, and displacement; E_i is the material elastic modulus; μ_i is the material Poisson's ratio; $G_i = E_i / ((1 + \mu_i)/2)$ is the material shear modulus; q_i , q_{i+1} were the interfacial pressure, positive in the radial increase direction, $i = 1, 2, 3$ represented the casing, cement

FIGURE 6: Stress induced by the uniform stress.

sheath, and formation, respectively; $A'_i = R_i^2 / (R_{i+1}^2 - R_i^2)$; $B'_i = R_{i+1}^2 / (R_{i+1}^2 - R_i^2)$; $C'_i = A'_i R_{i+1}^2$ are the constants, R_i ($i = 1, 2, 3, 4$) is the radii of internal casing wall, external casing wall, external cement sheath wall, and formation boundary, respectively.

3.1.1. Formation Stress. Before drilling the borehole, the initial geostress field already existed in the formation. When the rock was removed from the borehole, the wellbore stress field redistributed to produce a disturbance field, which only affected the near-wellbore zones [28]. So, the model was decomposed into two parts such as the original field and the disturbance field. The original field had initial stress and displacement. The disturbance field was induced by drilling a wellbore and mud pressure. In view of this, the actual stress field of F1 in the formation induced by the uniform stress, and internal pressure was decomposed into three parts as shown in Figure 7. They were the original stress field of A1, the excavation disturbance field of B1 induced by drilling of a wellbore and the interface disturbance field of C1 induced by the fluid column pressure.

In the polar coordinate system, the initial conditions of A1 were $\sigma_r^{30} = \sigma_\theta^{30} = -p_0$, $\epsilon_r^{30} = p_0 / 2G_3 (1 - 2\mu_3)$, and boundary stress conditions of B1 and C1 were $\sigma_r^{30}|_{r=R_3, B1} = p_0$ and $\sigma_r^{30}|_{r=R_3, C1} = -p_3'$. Substituting the initial and boundary conditions into equations (7) and (8), the displacement and stress in formation were obtained as shown below. Because R_4 approached to infinity, $A'_3 = 0$ and $C'_3 = R_3^2$ were obtained:

$$\begin{cases} u_r^{30}(r) = \frac{1}{2G_3} \frac{R_3^2}{r} (p_3' - p_0), \\ \sigma_r^{30}(r) = -p_0 - \frac{R_3^2}{r^2} (p_3' - p_0), \\ \sigma_\theta^{30}(r) = -p_0 + \frac{R_3^2}{r^2} (p_3' - p_0), \end{cases} \quad (9)$$

where u_r^{30} was the radial displacement in formation and u_r^{30} and σ_θ^{30} were the radial and tangential stresses in formation, respectively.

3.1.2. Casing and Cement Sheath Stress. The pressures at casing-cement sheath interface and cement sheath-formation interface were p_2' and p_3' , respectively, (Figure 8). The initial stresses of the casing and the cement sheath were $\sigma_r^{i0} = \sigma_\theta^{i0} = 0$ and $\epsilon_r^{i0} = 0$. The boundary stress conditions were $\sigma_r^{i2}|_{r=R_3} = -p_3'$ and $\sigma_r^{i2}|_{r=R_2} = -p_2'$, $\sigma_r^{i1}|_{r=R_1} = -p_i$.

Substituting these initial and boundary conditions in equations (7) and (8), the displacement and stress in casing and cement sheath were obtained as follows. Subscripts 1 and 2 represent the casing and cement sheath, respectively:

$$\begin{cases} u_r^{i1} = \frac{1}{2G_1} \left[(1 - 2\mu_1) A'_1 r + C'_1 \frac{1}{r} \right] p_i \\ \quad + \frac{1}{2G_1} \left[(1 - 2\mu_1) B'_1 r + C'_1 \frac{1}{r} \right] (-p_2'), \end{cases} \quad (10)$$

$$\begin{cases} u_r^{i2} = \frac{1}{2G_2} \left[(1 - 2\mu_2) A'_2 r + C'_2 \frac{1}{r} \right] p_2' \\ \quad + \frac{1}{2G_2} \left[(1 - 2\mu_2) B'_2 r + C'_2 \frac{1}{r} \right] (-p_3'), \end{cases}$$

$$\begin{cases} \sigma_r^{i1} = \left(A'_1 - C'_1 \frac{1}{r^2} \right) p_i + \left(B'_1 - C'_1 \frac{1}{r^2} \right) (-p_2'), \\ \sigma_\theta^{i1} = \left(A'_1 + C'_1 \frac{1}{r^2} \right) p_i + \left(B'_1 + C'_1 \frac{1}{r^2} \right) (-p_2'), \\ \sigma_r^{i2} = \left(A'_2 - C'_2 \frac{1}{r^2} \right) p_2' + \left(B'_2 - C'_2 \frac{1}{r^2} \right) (-p_3'), \\ \sigma_\theta^{i2} = \left(A'_2 + C'_2 \frac{1}{r^2} \right) p_2' + \left(B'_2 + C'_2 \frac{1}{r^2} \right) (-p_3'). \end{cases} \quad (11)$$

According to the hypotheses that cement sheath-formation interface and casing-cement sheath interface were perfectly bonded to each other, the interfacial displacement continuity conditions were expressed in the following equation:

$$\begin{cases} u_r^{i1}|_{r=R_2} = u_r^{i2}|_{r=R_2}, \\ u_r^{i2}|_{r=R_3} = u_r^{i3}|_{r=R_3}. \end{cases} \quad (12)$$

Substituting equation (10) into equation (12), the binary equations were obtained as

$$\begin{cases} AA p_2' - BB p_3' = CC p_i, \\ DD p_2' - EE p_3' = FF p_i, \end{cases} \quad (13)$$

FIGURE 7: Wellbore stress components under the condition of uniform stress.

FIGURE 8: Interface pressures induced by uniform stress.

where

$$\left\{ \begin{array}{l} AA = \frac{1}{2G_1} \left[(1 - 2\mu_1)B'_1R_2 + C_1\frac{1}{R_2} \right] \\ \quad + \frac{1}{2G_2} \left[(1 - 2\mu_2)A'_2R_2 + C'_2\frac{1}{R_2} \right], \\ BB = \frac{1}{2G_1} \left[(1 - 2\mu_1)B'_1R_2 + C_1\frac{1}{R_2} \right], \\ CC = \frac{1}{2G_1} \left[(1 - 2\mu_1)A'_1r + C'_1\frac{1}{R_2} \right], \\ DD = \frac{1}{2G_2} \left[(1 - 2\mu_2)A'_2R_3 + C'_2\frac{1}{R_3} \right], \\ EE = \left\{ \frac{1}{2G_1} \left[(1 - 2\mu_1)B'_1R_3 + C_1\frac{1}{R_3} \right] + \frac{1}{2G_3}R_3 \right\}, \\ FF = -\frac{R_3}{2G_3}. \end{array} \right. \quad (14)$$

The interfacial pressures p'_2 and p'_3 could be calculated by using equation (14). Substituting them into equation (11), the stresses induced by uniform stress were obtained subsequently.

3.2. Stress Induced by Deviator Stress. The deviator stress boundary conditions are shown in Figure 9. To calculate the stress distribution induced by deviator stress, the stress function was defined as

FIGURE 9: Stress induced by deviator stress. Outer stress, $s \cos 2\theta$; interface pressure, $p \cos 2\theta$.

$$\phi = \left(A_i''r^4 + B_i''r^2 + C_i'' + \frac{D_i''}{r^2} \right) \cos(2\theta). \quad (15)$$

The stress and strain under the condition of nonuniform stress are

$$\left\{ \begin{array}{l} \sigma_r^{ii} = -\left(2B_i'' + \frac{4C_i''}{r^2} + \frac{6D_i''}{r^4} \right) \cos 2\theta, \\ \sigma_\theta^{ii} = \left(12A_i''r^2 + 2B_i'' + \frac{6D_i''}{r^4} \right) \cos 2\theta, \\ \tau_{r\theta}^{ii} = \left(6A_i''r^2 + 2B_i'' - \frac{2C_i''}{r^2} - \frac{6D_i''}{r^4} \right) \sin 2\theta, \\ \varepsilon_r^{ii} = -\frac{1 + \mu_i}{E_i} \left[12v_iA_i''r^2 + 2B_i'' + (1 - \mu_i)\frac{4C_i''}{r^2} + \frac{6D_i''}{r^4} \right] \\ \quad \cdot \cos 2\theta - \varepsilon_r^{ii0}, \\ \varepsilon_\theta^{ii} = \frac{1 + \mu_i}{E_i} \left[12(1 - \mu_i)A_i''r^2 + 2B_i'' + \mu_i\frac{4C_i''}{r^2} + \frac{6D_i''}{r^4} \right] \\ \quad \cdot \cos 2\theta - \varepsilon_\theta^{ii0}, \end{array} \right. \quad (16)$$

where σ_r^{ni} and σ_θ^{ni} are the radial and tangential stresses; ε_r^{ni} and ε_θ^{ni} are the initial radial and tangential strains; and A_i'' , B_i'' , C_i'' , and D_i'' were the constants, $i = 1, 2, 3$ represented the casing, cement sheath, and formation.

From the geometric equations,

$$\begin{cases} \varepsilon_r^{ni} = \frac{\partial u_r^{ni}}{\partial r}, \\ \varepsilon_\theta^{ni} = \frac{1}{r} \frac{\partial u_\theta^{ni}}{\partial \theta} + \frac{u_r^{ni}}{r}. \end{cases} \quad (17)$$

The radial displacement u_r^{ni} and tangential displacement u_θ^{ni} were obtained as

$$\begin{cases} u_r^{ni} = -\frac{1+\mu_i}{E_i} \left[4\mu_i A_i'' r^3 + 2B_i'' r - (1-\mu_i) \frac{4C_i''}{r} - \frac{2D_i''}{r^3} \right] \\ \quad \cdot \cos 2\theta - r \varepsilon_r^{ni0}, \\ u_\theta^{ni} = \frac{1+\mu_i}{2E_i} \left[4(3-2\mu_i) A_i'' r^3 + 4B_i'' r - (1-2\mu_i) \frac{4C_i''}{r} + \frac{4D_i''}{r^3} \right] \\ \quad \cdot \sin 2\theta + r \int (\varepsilon_r^{ni0} - \varepsilon_\theta^{ni0}) d\theta, \\ \varepsilon_r^{ni0} = \frac{1+\mu_i}{E_i} \left[(1-\mu_i) \sigma_r^{ni0} - \mu_i \sigma_\theta^{ni0} \right], \\ \varepsilon_\theta^{ni0} = \frac{1+\mu_i}{E_i} \left[(1-\mu_i) \sigma_\theta^{ni0} - \mu_i \sigma_r^{ni0} \right], \end{cases} \quad (18)$$

where σ_r^{ni0} and σ_θ^{ni0} were the initial radial and tangential stresses.

3.2.1. Formation Stress. Similar to that of uniform stress, the actual stress field, $F2$ in the strata induced by the non-uniform stress, was decomposed into three parts: the original stress field, $A2$; the disturbance field, $B2$ induced by the wellbore excavation; and the interface pressure, $C2$ induced by the interface pressure (Figure 10).

In the polar coordinate system, initial stresses were $\sigma_r^{n30} = -s \cos(2\theta)$, $\sigma_\theta^{n30} = s \cos(2\theta)$, and $\tau_{r\theta}^{n30} = s \sin(2\theta)$; initial strains were $\varepsilon_r^{n30} = -(1+\mu_3)/E_3 \cdot s \cos(2\theta)$ and $\varepsilon_\theta^{n30} = (1+\mu_3)/E_3 \cdot s \cos(2\theta)$; and the boundary stresses were $\sigma_r^{n3}|_{r=\infty} = -s \cos(2\theta)$, $\sigma_\theta^{n3} = s \cos(2\theta)$, and $\tau_{r\theta}^{n3}|_{r=\infty} = s \sin(2\theta)$. Substituting the initial and boundary conditions into (14) and (15), it was obtained that $A_3'' = 0$ and $B_3'' = S/2$. The displacements and stresses in formation were expressed as shown in the following equations:

$$\begin{cases} u_r^{n3} = -\frac{1}{G_3} \left[-(1-\mu_3) \frac{2C_3''}{r} - \frac{D_3''}{r^3} \right] \cos 2\theta, \\ u_\theta^{n3} = \frac{1}{G_3} \left[-(1-2\mu_3) \frac{C_3''}{r} + \frac{D_3''}{r^3} \right] \sin 2\theta, \end{cases} \quad (19)$$

$$\begin{cases} \sigma_r^{n3} = -\left(s + \frac{4C_3''}{r^2} + \frac{6D_3''}{r^4} \right) \cos 2\theta, \\ \sigma_\theta^{n3} = \left(s + \frac{6D_3''}{r^4} \right) \cos 2\theta, \\ \tau_{r\theta}^{n3} = \left(s - \frac{2C_3''}{r^2} - \frac{6D_3''}{r^4} \right) \sin 2\theta. \end{cases} \quad (20)$$

3.2.2. Casing and Cement Sheath Stress. For casing and cement sheath in the polar coordinate system, initial stresses were $\sigma_r^{ni0} = \sigma_\theta^{ni0} = 0$ and initial strains were $\varepsilon_r^{ni0} = \varepsilon_\theta^{ni0} = 0$. Substituting the initial and boundary conditions into equations (14) and (15), the displacements and stresses were obtained as follows:

$$\begin{cases} u_r^{ni} = -\frac{1}{G_i} \left[2\mu_i A_i'' r^3 + B_i'' r - (1-\mu_i) \frac{2C_i''}{r} - \frac{D_i''}{r^3} \right] \cos 2\theta, \\ u_\theta^{ni} = \frac{1}{G_i} \left[(3-2\mu_i) A_i'' r^3 + B_i'' r - (1-2\mu_i) \frac{C_i''}{r} + \frac{D_i''}{r^3} \right] \sin 2\theta, \end{cases} \quad (21)$$

$$\begin{cases} \sigma_r^{ni} = -\left(2B_i'' + \frac{4C_i''}{r^2} + \frac{6D_i''}{r^4} \right) \cos 2\theta, \\ \sigma_\theta^{ni} = \left(12A_i'' r^2 + 2B_i'' + \frac{6D_i''}{r^4} \right) \cos 2\theta, \\ \tau_{r\theta}^{ni} = \left(6A_i'' r^2 + 2B_i'' - \frac{2C_i''}{r^2} - \frac{6D_i''}{r^4} \right) \sin 2\theta. \end{cases} \quad (22)$$

The interfacial displacement and stress continuity and boundary conditions were expressed in the following equation:

$$\begin{cases} \sigma_r^{n1}|_{r=R_1} = 0, \\ \tau_{r\theta}^{n1}|_{r=R_1} = 0, \\ \sigma_r^{n1}|_{r=R_2} = \sigma_r^{n2}|_{r=R_2}, \\ \tau_{r\theta}^{n1}|_{r=R_2} = \tau_{r\theta}^{n2}|_{r=R_2}, \\ \sigma_r^{n2}|_{r=R_3} = \sigma_r^{n3}|_{r=R_3}, \\ \tau_{r\theta}^{n2}|_{r=R_3} = \tau_{r\theta}^{n3}|_{r=R_3}, \\ u_r^{n1}|_{r=R_2} = u_r^{n2}|_{r=R_2}, \\ u_\theta^{n1}|_{r=R_2} = u_\theta^{n2}|_{r=R_2}, \\ u_r^{n2}|_{r=R_3} = u_r^{n3}|_{r=R_3}, \\ u_\theta^{n2}|_{r=R_3} = u_\theta^{n3}|_{r=R_3}. \end{cases} \quad (23)$$

FIGURE 10: Formation stress components under the nonuniform stress condition.

Substituting equations (20)–(23) into the following equation, equations were obtained as

$$\begin{cases} \left(-2B_1'' - \frac{4C_1''}{R_1^2} - \frac{6D_1''}{R_1^4} \right) \cos 2\theta = 0, \\ \left(6R_1^2 A_1'' + 2B_1'' - \frac{2}{R_1^2} C_1'' - \frac{6}{R_1^4} D_1'' \right) \sin 2\theta = 0, \end{cases} \quad (24)$$

$$\begin{cases} -\left(2B_1'' + \frac{4C_1''}{R_2^2} + \frac{6D_1''}{R_2^4} \right) + \left(2B_2'' + \frac{4C_2''}{R_2^2} + \frac{6D_2''}{R_2^4} \right) = 0, \\ \left(6A_1'' R_2^2 + 2B_1'' - \frac{2C_1''}{R_2^2} - \frac{6D_1''}{R_2^4} \right) \\ - \left(6A_2'' R_2^2 + 2B_2'' - \frac{2C_2''}{R_2^2} - \frac{6D_2''}{R_2^4} \right) = 0, \end{cases} \quad (25)$$

$$\begin{cases} -\left(B_2'' + \frac{2C_2''}{R_3^2} + \frac{3D_2''}{R_3^4} \right) + \frac{2C_3''}{R_3^2} + \frac{3D_3''}{R_3^4} = -s, \\ \left(3A_2'' R_3^2 + B_2'' - \frac{C_2''}{R_3^2} - \frac{3D_2''}{R_3^4} \right) + \frac{C_3''}{R_3^2} + \frac{3D_3''}{R_3^4} = s, \end{cases} \quad (26)$$

$$\begin{cases} -\frac{1}{G_1} \left[2\mu_1 R_2^3 A_1'' + R_2 B_1'' - (1-\mu_1) \frac{2}{R_2} C_1'' - \frac{1}{R_2^3} D_1'' \right] \\ + \frac{1}{G_2} \left[2\mu_2 R_2^3 A_2'' + R_2 B_2'' - (1-\mu_2) \frac{2}{R_2} C_2'' - \frac{1}{R_2^3} D_2'' \right] = 0, \\ \frac{1}{G_1} \left[(3-2\mu_1) R_2^3 A_1'' + R_2 B_1'' - (1-2\mu_1) \frac{1}{R_2} C_1'' + \frac{1}{R_2^3} D_1'' \right] - \frac{1}{G_2} \\ \cdot \left[(3-2\mu_2) R_2^3 A_2'' + R_2 B_2'' - (1-2\mu_2) \frac{1}{R_2} C_2'' + \frac{1}{R_2^3} D_2'' \right] = 0, \end{cases} \quad (27)$$

$$\begin{cases} -\frac{1}{G_2} \left[2\mu_2 R_3^3 A_2'' + R_3 B_2'' - (1-\mu_2) \frac{2}{R_3} C_2'' - \frac{1}{R_3^3} D_2'' \right] \\ + \frac{1}{G_3} \left[-(1-\mu_3) \frac{2}{R_3} C_3'' - \frac{1}{R_3^3} D_3'' \right] = 0, \\ \frac{1}{G_2} \left[(3-2\mu_2) R_3^3 A_2'' + R_3 B_2'' - (1-2\mu_2) \frac{1}{R_3} C_2'' + \frac{1}{R_3^3} D_2'' \right] \\ - \frac{1}{G_3} \left[-(1-2\mu_3) \frac{1}{R_3} C_3'' + \frac{1}{R_3^3} D_3'' \right] = 0, \end{cases} \quad (28)$$

where the constants of $A_1'', B_1'', C_1'', D_1'', A_2'', B_2'', C_2'', D_2'', C_3'',$ and D_3'' were calculated by the total 10 equations in equations (24)–(28). Then, wellbore stress distribution induced by deviator stress was obtained by substituting these 10 constants and A_3'' and B_3'' into equations (19)–(22).

3.3. Stress Induced by Shear Stress. The stress induced by shear stress was $u_r^{mi}, u_\theta^{mi}, \sigma_r^{mi}, \sigma_\theta^{mi},$ and $\tau_{r\theta}^{mi}, i = 1, 2, 3$ represented the casing, cement sheath, and formation, respectively, (Figure 11). The angle of Ω between σ_x and x -direction was calculated by using equation (29). Then, the principal stresses were obtained as follows [29]:

$$\Omega = \frac{1}{2} \arctan \left(-\frac{2\tau_{xy}}{\sigma_x - \sigma_y} \right) = \frac{\pi}{4}, \quad (29)$$

$$\begin{cases} \sigma_{\pi/4} = \tau_{xy}, \\ \sigma_{-\pi/4} = -\tau_{xy}. \end{cases} \quad (30)$$

It could be found that the stress distribution induced by shear stress was similar with that by deviator stress when counterclockwise rotating the angle of $\pi/4$. The stresses and displacements were obtained by substituting the reference variable $\theta = \theta' (-\pi/4)$ into the stress induced by deviator stress discussed in Section 3.2.

3.4. Stress Induced by Temperature Variation. The thermal field was obtained by using the steady temperature distribution model to calculate the thermal stress. When fracturing fluids were pumped into a wellbore with a high pump

FIGURE 11: Stress distribution induced by shear stress.

rate, they were always in the turbulent state. The heat transfer coefficient between casing and fluid was calculated using the Marshall model [30] shown in the following equation:

$$h = \frac{S_t k_m}{D} = 0.0107 \frac{k_m}{D} \left\{ \frac{\rho_a D_{\text{eff}} (4Q/\pi D^2)}{K ((3n+1)/4n)^n (32Q/\pi D^3)^{n-1}} \right\}^{0.67} \cdot \left[\frac{K ((3n+1)/4n)^n (32Q/\pi D^3)^{n-1} C_m}{k_m} \right]^{0.33}, \quad (31)$$

where h is the heat transfer coefficient ($\text{W} \cdot \text{m}^{-2} \cdot ^\circ\text{C}^{-1}$), S_t is the Stanton number, P_r is the Prandtl number, Re_g is the Reynolds number, $\mu_{w,\text{app}}$ is the fluid apparent viscosity, D is the inner diameter (m), D_{eff} is the equivalent diameter (m), ρ_a is the fluid density ($\text{kg} \cdot \text{m}^{-3}$), n is the liquidity index, K is the consistency coefficient ($\text{Pa} \cdot \text{s}^n$), v is the fluid velocity, Q is the fracturing pump rate ($\text{m}^3 \cdot \text{min}^{-1}$), k_m is the coefficient of heat conductivity ($\text{W} \cdot \text{m}^{-1} \cdot ^\circ\text{C}^{-1}$), and C_m is the fluid specific heat capacity ($\text{J} \cdot \text{kg}^{-1} \cdot ^\circ\text{C}^{-1}$).

The temperature distribution among casing, cement sheath, and formation is shown in Figure 12. In the cylindrical coordinate system of CCF, the differential equation of steady heat conduction of the cylinder is expressed as [31]

$$\frac{d^2 T}{dr^2} + \frac{1}{r} \frac{dT}{dr} = 0. \quad (32)$$

Temperature field distribution solutions were obtained according to integral and boundary conditions $k_i dT/dr = h_n (T_i - T_n)$, $T|_{r=R_i} = T_i$, $T|_{r=R_{i+1}} = T_{i+1}$ shown in the following equation:

$$T^i(r) = A_i^T \ln r + B_i^T, \quad (33)$$

$$\begin{cases} A_i^T = \frac{T_{i+1} - T_n}{\ln(R_{i+1}/R_i) + ((i^2/2) - (5i/2) + 3)k_i/R_i h}, \\ B_i^T = \frac{T_n \ln R_{i+1} - T_{i+1} \ln R_i + ((i^2/2) - (5i/2) + 3)(k_i/R_i h)T_{i+1}}{\ln(R_{i+1}/R_i) + ((i^2/2) - (5i/2) + 3)(k_i/R_i h)}, \end{cases} \quad (34)$$

where T^i is the temperature ($^\circ\text{C}$); T_i is the temperature at the interface ($^\circ\text{C}$); T_n is the fluid temperature ($^\circ\text{C}$); k_i is the material thermal conductivity ($\text{W} \cdot \text{m}^{-1} \cdot ^\circ\text{C}^{-1}$); R_i is the radius

FIGURE 12: The distribution of interface temperatures.

(m); and A_i^T and B_i^T were the constants, $i = 1, 2, 3$ represented casing, cement sheath, and formation, respectively.

The heat flow density continuity conditions were expressed as

$$k_i \frac{dT^i(r)}{dr} \Big|_{r=R_{i+1}} = k_{i+1} \frac{dT^{i+1}(r)}{dr} \Big|_{r=R_{i+1}}. \quad (35)$$

The temperatures at interfaces of casing-cement sheath and cement sheath-formation system were defined as T_2 and T_3 and were calculated by using the following equation:

$$\begin{cases} (1 + \beta_1)T_2 - \beta_1 T_3 = T_1, \\ -T_2 + (1 + \beta_2)T_3 = \beta_2 T_4, \end{cases} \quad (36)$$

where

$$\begin{cases} \beta_1 = \frac{k_2 \ln(R_2/R_1) + (k_1/R_1)h}{k_1 \ln(R_3/R_2)}, \\ \beta_2 = \frac{k_3}{k_2} \left[\frac{\ln(R_3/R_2)}{\ln(R_4/R_3)} \right]. \end{cases} \quad (37)$$

Interfacial temperature of T_i was obtained by solving equation (36). The steady-state temperature field around the wellbore could be calculated by substituting T_i into equations (33) and (34). According to thermal elastic mechanics, constitutive equations for a plane strain problem were expressed as

$$\begin{cases} \epsilon_r^T = \frac{1 + \mu_i}{E_i} [(1 - \mu_i)\sigma_r^T - \mu_i \sigma_\theta^T] + (1 + \mu_i)\alpha_i T, \\ \epsilon_\theta^T = \frac{1 + \mu_i}{E_i} [(1 - \mu_i)\sigma_\theta^T - \mu_i \sigma_r^T] + (1 + \mu_i)\alpha_i T, \\ \epsilon_z^T = 0, \\ \gamma_{r\theta}^T = \frac{2(1 + \mu_i)}{E_i} \tau_{r\theta}^T. \end{cases} \quad (38)$$

The actual thermal stress field, F_3 in the strata induced by the temperature changes, was decomposed into two

parts: the original stress field, A3, and the disturbance field, B3 induced by the temperature variation shown in Figure 13.

The initial stresses were $\sigma_r^{Ti0} = \sigma_\theta^{Ti0} = 0$, and the initial strains were $\varepsilon_r^{Ti0} = \varepsilon_\theta^{Ti0} = 0$. The stresses and displacements induced by thermal variations were expressed as

$$u_r^{Ti} = \frac{(1 + \mu_i)}{(1 - \mu_i)} \frac{\alpha_i}{r} \int_{R_i}^r r \Delta T^i dr + C_1^{Ti} r + \frac{C_2^{Ti}}{r} - r \varepsilon_r^{Ti0}, \quad (39)$$

$$\begin{cases} \sigma_r^{Ti} = -\frac{\alpha_i E_i}{1 - \mu_i} \frac{1}{r^2} \int_{R_i}^r r \Delta T^i dr + \frac{E_i}{1 + \mu_i} \left[\frac{C_1^{Ti}}{1 - 2\mu_i} - \frac{C_2^{Ti}}{r^2} \right], \\ \sigma_\theta^{Ti} = \frac{\alpha_i E_i}{1 - \mu_i} \frac{1}{r^2} \int_{R_i}^r r \Delta T^i dr + \frac{E_i}{1 + \mu_i} \left[\frac{C_1^{Ti}}{1 - 2\mu_i} + \frac{C_2^{Ti}}{r^2} \right] \\ - \frac{\alpha_i E_i}{1 - \mu_i} \Delta T^i, \end{cases} \quad (40)$$

where C_1^{Ti} and C_2^{Ti} are the constants, σ_r^{Ti} and σ_θ^{Ti} are the radial and tangential stresses (Pa), u_r^{Ti} is the radial displacement (m), ΔT^i is the temperature changes ($^\circ\text{C}$), p_i is the interface pressure (Pa), and α_i is the material thermal expansion coefficient, $i = 1, 2, 3$ represented casing, cement sheath, and formation, respectively.

The temperatures were known, and the boundary was free at internal casing and external formation. So, radial stress at inner and outer boundaries equals to zero, and radial displacement at the outer boundary equals to zero as well. The boundary and interfacial displacement continuity conditions were expressed as

$$\begin{cases} u_r^{T1}|_{r=R_2} = u_r^{T2}|_{r=R_2}, \\ u_r^{T2}|_{r=R_3} = u_r^{T3}|_{r=R_3}, \\ \sigma_r^{T1}|_{r=R_2} = \sigma_r^{T2}|_{r=R_2}, \\ \sigma_r^{T2}|_{r=R_3} = \sigma_r^{T3}|_{r=R_3}, \\ \sigma_r^{T1}|_{r=R_1} = 0, \\ \sigma_r^{T3}|_{r=R_4} = 0. \end{cases} \quad (41)$$

Substituting equations (39) and (40) into the following equation, the equations were obtained as

$$\begin{cases} C_1^{T1} R_2 + \frac{C_2^{T1}}{R_2} - C_1^{T2} R_2 - \frac{C_2^{T2}}{R_2} = -\frac{(1 + \mu_1)}{(1 - \mu_1)} \frac{\alpha_1}{R_2} \int_{R_1}^{R_2} r T^1 dr, \\ C_1^{T2} R_3 + \frac{C_2^{T2}}{R_3} - C_1^{T3} R_3 - \frac{C_2^{T3}}{R_3} = \frac{(1 + \mu_2)}{(1 - \mu_2)} \frac{\alpha_2}{R_3} \int_{R_2}^{R_3} r T^2 dr, \end{cases} \quad (42)$$

$$\begin{cases} \frac{E_1}{1 + \mu_1} \left[\frac{C_1^{T1}}{1 - 2\mu_1} - \frac{C_2^{T1}}{R_2^2} \right] - \frac{E_2}{1 + \mu_2} \left[\frac{C_1^{T2}}{1 - 2\mu_2} - \frac{C_2^{T2}}{R_2^2} \right] \\ = \frac{\alpha_1 E_1}{1 - \mu_1} \frac{1}{R_2^2} \int_{R_1}^{R_2} r \Delta T^1 dr, \\ \frac{E_2}{1 + \mu_2} \left[\frac{C_1^{T2}}{1 - 2\mu_2} - \frac{C_2^{T2}}{R_3^2} \right] - \frac{E_3}{1 + \mu_3} \left[\frac{C_1^{T3}}{1 - 2\mu_3} - \frac{C_2^{T3}}{R_3^2} \right] \\ = \frac{\alpha_2 E_2}{1 - \mu_2} \frac{1}{R_3^2} \int_{R_2}^{R_3} r \Delta T^2 dr, \end{cases} \quad (43)$$

$$\begin{cases} C_1^1 R_1 + \frac{C_2^1}{R_1} = 0, \\ C_1^3 R_4 + \frac{C_2^3}{R_4} = -\frac{(1 + \mu_2)}{(1 - \mu_2)} \frac{\alpha_2}{R_4} \int_{R_3}^{R_4} r T dr. \end{cases} \quad (44)$$

The constants of $C_1^{T1}, C_2^{T1}, C_1^{T2}, C_2^{T2}, C_1^{T3}$, and C_2^{T3} were obtained by equations (42)–(44). The wellbore stress was obtained by substituting these constants into equation (40).

The total stresses were obtained using the following equation:

$$\begin{cases} \sigma_r^i = \sigma_r^i + \sigma_r^{\prime\prime i} + \sigma_r^{\prime\prime\prime i} + \sigma_r^{Ti}, \\ \sigma_\theta^i = \sigma_\theta^i + \sigma_\theta^{\prime\prime i} + \sigma_\theta^{\prime\prime\prime i} + \sigma_\theta^{Ti}, \\ \sigma_z^i = \mu_i (\sigma_r^i + \sigma_\theta^i), \\ \tau_{r\theta}^i = \tau_{r\theta}^{\prime\prime i} + \tau_{r\theta}^{\prime\prime\prime i}, \end{cases} \quad (45)$$

where σ_r^i is the radial stress, σ_θ^i is the tangential stress, σ_z^i is the axial stress, and $\tau_{r\theta}^i$ is the shear stress.

3.5. Estimation of Wellbore Integrity. It is generally accepted that the yield of isotropic material such as casing has nothing to do with hydrostatic pressure, while hydrostatic pressure is not considered in von Mises yield criterion. So, this criterion was adopted to determine the casing failure:

$$f(J_2, k) = \sqrt{J_2} - k = 0,$$

$$\begin{aligned} J_2 = \frac{1}{6} [(\sigma_{11} - \sigma_{22})^2 + (\sigma_{22} - \sigma_{33})^2 + (\sigma_{33} - \sigma_{11})^2] \\ + \sigma_{12}^2 + \sigma_{23}^2 + \sigma_{31}^2, \end{aligned} \quad (46)$$

where J_2 is the second stress partial tensor; k is the critical value of failure; and σ_{ij} is the stress components, $i, j = 1, 2, 3$ represented the three directions of the system, respectively.

For uniaxial tension, $\sqrt{J_2} = \sigma/\sqrt{3}$, the von Mises stress could be expressed as follows in the polar coordinate:

FIGURE 13: Thermal stress field. (a) Formation stress components. (b) Interface pressures. p_i is the interface pressure, $i = 2, 3$ represented the casing-cement sheath interface and cement sheath-formation interface.

$$\sigma_{\text{Mises}} = \sqrt{\frac{1}{2} [(\sigma_r - \sigma_\theta)^2 + (\sigma_\theta - \sigma_z)^2 + (\sigma_z - \sigma_r)^2] + 3\tau_{r\theta}^2 + 3\tau_{\theta z}^2 + 3\tau_{zr}^2}. \quad (47)$$

4. Model Validation

From 2009 to 2017, PetroChina has drilled 141 fracturing wells (including 112 horizontal wells) in the Changning-Weiyuan National Shale Gas Demonstration Area. The geometrical dimensions of the CCF model were a wellbore diameter of 8.5 in, casing diameter of 5.5 in, and casing thickness of 9.17 mm. According to the Saint-Venant principle, a formation boundary dimension should be five to six times larger than that of the wellbore geometry to avoid the influence of boundary effect on wellbore stress. In view of this, the model geometry was $2,000 \times 2,000$ mm, while the corresponding wellbore diameter was 215.9 mm. The direction of horizontal in situ stress was N120°E. The well deviation angle was 90°, and the wellbore azimuth was N30°E, indicating that the horizontal trajectory was along the minimum in situ stress direction. The internal casing pressure was calculated from the pump pressure plus the downhole hydrostatic fluid pressure. The external boundary stress was obtained from the geostress data of the shale reservoir. The thermal and mechanical properties of different materials are presented in Table 1. The casing stress and displacement were calculated and analyzed considering thermal-pressure coupling.

The applied maximum horizontal stress σ_H was 82 MPa, the minimum horizontal stress σ_h was 55 MPa, the vertical stress σ_v was 57 MPa, the inner casing pressure P_i was 75 MPa, the boundary temperature T_4 was 100°C, the fluid temperature T_a was 20°C, and the convective heat transfer coefficient was obtained by using equation (20) ($1890 \text{ W} \cdot \text{m}^{-2} \cdot ^\circ\text{C}^{-1}$) at the pump rate of $20 \text{ m}^3/\text{min}$.

The finite element analysis method was adopted to validate the results of the analytical models. A steady-state thermal analysis followed by a static structural analysis was conducted to calculate the stress considering thermal-pressure coupling. The solutions of radial stress, circumferential stress, and Mises stress are compared in Figure 14.

The analytical solutions of radial stress, circumferential stress, and Mises stress were in good agreement with the results obtained by a finite element method, which indicates

the validity of the analytical method. The maximum deviation between analytical and finite element results was 1.4–13.9%, indicating that the analytical model could provide an accurate calculation of stress distribution for the CCF system.

From Figures 14(a) and 14(b), the radial stress increased with the increase of radius in casing and cement sheath, but decreased in the formation. The absolute value of radial stress calculated by the new model was smaller than that of the existing model. This was mainly because the new model excluded the strain induced by the initial stress. From Figures 14(c) and 14(d), the circumferential stress decreased with the increase of radius in the casing and cement sheath and increased slowly to a constant value in the formation. The interfacial stress at the internal casing wall was larger than that at the external casing wall. The solutions calculated by the new model were larger than those by the existing model. From Figure 14(e), casing Mises stress obtained by the new model was larger than that of the existing model. It could be explained that circumferential stress was larger than radial stress and had a main influence on Mises stress.

The radial displacements along the 0° direction calculated by the new model and existing model under the same conditions were shown in Figure 15. There was an obvious difference for two models, especially at the outer boundary. The displacements of new model approached zero when the outer boundary was infinite, which reached an agreement with the actual boundary condition. However, the displacements obtained by the existing model increased linearly in the formation. So, only the new model could reflect the actual situation.

5. Sensitivity Analysis

The sensitivity analyses were carried out to study the influences of cement sheath properties, geostress, fracturing pressure, fluid temperature, casing thickness, and cement sheath thickness on casing stress. During analyzing, only one parameter was variable and others were constants. Unless otherwise mentioned, the parameters were set as mentioned in Section 4.

TABLE 1: Thermal and mechanical Parameters of fluid-casing-cement sheath-formation system.

Property	Casing	Cement sheath	Formation	Fluid
Elastic modulus, E_i (GPa)	210	5	35	—
Poisson's ratio, μ_i	0.3	0.15	0.25	—
Coefficient of thermal expansion, α_i ($10^{-5} \cdot ^\circ\text{C}^{-1}$)	1.5	1.0	1.0	—
Thermal conductivity, k_i ($\text{W} \cdot \text{m}^{-1} \cdot ^\circ\text{C}^{-1}$)	58.2	1.0	1.0	1.73
Specific heat, C_{pi} ($\text{J} \cdot \text{kg}^{-1} \cdot ^\circ\text{C}^{-1}$)	460	1830	1043	3935
Density, ρ_i ($\text{kg} \cdot \text{m}^{-3}$)	7850	1800	2500	1080

Note: properties in parenthesis were used in the parametric study.

FIGURE 14: Continued.

FIGURE 14: Comparison of numerical and analytical solutions. (a) Radial stress along the radial directions of 0° and 90° . (b) Radial stress at the internal casing wall. (c) Circumferential stress along the radial directions of 0° and 90° . (d) Circumferential stress at the internal casing wall. (e) Mises stress at inner and outer casing walls.

FIGURE 15: Radial displacements of the wellbore assembly along the 0° direction.

5.1. Influence of Elastic Modulus. Cement sheath properties is crucial for casing safety. To evaluate the effect of elastic modulus on casing stress, the cement sheath elastic modulus of E_2 was set at the range from 2 GPa to 50 GPa, and the formation elastic modulus of E_3 was set as 5 and 35 GPa to simulate a soft and hard formation. The Mises stresses at internal casing are shown in Figure 16.

From Figures 16(a) and 16(c), the maximum Mises stress appeared at the angles of 0° and 180° for the new model and 90° and 270° for the existing model when the formation

modulus was small. However, the maximum stress all appeared at the angles of 0° and 180° for the new and existing models when the formation modulus was large. From Figure 16(b), in a soft formation (a modulus of 5 GPa), with the increase of the cement sheath modulus, the maximum casing stress increased first, and then decreased for existing model, while decreasing all the time for the new model. From Figure 16(b), in a hard formation (modulus of 35 GPa), the maximum casing stress always decreased with the increase of the cement sheath modulus for two models. In the soft

FIGURE 16: Casing Mises stress. (a, b) $E_3 = 5$ GPa and (c, d) $E_3 = 35$ GPa.

formation, the stress calculated by the new model was smaller than that by the existing model. However, the stress obtained by the new model was larger than that by the existing model in a hard formation. According to the fact that shale formation had a large elastic modulus, the existing model underestimated casing stress during the fracturing operation.

5.2. Influence of Poisson's Ratio. To evaluate the effect of Poisson's ratio on casing stress, cement sheath Poisson's ratio, μ_2 , with a range from 0.05 to 0.45, was adopted and the formation Poisson's ratio, μ_3 , was set as 0.05 and 0.45 to

simulate a hard and soft formation. The casing Mises stresses are shown in Figure 17.

From Figures 17(a) and 17(b), the maximum Mises stress decreased with the increase of cement sheath Poisson's ratio for two models. In a hard formation (Poisson's ratio of 0.05), the maximum stress obtained by the new model was larger than that by the existing model. However, in a soft formation (Poisson's ratio of 0.45), it was a little smaller than that by the existing model. According to the fact that shale formation had a small Poisson's ratio, the existing model underestimated casing stress during the fracturing process.

FIGURE 17: Casing Mises stresses for different Poisson's ratios. (a) $\mu_3 = 0.05$. (b) $\mu_3 = 0.45$.

5.3. Influence of In Situ Stress Nonuniformity. During the multifracturing operation for shale gas wells, the fracturing fluid was pressed into the formation and the in situ stress field changed abruptly to increase the nonuniformity of the stress around the wellbore. To evaluate the effect of in situ stress nonuniformity on casing stress, the nonuniformity index was defined as $\delta = \sigma_H / \sigma_v$. Different δ with a range of 0.1–3.0 was adopted. The casing Mises stresses calculated by two models are shown in Figure 18.

As seen from Figure 18(a), for δ smaller than 1.0, the maximum Mises stresses appeared at 90° and 270° directions. However, for δ larger than 1.0, the maximum Mises stresses appeared at 0° and 180° directions. For δ of 1.0, the casing Mises stress around the wellbore was at a uniform state. From Figure 18(b), the maximum casing stress increased dramatically with the increase of δ from 1.0 or decrease of δ from 1.0. The solution obtained by the new model was larger than that by the existing model for δ between 0.5 and 2.25. When δ was larger than 2.25 or smaller than 0.5, the casing stress obtained by the existing model was larger than that by the new model.

5.4. Influence of Fracturing Pressure. A fracturing fluid with high pressure was used to fracture a shale formation. The high pressure depended on the formation regional tectonic stress; the larger the tectonic stress, the higher the pressure. Moreover, a high fracturing pressure posed a great potential challenge to casing failure. Different fracturing pressures with a range of 5–105 MPa were adopted to evaluate the effect of fracturing pressure on casing stress. The maximum casing Mises stresses are shown in Figure 19.

It can be seen from Figure 19 that the casing stresses calculated by the two models decreased first and then increased with the increase of fracturing pressure. The minimum stress appeared at 15 MPa for the new model; however, it appeared at about 25 MPa for the existing model.

In addition, the casing Mises stress obtained by the existing model was larger than that by the new model for pressure lower than 25 MPa and smaller than that by the new model for pressure higher than 25 MPa. During fracturing operation, pressure must be large enough to fracture the formation, so the existing model underestimated the casing stress.

5.5. Influence of Fluid Temperature. During the cycle injection of fracturing fluid, the heat transfer coefficient h was calculated using equation (20) with a pump rate of $20 \text{ m}^3/\text{min}$. The corresponding casing internal Mises stress was calculated under different fluid temperatures at a range of $10\text{--}100^\circ\text{C}$ to evaluate the effect on casing stress. Figure 20 presented the maximum casing stress over temperature and the comparison of the results obtained by the existing model and new model.

From Figure 20, the maximum Mises stress decreased with the increase of the injection fluid temperature, indicating that a fracturing fluid with high temperature was effective to decrease casing stress. Furthermore, the stress obtained by the existing model was smaller than that by the new model. It revealed that the existing model underestimated the casing Mises stress.

5.6. Influence of Thickness. The thickness of cement sheath and casing was curial for casing safety. To evaluate the effect of thickness on the casing stress, the cement thickness was set at a range of 2–50 mm and the casing thickness was set at a range of 5–15 mm. The comparisons of maximum casing Mises stress obtained by the two models are shown in Figure 21.

As shown in Figure 21, the maximum casing Mises stress increased with the increase of cement sheath thickness and, however, decreased with the increase of casing thickness. So, a thicker casing wall and thinner cement sheath were

FIGURE 18: Casing Mises stress for different stress nonuniformity indexes: (a) casing internal stress and (b) maximum casing stress.

FIGURE 19: Maximum casing Mises stress for different pressures.

effective to ensure the casing integrity. Meanwhile, casing stress obtained by the existing model was smaller than that by the new model indicating that the existing model underestimated casing stress.

6. Conclusions

A new analytical model considering drilling construction was established to assess the casing stress under different conditions considering thermal-pressure coupling. The solutions were obtained by dividing the model into three parts such as initial stress field, wellbore disturbance field, and thermal stress field. Sensitivity analyses of different factors were conducted to evaluate the influences on casing stress.

FIGURE 20: Casing Mises stress for different temperatures.

Some conclusions were drawn from the comparisons between new model and existing model:

- (1) The results of radius stress, tangential stress, and casing Mises stress calculated by the analytical method were in good agreement with the solutions by a finite element solution. The minor deviations did not exceed 13.9%. Moreover, the analytical solutions were in-line with the actual boundary conditions of shale gas wells.
- (2) The casing stress calculated by the existing model was smaller than that by the new model for hard

FIGURE 21: Casing Mises stress for different thicknesses of casing and cement sheath. (a) Different cement thickness. (b) Different casing thickness.

formation with larger modulus or low Poisson's ratio, geostress heterogeneity index at a range of 0.5–2.25, and fracturing pressure larger than 25 MPa.

- (3) The casing stress increased with the increase of the in situ stress nonuniformity index. With the increase of fracturing pressure, casing stress decreased first and then increased.
- (4) Cement sheath with appropriate modulus and larger Poisson's ratio, thinner cement sheath, thicker casing, and higher fluid temperature were effective to decrease the casing stress.

In conclusion, the new analytical model can accurately predict casing stress and become an alternative method of casing integrity evaluation for shale gas wells. It is a useful and efficient method for a preliminary design, being capable of simulating the actual situations in order to assess the casing stresses and integrity.

Data Availability

The data of each figure used to support the findings of this study are available from the corresponding author upon request.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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