Research Article

A Numerical Analysis of Ground Vibration Induced by Typical Rail Corrugation of Underground Subway

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A two-step approach is used to establish a numerical prediction model to study the impact of typical rail corrugation on ground vibration from an underground subway. In the first step, a vehicle-track-tunnel rigid-flexible coupling subsystem is established based on a lumped mass model dynamics and finite element analysis cosimulation method to simulate the generation of vibration. In the second step, a track-tunnel-soil three-dimensional (3D) finite element subsystem is built to simulate the propagation of the vibration. The ground vibration response is obtained by applying the wheel-rail force calculated from the first step. A section of Chengdu Metro Line 3 is studied, and the accuracy of the numerical prediction model is then verified by comparison with in situ measurement. Based on that, the impact of corrugation on wheel-rail interaction and ground vibration is investigated by taking rail corrugation in typical subway sections and track geometry irregularities as system input excitation. In addition, to further analyze the sensitivity between different wavelength components in the rail corrugation samples and ground vibration, the measured rail corrugation is decomposed into five kinds with different wavelength components by filtering. The results show that the typical rail corrugation has a large impact on ground vibration response, which increases significantly in the range 8–16 Hz and 50–80 Hz, and the impact decreases with the distance from the vibration source. For typical subway rail corrugation with the significant wavelength of 125 mm and the secondary significant wavelength of 63 mm, the ground vibration response is sensitive to two wavelength components at 40–60 mm and 60–100 mm. Rail corrugation with the short wavelength of 60–100 mm significantly affects ground vibration levels.

1. Introduction

Rail transit has developed rapidly in recent years. The construction of urban rail transit, especially subways, helps alleviate pressure on urban traffic, improving urban air quality and facilitating the life and travel of urban residents. However, as the density of rail transit networks increases rapidly, ground vibration caused by subway trains has also become increasingly prominent, which has brought new challenges to residents’ quality of work and life, normal use of vibration-sensitive equipment, and protection of ancient structures, and cultural and historical relics. In response, state authorities have been paying great attention to environmental protection. Nevertheless, due to many uncertainties, China still lacks ground vibration standards, the evaluation system and engineering practice are inconsistent, and prediction methods are limited. Thus, residents have lodged more and more complaints about vibration noise and have even held civic protests after the construction and operation of almost all new lines. In addition, there have been cases in which underground railways have been relocated to ensure that ancient buildings would not be damaged, or that accurate equipment could be used normally. These problems have seriously affected the harmonious and rapid development of urban rail transit.

Damper fasteners, ladder sleepers, steel-spring floating slabs, and other track reduction measures in different levels have been extensively used in newly built lines in various cities to control vibration from rail transit, and the vibration...
reduction sections of some underground railways have exceeded 40% of the total track mileage [1, 2]. In practical application, however, the various types of vibration reduction measures seem to be ineffective, or they lose their effectiveness after a short period of service, or even cause or amplify the vibration, potentially damaging track structure. The key to this problem is to predict the environmental impact of train vibration, and put forward durable control measures considering engineering practice. In fact, the primary prediction step is to establish an accurate and effective method to predict ground vibration. Prediction of ground vibration from underground railways is an urgent task.

Numerical simulation is one of the important means of ground vibration prediction. Gardien has proposed a three-dimensional dynamic finite element method composed of static deformation, track structure, and propagation structure submodels to analyze the problem of vibration from trains in railway tunnels, and studied parameters such as grid size, soil stiffness, damping, and boundary conditions [3]. Degrange et al. described tunnels using finite elements and soil using boundary elements to solve the three-dimensional coupling problem between the soil and tunnel using subregion algorithms and predict tunnel vibration under the action of subway train operation [4]. Gupta et al. used coupling periodic finite element and boundary element methods, combined with a pipe-in-pipe model, to numerically analyze the problem of vibration caused by the operation of trains in tunnels and verified the model through field observation [5, 6]. Kouroussis and Verlinden built vehicle-track subsystem and tunnel-soil subsystem models using a two-step approach and developed a three-dimensional ground vibration prediction model by coupling the two subsystems. To verify the prediction model, they measured the ground vibration caused by Brussels trains onsite [7, 8]. Lopes predicted the source strength and the vibration received by surrounding buildings using a numerical method and adopted the method to research the vibration impact from subway operation and the induced vibration on houses. The trains were modeled as rigid multibodies, the track-tunnel-soil system used a 2.5D model, and the buildings used a 3D model. The study showed that the soil parameters have a great impact on the results [9]. In these studies, the vehicle load often comes from theoretical analysis or test data, as an external load on the tunnel structure, but few studies take the vehicle as a rigid multibody system, directly linking it with the track structure as an integral coupling system. Moreover, the vibration source caused by train operation and its propagation form a large coupled system; simplification of the tunnel structure and step-by-step modeling of the train, track, and tunnel neglect the partial coupling within the system. Wheel-rail interaction is the excitation force of the ground dynamic response, so the impact of train response, track, and tunnel structure must be taken into account. Therefore, a numerical method is necessary to predict underground railway vibration, to establish the vehicle-track-tunnel-soil rigid-flexible large coupling system and to analyze the environmental vibration caused by the train.

Rail corrugation is a major type of damage in China’s underground subways. Since subway rail corrugation has a relatively short wavelength, mainly 20–500 mm [10], the excitation vibration energy is strong, and the frequency is high, which not only aggravates the interaction of the wheel-rail system and leads to fatigue damage to the vehicle and track components, but also causes excessive vehicle and track structure vibration, resulting in noise and vibration pollution. Rail corrugation has been widely studied in the field of wheel and rail vibration and noise. Thompson found a linear relationship between rail transit vibration noise and the depths of rail surface short wave irregularities within the range from 500–2500 Hz [11]. Nielsen et al. asserted that wheel-rail rolling noise is mainly caused by short-wavelength random irregularities on the wheel-rail surface [12]. Nordborg explained the generation of wheel-rail noise when the rail is very smooth or has a corrugation/wavelength corresponding to the pinned-pinned frequency [13]. In addition, rail corrugation is also applied in the study of train-track system dynamic characteristics. Ali Zakeri et al. studied the influence of rail corrugation on the track and vehicle component [14]. In fact, track irregularity is an important source of excitation for ground vibration caused by trains. Although many scholars have considered track irregularity when studying train-track dynamic characteristics [15–18], most of them have only considered measured irregularities or medium-to-long wave random irregularities with wavelengths greater than 1 m. Few have used short-wavelength rail corrugation to study the dynamic characteristics of the train-track-tunnel-soil coupling system to analyze ground vibration that may be caused by rail corrugation excitation. Therefore, it is necessary to superimpose rail corrugation (data on typical rail corrugation is obtained from the measurements in [10]) with initial geometric track imperfections to research the impact of typical rail corrugation on ground vibration from underground subways.

A numerical model for predicting ground vibration from underground railways is established, and the impact of rail corrugation on environmental response is evaluated. The research is carried out through the following main steps. First, a two-step approach is applied to establish the numerical model to predict ground vibration from underground railways. The two steps are coupled through wheel-rail force to predict the vibration source and the vibration induced by trains. Second, a section of Chengdu Metro Line 3 is studied to compare the theoretical calculation results with measured data, verifying the reasonableness, higher prediction accuracy, and comprehensiveness of the vehicle-track-tunnel-soil numerical prediction model and its parameters. On that foundation, typical rail subway section corrugation is superposed with track geometry irregularity as system excitation input to study the impact of rail corrugation on wheel-rail interaction and ground vibration. Finally, the measured typical rail corrugation is decomposed into five
components with different wavelengths by filtering, which is realized through setting the unnecessary frequency band to zero after Fourier transform [19], and the sensitivity between the different wavelength components in the rail corrugation samples and ground borne vibration is analyzed.

2. Establishment of Prediction Models

The vibration and propagation caused by trains in underground railways is shown in Figure 1(a). The system consists of four parts from top to bottom: the vehicle, track structure, tunnel, and soil, all coupled with each other. Subway train vibration is affected by multiple factors, such as track irregularity, tunnel structure, and the nonuniform discontinuity characteristics of the soil, making the vibration caused by underground railways complex. The induced vibration may propagate through multiple paths. Kouroussis and Verlinden used a two-step approach to study ground vibration from subways [7], and this approach will also be used in the present paper. The system will be divided into two subsystems: the vehicle-track-tunnel rigid-flexible coupling subsystem (vibration generation) and the track-tunnel-soil three-dimensional finite element subsystem (vibration propagation), as shown in Figure 1(b). First, subsystem 1 is used to calculate the wheel-rail interaction, which is then applied to the rail of subsystem 2 as excitation to solve the ground vibration response. Cosimulation can be achieved between these two subsystems through wheel-rail force, and each subsystem has high computation accuracy.

2.1. Vertical Wheel-Rail Interaction. Based on lumped mass model dynamics and finite element method, the vehicle-track-tunnel coupling system is established, and cosimulation is achieved in dynamics software. The vehicle is considered as a lumped mass model, and the track and tunnel are considered as a flexible body model, which are coupled by nonlinear wheel-rail contact.

2.1.1. Vertical Wheel-Rail Contact Force Distribution. A lumped mass vehicle model is established, ignoring the structural elastic deformation of all of its components and assuming that the vehicle runs at a constant speed, that is, the longitudinal acceleration and deceleration of trains are not taken into account. Each vehicle is considered to have a car body, two bogie frames, and four wheelsets, totaling seven space rigid body parts, of which wheelsets and bogie frames are connected by primary suspension and car body and bogie frames are connected by secondary suspension. Vehicle system’s degree of freedom is shown in Figure 2. Each rigid body shall consider the degree of freedom in six directions such as longitudinal, transverse, vertical, rolling, nodding, and shaking directions. The components shall be connected by spring-damper system, the suspension system is treated nonlinearly, and the vehicle system is simplified into a model with 42 degrees of freedom. The parameters of vehicle model are shown in Table 1.

2.1.2. Flexible Track-Tunnel Modeling. Based on the method of combining finite element method and lumped mass model dynamics, the flexible track-tunnel model is established. In the model, rigid body motion is obtained by the theory of lumped mass model dynamics, and structural deformation is solved using the finite element and mode superposition method. In this paper, the fixed track is studied, track-tunnel model is established, and rail flexibility, fastener elasticity, deformation of track slab and tunnel, and elastic support are considered, so that nonlinear Hertz contact of wheel and rail can be taken into account while the simulated vehicle running on the flexible track. Rail is regarded as the Timoshenko beam model supported by continuous discrete points with an interval of 0.6 m; solid element simulation is applied to track slab and tunnel; fasteners are considered as linear spring-damper element to simulate the connection between rail and track slab; the elastic support of soil is simplified as uniformly distributed viscoelastic element with the support stiffness of 60 MPa/m [20].

(1) Rail Model. Based on the theory of Timoshenko beam, rail bending deformation includes translational motion and rotation when the vehicle runs on a track at the speed of v. To facilitate the numerical analysis, the canonical mode shape coordinate of rail is usually introduced, Ritz method is used to transform the vibration equations into second-order ordinary differential equations, and the explicit integration algorithm is used to integrate them, so as to obtain the dynamic response of rail for all nodes at any time.

Vertical vibration equation of the rail is as follows:

\[ \ddot{q}_{ik}(t) + \frac{EI}{m_r} \left( \frac{k\pi}{l} \right)^4 q_{ik}(t) = \sum_{j=1}^{N} F_{yi,i} Y_i(x_i) + \sum_{j=1}^{4} P_{yi,i} Z_{ij}(x_{pj}), \]

\[ k = 1-NMZ. \]

(1)

Transverse vibration equation of the rail is as follows:

\[ \ddot{q}_{jk}(t) + \frac{EI}{m_r} \left( \frac{k\pi}{l} \right)^4 q_{jk}(t) = \sum_{i=1}^{N} F_{yi,j} Y_i(x_i) + \sum_{j=1}^{4} Q_{yi,j} Z_{ij}(x_{pj}), \]

\[ k = 1-NMY. \]

(2)

Torsional vibration equation of the rail is as follows:

\[ \ddot{q}_{Tk}(t) + \frac{GK}{\rho l_0} \left( \frac{k\pi}{l} \right)^4 q_{Tk}(t) = \sum_{i=1}^{N} M_{yi,i} \Theta_i(x_i) + \sum_{j=1}^{4} G_{yi,j} \Theta_{ij}(x_{pj}), \]

\[ k = 1-NMR, \]

(3)

where l refers to the calculated length of rail, NMZ, NMY, and NMR refer to the cutoff mode orders of vertical,
Figure 1: (a) Schematic diagram of vibration transmission path and (b) schematic diagram of numerical simulation procedure.

Figure 2: Dynamic model of a train: (a) side view, (b) plan view, and (c) elevation view.
transverse, and torsional vibration modes of rail, respectively, \( Z_k, Y_k, \) and \( \Theta_k \) refer to the functions of vertical, transverse, and torsional vibration modes of rail, respectively.

(2) Track Slab-Tunnel Model. When analyzing wheel-rail coupling dynamics, the numerical model of track slab and tunnel is usually established based on finite element theory and mode superposition method, which not only can improve calculation efficiency and ensure calculation accuracy, but can simulate the coupling effect of three-dimensional spatial vibration response of track slab and tunnel. In the finite element model, the single track slab is 5.4 × 2.5 × 0.4 m\(^3\), including 1705 nodes and 1204 hexahedral elements. As for tunnel, it has an outer diameter of 3.0 m and a lining thickness of 0.3 m, including 92672 nodes and 69120 elements. Based on the principle of mode superposition, the canonical mode shape coordinate of track slab can be introduced to derive the vibration differential equation of the track slab associated with its anterior NMS–order mode, so as to calculate the dynamic response for all nodes of the track slab at any time, as shown in the equation below:

\[
M_{sik} \ddot{q}_{sik}(t) + C_{sik} \dot{q}_{sik}(t) + K_{sik} q_{sik}(t) = F_{fsik}(t), \quad i = 1\sim N_{slab}, k = 1\sim N_{MS},
\]

(4)

where \( \ddot{q}_{sik}(t), \dot{q}_{sik}(t), \) and \( q_{sik}(t) \) refer to the displacement, velocity, and acceleration matrix corresponding to the kth order mode of the ith track slab, respectively; \( M_{sik}, C_{sik} \) and \( K_{sik} \) refer to the mass matrix, stiffness matrix, and damping matrix of the track slab, respectively; \( F_{fsik} \) refers to the external force matrix; \( N_{slab} \) refers to the number of track slabs, that is 20; NMS refers to the mode order of the track slab considered in the calculation model, that is 20, which includes 6 rigid modes and 14 flexible modes, the highest modal frequency is 522.2 Hz.

Similarly, the vibration differential equation of tunnel can be derived that is associated with its anterior NMT–order mode, so as to calculate the dynamic response for all nodes of the track slab at any time, as shown in the equation below:

\[
M_{tik} \ddot{q}_{tik}(t) + C_{tik} \dot{q}_{tik}(t) + K_{tik} q_{tik}(t) = F_{ftik}(t), \quad i = 1\sim N_{tunnel}, k = 1\sim N_{MT},
\]

(5)

where \( \ddot{q}_{tik}(t), \dot{q}_{tik}(t), \) and \( q_{tik}(t) \) refer to the displacement, velocity, and acceleration matrix corresponding to the kth order mode of the ith tunnel, respectively; \( M_{tik}, C_{tik} \) and \( K_{tik} \) refer to the mass matrix, stiffness matrix, and damping matrix of the tunnel, respectively; \( F_{ftik} \) refers to the external force matrix; \( N_{tunnel} \) refers to the number of track slabs, that is 1. NMT refers to the mode order of the tunnel considered in the calculation model, that is 30, which includes 6 rigid modes and 24 flexible modes, the highest modal frequency is 49 Hz.

2.1.3. Wheel-Rail Contact. Wheel-rail contact is used to connect vehicle system and track structure, which constitutes an important part of the model. Previous studies have shown that there is a weak coupling relationship between vertical and transverse wheel-rail vibration, which is unidirectional, that is, vertical vibration can be regarded as independent, while transverse vibration is affected by vertical vibration [21]. At present, the Hertzian contact theory is usually applied to wheel-rail contact analysis in domestic

<table>
<thead>
<tr>
<th>Notation Item</th>
<th>Item</th>
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<tbody>
<tr>
<td>( M_s )</td>
<td>Mass of car body</td>
</tr>
<tr>
<td>( I_{cx} )</td>
<td>Mass moment of inertia of car body roll</td>
</tr>
<tr>
<td>( I_{cy} )</td>
<td>Mass moment of inertia of car body pitch</td>
</tr>
<tr>
<td>( I_{cz} )</td>
<td>Mass moment of inertia of car body yaw</td>
</tr>
<tr>
<td>( M_b )</td>
<td>Mass of bogie</td>
</tr>
<tr>
<td>( I_{bx} )</td>
<td>Mass moment of inertia of bogie roll</td>
</tr>
<tr>
<td>( I_{by} )</td>
<td>Mass moment of inertia of bogie pitch</td>
</tr>
<tr>
<td>( I_{bz} )</td>
<td>Mass moment of inertia of bogie yaw</td>
</tr>
<tr>
<td>( M_w )</td>
<td>Mass of wheelset</td>
</tr>
<tr>
<td>( I_{wx} )</td>
<td>Mass moment of inertia of wheel roll</td>
</tr>
<tr>
<td>( I_{wy} )</td>
<td>Mass moment of inertia of wheel pitch</td>
</tr>
<tr>
<td>( I_{wz} )</td>
<td>Mass moment of inertia of wheel yaw</td>
</tr>
<tr>
<td>( k_{sz} )</td>
<td>Vertical stiffness of secondary suspension</td>
</tr>
<tr>
<td>( k_{sx} )</td>
<td>Longitudinal stiffness of secondary suspension</td>
</tr>
<tr>
<td>( c_{sz} )</td>
<td>Vertical damping of secondary suspension</td>
</tr>
<tr>
<td>( c_{sx} )</td>
<td>Lateral damping of secondary suspension</td>
</tr>
<tr>
<td>( k_{sv} )</td>
<td>Vertical stiffness of primary suspension</td>
</tr>
<tr>
<td>( k_{px} )</td>
<td>Longitudinal stiffness of primary suspension</td>
</tr>
<tr>
<td>( c_{sv} )</td>
<td>Lateral damping of primary suspension</td>
</tr>
<tr>
<td>( h_s )</td>
<td>Vertical distance between center of gravity of car body and lateral secondary suspension system</td>
</tr>
<tr>
<td>( h_b )</td>
<td>Vertical distance between lateral secondary suspension system and center of gravity of bogie</td>
</tr>
<tr>
<td>( h_p )</td>
<td>Vertical distance between center of gravity of bogie and lateral primary suspension system</td>
</tr>
<tr>
<td>( l_k )</td>
<td>Half-distance between two bogies of a vehicle</td>
</tr>
<tr>
<td>( l_t )</td>
<td>Half-distance between wheelsets of a bogie</td>
</tr>
<tr>
<td>( b_s )</td>
<td>Half of transverse distance between vertical secondary suspension systems</td>
</tr>
<tr>
<td>( b_p )</td>
<td>Half of transverse distance between vertical primary suspension systems</td>
</tr>
<tr>
<td>( V )</td>
<td>Forward speed of vehicle</td>
</tr>
</tbody>
</table>

Table 1: Parameters of the train model.
and foreign calculation theories and simulation models, and the theoretical calculation equation is as follows:

\[ p(t) = \left[ \frac{1}{G} \delta z(t) \right]^{3/2}, \]  

where \( G \) refers to the wheel-rail contact constant in \((m/N)^{2/3}\); \( \delta z(t) \) refers to the elastic compression between wheel and rail in meters. According to dynamics software simulation, vertical wheel-rail force is calculated based on Hertzian nonlinear elastic contact theory and transverse wheel-rail force is established based on the Hertzian contact theory, and Kalker rolling contact is used to nonlinearly simplify the theoretical Fastsim algorithm [15].

2.1.4. Track Irregularity. Track irregularity is an important factor to excite random wheel-rail vibration, so that it is necessary to consider the ground vibration induced by running trains. The commonly used track spectra include the US spectrum, German railway spectrum of high-low irregularity, Qinhuangdao-Shenzhen railway line spectrum researched and summarized in China regarding the irregularity power spectrum of trunk railway, and the irregularity spectrum of three trunk railways [22], but the irregularity power spectrum of underground railways has not yet been formed at this stage. Subway-induced ground vibration is mainly vertical, track vertical profile irregularity is considered only in this paper. To reflect the track regularity more truthfully, the track irregularity used in the dynamic simulation is the typical rail corrugation superimposed with the geometric defect of initial track irregularity, as shown in Figure 3. The wavelength range of rail corrugation is 0.02–0.5 m, the wave depth is 0.05–0.10 mm, and the significant wavelength is 125 mm and the secondary significant wavelength is 63 mm. The 1/3 octave spectrum of rail corrugation is shown in Figure 4. The track geometry irregularity is based on the fifth power spectral density formula proposed by U.S. railways, and the wavelength range is 0.5–200 m [23].

2.2. Track-Tunnel-Soil Three-Dimensional Finite Element Subsystem Modeling. The track-tunnel-soil three-dimensional finite element ground vibration prediction model is established, as shown in Figure 5. The model, from top to bottom, includes rail, fasteners, track slabs, tunnel, and surrounding soil, of which the rail model was simulated with beam elements, the track slab, tunnel, and soil were simulated with solid elements, and the connection between rail and track slab was simulated with spring-damper elements. In the fixed track, the track slab and the tunnel base are consolidated. The model extends 80 m along the line vertically, the width perpendicular to the central line of the line is 100 m, the depth of soil layer is 60 m, the beam element grid of rail is 0.1 m, and the solid element grid of track slab and tunnel is 0.5 m. To meet the frequency range of analysis requirements [24], the solid element of soil is divided into three regions, and the grid size of the middle region is 0.5 m, that perpendicular to two sides along the line is 0.8 m, and that of the bottom region is 1.0 m. The finite element model has a total of 1260560 elements and 1341516 nodes. In this paper, the integral step adopted in calculation is 0.002 s, and the total integral time lasts for 4 s.

To reflect the semi-infinite spatial properties of soil and in view of the reflection and superposition effect of stress wave in soil, the cross sections on both sides and the bottom of soil in the track-tunnel-soil three-dimensional finite element model parallel to line direction are separately provided with three-dimensional uniform viscoelastic artificial boundary [25], so as to reduce the reflection of wave above soil boundary, obtain the calculation results of dynamic response more accurately, and shorten the calculation time. To establish three-dimensional uniform viscoelastic artificial boundary, finite elements shall be firstly discretized. Normal and tangential artificial spring damper elements are evenly distributed on the plane determined by four boundary nodes, as a result of which stiffness matrix and damping matrix are formed. By combining stiffness and damping matrices into the stiffness matrix of soil, the three-dimensional uniform viscoelastic artificial boundary of soil is applied. The equivalent viscoelastic artificial boundary element is achieved by extending a layer of solid element with a certain thickness along the normal direction on the

\textbf{Figure 3:} Power spectral density: simulated vertical track irregularity is composed of track irregularity with the wavelengths of 0.5–200 m (the left of red line) and measured rail surface roughness with the wavelengths of 0.02–0.5 m (the right of red line).

\textbf{Figure 4:} Roughness level spectra based on the measurement in typical underground subway [10].
boundary of the three-dimensional finite element model, assigning the material properties of the boundary element to the solid element, and fixing the outermost boundary. The equivalent shear modulus, equivalent elasticity modulus, and equivalent Poisson ratio of solid boundary element are shown in the following equation:

\[ \tilde{G} = h K_{BT} = \alpha_r h \frac{G}{R} \]

\[ \tilde{E} = \frac{(1 + \tilde{\nu})(1 - 2\tilde{\nu})}{(1 - \tilde{\nu})} h K_{BN} = \alpha_r h \frac{E}{R} \left( \frac{(1 + \tilde{\nu})(1 - 2\tilde{\nu})}{(1 - \tilde{\nu})} \right) \]

\[ \tilde{\nu} = \frac{\alpha - 2}{2(\alpha - 1)}, \quad (\alpha \geq 2), \]

\[ \alpha = \frac{\alpha_N}{\alpha_T} \]

where \( G \) refers to the shear modulus of the adjacent element of the boundary element, \( K_{BT} \) and \( K_{BN} \) refer to the normal and tangential stiffness of the viscoelastic artificial boundary spring, \( h \) refers to the thickness of the equivalent element, and \( \alpha_N \) and \( \alpha_T \) refer to the correction coefficient of normal and tangential viscoelastic boundaries, of which \( \alpha_N = 4/3 \) and \( \alpha_T = 2/3 \).

2.3. Ground Vibration Evaluation Indicator. To analyze the impact of the operation of running trains on the environment, its bearing surface is the ground or building floor for the induced vibration-affected, that is, human body, instrument, and equipment. On the basis of ISO2631 [26, 27] and in combination with a large number of domestic scientific achievements, China has promulgated GB/T 13441 series standards [28, 29], serving as a guiding criterion for China’s ground vibration research industry. In this paper, the author intends to evaluate ground vibration with peak time-domain acceleration and vibration acceleration level.

The peak time-domain acceleration (PPA, unit: m/s²) refers to the maximum amplitude of the acceleration signal measured within the prescribed time, as shown in the following equation:

\[ PPA = \max |a(t)|. \]

The vibration acceleration level (VAL, unit: dB) is a vibration evaluation indicator adopted to facilitate the observation, operation, and comparison of acceleration, and the calculation formula is shown in the following equation:

\[ VAL = 20 \log \frac{a}{a_0} \]

where \( a \) refers to the mean square root of weighted acceleration, and \( a_0 \) refers to the reference acceleration, which is usually \( 10^{-6} \) m/s².

3. Validation of the Prediction Methods with Experimental Data

3.1. Determination of Calculation Parameters. The simulation results of the numerical model are verified by the measured data of ground vibration of Chengdu Metro Line 3. The vehicle is B-type subway, and the specific parameters are shown in Table 2. The average running speed of the train in the test section is about 72 km/h. The track structure consists of 60 kg/m rail, DZIII-type fasteners, and short sleeper-embedded ballastless track. The tunnel is a circular shield tunnel, and the detailed parameters are shown in Table 3. The site soil layer is divided into 4 layers from top to bottom, and the detailed parameters are shown in Table 4. In this paper, the widely used Rayleigh damping is adopted to reflect dissipation of vibration in soil materials [30, 31]. Assuming that the damping matrix of the system is a linear combination of mass matrix and stiffness matrix, the commonly used Rayleigh damping is expressed as

\[ C = \alpha M + \beta K, \]

\[ \alpha = 2\omega_1\omega_2 \frac{\xi_1\omega_1 - \xi_2\omega_2}{\omega_1^2 - \omega_2^2} \]

\[ \beta = 2\xi_2\omega_1 - \xi_1\omega_2 \frac{\omega_1^2 - \omega_2^2}{\omega_1^2 - \omega_2^2} \]

where \( \omega_1, \omega_2, \xi_1, \) and \( \xi_2 \) refer to first-order and second-order natural frequency and appropriate damping ratio of the structure. As the Rayleigh damping coefficients \( \alpha \) and \( \beta \) are determined, the damping matrix can be solely obtained. Assuming that the damping ratio is 0.03 and characteristic frequency is 5 Hz and 80 Hz, respectively, the damping coefficient of soil is calculated according to equation (7), of which \( \alpha = 1.773, \beta = 0.000112. \)

To analyze the characteristics of ground vibration caused by running trains, the fixed track sections are selected for experimental study, ground observation points are arranged as shown in Figure 6, and three ground observation points of ground vibration are separately arranged at 0 m, 20 m, and 40 m from the central line of the line along the line.
perpendicular to the running direction of train, and the observation points are numbered as P1–P3.

3.2. Observation Results and Discussion. The vertical vibration acceleration of the observation point in the finite element model is extracted according to the location of the field observation point, so as to compare the time interval results. Figure 7 compares the results of vertical acceleration of the top soil layer at 0 m and 20 m from the central line of the line, from which it can be seen that the two have fairly consistent time interval form and amplitude. Therefore, the numerical model and calculation method established in this paper are effective in predicting the ground vibration caused by underground railways.

The 1/3 octave vertical frequency-weighted vibration acceleration level (VAL) obtained by processing the time-domain data in Figure 7 shall be analyzed comparatively, as shown in Figure 8. From the measured results, it can be seen that the main frequency of vertical vibration of soil at the observation point is 4–100 Hz, and there are two peaks in total. The smaller peak is located at 10 Hz of the central frequency, and the larger one is located at 63 Hz of the central frequency, which is caused by inducing wheel-rail resonance because the wheelset vibration frequency is similar to the vibration frequency of track structure [10]. Similarly, the theoretical calculation results show that the vertical vibration acceleration level of the ground has a good coincidence in the whole frequency domain, and the difference between the two peaks at the central frequency of 63 Hz is 1.2 dB, which indicates that the track irregularity and calculation conditions of the measured sections are close, and the numerical model has good calculation accuracy.

### Table 2: Key parameters of China’s B-type subway vehicle.

<table>
<thead>
<tr>
<th>Element</th>
<th>Parameter</th>
<th>Notation</th>
<th>Unit</th>
<th>Value</th>
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<tr>
<td></td>
<td>Mc (kg)</td>
<td>M_b (kg)</td>
<td>M_w (kg)</td>
<td>I_xx (kg.m²)</td>
</tr>
<tr>
<td>Mc</td>
<td>39000</td>
<td>3600</td>
<td>1600</td>
<td>1400000</td>
</tr>
</tbody>
</table>

### Table 3: Calculation parameters of track and tunnel.

<table>
<thead>
<tr>
<th>Element</th>
<th>Parameter</th>
<th>Notation</th>
<th>Unit</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rail</td>
<td>Mass per unit length</td>
<td>M_r</td>
<td>kg/m</td>
<td>60.64</td>
</tr>
<tr>
<td></td>
<td>Young’s modulus</td>
<td>E_r</td>
<td>GPa</td>
<td>210</td>
</tr>
<tr>
<td></td>
<td>Moment of inertia</td>
<td>I_l</td>
<td>m⁴</td>
<td>3.22 x 10⁻⁵</td>
</tr>
<tr>
<td>Rail pad</td>
<td>Support spacing</td>
<td>L</td>
<td>m</td>
<td>0.625</td>
</tr>
<tr>
<td></td>
<td>Stiffness</td>
<td>K_p</td>
<td>MN/m</td>
<td>35</td>
</tr>
<tr>
<td></td>
<td>Damping</td>
<td>C_p</td>
<td>kN·m/s</td>
<td>20</td>
</tr>
<tr>
<td>Slab</td>
<td>Length</td>
<td>L</td>
<td>m</td>
<td>5.6</td>
</tr>
<tr>
<td></td>
<td>Width</td>
<td>w</td>
<td>m</td>
<td>2.5</td>
</tr>
<tr>
<td>Slab</td>
<td>Thickness</td>
<td>h</td>
<td>m</td>
<td>0.4</td>
</tr>
<tr>
<td></td>
<td>Density</td>
<td>ρ_r</td>
<td>kg/m³</td>
<td>2500</td>
</tr>
<tr>
<td>Slab</td>
<td>Young’s modulus</td>
<td>E_s</td>
<td>GPa</td>
<td>35</td>
</tr>
<tr>
<td>Tunnel</td>
<td>Outside diameter</td>
<td>R</td>
<td>m</td>
<td>3.0</td>
</tr>
<tr>
<td></td>
<td>Inside diameter</td>
<td>R</td>
<td>m</td>
<td>2.7</td>
</tr>
<tr>
<td>Tunnel</td>
<td>Density</td>
<td>ρ_t</td>
<td>kg/m³</td>
<td>2500</td>
</tr>
<tr>
<td>Tunnel</td>
<td>Young’s modulus</td>
<td>E_t</td>
<td>GPa</td>
<td>32.5</td>
</tr>
</tbody>
</table>

### Table 4: Parameters of soil in test site.

<table>
<thead>
<tr>
<th>Soil type</th>
<th>Thickness (m)</th>
<th>Young’s modulus (MPa)</th>
<th>Density (kg/m³)</th>
<th>Damping ratio</th>
<th>Poisson’s ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td>Artificial soil</td>
<td>2.0</td>
<td>110</td>
<td>1755</td>
<td>0.03</td>
<td>0.36</td>
</tr>
<tr>
<td>Silty clay</td>
<td>3.5</td>
<td>276</td>
<td>1979</td>
<td>0.034</td>
<td>0.37</td>
</tr>
<tr>
<td>Medium sand</td>
<td>20.0</td>
<td>202</td>
<td>2397</td>
<td>0.028</td>
<td>0.37</td>
</tr>
<tr>
<td>Cobblestone</td>
<td>&gt;16</td>
<td>579</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Figure 6: Position of observation points P1–P3 at ground surface.
4. Impact of Rail Corrugation on Ground Vibration

Firstly, to analyze the ground vibration response caused by typical rail corrugation, case 1 (measured rail corrugation together with track geometry irregularity) and case 2 (track geometry irregularity) are separately provided based on the numerical prediction model established above as system input excitation for simulation calculation, so as to study the impact of short-wavelength rail corrugation on wheel-rail interaction and ground vibration. Secondly, to further explore the impact of different wavelengths of typical rail corrugation on ground vibration, the typical rail corrugation is decomposed into five corrugations with different wavelength components by filtering, so as to analyze the sensitivity relationship between different wavelength components and ground vibration response in rail corrugation samples. When the train is at allowable speed $v$ (km/h) and passes through the rail corrugation with a wavelength of $\lambda$, then the corresponding passing frequency $f$ is expressed as,

\[ f = \frac{(1000v)}{(3.6\lambda)} \text{ (Hz)} \]

Since five corrugations with different wavelength components derive from the measured typical rail corrugation, the corresponding wave depths are also inconsistent, and the specific characteristics are shown in Table 5. Based on the measured data, the case of the same wave depth and different wavelengths are not considered temporally. There are 5 cases for track irregularities, which are as follows:

Case 3: rail corrugation of wavelength 20–40 mm together with track geometry irregularity
Case 4: rail corrugation of wavelength 40–60 mm together with track geometry irregularity
Case 5: rail corrugation of wavelength 60–100 mm together with track geometry irregularity
Case 6: rail corrugation of wavelength 100–200 mm together with track geometry irregularity
Case 7: rail corrugation of wavelength 200–500 mm together with track geometry irregularity

![Figure 7: Ground response-predicted and experimental acceleration time histories: (a) 0 m and (b) 20 m.](image)

![Figure 8: 1/3 octave vertical frequency-weighted vibration acceleration: (a) 0 m and (b) 20 m.](image)
4.1. Impact of Typical Rail Corrugation on Ground Vibration.

The track irregularity of case 1 and case 2 is taken as input excitation, so as to obtain the wheel-rail force, as shown in Figure 9. From wheel-rail force in the time domain, it can be seen that short-wavelength rail corrugation has a great impact on the maximum vertical force of wheel-rail, and its impact is far greater than that of track geometry random irregularity. If rail corrugation is not considered, the maximum vertical force of wheel and rail is only 70.4 kN; if rail corrugation is considered, the maximum vertical force of wheel and rail can reach 106.0 kN. From the wheel-rail frequency-domain result, it can be seen that the significant wavelength and secondary significant wavelength of rail corrugation, as the short-wavelength irregularity, are 125 mm and 63 mm, respectively, and the passing frequency is 160–317 Hz at the speed of 75 km/h. The effect of the corrugation excitation is mainly reflected in the middle and high frequencies, but it still has a certain impact on the low frequencies within 100 Hz. The fastener system has a weak control effect on wheel-rail low-frequency vibration and is easy to transmit to the earth through track structure and tunnel, which causes low-frequency ground vibration with the main frequency of 50–80 Hz.

The ground vibration response at 0 m, 20 m, and 40 m is shown in Table 6 and Figure 10. According to time-domain results, if the rail corrugation is not considered, the peak acceleration at 0 m from the observation point is 0.018 m/s² and the total vibration level is 72.8 dB; the peak acceleration at 20 m from the observation point is 0.0058 m/s² and the total vibration level is 64.5 dB; the peak acceleration at 40 m from the observation point is 0.0032 m/s² and the total vibration level is 60.5 dB; if the rail corrugation is considered, the peak acceleration at 0 m from the observation point is 0.055 m/s² and the total vibration level is 83.5 dB; the peak acceleration at 20 m from the observation point is 0.0134 m/s² and the total vibration level is 72.2 dB; and the peak acceleration at 40 m from the observation point is 0.0059 m/s² and the total vibration level is 65.9 dB. It thus can be found that short-wavelength rail corrugation will lead to the increase of ground vibration response, and the impact decreases with distance. The observation point near the vibration source is greatly affected by rail corrugation, while the point far from the vibration source is less affected by rail corrugation.

### Table 5: Rail corrugation with different wavelength components (hmax is the maximum wave depth).

<table>
<thead>
<tr>
<th>Case 3</th>
<th>Case 4</th>
<th>Case 5</th>
<th>Case 6</th>
<th>Case 7</th>
</tr>
</thead>
<tbody>
<tr>
<td>λ (mm)</td>
<td>20–40</td>
<td>40–60</td>
<td>60–100</td>
<td>100–200</td>
</tr>
<tr>
<td>hmax (mm)</td>
<td>0.01</td>
<td>0.02</td>
<td>0.04</td>
<td>0.1</td>
</tr>
<tr>
<td>v (km/h)</td>
<td>20</td>
<td>20</td>
<td>20</td>
<td>20</td>
</tr>
<tr>
<td>f (Hz)</td>
<td>500–1000</td>
<td>333–500</td>
<td>200–333</td>
<td>100–200</td>
</tr>
</tbody>
</table>

Figure 9: Wheel-rail force: (a) response and (b) spectrum.

If the rail corrugation is not considered, the ground observation point peaks only at 32 Hz and 63 Hz by analyzing the frequency-domain results, and the amplitude is smaller. The peak at 32 Hz corresponds to the vibration frequency of sleepers with the speed of 75 km/s and fastener spacing of 0.625 m. If the rail corrugation is considered, the vibration response of ground observation point at 8–16 Hz and 50–80 Hz significantly increases, and the main frequency range of ground vibration...
Table 6: Ground vibration results for Cases 1–2: Case 2 served as controls.

<table>
<thead>
<tr>
<th>Observation points</th>
<th>0 m</th>
<th>20 m</th>
<th>40 m</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Case 1</td>
<td>Case 2</td>
<td>Reduced value</td>
</tr>
<tr>
<td>PPA ($\times 10^{-3} \text{m/s}^2$)</td>
<td>54.9</td>
<td>17.9</td>
<td>37.0</td>
</tr>
<tr>
<td>VAL$_{total}$ (dB)</td>
<td>83.5</td>
<td>72.8</td>
<td>10.7</td>
</tr>
</tbody>
</table>

![Figure 10](image_url)

**Figure 10:** Surface accelerations for Case 1 and Case 2: (a) response at 0 m, (b) spectrum at 0 m, (c) response at 20 m, (d) spectrum at 20 m, (e) response at 40 m, and (f) spectrum at 40 m.
response is also widened. Therefore, the impact of short-wavelength rail corrugation cannot be neglected while predicting and analyzing the ground vibration caused by running trains.

4.2. Sensitivity Analysis of Ground Vibration and Different Wavelength Components of Rail Corrugation. Five different track irregularities are used as system input excitation to obtain the time-domain and frequency-domain results of wheel-rail force, as shown in Figures 11–15. The ground vibration response under different cases is obtained by the input of wheel-rail force into subsystem 2, as shown in Figure 16.

Figure 16(a) compares the results of peak time-domain acceleration to each ground observation point in the central line of track. In case 4, the peak acceleration at 0 m, 20 m, and 40 m from the observation point is 21.8 mm/s², 6.5 mm/s², and 3.4 mm/s², respectively. Compared with the case without rail corrugation, the time-domain amplitude has increased by 21.8% at 0 m, 12.1% at 20 m, and 6.3% at 40 m. It thus can be seen that the rail corrugation with a wavelength of 40–60 mm has a certain impact on the time-domain response. In case 5, the peak acceleration at 0 m, 20 m, and 40 m from the observation point is 28.5 mm/s², 8.3 mm/s², and 3.6 mm/s², respectively. Compared with the case without rail corrugation, the time-domain amplitude has been seriously affected, which increases by 59.2% at 0 m, 43.1% at 20 m, and 12.5% at 40 m. It thus can be seen that the rail corrugation with wavelength 60–100 mm has a great impact on the time-domain response than the other wavelength components.

Figure 16(b) compares the results of total acceleration level (VAL_total) of ground vibration, and the variation law is basically consistent with the peak time-domain acceleration. In case 4, the total acceleration level at 0 m increases from 72.8 dB to 74.2 dB; that at 10 m increases from 64.5 dB to 65.5 dB; that at 20 m increases from 60.5 dB to 60.9 dB. In case 5, the total acceleration level at 0 m increases from 72.8 dB to 78.7 dB; that at 10 m increases from 64.5 to 68.1 dB; that at 20 m increases from 60.5 dB to 63.1 dB.

According to the above analysis, the impact of rail corrugation on ground vibration is mainly caused by two wavelength components, that is, rail corrugation with the wavelength of 40–60 mm and 60–100 mm, respectively, of which that with the wavelength 60–100 mm has a greater impact. The specific reason may be divided into the following two aspects. First, the significant wavelength and secondary significant wavelength of typical rail corrugation are 125 mm and 63 mm, respectively, and 40–60 mm and 60–100 mm are the main wavelength components of rail corrugation. Their vibration frequencies are easily stimulated in the vibration of various components of track and substructure and will cause greater vibration. Second, the wheel-rail excitation frequency corresponding to the rail corrugation with a wavelength of 60–100 mm is about 200–333 Hz, while the first fourth-order vibration modes related to the vertical vibration of the track slab are shown in Figure 17. The vibration frequency components caused by rail corrugation attenuate downward along the track structure, which may resonate with the third- and fourth-order vertical bending frequencies of the track slab due to similar frequency, increasing ground vibration response. Therefore, it is suggested that the rail corrugation limit of underground railways should take into account the impact of ground vibration since the ground vibration response is most sensitive to the rail corrugation with the wavelength component of 60–100 mm for typical rail corrugation and strictly control the wave depths of rail corrugation with a short wavelength of 60–100 mm, so as to avoid severe ground vibration.

5. Conclusions

To study the impact of typical rail corrugation on the ground vibration of underground subway, a two-step approach is used to establish the numerical prediction model for ground vibration response. The effect of car body, track, and tunnel structure on wheel-rail interaction is considered in detail, and the wheel-rail force is used as excitation source input to study ground dynamic response, the following conclusions are drawn:

(1) Combined with a two-step approach, the 3D rigid-flexible coupling system of underground subway to predict ground vibration is established. By validating the numerical simulation results in free-field case, the numerical prediction model is presented as an effective tool with good calculation accuracy.

(2) Considering the excitation of rail corrugation to wheel-rail system, according to time domain and frequency-domain analysis results, the peak acceleration of ground observation points when a train runs are substantially more than those without rail corrugation, and the dominant frequency range is widened. Therefore, the impact of short-wavelength rail corrugation cannot be neglected in environmental vibration studies, and proper maintenance on mitigating rail corrugation for underground subway helps reduce ground vibration.

(3) The observation point near the vibration source will be greatly affected by rail corrugation than the point far from the vibration source; as the short-wavelength rail corrugation will lead to the increase of ground vibration response where the impact decreases with distance, it is necessary to model rail corrugation in the near-field ground vibration studies.

(4) For the typical rail corrugation of underground subway, the impact of rail corrugation on ground vibration is mainly caused by two wavelength components, 40–60 mm and 60–100 mm. As the ground vibration response is most sensitive to the rail corrugation with the wavelength component of 60–100 mm, the depths of rail corrugation with a short
Figure 11: Wheel-rail force for Case 3: (a) response and (b) spectrum.

Figure 12: Wheel-rail force for Case 4: (a) response and (b) spectrum.

Figure 13: Wheel-rail force for Case 5: (a) response and (b) spectrum.
Figure 14: Wheel-rail force for Case 6: (a) response and (b) spectrum.

Figure 15: Wheel-rail force for Case 7: (a) response and (b) spectrum.

Figure 16: Ground response for case 2–case 7 at all 3 observation points: (a) PPA; (b) VAL_total.
wavelength of 60–100 mm shall be strictly controlled, so as to avoid causing serious ground vibration.

Data Availability

The data used to support the findings of this study are available from the corresponding author upon request.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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