Review Article

A Review on Model and Control of Electromagnetic Active Engine Mounts

Henghai Zhang,1,2 Wenku Shi,1 Jun Ke,3 Guoyu Feng,4 Junlong Qu,1 and Zhiyong Chen1

1State Key Laboratory of Automotive Simulation and Control, Jilin University, Changchun 130022, China
2School of Automotive Engineering, Shandong Jiaotong University, Jinan 250023, China
3Zhejiang Provincial Key Laboratory of Modern Textile Machinery, Zhejiang Sci-tech University, Hangzhou 310018, China
4School of Aviation Operations and Services, Aviation University of Air Force, Changchun 130022, China

Correspondence should be addressed to Henghai Zhang; zhanghh13@mails.jlu.edu.cn and Zhiyong Chen; chen_zy@jlu.edu.cn

Received 21 December 2019; Accepted 5 February 2020; Published 22 June 2020

1. Introduction

To meet the requirements of low emissions and low fuel consumption, downsizing, on-demand cylinders (COD), turbochargers, and active fuel management are applied in vehicles [1–5], which results in changes in the engine vibration excitation level and dominant engine orders shown in Figures 1(a) and 1(b). Attenuation of the vibration from the engine is the most challenging and disruptive vibrational problem. Compared with passive engine mounts or semi-active engine mounts, the active engine mount (AEM) achieves significant noise, vibration, and harshness (NVH) performance improvements [5, 6]. The actuator is the key component of the AEM. Different actuators, such as the pneumatic actuator [7–23], the magnetostrictive actuator [24, 25], the piezoelectric actuator [26–49], and the electromagnetic actuator [4, 50], have been applied to AEMs by scholars and researchers. This present work is focused on an AEM with an electromagnetic actuator, named the electromagnetic AEM. The electromagnetic AEM has attracted the attention of suppliers and automobile manufacturers. Researchers from Avon VMS [4, 51–53], Continental [1, 54, 55], Nissan [56, 57], Isuzu [58–60], Honda [61], Hyundai Motor [62–66], Paulstra [49, 67], and GM [68] have successively studied electromagnetic AEMs with a conventional passive hydraulic engine mount (HEM) extended by an electromagnetic actuator.

One of the main objectives of the present work is to summarize and show general information about the model of AEMs. Models can be categorized as theoretical models, finite-element models, and identification [1], which can provide deep comprehension of the dynamic behaviour of electromagnetic actuator [4, 50], have been applied to AEMs by scholars and researchers. This present work is focused on an AEM with an electromagnetic actuator, named the electromagnetic AEM. The electromagnetic AEM has attracted the attention of suppliers and automobile manufacturers. Researchers from Avon VMS [4, 51–53], Continental [1, 54, 55], Nissan [56, 57], Isuzu [58–60], Honda [61], Hyundai Motor [62–66], Paulstra [49, 67], and GM [68] have successively studied electromagnetic AEMs with a conventional passive hydraulic engine mount (HEM) extended by an electromagnetic actuator.

One of the main objectives of the present work is to summarize and show general information about the model of AEMs. Models can be categorized as theoretical models, finite-element models, and identification [1], which can provide deep comprehension of the dynamic behaviour of
AEMs and improve the control performance in designing AEM controllers. A second aim of this present work is a quantitative comparison of the various control strategies of AEMs regarding their weighting functions, the order of controllers, the sampling frequency, and the step size filter length. Based on this comparison, the merits and weaknesses are discussed. The goal is to identify the focus of the literature on the dynamic modelling and control of AEMs.

This article is organized as follows: General Considerations of Electromagnetic AEMs relates the main considerations and parameters needed for a deep comprehension of AEMs. Model describes a review of the different models of AEMs: Section 3.1 reviews the theoretical model, Section 3.2 reviews the finite-element model, and Section 3.3 reviews identification (or the experimental model). In Control Strategies, classical control problems of AEMs are discussed, and adaptive control, such as LMS, filtered-x least mean squares, minimal controller synthesis (MCS), robust control, and 2DOF control, are reviewed. Finally, concluding remarks are presented in Conclusions.

2. General Considerations of Electromagnetic AEMs

2.1. Actuator. Electromagnetic actuators, such as solenoids or voice coil (moving coil) actuators, are shown in Figure 2 and have the characteristics of compact structure, low energy consumption, sensitive response, work densities, easily control, and good force.

2.1.1. Electromagnetic AEMs with a Voice Coil Actuator. The passive HEM and the voice coil actuator are designed as the passive components and active components of the electromagnetic AEM, respectively. As shown in Figure 4, the excitation plate is actuated by the solenoid actuator, which changes the liquid pressure of the chamber. Mansour et al. [71–73] proposed an electromagnetic AEM with a solenoid actuator for testing on vehicles. The electromagnetic AEM with a solenoid actuator shown in Figure 4 was applied on the Honda INSPIRE [61, 70, 74]. Kitayama et al. [75–83] studied the linear electromagnetic actuator for AEM. The permanent magnet is bonded on a diaphragm. A magnetic field is produced by the electric current in the solenoid coil, which can produce mechanical force and alter the pressure of fluid in the chamber [84].

From the above discussion, the passive HEM and actuator are designed as the passive components and active components of the electromagnetic AEM, respectively. The voice coil actuator subsystem consists of a permanent magnet and a coil, as shown in Figure 5; the electrical differential equation of voltage applied on the voice coil actuator can be expressed as

\[ u(t) = K_M x_a(t) + R x(t) + L \frac{dx(t)}{dt}, \]

where \( u(t) \), \( R \), \( L \), and \( x_a(t) \) are the input voltage applied on the voice coil, the electrical resistance of the voice coil, the
The inductance of the voice coil, and the position of the moving diaphragm, respectively.

The magnetic flux generated by the permanent magnet interacts with the current in the coil, and the actuator produces a Lorentz force, which can be expressed as

$$F = B l w i,$$

where $B$, $l_w$, and $i$ denote the field density, wire length, and current flowing in the wire, respectively.

The decoupling membrane of the HEM is replaced by the voice coil actuator or the solenoid actuator. The actuator acting on the diaphragm is usually regarded as a mass-spring-damper system, as shown in Figure 6, which can be expressed by

$$f_a(t) - p_u(t)A_a = m_a \ddot{x}_a(t) + d_a(\dot{x}_a(t) - \dot{x}_c(t)) + k_a(x_a(t) - x_c(t)),$$

where $f_a$ is the actuator force, $p_u$ is the pressure inside the upper fluid chamber, $m_a$ is the actuator mass, $d_a$ is the damping coefficient of the actuator, and $k_a$ is the spring constant of actuator. When the actuator is turned on, the displacement of the chassis is small. If it is supposed that the chassis is fixed, then $x_c(t)$ is zero.

2.2. Fluid. The fluid in an AEM is assumed to be incompressible. The continuity equation of the upper chamber is expressed as

$$A_i x_i(t) + A_n (x_n(t) - x_c(t)) = A_m (x_c(t) - x_c(t)) + \Delta V_b,$$

where $A_i$, $A_n$, and $A_m$ denote the actuator diaphragm area, the cross-sectional area of the inertia track, and the equivalent piston area of the main rubber spring, respectively. $x_n$, $x_i$, and $x_c$ denote the displacement of the actuator, the fluid in the inertia track, the AEM at the engine side, and the AEM at the chassis side, respectively. The volumetric compliance element is expressed as
Frequency-dependent stiffness and damping of the fluid are generated in the inertia track of the HEM [86]. The static pressure of the AEM upper chamber is relieved only in the inertia track [65]. A column of fluid is created in the inertia track [65]. The friction generated by the fluid flow in the track is not directly transmitted to the chassis. The inertia track is assumed to be connected to the absolute reference frame [66]. The fluid in the inertial track is forced to flow by the pressure in the upper chamber, which can be represented by

\[ -p_u(t)A_i = m_i\dot{x}_i(t) + d_i\ddot{x}_i(t) + k_i x_i(t), \]

where \( m_i \) and \( d_i \) are the equivalent mass and damping coefficient of the fluid in the inertia track, respectively. The stiffness of the lower chamber is smaller than that of the upper chamber [87–89], and the stiffness of the lower chamber \( k_i \) can be neglected.

2.3. Elastomeric. The AEM can be designed by incorporating a conventional passive HEM [88–92] with the actuator. The dynamic stiffness of a passive engine mount depends on the frequency, temperature, amplitude, and types of external excitation [93, 94]. Meanwhile, the main rubber spring or elastomer rubber element of the passive engine mount has amplitude- and frequency-related behaviours [95]. The damping, stiffness, and bulking properties of the main rubber spring or elastomer rubber element of the AEM contribute substantially to the dynamic characteristics of the AEM. The frequency-dependent dynamics model of the elastic coupling unit can be traced back to the 18th century; Maxwell et al. studied the behaviour of viscoelastic materials [96]. Some viscoelastic material models have been widely used in simulations, such as the Maxwell [97] and Kelvin–Voigt models [98]. In the AEM mathematical model, the main rubber springs or elastomer rubber elements are typically modelled using linear spring elements [99, 100] or a spring in parallel to a damper (the Kelvin–Voigt model shown in Figure 6(a)) [101–103], which overestimates both stiffness and damping at higher frequencies.

To settle the problem of the Kelvin–Voigt model, as shown in Figure 6(b), the main rubber spring and its bulking properties are modelled as one spring in parallel to two dampers and another spring. Lambertz et al. [104–108] applied this approach in studying conventional HEMs, and the transfer function of the elastomer element in the Laplace domain is expressed as

\[ K_{\text{dyn}}(s) = \frac{F_c(s)}{X_c(s)} = k_1 + \frac{d_1 k_2 s + d_2 s^2}{k_2 + (d_1 + d_2) s^2}, \]

where \( K_{\text{dyn}}(s) \) is the dynamic properties of the main rubber spring or elastomer rubber. The damping and stiffness of the high frequency simulation are in good agreement with those of the test [85]. The Kelvin–Voigt model is consistent with the measured loss angle at only one design frequency [109].

2.4. Force Transmitted to the Engine and the Chassis. The force transmitted to the engine, chassis, and body through the AEM, \( F_e(t) \), and \( F_c(t) \), respectively, can be represented by

\[
\begin{align*}
F_e(t) &= -k_c(x_e(t) - x_c(t)) + p_u(t)A_m, \\
F_c(t) &= k_c(x_e(t) - x_c(t)) - p_u(t)(A_m - A_a) \\
&\quad + d_a(x_a(t) - x_c(t)) + k_a(x_a(t) - x_c(t)) - f_a(t).
\end{align*}
\]

Similar equations [66, 85, 110] are used to deduce the transfer functions to study the dynamic characteristics of electromagnetic AEMs.

3. Model

As discussed in Section 2, the dynamic behaviour of AEMs is nonlinear. Accurate AEM models can improve the controller performance in designing model-based controllers and facilitate the accurate study of the dynamic behaviour of AEMs [111–114]. There are three general models, namely, theoretical models, finite-element models, and identification [1]. The theoretical model is obtained by applying methods from calculus to equations derived from physics. The finite-element models make use of a virtual development
environment, such as ADAMS. Identification, which is also named experimental models, is a mathematical model derived from measurements.

3.1. Theoretical Model. The theoretical model describes the dynamic characteristics of the AEM, which is expressed by the transmission performance of force between the chassis and the engine, and the secondary path transfer function of the AEM between the actuator input signal and the output force (displacement or acceleration) on the chassis (or engine) side. It is generally assumed that (1) the forces transmitted to the engine and the chassis are always equal to each other, (2) the displacement or acceleration of the chassis is zero, (3) the stiffness or damping of the elastomeric rubber is independent of the frequency or preload, and (4) the dynamics of the chassis or the mass of the engine are ignored, and the characteristics of the AEM are derived therefrom [88, 115].

Assuming that the chassis displacement is zero and the dynamics of the engine, Lee and Lee [65] proposed a theoretical model to describe the dynamic characteristics of AEMs, which is expressed by the secondary path transfer function between the control voltage and the chassis, and the transmitted force in terms of the engine excitation force and the actuator motion. The theoretical model is verified by experiments, and the analytical results based on the proposed theoretical model agree well with the experimental results and confirm that the proposed theoretical model accurately describes the dynamic behaviour of an AEM. Considering the dynamics of actuator, the dynamics of the fluid in the inertia track, the structural parameter of the AEM, the chassis displacements, the frequency-dependent characteristics of the volumetric stiffness of the main liquid chamber, and the complex stiffness of the main rubber spring, Hausberg [103] proposed a theoretical model expressed in the Laplace domain to describe the dynamic characteristics of an AEM. The proposed theoretical model is expressed by the cross point and driving point dynamic stiffness of the AEM at the engine side and the secondary path transfer function between the control voltage and the chassis (engine). The simulated curves of theoretical model proposed by Hausberg are in good agreement with the experimental results.

3.2. Finite-Element Model. Olsson [116] studied a three-point suspended 5-cylinder combustion diesel engine, as shown in Figure 7, which is attached to the vehicle body on the LHS (left-hand side) and RHS (right-hand side) via two rubber engine mounts and connected to the subframe via a TR (torque rod) with rubber bushings at both ends. Then, a finite-element model was proposed by ADAMS. The engine and torque rod are modelled with 12 kinematical degrees of freedom (DOFs) using rigid body representation. The LHS engine mount, RHS engine mount, and rubber bushings are modelled using 6 DOFs. Finally, the body and subframe attachment points are regarded as rigid in all directions. The finite-element model was not compared with the theoretical model or experimental model by Olsson in this article.

3.3. Identification (Experimental Model). The identification of the AEM shown in Table 1 is significant in the verification of the theoretical model, the controller design, and the dynamic characteristics of the AEM. The primary path is the transfer path between the error sensor and the disturbance source. The transfer path between the error sensor and the controller output is named the secondary path. The identification of the primary path or the secondary path is required for model-based control. Identification techniques include harmonic identification [57, 66], LMS- or FBLMS-based finite impulse response (FIR) identification [69],
Table 1: Summary of AEM model and control.

<table>
<thead>
<tr>
<th>Ref.</th>
<th>Year</th>
<th>Model</th>
<th>Controller</th>
<th>Model verification</th>
<th>Control verification</th>
</tr>
</thead>
<tbody>
<tr>
<td>Riley et al. [117]</td>
<td>1995</td>
<td>Identification</td>
<td>FXLMS/AC</td>
<td>—</td>
<td>V</td>
</tr>
<tr>
<td>Nakaji et al. [57]</td>
<td>1999</td>
<td>FRT identification</td>
<td>SFX/AC</td>
<td>—</td>
<td>R/V</td>
</tr>
<tr>
<td>Aoki et al. [9]</td>
<td>1999</td>
<td>—</td>
<td>SFX/AC</td>
<td>—</td>
<td>V</td>
</tr>
<tr>
<td>Lee et al. [64]</td>
<td>2000</td>
<td>Theoretical model</td>
<td>Normalized FXLMS/AC</td>
<td>E</td>
<td>R</td>
</tr>
<tr>
<td>Fursdon et al. [51]</td>
<td>2000</td>
<td>—</td>
<td>Self-tuning cancellation/AC</td>
<td>—</td>
<td>V</td>
</tr>
<tr>
<td>Yang et al. [118]</td>
<td>2001</td>
<td>Theoretical model</td>
<td>FXLMS/AC &amp; RC</td>
<td>E</td>
<td>S/R</td>
</tr>
<tr>
<td>Lee and Lee [65]</td>
<td>2002</td>
<td>Theoretical model</td>
<td>FXLMS</td>
<td>E</td>
<td>R</td>
</tr>
<tr>
<td>Togashi and Ichiryu</td>
<td>2003</td>
<td>Theoretical model</td>
<td>FXLMS/AC</td>
<td>—</td>
<td>R</td>
</tr>
<tr>
<td>Kowalczyk et al. [1]</td>
<td>2004</td>
<td>FRT identification</td>
<td>FXLMS/AC and disturbance observer</td>
<td>—</td>
<td>V</td>
</tr>
<tr>
<td>Hillis et al. [52]</td>
<td>2005</td>
<td>Identification</td>
<td>Narrow-band FXLMS/AC</td>
<td>—</td>
<td>V</td>
</tr>
<tr>
<td>Hillis et al. [119]</td>
<td>2005</td>
<td>Theoretical model</td>
<td>Er-MCSI/AC</td>
<td>—</td>
<td>V</td>
</tr>
<tr>
<td>Hillis et al. [52]</td>
<td>2005</td>
<td>Theoretical model</td>
<td>Er-MCSI/AC</td>
<td>—</td>
<td>R/V</td>
</tr>
<tr>
<td>Hillis et al. [119]</td>
<td>2005</td>
<td>Theoretical model</td>
<td>NBMCS/AC</td>
<td>—</td>
<td>R/V</td>
</tr>
<tr>
<td>Bouzid et al. [120]</td>
<td>2005</td>
<td>SI identification</td>
<td>FXLMS/AC</td>
<td>—</td>
<td>S</td>
</tr>
<tr>
<td>Olsson [116]</td>
<td>2006</td>
<td>Finite-element model</td>
<td>Gain scheduled H₂/RC</td>
<td>—</td>
<td>Co-S</td>
</tr>
<tr>
<td>Shi Wenku and Chai</td>
<td>2006</td>
<td>Theoretical model</td>
<td>FXLMS/AC</td>
<td>—</td>
<td>S</td>
</tr>
<tr>
<td>Karimi and Lohmann</td>
<td>2007</td>
<td>Theoretical model</td>
<td>Haar wavelet-based H₉₀/RC</td>
<td>—</td>
<td>C</td>
</tr>
<tr>
<td>Shin [123]</td>
<td>2007</td>
<td>—</td>
<td>OL&amp;STAFC</td>
<td>—</td>
<td>S</td>
</tr>
<tr>
<td>Darsivan et al. [124]</td>
<td>2008</td>
<td>Identification</td>
<td>NARMA-L2 neural/IC</td>
<td>—</td>
<td>S</td>
</tr>
<tr>
<td>Lee &amp; Lee [66]</td>
<td>2009</td>
<td>Theoretical model</td>
<td>Current shaping control</td>
<td>FRT identification</td>
<td>R</td>
</tr>
<tr>
<td>Darsivan et al. [125]</td>
<td>2009</td>
<td>Identification</td>
<td>NARMA-L2 neural network/IC</td>
<td>—</td>
<td>S</td>
</tr>
<tr>
<td>Fakhari et al. [126]</td>
<td>2010</td>
<td>Theoretical model</td>
<td>H₂ and H₉₀/RC</td>
<td>—</td>
<td>S</td>
</tr>
<tr>
<td>Hillis [127]</td>
<td>2011</td>
<td>Identification</td>
<td>Narrow-band FXLMS/AC</td>
<td>—</td>
<td>V</td>
</tr>
<tr>
<td>Mahil et al. [128]</td>
<td>2011</td>
<td>Theoretical model</td>
<td>PID/CC</td>
<td>—</td>
<td>S</td>
</tr>
<tr>
<td>Mahil et al. [128]</td>
<td>2011</td>
<td>Theoretical model</td>
<td>LQR/CC</td>
<td>—</td>
<td>S</td>
</tr>
<tr>
<td>Togashi et al. [59]</td>
<td>2011</td>
<td>—</td>
<td>Modified LMS/AC</td>
<td>—</td>
<td>S/V</td>
</tr>
<tr>
<td>Mansour et al. [71]</td>
<td>2012</td>
<td>Theoretical model</td>
<td>CL</td>
<td>—</td>
<td>R</td>
</tr>
<tr>
<td>Fok et al. [129]</td>
<td>2012</td>
<td>FRT identification</td>
<td>H₉₀/RC</td>
<td>—</td>
<td>R</td>
</tr>
<tr>
<td>Fakhari and Ohadi</td>
<td>2012</td>
<td>Theoretical model</td>
<td>H₂ and H₉₀/RC</td>
<td>—</td>
<td>S</td>
</tr>
</tbody>
</table>
subspace identification [120], and neural network identification [138], all of which have been implemented in the identification of AEMs.

3.3.1. Frequency Response Measurements. Scholars have estimated AEM models by harmonic identification [129, 139]. For the identification of the actuator dynamics, the harmonic identification tests shown in Figure 8 were applied using a commercial rubber testing machine by Lee et al. [100].

Fakhari et al. [135] carried out the identification test shown in Figure 9(a), which identified the transfer function of the passive parts between the control force of the electromagnetic actuator and the output displacement of the AEM on the engine side. As shown in Figure 9(b), the transfer function of the active parts between the control force of the electromagnetic actuator is identified. Then, the secondary path transfer function of the AEM is proposed through the transfer function of the passive parts between the control force of the electromagnetic actuator and the output displacement of the AEM on the engine side. As shown in Figure 9(b), which identified the transfer function of the active parts between the control force of the electromagnetic actuator and the output displacement of the AEM on the engine side.

3.3.2. LMS and FBLMS Identification. A finite impulse response (FIR) filter can be modelled as the estimate of the secondary path. Modelling errors between the secondary path and its estimate may lead to instability or serious performance degradation. The control system may be unstable when the phase error between the secondary path and its estimate is not less than 90° [140]. Broadband white noise can be used as an input signal in dynamical models, and the transfer function of the secondary path can be identified by the LMS identification algorithm. As shown in Figure 10, an LMS filter can identify the secondary path.

However, the LMS identification algorithm cannot cancel interference from the input engine vibration, which can be resolved by a fast-block LMS (FBLMS) in the frequency domain. FBLMS identification is robust to large system parameter variations, unknown or unmodelled dynamics, and nonlinear effects. Shynk [142] explained the general structure of FBLMS. As shown in Figure 11, the FBLMS identification algorithm is applied to a MIMO AEM system. The MIMO FBLMS filter updates the equation of the $K^{th}$ length $L$ data block [127], which is expressed as

$$\tilde{y}_{11}(K + 1) = v\tilde{s}_{11}(K) + \mathcal{F}^{-1}[u_1(2)(z_k)B_1^*(z_k)F(z_k)]_{1:L},$$

$$\tilde{y}_{21}(K + 1) = v\tilde{s}_{21}(K) + \mathcal{F}^{-1}[u_2(2)(z_k)B_2^*(z_k)F(z_k)]_{1:L},$$

$$\tilde{y}_{12}(K + 1) = v\tilde{s}_{12}(K) + \mathcal{F}^{-1}[u_1(2)(z_k)B_1^*(z_k)F(z_k)]_{1:L},$$

$$\tilde{y}_{22}(K + 1) = v\tilde{s}_{22}(K) + \mathcal{F}^{-1}[u_2(2)(z_k)B_2^*(z_k)F(z_k)]_{1:L},$$

where $B_1(z_k)$ and $B_2(z_k)$ can be expressed by

$$\tilde{y}_{11}(K + 1) = v\tilde{s}_{11}(K) + \mathcal{F}^{-1}[u_1(2)(z_k)B_1^*(z_k)F(z_k)]_{1:L},$$

$$\tilde{y}_{21}(K + 1) = v\tilde{s}_{21}(K) + \mathcal{F}^{-1}[u_2(2)(z_k)B_2^*(z_k)F(z_k)]_{1:L},$$

$$\tilde{y}_{12}(K + 1) = v\tilde{s}_{12}(K) + \mathcal{F}^{-1}[u_1(2)(z_k)B_1^*(z_k)F(z_k)]_{1:L},$$

$$\tilde{y}_{22}(K + 1) = v\tilde{s}_{22}(K) + \mathcal{F}^{-1}[u_2(2)(z_k)B_2^*(z_k)F(z_k)]_{1:L}.$$
4. Control Strategies

To improve the vibration control performance of AEM, many investigators have implemented various control algorithms, such as PID [50], adaptive control [121], robust control [132], and 2DOF control. Table 1 shows a summary of the control strategies applied during the last 2 decades, which are discussed in the following section.

4.1. Classical Control. PID control applied in AEMs continuously calculates an error value between a desired set point and the measured variable [124, 125, 128, 145]. The PID controller is mainly applied in the control of single-input-single-output (SISO) systems. It is difficult to control MIMO systems with the PID controller based on the transfer function.

The linear quadratic regulator (LQR) can overcome the aforementioned drawbacks of PID [126, 128, 146]. The weights of Q and R are adjusted, and the value of the gain K is calculated. The optimal results are obtained when R and Q are determined by various iterations. With $Q = \text{diag}(10^9, 10^6, 10^6, 10^6)$, the LQR controller $R$ was set to be unity by Mahil et al. [128] to obtain a better AEM performance. Classical control has been assessed but found to be deficient.

$\begin{align*}
\text{diag}[B_1(z_k)] &= \mathcal{F}^{-1} \begin{bmatrix} b_1(K-1) \\ b_1(K) \end{bmatrix}, \\
\text{diag}[B_2(z_k)] &= \mathcal{F}^{-1} \begin{bmatrix} b_2(K-1) \\ b_2(K) \end{bmatrix}, \\
F_1(z_k) &= \mathcal{F}^{-1} \begin{bmatrix} 0_{L \times 1} \\ -e_1(K) - f_{11} - f_{12} \end{bmatrix}, \\
F_2(z_k) &= \mathcal{F}^{-1} \begin{bmatrix} 0_{L \times 1} \\ -e_2(K) - f_{22} - f_{21} \end{bmatrix},
\end{align*}$

where

$\begin{align*}
f_{11} &= \mathcal{F}^{-1} \left. B_1(z_k) \mathcal{F} \begin{bmatrix} 3_{11}(K) \\ 0_{L \times 1} \end{bmatrix} \right|_{L+1:2L}, \\
f_{12} &= \mathcal{F}^{-1} \left. B_2(z_k) \mathcal{F} \begin{bmatrix} 3_{12}(K) \\ 0_{L \times 1} \end{bmatrix} \right|_{L+1:2L}, \\
f_{22} &= \mathcal{F}^{-1} \left. B_1(z_k) \mathcal{F} \begin{bmatrix} 3_{22}(K) \\ 0_{L \times 1} \end{bmatrix} \right|_{L+1:2L}, \\
f_{21} &= \mathcal{F}^{-1} \left. B_1(z_k) \mathcal{F} \begin{bmatrix} 3_{21}(K) \\ 0_{L \times 1} \end{bmatrix} \right|_{L+1:2L},
\end{align*}$

where $\mathcal{F}$ represents the fast Fourier transform (FFT).

$u(k) = \frac{y_k(k+d) - f(y(k), y(k-1), \ldots, y(k-n+1), u(k-1), \ldots, u(k-m+1))}{g(y(k), y(k-1), \ldots, y(k-n+1), u(k-1), \ldots, u(k-m+1))}$

3.3.3. Other Identification. Subspace identification [143, 144] was carried out to achieve the frequency responses from the input to the output of the system, as shown in Figure 12(a). The transfer functions $G_{hyd}$ represent the frequency response between the disturbance and the response. In general, $G_{io}$ represents the transfer function from the input $i$ to the output $o$. The transfer function between the secondary source (the input $u$) and the error signal (the output $y_o$) and the transfer function between the disturbance signal and the error signal were identified by Seba et al. [120].

The neural network identification shown in Figure 12(b) can be defined mathematically for the NARMA-L2 neural network controller as the following equation:

$\tilde{y}(k+d) = f[y(k), y(k-1), \ldots, y(k-n+1), u(k-1), \ldots, u(k-m+1)] + g[y(k), y(k-1), \ldots, y(k-n+1), u(k-1), \ldots, u(k-m+1)] \cdot u(k),$

where $u(k)$ is the system input, $y(k)$ is the system output, and $d$ is the delay of the parameters. The NARMA-L2 neural network controller identifies the inverse of the control plant. The control input to the plant can be determined by the equation expressed as

$4.2. \text{Adaptive Control.}$ To tackle the high uncertainty of complex and variable environments, modern adaptive control is increasingly being applied. The AEM system is a time-varying system, which requires adaptive control. This control method can attenuate the vibration by adjusting the frequency and amplitude of the AEM actuator.

4.2.1. LMS. In 1960, a highly simplified recursive algorithm was developed by Widrow and Hoff [147] to calculate the optimal filter, named the LMS algorithm. To simplify the control strategies, as shown in Figure 13, the LMS algorithm can be verified by setting $G$ to 1 and by setting error signal as the input signal for controlling the AEM.

The error signal $e[n]$ and the weight vector $w_i$ of the modified LMS algorithm are expressed by

$e[n] = d[n] - y[n] = d[n] - \sum_{i=1}^{N} w_i e[n-i],$

$w_{i+1} = w_i + 2\mu e[n-i], \quad i = 1, 2, \ldots, N.$

Vehicle test results show that the transmitted force, seat rail vibration, and interior noise can be simultaneously attenuated by the modified LMS controller over a wide frequency band. The LMS controller only can reduce the amplitude at a certain frequency corresponding to a
reference signal and cannot simultaneously attenuate the amplitude over a wide frequency band.

4.2.2. Filtered-X Least Mean Squares. Conover et al. [148, 149] proposed the filtered-x least mean squares (FXLMS) algorithm, which has been described in detail by other scholars [111–114]. The FXLMS control algorithm has become the primary tool to reduce active vibration or noise. As shown in Table 1, the FXLMS algorithm is the most common control strategy for controlling AEMs.

Figure 14 shows the block diagram of the normalized FXLMS algorithm for AEM control, which uses the engine RPM as the reference signal and the transmitted force from the engine to the chassis or body as the error signal for the generation of the actuator signal. The main design parameters of the FXLMS algorithm are the step size or the convergence coefficient, filter length, and leaky factor. The convergence speed of the FXLMS algorithm is influenced by the step size. When the filter converges to a steady state, the step size has a slight influence on the performance. The overall system stability is influenced by the leaky factor. The sampling rate and available computer specifications are influenced by the filter length. Table 2 shows that the sampling frequency, step size, filter length, and leaky factor vary in different studies. The FXLMS algorithm requires an
estimate of the secondary path of the AEM because model errors can cause performance degradation or instability of the FXLMS algorithm. To reduce the errors of model, an online system identification scheme is employed by Bao et al. [133]. In addition, the robustness of algorithm [150–152] must be improved to reduce the impact of errors on the control algorithm.

To control more than one AEM, Hillis [127] applied a MIMO narrow-band FXLMS algorithm to a system of two AEMs. The control equation is defined as

\[
u[k] = w_1[k]x_1[k] + w_2[k]x_2[k].\]

(16)

The reference signal is defined by

\[
\begin{align*}
x_1[k] &= \sin(i\omega_e k\Delta), \\
x_2[k] &= \cos(i\omega_e k\Delta),
\end{align*}
\]

(17)

where \(\omega_e\) is the engine crankshaft rotation frequency, \(i\) is the engine order, and \(\Delta\) is the sampling interval.

The weight vectors \(w_1(k) \in \mathbb{R}^{q_1}\), \(w_2(k) \in \mathbb{R}^{q_2}\) are updated according to

\[
\begin{align*}
S_{11}(z) & \quad S_{21}(z) \\
S_{11}(z) & \quad S_{12}(z) \\
S_{21}(z) & \quad S_{22}(z) \\
\end{align*}
\]

Figure 11: MIMO online system identification [127].

\[
\begin{align*}
d & \quad \text{disturbance} \\
G_{d,yd} & \quad \text{Learning algorithm} \\
G_{d,yu} & \quad \text{Neural network model} \\
G_{yp} & \quad \text{Error} \\
\end{align*}
\]

(a)

Figure 12: Block diagram of identification: (a) subspace identification [120]; (b) neural network identification [124, 125, 138].

\[
\begin{align*}
d[n] & \quad \text{disturbance} \\
G & \quad \text{Error signal} \\
\end{align*}
\]

(a)

(b)

Figure 13: Block diagram of the LMS in the AEM [59]: (a) LMS; (b) modified LMS.
where $\mu$ and $\nu$ are the step size and leakage factor, respectively. To avoid the accumulation of numerical rounding errors, the leakage factor is applied to the tap weights. 

$$
\begin{align*}
  \omega_1 [k + 1] &= \nu \omega_1 [k] - 2 \mu x_1^T (k) e (k), \\
  \omega_2 [k + 1] &= \nu \omega_2 [k] - 2 \mu x_2^T (k) e (k),
\end{align*}
$$

where $\mu$ and $\nu$ are the step size and leakage factor, respectively. 

The proposed MIMO FXLMS algorithm was applied to a two-mount/two-sensor system fitted to a saloon car equipped with a four-cylinder two-litre turbo-diesel engine. Vehicle tests indicate that the controller typically reduces chassis vibration by 50 percent to 90 percent under normal driving conditions.

Synchronized filtered-x least mean square (SFX) is a modified form of the FXLMS algorithm with a limited application to cyclic phenomena [56, 57]. Under this application environment, SFX has a computational advantage compared with the FXLMS algorithm. SFX has the ability to control higher order components and straight convergence. The control output of SFX is expressed as

$$
 y (n) = \sum_{i=0}^{l-1} \omega_n (i) \sum_{a=-\infty}^{\infty} \delta (n - a l - i). 
$$

The filter weights and reference signal are, respectively, expressed by

$$
\begin{align*}
  \omega_i (n + 1) &= \omega_i (n) - \mu e (n) r (n - i), \\
  r (n) &= \sum_{j} \hat{e}_j x (n - j).
\end{align*}
$$

Vehicle tests show that reductions in the vehicle idle vibration, the boom noise during driving, and the higher order boom noise during idling are achieved by the proposed SFX controller [56].

Lee et al. [3, 134, 153] proposed the Newton FXLMS algorithm, the filter weight of which is expressed by

$$
\begin{align*}
  w(n + 1) &= w(n) - \mu \tilde{S}^{-1} (r \omega)e^{-j \omega n l} \xi (n),
\end{align*}
$$

Simulations show that the Newton FXLMS control has an equal convergence rate at different engine speeds, while the FXLMS control has different convergence rate at different engine speeds. To tackle the limited convergence time and the tracking speed, as shown in Figure 15, online adapted look-up tables were incorporated as parameter-maps or parallel-maps, respectively, into the proposed Newton FXLMS algorithm. The look-up tables store the initial conditions of the Newton FXLMS, which is the filter weight vector $w(0)$ that is passed from the look-up tables to

### Table 2: Parameters in the FXLMS algorithm.

<table>
<thead>
<tr>
<th>Ref.</th>
<th>Year</th>
<th>Sampling frequency</th>
<th>Step size</th>
<th>Filter length</th>
<th>Leaky factor</th>
</tr>
</thead>
<tbody>
<tr>
<td>LEE et al. [64]</td>
<td>2000</td>
<td>4 kHz</td>
<td>Conservatively small</td>
<td>150</td>
<td>—</td>
</tr>
<tr>
<td>Hillis et al. [52]</td>
<td>2005</td>
<td>4 kHz</td>
<td>—</td>
<td>32</td>
<td>—</td>
</tr>
<tr>
<td>Bouzid et al. [120]</td>
<td>2005</td>
<td>2 kHz</td>
<td>Normalized 0–2</td>
<td>—</td>
<td>0.9995</td>
</tr>
<tr>
<td>Hillis [127]</td>
<td>2011</td>
<td>4 kHz</td>
<td>—</td>
<td>128</td>
<td>—</td>
</tr>
<tr>
<td>Raoofy et al. [131]</td>
<td>2013</td>
<td>2 kHz</td>
<td>$10^{-6}$</td>
<td>10</td>
<td>—</td>
</tr>
<tr>
<td>Vahdati and Heidari [69]</td>
<td>2015</td>
<td>5 KHz</td>
<td>0.0002</td>
<td>120</td>
<td>—</td>
</tr>
<tr>
<td>Guo-Rong et al. [137]</td>
<td>2017</td>
<td>—</td>
<td>$10^{-6}$</td>
<td>50</td>
<td>—</td>
</tr>
</tbody>
</table>
the Newton FXLMS before activating the Newton FXLMS with the parameter-map. The look-up table is applied in parallel to the Newton FXLMS, which is the Newton FXLMS with the parallel-map. The output $w_{LU}$ of the parallel-map can be expressed as

$$w_{LU} = \sum_{k=1}^{K} \sum_{l=1}^{L} \theta_{kl} \Phi_{kl}(y, c).$$  \hfill (24)

The vector of the table data is adapted online with the algorithm, which is expressed by

$$v_i(n+1) = v_i(n) + \mu_{LU} e_{LU}(n)(\Phi_i(y(n), c) \sum_{j=1}^{K} \sum_{k=1}^{L} \Phi_{kl}^{H}(y(n), c), i = 1, 2, \ldots, K, L.$$  \hfill (25)

The step size $\mu_{LU}$ is different from the step size $\mu$ of the FXLMS. $\mu$ is set to 0.001 and $\mu_{LU} < 0.0005$, which will suppress short-term variations of the training signal.

Vehicle tests show that the proposed control strategy applied on the AEM can reduce the convergence time. When $\mu$ is set to 0.001, the Newton FXLMS with the parameter-map or the parallel-map completely attenuates the vibration with almost no convergence time. The tracking behaviour is improved by the Newton FXLMS algorithm with the parallel-map control applied on the AEM, while the tracking performance remains unchanged for the Newton FXLMS algorithm with the parameter-map.

As shown in Figure 16, body input point inertance (IPI) and extended FXLMS were proposed by Guo et al. [137]. The transmitted force to the chassis is converted to an acceleration, which is set to the error signal. The IPI is expressed as

$$H_{iip}(\omega) = \frac{\dot{X}}{F},$$  \hfill (26)

where $\dot{X}$, $F$, and $\omega$ are the acceleration, transmitted force to the chassis, and angular frequency of the engine crankshaft angle, respectively. The weight coefficient can be expressed as

$$W(n+1) = (1 - \alpha \mu)W(n) - 2\mu e(n)R(n),$$  \hfill (27)

where $\alpha$ is the leakage coefficient, $\mu$ is the step size, and $R'(n)$ is the “filtered” reference signal vector.

Compared with the uncontrolled AEM, the simulation results show that the acceleration on the AEM has decreased by 80 percent for most of the time domain after applying the extended FXLMS controller.

4.2.3. Minimal Controller Synthesis (MCS). The minimal control synthesis (MCS) requires no knowledge of the parameters and achieves stability and robustness [154]. The Er-MCSI controller shown in Figure 17 is a derivation of the MCS controller. The control law of the Er-MCSI is expressed as

$$u(n) = K_e(n)x_e(n) + K_f(n)x_f(n),$$  \hfill (28)

where $x_e(n)$ is the state error and $x_f$ is the scalar discrete-time integral of the output error signal. The update law of the controller gains can be expressed as

$$\begin{cases} K_e(t) = K_e(n-1) + \beta q_e(n) - \alpha q_e(n-1), \\ K_f(t) = K_f(n-1) + \beta q_f(n) - \alpha q_f(n-1), \end{cases}$$  \hfill (29)

where

$$q_e = y_eX_e^T, \quad q_f = y_fX_f, \quad \alpha = \beta - a\Delta,$$

where $\alpha$ and $\beta$ are weights and $y_e$ and $\Delta$ are the output error and the sampling interval, respectively.

The proposed Er-MCSI controller was applied on an AEM, which is fitted to a car equipped with a four-cylinder engine. Test results show that the Er-MCS algorithm performed similarly to the FXLMS algorithm in terms of the cancellation level and convergence speed, and the Er-MCS algorithm has a significant computational advantage compared with the FXLMS algorithm. When the Er-MCSI is applied to control multiple AEMs and multiple sensor systems, the computational advantage will become more significant.

To tackle narrow-band error signals or the narrow-band component of broadband signals, the narrow-band MCS (NBMCS) controller shown in Figure 18 was developed based on the Er-MCSI by Hillis et al. [119]. For the MCS algorithm, the plant parameters are unknown or
time-varying. For the NBMCS, the disturbance frequency must be known or measured.

The control equation is defined as

$$u(t) = K_s(t)x_s(t) + K_c(t)x_c(t),$$

where \(x_s(t)\) and \(x_c(t)\) are described by

$$\begin{cases} x_s(t) = \sin(\omega t), \\ x_c(t) = \cos(\omega t). \end{cases}$$

The proposed NBMCS algorithm was compared with the Er-MCSI algorithm using simulations and implementation with an AEM fitted to a diesel engine saloon car [119]. When the frequency of the disturbance is known or measured, the simulations and vehicle tests show that the NBMCS algorithm outperforms the broadband Er-MCSI algorithm in narrow-band applications and overcomes the gain windup problem in the Er-MCSI.

4.3. Robust Control. Accurate models of the AEM system are difficult in model-based control, and controllers may not achieve the desired performance due to model uncertainties, disturbances, and noise. Robust controllers have robust performance in the presence of perturbations, such as disturbance, noise, parametric uncertainties, and unmodelled dynamics [155, 156].

To attenuate transmitted forces over a large frequency band from the engine to the chassis, the gain scheduled H\(_2\) controllers applied on an AEM by Olsson [116] can work well with system nonlinearities and engine vibration, but they do not work well at extremely high ramping speeds and nominal engine torque.

To isolate the engine vibration and prevent actuator saturation in the case of perturbations, Fakhari et al. [110, 126, 130] applied the H\(_2\) and H\(_{\infty}\) controllers shown in Figure 19. The weighting functions \(W_n\), \([W_d]_{6 	imes 6}\), \(W_F\), and \(W_u\) are the sensor noise, disturbances, transmitted force, and
input current to the actuator, respectively. $\mu$-analysis was used to evaluate the robust stability of the controllers. Using the Hankel-norm approximation method, the order of the $H_2$ and $H_\infty$ controllers can be set to 16 and 14, respectively, for low cost, easy commissioning, high reliability, and the maintenance of the controller.

Simulations were carried out by Fakhari and Ohadi [130] to evaluate the effectiveness of an AEM in vibration suppression of a four-cylinder engine using the closed-loop system with $H_2$ and $H_\infty$. Compared with the $H_\infty$ control, the simulation shown in Figure 20 indicates that the $H_2$ control requires a greater control effect to achieve the control performances. Meanwhile, robust performance and stability of the closed-loop system are achieved with $H_2$ and $H_\infty$.

4.4.2 DOF Control. To solve the disadvantage of nonadaptive control and adaptive control, researchers have studied 2DOF controllers, such as integrating open- and closed-loop control methods for AEMs [157]. In the case of perturbations, such as disturbance, noise, and unmodelled dynamics, the conventional adaptive control system may become unstable, while the robust approaches are still robust [155, 156]. When there are nonparametric uncertainties, the robust adaptive strategy can conquer the behaviour of the adaptive controller. The robust adaptive controller can overcome the conservative behaviour of the robust controller.

By selecting a proper reference model, the robust model reference adaptive control (MRAC) method based on the modified gradient method was proposed by Ioannou et al. [135] and can be expressed as

$$ u_p = \theta^T \omega, $$

where $\theta$ is the controller parameters and $\omega = [\omega_1^T \omega_2^T y_p^T]^T$ is the state variables of the controller. The adaptive law can be expressed as

$$ \dot{\theta} = -[\Gamma] e \phi \operatorname{sgn}(\rho^*), $$

$$ \dot{\rho} = \gamma e \xi, $$

where $\gamma > 0$, $[\Gamma] = [\Gamma]^T > 0$, $[\Gamma]$ and $\gamma$ denote the adaptive gains. $\operatorname{sgn}$ and $\rho$ are the sign function and the estimate of $\rho^*$, respectively. $\rho^*$ can be expressed as $\rho^* = k_p/k_m$.
An experimental setup was carried out by Ioannou et al. [135] to evaluate the vibration performance of an AEM with the proposed robust MRAC. Tests show that the proposed robust MRAC provides a better control performance than the P control with a phase shift in the case of large uncertainties. However, in the case of no uncertainties, the P control with a phase shift provides a better control performance than the proposed robust MRAC in a certain high frequency range.

To control the vibrations of the engine-chassis system, Yang [118] proposed a 2DOF control strategy applied on an AEM, which is formed of an \( H_{\infty} \) robust feedback controller and a FXLMS feedforward controller. The weight is represented as

\[
W(n + 1) = W(n) + \mu E(n)X_c(n),
\]

(35)

where \( X_c(n) \) is the filtered input and \( E(n) \) is the error. The robust feedback controller is designed by \( \mu \)-synthesis. To achieve control through a personal computer, the order of the robust controller was reduced from 23 to 7, which attenuated the vibration from the engine to the chassis, and robustness was achieved.

As shown in Figure 21, open-loop control is designed based on the manifold absolute pressure and crank speed. Based on the single-tone adaptive feedforward control, closed-loop control is designed. The update algorithm in discrete-time implementation is expressed as

\[
\tilde{u}_{c,new} = \tilde{u}_{c,old} - \gamma \bar{G}^*(jp\omega)Q\bar{r}_{c,old}.
\]

(36)

Simulations show that fast response times and robustness to vehicle variations can be achieved at the same time by integrating open-loop and closed-loop controls.

5. Conclusions

Based on the above review, the difficulties and trends in the research of modelling and controlling AEMs are summarized as follows:

(1) Most theoretical models are based on assumptions that are not always consistent with the working conditions of AEMs. The relative displacements (actuator, engine attachment point, and chassis attachment point), the frequency-dependent and amplitude-dependent characteristics of the elastomeric stiffness, and the damping should be considered. Similarly, theoretical models are oversimplified and do not consider complex interactions between the preload and the mass of the chassis, which can yield a vibration solution for one AEM fitted to different vehicles equipped with different engines.

(2) The method of AEM identification should be proposed to verify the above complex theoretical models and finite-element models of AEMs. Meanwhile, the effects of the AEM parameters (structural parameters and performance parameters) on the AEM dynamics should be carried out to design, manufacture, and control AEMs.

(3) The proposed closed-loop control system of AEM must be stable or robust in the case of high uncertainty in complex and variable environments, such as disturbance, noise, and unmodelled dynamics. The implementation of some controllers is sometimes costly and time-consuming. Thus, the demand of AEM control is the development and application of more advanced control approaches based on the principles of stability, robustness, intelligence, and optimality.

(4) The goal of the next generation of electromagnetic AEMs will be a compromise among a small volume, light weight, large actuator force, low cost, intelligence, and effectiveness under all working conditions.

Conflicts of Interest

The authors declare no potential conflicts of interest with respect to the research, authorship, and/or publication of this article.

Acknowledgments

The authors are grateful for the assistance of staff of NVH in the State Key Laboratory of Automotive Simulation and
References


Shock and Vibration


