Study on Dynamic Mechanical Properties and Damage Evolution Model of Siltstone

Guoliang Zhang,1,2 Haipeng Jia,1 and Shuaifeng Wu3

1School of Mechanics and Civil Engineering, China University of Mining and Technology (Beijing), Beijing 100083, China
2Public Order Administration Department of the Ministry of Public Security, Beijing 100741, China
3State Key Laboratory of Simulation and Regulation of Water Cycle in River Basin, China Institute of Water Resources and Hydropower Research, Beijing 100038, China

Correspondence should be addressed to Haipeng Jia; bqt1700603024@student.cumtb.edu.cn

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This paper is focused on exploring the dynamic mechanical properties and damage process of siltstone. For this purpose, different stress wave wavelengths (0.5 m–2.0 m) and different strain rates (25 s−1–120 s−1) were applied to siltstone specimens in the SHPB dynamic impact test. The experimental results show that the dynamic compressive strength of siltstone is linearly positively correlated with the strain rate, and the dynamic increase factor is linearly positively correlated with the natural logarithm of strain rate; the peak strain is linearly positively correlated with the strain rate, and the increase in wavelength causes the peak strain to increase. Through multiple impact tests, it is concluded that the cumulative damage to siltstone increases with the number of impacts. The cumulative damage curve exhibits an initial rapid rise, followed by a stable development, followed by another rapid rise. With increasing wavelength of the stress wave, the stable development of the curve gradually decreases, the cumulative damage to the siltstone is intensified, and the number of repeated impacts is reduced. Meanwhile, a model for damage evolution is established based on the inverse of the Gompertz function, and the physical meanings of the model parameters are determined. The model can reflect the influence of both stress wave parameters and impact times. Verification of the model demonstrates the rationality of the model and the correctness of the physical meaning of the parameters. The model could be applied in future studies of damage to sedimentary rocks.

1. Introduction

In many engineering constructions, such as tunnels, water conservancy, mining, and others, blasting is an efficient, economical, and reliable method, which involves the dynamics of rock under impact load. The mechanical properties of rock materials under impact loads are different from those under static loads; this is due to the transient response characteristics of the rock caused by the transient characteristics of the impact load. Studying the mechanical properties of rock materials under impact loading is the basic content of rock dynamics. Many scholars have conducted in-depth research on this [1–4], and many important achievements have been made in the overall comparison of the mechanical properties of rock under different strain rates. Zhi et al. uniaxial impact compression test on siltstone shows that the strain rate sensitivity of siltstone is stronger than other brittle rocks [5–7]. When the strain rate ranges from 50−1 to 100 s−1, the dynamic increase factor is 1.3–2.4. The deformation morphology changes as the strain rate increases, from the peripheral spalling radial tensile failure to the multilobed fracture to the comminution failure mode. Ping et al. [8–10] conducted an impact compression test of siltstone with a strain rate of 5−1–200 s−1. It was found that the dynamic increase factor was 1.0–1.4, and the dynamic strength and strain rate were in a power function relationship. The failure mode is converted from tensile failure to splitting comminuted damage. Li et al. [11–15] studied the stress balance and constant strain rate of the rock samples by changing the incident loading waveform. It is considered that the half sine wave has almost no dispersion effect, and its wave head rises gently and is not affected by changes in rod.
diameter. Mao et al. [16–18] summarized the constitutive relationship of the red siltstone after the impact test and divided the failure process into five typical characteristic stages. It was found that the siltstone had the characteristics of strain softening under high strain rate loading.

In the practice of a large number of blasting engineers, the surrounding rock far from the blasting working face will be deformed, unstable, and collapsed. This is because the stress wave generated by one explosion cannot cause the surrounding rock damage from the far-field region of the explosion source, but under the action of multiple low-amplitude stress waves, the rock mass joints will expand, cracks will be formed and gradually penetrated, and eventually the failure will occur [19], which is the process of damage accumulation, and at the macroscopic level, it shows a deterioration in mechanical properties and a decrease in bearing capacity and stability. Therefore, it is particularly important to study the cumulative damage evolution of rocks under multiple stress waves. The core of the research is attributed to the establishment of damage evolution models. Jin et al. [20], Liu et al. [21], and Liu et al. [22] pioneered the theory of damage to rocks under confining pressure and the model of rock confining damage. On this basis, scholars conducted more in-depth research. Based on the results of the brittle rock dynamic load test, Gao and Liu [23] introduced the relationship between energy dissipation rate and acoustic attenuation coefficient and established the dynamic damage evolution equation of rock. Zhu et al. [19] carried out an impact test on siltstone and found that the siltstone damage changes in the vicinity of the dynamic stress peak and the internal crack of the rock enters the development stage. Li et al. [11] conducted a repeated impact test on the granite and found that when the peak pressure of the impact load is less than the static compressive strength of the rock, multiple impacts will not cause rock damage. Based on the logistic function, Jin et al. [24, 25] established a rock damage model suitable for confining pressure and cyclic impact.

Although many scholars have carried out more dynamic properties of rock from the aspects of constitutive [26, 27], loading mode [28], energy dissipation [28–31], and fracture [32, 33], they did not consider the effect of different stress wavelengths on dynamic mechanical properties, especially the sensitivity of damage evolution to stress wavelength. The stress wavelength determines the action time of the load, which in turn affects the morphology of the constitutive curve and the cumulative damage evolution process. Based on this, the impact test of siltstone under different combinations of impact rod length and impact velocity is carried out by using the Split Hopkinson Pressure Bar device. The mechanical properties and damage evolution of siltstone under different stress wave wavelengths and amplitudes are studied, and the corresponding damage model is established.

2. Siltstone SHPB Test Device and Test Plan

In the SHPB test, the test process should satisfy the one-dimensional wave assumption and the stress uniformity assumption [34], which requires the specimen to be selected to a reasonable size. Under this premise, the design of the specimen is a cylinder with the diameter of 50 mm and the length of 40 mm, the end faces are precisely polished, and the flatness is controlled within 0.02 mm, at the same time, and in order to minimize the friction effect of the end face, the lubricant (vaseline) is applied to both ends of the specimen during the test. The siltstone selected in this paper was taken from Weiyuan City of Sichuan Province in the People’s Republic of China. The selected siltstone was defined as siliceous siltstone after mineral composition analysis. To ensure the consistency of the test, each small specimen with the sound velocity difference within 5% after ultrasonic testing was selected. The basic physical and mechanical index values of siltstone are obtained based on the reliability statistics: the longitudinal wave velocity of 3291 m/s, the elastic modulus of 25.6 GPa, the uniaxial compressive strength of 76.13 MPa, and Poisson’s ratio of 0.22.

The siltstone has a particle size of 0.0039 mm to 0.0625 mm, which accounts for 85% of the particles, and the diameter of the SHPB system is 50 mm. According to the requirement of satisfying the stress uniformity assumption, the wave impedance of the specimen and the wave impedance of the incident and transmission rods of the SHPB system are as close as possible. The standard value of the wave impedance of the siltstone selected in this test is 8.59 MPa·m−1·s−1; after comparison, the aluminum alloy rod was selected as the test system. The wave impedance of the aluminum alloy rod is 13.7 MPa·m−1·s−1, the wave impedance ratio of the siltstone to the aluminum alloy rod is 0.63, the stress wave is reciprocated three times in the specimen to achieve uniform stress [35], and the time elapsed is 24 μs. The test system is shown in Figure 1.

The wavelength of the stress wave is determined by the length of the impact rod, and the amplitude is determined by the impact rod velocity. The test covers five wavelengths: 0.5 m, 0.8 m, 1.2 m, 1.6 m, and 2.0 m, respectively, and 5 impact rod velocity grades, a total of 25 groups, and each group of experiments repeated more than 3 times. The choice of wavelength is mainly to cover the wavelength of the stress wave provided by the more common SHPB test and to make certain differences between the groups. The selection of the impact velocity in the study of dynamic mechanical properties covers the range of velocities from the gradual increase of no damage to the occurrence of comminuted damage. In the study of damage evolution, the impact velocity is slightly larger than the damage threshold velocity corresponding to each wavelength, and the impact is repeated with this constant value.

3. Study on Dynamic Mechanical Properties of Siltstone

3.1. Analysis of Strain Rate Effect of Dynamic Mechanical Properties of Siltstone

According to the above impact test program, the impact test results of siltstone under different stress wavelengths and impact velocities are shown in Table 1.

Figure 2 shows the relationship between dynamic strength and strain rate of siltstone. From Figure 2, it can be
seen that the dynamic compressive strength of siltstone is linearly positively correlated with strain rate.

Dynamic increase factor (DIF) is a measure of the strength of rock under dynamic load conditions and is the ratio of dynamic compressive strength to static compressive strength. It can be expressed as follows:

$$D = \frac{f_d}{f_s}, \quad (1)$$

where $f_d$ and $f_s$ are the dynamic compressive strength and static compressive strength of the rock, respectively.

Figure 3 shows the natural logarithmic relationship between the dynamic increase factor of the siltstone and the strain rate, reflecting the sensitivity of the dynamic strength growth of the siltstone to the change of the stress wave parameters. A comprehensive analysis of Figures 2 and 3 shows that when the stress wave amplitude exceeds the static compressive strength of siltstone (76.13 MPa in this paper), the change of wavelength does not cause the change of dynamic strength; the decisive factor affecting the dynamic strength is the change of strain rate. The specific performance is as follows: the greater the impact velocity, the larger the amplitude and the greater the strain rate, resulting in an increase in dynamic strength. The DIF is linearly positively correlated with the natural logarithm of the strain rate and can be expressed by using formula (2), and the correlation coefficient $R$ is 0.89:

$$\text{DIF} = -5.984 + 1.565 \ln \dot{\varepsilon}, \quad 60 < \dot{\varepsilon} < 160. \quad (2)$$

### Table 1: Results of the SHPB test on siltstone.

<table>
<thead>
<tr>
<th>Wavelength (m)</th>
<th>Specimen number</th>
<th>Impact velocity (m·s$^{-1}$)</th>
<th>Strain rate (s$^{-1}$)</th>
<th>Peak stress (MPa)</th>
<th>Peak strain ($\times10^{-3}$)</th>
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<td>145.15</td>
<td>13.35</td>
<td>1.91</td>
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</table>

3.2. Wavelength Effect Analysis of Dynamic Mechanical Properties of Siltstone. Under the impact, the deformation and failure state of the siltstone shows a strong correlation with the wavelength of the stress wave.

When the strain rate range is basically the same, and the wavelength is increased from 0.5 m to 2.0 m, the peak strain...
corresponding to the dynamic strength of the siltstone gradually increases. Figure 4 shows the relationship between the peak strain and the stress wave wavelength at the same strain rate level, indicating that the strain rate is linearly positively correlated within \(10^2\) s\(^{-1}\). The relationship between the wavelength and the peak strain can be expressed as equations (3) to (7), the strain rates are all in the range of 69 s\(^{-1}\) to 160 s\(^{-1}\), and the \(R\) is the fitting correlation coefficient.

When the wavelength is 0.5 m, \(R = 0.981\):

\[
\varepsilon_p = (0.066\dot{\varepsilon} - 2.129) \times 10^{-3}.
\]

(3)

When the wavelength is 0.8 m, \(R = 0.99\):

\[
\varepsilon_p = (0.076\dot{\varepsilon} - 2.46) \times 10^{-3}.
\]

(4)

When the wavelength is 1.2 m, \(R = 0.980\):

\[
\varepsilon_p = (0.078\dot{\varepsilon} - 2.76) \times 10^{-3}.
\]

(5)

When the wavelength is 1.6 m, \(R = 0.986\):

\[
\varepsilon_p = (0.111\dot{\varepsilon} - 3.72) \times 10^{-3}.
\]

(6)

When the wavelength is 2.0 m, \(R = 0.991\):

\[
\varepsilon_p = (0.112\dot{\varepsilon} - 3.047) \times 10^{-3}.
\]

(7)

In summary, the dynamic mechanical properties of siltstone exhibit strain rate effects and wavelength effects. On the one hand, at the medium strain rate level, the dynamic strength of siltstone is only related to the strain rate and is not affected by the wavelength change. The specific performance is as follows: the dynamic strength is linearly positively correlated with the strain rate, and when the strain rate is 140 s\(^{-1}\), the dynamic strength is increased by about 80% compared with static strength; the dynamic increase factor is linearly positively correlated with the natural logarithm of strain rate. On the other hand, the peak strain corresponding to the dynamic strength of siltstone is linearly positively correlated with the strain rate, and the increase of the wavelength causes the overall trend of peak strain to rise. This is because the increase of the wavelength of the stress wave causes the siltstone to receive more energy at the same amplitude, which causes the peak strain to increase correspondingly.

4. Dynamic Damage Evolution Law of Siltstone

4.1. Siltstone Impact Damage Test. In order to study the damage evolution law of siltstone under stress wave, the stress wave parameters are also taken as variables and the
influence of multiple impacts on cumulative damage is considered. The wavelengths use the same five wavelengths as in Section 3, but the damage evolution is for the development of rock damage rather than the failure. Therefore, the siltstone impact test is carried out with the impact rod velocity increment of 0.3 m/s to find out the impact velocities with only a decrease in sound velocity but no visible crack in the siltstone at the five wavelengths using the ultrasonic detector to measure the wave velocity of the specimen. The amount of damage is also characterized by sonic measurement, as shown in the following equation:

\[ D = 1 - \left( \frac{\nabla_p - \nabla^{0}}{V_p} \right)^2. \]  \hspace{1cm} (8)

In the formula, \( \nabla_p \) is the longitudinal wave velocity of the rock before the impact and \( V_p \) is the longitudinal wave velocity of the rock after the impact.

The final determination of the impact velocity at five wavelengths and the number of repeated impacts that the siltstone can withstand before the rupture at the corresponding wavelength are shown in Table 2.

### 4.2. Siltstone Damage Evolution Law

In this test, 3n specimens were selected at each wavelength for the first impact. After completing the first impact, 3 specimens were reserved for static mechanical testing. Then, the remaining \( 3n - 3 \) specimens are subjected to the second impact, and after the second impact, three specimens are left to be subjected to the static mechanical test. And so on, until the entire repeated impact process is completed. In this way, it is possible to ensure that there are 3 parallel experiments in each state. Table 3 shows the test results of the siltstone at the wavelength of 1.2 m with 9 stress waves.

The test specimens were taken from the same rock specimen, and the sound velocity difference of each test specimen after ultrasonic testing was within 5%, so it can be approximated that there is no difference between each test specimen. Therefore, under the same stress wave length and impact rod velocity, the cumulative damage of siltstone is only related to the number of impacts. The evolution graph is drawn based on the cumulative damage statistics at each of the five wavelengths, as shown in Figure 5.

As can be seen from Figure 5: Siltstone exhibits a form of "rapid rise-stable development-rapid rise" under repeated impacts, especially at short wavelengths. When the wavelength is 0.5 m, these three stages are most obvious; as the wavelength increases, the stable development of the damage gradually decreases. When the wavelength is 0.5 m, the stable development segment corresponds to 6 repeated impacts. When the wavelength is 0.8 m, the stable development segment corresponds to 5 repeated impacts. When the wavelength is 1.2 m, the stable development segment corresponds to 3 repeated impacts. When the wavelength is 1.6 m, the stable development segment corresponds to 2 repeated impacts. When the wavelength is 2.0 m, there is basically no stable development segment, and the cumulative damage increases rapidly with the increase of the number of impacts.

| Table 2: Experimental parameters of circular impact. |
|-----------------|-----------------|-----------------|
| Stress wavelength (m) | Impact velocity (ms\(^{-1}\)) | Number of circular impacts |
| 0.5              | 9.45            | 14              |
| 0.8              | 7.53            | 12              |
| 1.2              | 6.50            | 9               |
| 1.6              | 5.25            | 6               |
| 2.0              | 4.71            | 4               |

Figure 6 shows the damage evolution of siltstone under repeated impacts. The relative number of impacts is normalized by the number of repeated impacts at the five wavelengths, allowing direct comparison of the different datasets. It can be seen from Figure 6 that when the relative number of impacts is an independent variable, all the data are still in the same development trend, that is the form of "rapid rise-stable development-rapid rise."

### 4.3. Siltstone Damage Evolution Model

#### 4.3.1. Model Construction

In the social and economic fields, there are many function curves that indicate "slow increase-rapid development-development maturity," such as the Gompertz function and logistic function [36–38], which is also a well-known S-shaped curve in the field of mathematics. The main difference between the Gompertz function and the logistic function is that the initial portion of the Gompertz function is shorter, the distal portion develops longer, and the logistic function has the characteristics of intermediate symmetry. The development trend of siltstone damage is consistent with the characteristics of the Gompertz function curve. The form of the Gompertz function curve is shown in Figure 7.

The Gompertz function expression is as follows:

\[ y = A \cdot e^{-e^{B(x-C)}}. \]  \hspace{1cm} (9)

Comparing the similarities and differences between the Gompertz function and the siltstone damage, the characteristics of the three stages of siltstone cumulative damage can be expressed by inversely transforming the Gompertz function. The inverse function of the Gompertz function is expressed as follows:

\[ y = A - B \cdot \ln \left( \frac{C}{x} \right). \]  \hspace{1cm} (10)

where \( A, B, \) and \( C \) are the parameters to be determined; \( x \) is the independent variable, indicating the number of repeated impacts; \( y \) is the dependent variable, indicating the cumulative damage value.

Therefore, the expression of the damage evolution model of siltstone under repeated impact can be expressed as follows:

\[ D = \delta - \alpha \cdot \ln \left( \frac{\beta}{n} \right). \]  \hspace{1cm} (11)

For the assignment of equation (11), \( \alpha, \beta, \) and \( \delta \) are taken as 0.15, 14, and 0.25, respectively, and the three-stage function graph of "rapid rise-stable development-rapid rise" can be obtained, as shown in Figure 8.
4.3.2. Influence of Parameters and Physical Meaning. It can be seen from Table 3 that the value of the damage $D$ is $0 \sim 1$, the number of impacts is a natural number, and the above value range can be obtained by substituting formula (11): the value of $\alpha$ is in the range of $0.1$ to $1$, the range of $\beta$ is the number of impacts, and $\delta$ affects the up and down movement of the overall model curve. By fixing the two parameter values and changing the other parameter values, the influence of the parameter changes on the model curve is analyzed to estimate the physical meaning of the $\alpha$ and $\beta$ parameters.

When $\beta$ and $\delta$ are fixed and $\alpha$ values are 0.15, 0.25, 0.35, 0.45, and 0.55, respectively, the effect on the model curve is shown in Figure 9. It can be seen from the figure that the smaller the $\alpha$ value, the longer the curve lasts in the stationary development segment; as $\alpha$ increases, the stationary development segment gradually shortens and gradually evolves from the obvious 3-stage characteristic to the linear exponential function type, that is the $\alpha$ controls the length of the smooth development segment. According to the experimental data, the three-stage regularity is most obvious when the wavelength is 0.5 m. As the wavelength increases, the smooth development of the damage gradually decreases. When the wavelength is 2.0 m, there is basically no stable development, and cumulative damage increases rapidly throughout the repeated impact process. Therefore, the effect of wavelength on cumulative damage is manifested by the parameter $\alpha$, and the shorter the wavelength, the smaller the $\alpha$ and the greater the opposite. In summary, the physical

<table>
<thead>
<tr>
<th>Wave length (m)</th>
<th>Number of impacts</th>
<th>Impact velocity (m s$^{-1}$)</th>
<th>Strain rate (s$^{-1}$)</th>
<th>Longitudinal wave velocity (m s$^{-1}$)</th>
<th>Cumulative damage</th>
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Table 3: Damage of siltstone under nine times stress waves loadings at 1.2 m wavelength.

Figure 5: Damage evolution of siltstone under cyclical impact.

Figure 6: Damage evolution of siltstone under cyclical impact.

4.3.2. Influence of Parameters and Physical Meaning. It can be seen from Table 3 that the value of the damage $D$ is $0 \sim 1$, the number of impacts is a natural number, and the above value range can be obtained by substituting formula (11): the value of $\alpha$ is in the range of $0.1$ to $1$, the range of $\beta$ is the number of impacts, and $\delta$ affects the up and down movement of the overall model curve. By fixing the two parameter values and changing the other parameter values, the influence of the parameter changes on the model curve is analyzed to estimate the physical meaning of the $\alpha$ and $\beta$ parameters.

When $\beta$ and $\delta$ are fixed and $\alpha$ values are 0.15, 0.25, 0.35, 0.45, and 0.55, respectively, the effect on the model curve is shown in Figure 9. It can be seen from the figure that the smaller the $\alpha$ value, the longer the curve lasts in the stationary development segment; as $\alpha$ increases, the stationary development segment gradually shortens and gradually evolves from the obvious 3-stage characteristic to the linear exponential function type, that is the $\alpha$ controls the length of the smooth development segment. According to the experimental data, the three-stage regularity is most obvious when the wavelength is 0.5 m. As the wavelength increases, the smooth development of the damage gradually decreases. When the wavelength is 2.0 m, there is basically no stable development, and cumulative damage increases rapidly throughout the repeated impact process. Therefore, the effect of wavelength on cumulative damage is manifested by the parameter $\alpha$, and the shorter the wavelength, the smaller the $\alpha$ and the greater the opposite. In summary, the physical
The meaning of the $\alpha$ parameter is the cumulative damage influencing factor affected by the wavelength, and the value ranges from 0 to 1.

When $\alpha$ and $\delta$ are fixed and $\beta$ values are taken as 14, 13, 12, 11, and 10, respectively, the influence on the model curve is shown in Figure 10. It can be seen from the graph that the asymptote of the end point of the curve is almost the same as the value of $\beta$, and $\beta$ only affects the end point of the independent variable in the late fast growth stage, which has little effect on the early development; that is, the $\beta$ controls the value of the independent variable in the late fast growth segment. According to the test data, when the wavelength is 0.5 m, the number of repeated impacts is the most (14 times); as the wavelength increases, the number of repeated impacts gradually decreases. When the wavelength is 2.0 m, the number of repeated impacts is only 5 times. Therefore, the effect of the number of repeated impacts on the cumulative damage is reflected by the parameter $\beta$; the shorter the wavelength, the more the number of impacts, the larger the $\beta$, and the smaller the opposite. Therefore, the physical meaning of $\beta$ is the cumulative damage impact factor affected by the number of impacts. When the number of impacts is an independent variable, the value is approximately the number of impacts under a certain condition. When the relative number of impacts is used as an independent variable, $\beta$ is a fixed value, and the value is taken as 1.

When $\alpha$ and $\beta$ are fixed, the $\delta$ values are 0.25, 0.35, 0.45, 0.55, and 0.65, respectively, and the effect on the model curve is shown in Figure 11. It can be seen from the figure that the value of $\delta$ affects the initial development of the cumulative damage, and the larger the value, the larger the damage value in the initial stage. Therefore, the physical meaning of the $\delta$ parameter is the cumulative damage model adjustment factor.

### 4.4 Model Verification

The model was used to fit the cumulative damage of siltstone under repeated impact, and the results are shown in Figure 12. At the same time, the corresponding fitting data are obtained, as shown in Table 4.

As can be seen from Figure 12, the model can simultaneously reflect the effects of changes in wavelength and changes in the number of impacts on cumulative damage. At 0.5 m wavelength, the damage evolution has obvious three-stage characteristics, and the cumulative damage data points are the most dense; as the wavelength increases, the development of cumulative damage gradually increases with the increase of wavelength. When the wavelength is 2.0 m, the cumulative damage changes the fastest with the increase of the number of impacts and the damage data points are the most sparse. This behavior can be fully expressed by the cumulative damage model constructed in this paper, and the
Fitting correlation of the experimental data at each wavelength is very good, and the fitting effect is ideal.

Figure 13 shows the results of fitting the cumulative damage using the model constructed in this paper when the relative number of repeated impacts is independent. It shows that the model can also express the damage evolution of siltstone under normalized impact times. The values of $\alpha$, $\beta$, and $\delta$ were 0.344, 1.0, and 0.178, respectively, and the correlation coefficient was 0.967.

Table 4 shows the values of the parameters for fitting the cumulative damage of siltstone under different stress wave parameters using the damage evolution model constructed in this paper.

As can be seen from Table 4, as the wavelength increases and the number of impacts decreases, $\alpha$ and $\delta$ increase and the $\beta$ value decreases. According to the physical meanings of the parameters $\alpha$ and $\beta$, it is shown that with the increase of the wavelength, the stationary development of the cumulative damage is gradually shortened, the damage development in the initial stage is also aggravated with the increase of the wavelength, and the number of repeated impacts decreases with the increase of the wavelength. The changes of the parameters $\alpha$, $\beta$, and $\delta$ are consistent with their physical meanings.

5. Conclusion

(1) The dynamic strength and strain rate are linearly positively correlated with the different wavelengths and different amplitudes of the siltstone under the strain rate of $10^{-2}$ s$^{-1}$. When the strain rate is 140 s$^{-1}$, the dynamic strength increases by about 80%. The dynamic intensity factor is linearly positively correlated with the natural logarithm of the strain rate, but the dynamic intensity is not affected by the wavelength change. The peak strain and the strain rate are also linearly positively correlated, and the increase of the wavelength causes the peak strain to develop as a whole.

(2) Under the multiple impacts of siltstone, the cumulative damage and impact times show a form of “rapid rise-stable development-rapid rise”; the increase of the stress wave causes the cumulative...
damage to increase with multiple impacts, and the number of repeated impacts gradually decreases meanwhile.

(3) Based on the cumulative damage development of siltstone, a cumulative damage evolution model based on the inverse function of the Gompertz function is established, which can simultaneously reflect the stress wave wavelength and the number of impacts. The physical parameters of the model parameters $\alpha$, $\beta$, and $\delta$ are cumulative damage influencing factors affected by wavelength, cumulative damage influencing factors affected by the number of impacts, and cumulative damage model adjustment factors.

(4) With the increase of the wavelength and the decrease of the number of impacts, $\alpha$ and $\delta$ increase, while the $\beta$ value decreases. The stable development of the cumulative damage is gradually shortened, and the damage development in the initial stage is also intensified.

Data Availability

The numerical data used to support the findings of this study are available from the corresponding author upon request.

Conflicts of Interest

The authors declare that there are no conflicts of interest regarding the publication of this article.

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