Research Article

Stiffness Model and Experimental Study of Hydrostatic Spindle System considering Rotor Swing

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For high-end CNC machine tools, the stiffness of the spindle system is one of the most important performance indicators. In this paper, the hydrostatic motorized spindle system of a grinding machine is taken as the research object, and a two-degree-of-freedom stiffness model of the spindle system considering rotor swing is proposed. The stiffness of the spindle system under different excitation frequencies is analyzed, and the contributions of the stiffness of two bearings to the stiffness of the spindle system are evaluated. The vibration test on the spindle system is implemented through the hammering method. The vibration responses of the spindle system are obtained, and the stiffness of the spindle system is identified. The results show that the test results of the stiffness of the spindle system are in good agreement with the theoretical calculation, with an average error of about 14.21%. The research in this paper can provide theoretical and data support for bearing design and stiffness evaluation of a hydrostatic spindle system.

1. Introduction

With the development of high precision machine tools to high efficiency and high energy [1, 2], precision machine tools require high precision, high efficiency, and high-speed heavy load [3]. Because of the advantages of stable operation, compact structure, high speed, and heavy load, the hydrostatic spindle is more and more widely used in high-end machine tools [4, 5]. As an important performance parameter of the hydrostatic spindle system, the stiffness of the spindle system plays a significant role in determining the service performance of the spindle system [6–8]. Therefore, it is necessary to study the theoretical model of spindle system performance and to analyze the influence rules of the spindle system stiffness, so as to optimize the structure of the spindle system in the design stage of machine tools.

At present, scholars at home and abroad have done a lot of research in this field. Xia et al. [9] used FLUENT, a computational fluid dynamics software based on the finite

element volume method, to predict the bearing capacity, stiffness, and other performances of the hydrostatic spindle. Guo et al. [10] established a high-speed hydrostatic built-in motorized spindle system model and proposed a multiparameter optimization method of the motorized spindle system based on genetic algorithm. Chen et al. [11] presented a design system for a hydrostatic spindle based on the laws of the fluid mechanics, engineering thermodynamics, and rotor dynamics. Gaber and Hashemi [12] developed a calibrated dynamics stiffness matrix (CDSM) method, which could be effectively applied to predict the vibration characteristics of spindle systems.

The test methods of spindle system performance mainly include the time domain analysis method and frequency domain analysis method [13, 14]. The method of time domain analysis is to identify stiffness and damping coefficients by using time domain vibration data of the spindle system. Through exerting sinusoidal excitation to the spindle system, the frequency domain analysis method identifies dynamic

characteristics according to the vibration responses at different frequencies [15–17]. The most typical time domain analysis is the hammering method. Through exerting an impulse excitation to the rotor bearing system by a hammer, the stiffness coefficient, damping coefficient, and inherent frequency of the rotor bearing system can be identified by analyzing the free vibration response of the rotor bearing system [3, 18].

Taking the motorized system of a grinding machine as the research object, the theoretical stiffness model of the spindle system considering rotor swing is established in this paper, and the contribution degrees of bearings to spindle system stiffness are evaluated. The stiffness of the spindle system is tested by the hammering method, and the test results are compared with the theoretical calculation.

2. Stiffness Model of Spindle System considering Rotor Swing

For the motorized spindle system of a grinding machine, the rotor is supported by two hydrostatic bearings, which is shown in Figure 1. The mass of the spindle rotor is m , and the rotational inertia round the mass center is J . The stiffness coefficients of two support bearings are k_2 and k_1 . The translational displacement of the mass center of the motorized spindle rotor is x_c under the external load F , and the rotation angle is θ . The lengths of the mass center to the bearing at the front and back ends of spindle are L_2 and L_1 , respectively. The length of the overhang end is L_3 .

When the spindle gravity and bearing damping of the spindle are ignored, the two-degree-of-freedom motion differential equations of the spindle system, which include the translation and swing, are as follows:

$$\begin{bmatrix} m & 0 \\ 0 & J \end{bmatrix} \begin{bmatrix} \ddot{x}_c \\ \ddot{\theta} \end{bmatrix} + \begin{bmatrix} k_1 + k_2 & k_2 L_2 - k_1 L_1 \\ k_2 L_2 - k_1 L_1 & k_1 L_1^2 + k_2 L_2^2 \end{bmatrix} \begin{bmatrix} x_c \\ \theta \end{bmatrix} = \begin{bmatrix} F \\ F(L_2 + L_3) \end{bmatrix}. \quad (1)$$

If the external load is $F = \bar{F} \cos \omega t$, where $\bar{F}$ is the amplitude of the external load, the system responses are as follows:

$$\begin{cases} x_c = \bar{x}_c \cos(\omega t + \varphi_c), \\ \theta = \bar{\theta} \cos(\omega t + \varphi_\theta). \end{cases} \quad (2)$$

The amplitudes of the responses are shown as follows by solving equations (1) and (2), simultaneously.

$$\begin{cases} \bar{x}_c = \frac{\bar{F}[(k_1 L_1^2 + k_2 L_2^2 - J\omega^2) - (k_2 L_2 - k_1 L_1)(L_2 + L_3)]}{(k_1 + k_2 - m\omega^2)(k_1 L_1^2 + k_2 L_2^2 - J\omega^2) - (k_2 L_2 - k_1 L_1)^2}, \\ \bar{\theta} = \frac{\bar{F}[(k_1 + k_2 - m\omega^2)(L_2 + L_3) - (k_2 L_2 - k_1 L_1)]}{(k_1 + k_2 - m\omega^2)(k_1 L_1^2 + k_2 L_2^2 - J\omega^2) - (k_2 L_2 - k_1 L_1)^2}. \end{cases} \quad (3)$$

If $\bar{x}$ is the amplitude of displacement of shaft end, then $\bar{x} = \bar{x}_c + \bar{\theta}(L_2 + L_3)$. The displacement of shaft end under external load F can be obtained as follows:

$$\bar{x} = \frac{\bar{F}[(k_1 + k_2 - m\omega^2)(L_2 + L_3)^2 + (k_1 L_1^2 + k_2 L_2^2 - J\omega^2) - 2(k_2 L_2 - k_1 L_1)(L_2 + L_3)]}{(k_1 + k_2 - m\omega^2)(k_1 L_1^2 + k_2 L_2^2 - J\omega^2) - (k_2 L_2 - k_1 L_1)^2}. \quad (4)$$

The stiffness of the spindle system is

$$K = \frac{\bar{F}}{\bar{x}} = \frac{(k_1 + k_2 - m\omega^2)(k_1 L_1^2 + k_2 L_2^2 - J\omega^2) - (k_2 L_2 - k_1 L_1)^2}{[(k_1 + k_2 - m\omega^2)(L_2 + L_3)^2 + (k_1 L_1^2 + k_2 L_2^2 - J\omega^2) - 2(k_2 L_2 - k_1 L_1)(L_2 + L_3)]}. \quad (5)$$

3. Analysis of Influence Factors of Spindle System Stiffness

3.1. Parameters of Spindle System. The motorized spindle system of a grinding machine supported by two hydrostatic bearings and the spindle rotor is a hollow shaft. The structural parameters of the rotor of the spindle system are shown in Table 1. The hydrostatic bearing parameters at the front and back ends are shown in Table 2.

For the hydrostatic bearing with an orifice-type restrictor in the spindle system, the oil film stiffness of the hydrostatic cavity can be calculated as follows [19]:

$$K_j = \frac{12P_s A_e (\beta - 1) \cos \theta_1}{c\beta(2\beta - 1)}, \quad (6)$$

where A_e is the effective bearing area, and its calculation equation is as follows [20]:

$$A_e = D(L - l_1) \sin \frac{\theta_1 + \theta_2}{2}, \quad (7)$$

where β is the throttling ratio of the orifice-type restrictor, which can be calculated by the following equation [19]:

$$\beta = 0.5 + \frac{1}{2} \sqrt{\frac{8\rho P_s R^2 c^6 ((l_1/Rb_1) + 2\theta_1)^2}{9\alpha^2 \pi^2 \eta^2 l_1^4 d_0^4} + 1}, \quad (8)$$

FIGURE 1: Schematic diagram of force analysis of the spindle system.

TABLE 1: Structure and working condition parameters of the spindle system rotor.

Item	Symbol	Unit	Value
Mass of rotor	m	kg	13.8
Length of center of mass to front-end bearing	L_2	mm	140.9
Length of center of mass to back-end bearing	L_1	mm	289.1
Length of the overhang end	L_3	mm	130
Inner diameter of hollow spindle	r	mm	55
External load at shaft end	F	N	145
Speed	n	r/min	2600
Rotational inertia	J	$\text{kg}\cdot\text{m}^2$	0.059685

TABLE 2: Structure and working condition parameters of hydrostatic bearing.

Item	Symbol	Unit	Front-end bearing	Back-end bearing
Diameter of bearing	D	mm	85	69
Width of bearing	L	mm	100	120
Axial width of oil seal surface	l_1	mm	8.5	6.9
Circumferential width of oil seal surface	b_1	mm	8.5	6.9
Width of oil return groove	b_2	mm	4	4
Radial gap of hydrostatic cavity	c	μm	25	18
Diameter of throttling orifice	d_0	mm	1.2	1
Wrap angle of hydrostatic cavity and oil seal surface	$2\theta_2$	$^\circ$	84.6	83.3
Wrap angle of hydrostatic cavity	$2\theta_1$	$^\circ$	61.7	60.4
Dynamic viscosity of lubricating oil	η	$\text{pa}\cdot\text{s}$	0.0132	
Density of lubricating oil	ρ	$\text{kg}\cdot\text{m}^{-3}$	882	
Supply pressure of lubricating oil	P_s	pa	2.5×10^6	

where α is the flow coefficient of the orifice-type restrictor, which is generally selected from 0.6 to 0.7. In this paper, α is set as 0.7. R is the bearing radius and $R = D/2$. l is the length of hydrostatic cavity, and $l = L - 2l_1$. The meanings of other symbols are shown in Table 2.

Through simultaneous equations from (6) to (8), it can be obtained that the front-end bearing stiffness k_2 of the spindle system is $5.40 \times 10^7 \text{ N/m}$, and the back-end bearing stiffness k_1 is $7.08 \times 10^7 \text{ N/m}$.

3.2. Amplitude and Stiffness of Spindle System under Different Excitation Frequencies. By taking the structural parameters of the spindle rotor in Table 1 and the bearing stiffness of the front and back ends into equation (5), the stiffness and amplitude of the spindle system under different excitation frequencies can be obtained, which is shown in Figure 2. The stiffness of the spindle system is not very sensitive to the excitation frequency. In the range of excitation frequency from 0 to 50 Hz, the stiffness of the spindle system decreases slightly with the increase of the excitation frequency, and the reduction amplitude is about 0.66%.

3.3. Influence Rules of Bearing Stiffness on Spindle System Stiffness. The stiffness of the spindle system is greatly affected by the bearing stiffness, and it is also related to the position distribution of the bearing. Under the different bearing stiffness, the change rules of spindle system stiffness are shown in Figure 3, where point A represents the corresponding position in the figure of the bearing stiffness in this case. For the motorized spindle system studied in this paper, the stiffness of the spindle mainly depends on the stiffness of the front-end bearing and almost linearly increases with the increase of the front-end bearing stiffness. The stiffness value of the spindle is about 55% of the front-end bearing in this case.

4. Vibration Test and Stiffness Analysis of Spindle System

4.1. Test Method and Scheme of Spindle System Stiffness. In general, the hydrostatic bearing rotor system is underdamped. The time domain waveform of its free vibration response is shown in Figure 4. The stiffness coefficient,

FIGURE 2: Stiffness and amplitude of the spindle system under different excitation frequencies.

FIGURE 3: Change rules of spindle system stiffness with different bearing stiffness (2600 r/min).

FIGURE 4: Damped free vibration response of the rotor bearing system.

FIGURE 5: Testing scheme of spindle system stiffness by the hammering method.

FIGURE 6: Layout of eddy current displacement sensors at the front and back ends of the spindle.

damping coefficient, and inherent frequency of the system can be calculated according to the following equation.

$$\left\{ \begin{array}{l} \omega_n = 2\pi \cdot f = \sqrt{\frac{K}{m}}, \\ \omega_d = \omega_n \sqrt{1 - \zeta^2}, \\ \delta = \ln\left(\frac{M_i}{M_{i+1}}\right), \\ \zeta = \sqrt{\frac{\delta^2}{\delta^2 + 4\pi^2}}, \\ C = 2m \cdot \zeta \cdot \omega_n, \end{array} \right. \quad (9)$$

where δ is the logarithmic decrement, M_i is the amplitude, ζ is the damping ratio, ω_d is the damped inherent frequency, ω_n is the undamped inherent frequency, C is the damping coefficient, and K is the stiffness coefficient.

The hydrostatic spindle system test bed used in this paper is shown in Figure 5. The eddy current displacement sensors are installed in the horizontal and vertical directions of the front and back ends of the spindle. The installation layout of eddy current displacement sensors is shown in Figure 6. The eddy current displacement sensors are installed on an annular bracket connected with the shell, and the installation angle is 90° . The horizontal and vertical vibrations of the front and back ends of the spindle are tested, respectively.

An impulse excitation is applied to the overhanging end of the spindle system through the force hammer in the stiffness experiment. According to the data of the eddy current displacement sensor, the errors caused by the spindle deviation can be eliminated. The stiffness of the spindle system can be identified by analyzing the free vibration response of the spindle system.

4.2. Results and Analysis of Stiffness Test

4.2.1. Vibration Test of Shaft End of Spindle System. When the spindle speed is 2600 r/min, the vibration test results of the spindle front end are shown in Figures 7–9. The

FIGURE 7: Vertical vibration test results of spindle front end (2600 r/min). (a) Time domain oscillogram of vertical vibration. (b) Frequency spectrogram of vertical vibration.

FIGURE 8: Horizontal vibration test results of spindle front end (2600 r/min). (a) Time domain oscillogram of horizontal vibration. (b) Frequency spectrogram of horizontal vibration.

FIGURE 9: Free attenuation curve in the vertical direction of the spindle front end (2600 r/min).

FIGURE 10: Stiffness and amplitudes of the frequency multiplication of the spindle front end via the speed.

FIGURE 11: Test value and theoretical value of spindle stiffness via the speed.

time domain waveform in the vertical direction is shown in Figure 7(a). The maximum amplitude in the vertical direction of the spindle is about $5 \mu\text{m}$. The vibration frequency spectrum of the vertical direction is shown in Figure 7(b). The maximum amplitude corresponding to the rotation frequency of the spindle is about $2.2 \mu\text{m}$.

The maximum amplitude of the horizontal direction of the spindle front end shown in Figure 8(a) is about $7 \mu\text{m}$, and the maximum amplitude corresponding to the spindle rotation frequency shown in Figure 8(b) is about $2.5 \mu\text{m}$. The stiffness of the front end of the spindle can be identified as $3.62 \times 10^7 \text{ N/m}$ by using the hammering method shown in Figure 9.

4.2.2. Identification of Supporting Stiffness of Spindle System. By changing the spindle speed through the frequency converter, the vibration data of the spindle at different speeds are as shown in Figure 10.

The amplitudes of the frequency multiplication in both directions of the front end of the spindle decrease with the increase of the speed, and the stiffness of the spindle front end also tends to decrease. At about 2300 r/min, the stiffness is about $3.0 \times 10^7 \text{ N/m}$, and the corresponding vertical and horizontal amplitudes are $2.5 \mu\text{m}$ and $2.9 \mu\text{m}$, respectively.

4.2.3. Analysis of Test Results. The stiffness value of the spindle system can be obtained as $3.41 \times 10^7 \text{ N/m}$ by averaging the 9 groups of the stiffness values under the speed from 0 to 2600 r/min. Similarly, the average value of the corresponding theoretical stiffness is $3.89 \times 10^7 \text{ N/m}$, as shown in Figure 11. The error between the test identification value and the theoretical calculation value is 14.21%, which means that the two values match each other well.

It should be noted that the natural frequencies of the two-degree-of-freedom spindle system are 427.543 Hz and 1735.68 Hz. The maximum test frequency is 43.33 Hz, so the respective frequency ratios are about 0.101 and 0.024. Therefore, it is shown that the stiffness test values in the lower frequency range are in agreement with the theoretical values, which also indicates that the model established in this paper can better reflect the performance of the spindle in the lower frequency range.

5. Conclusion

- (1) The force analysis of the hydrostatic spindle system is implemented, and a stiffness model of the spindle system considering rotor swing is established.
- (2) The stiffness and amplitude of shaft end under different excitation frequencies are analyzed. The results show that the stiffness of the front-end bearing has a greater impact on the stiffness of the spindle system.
- (3) The stiffness of the spindle system is identified by the hammering method and compared with the theoretical calculation stiffness, so that the theoretical model can be preliminarily verified. In the low frequency range, the research results can provide theoretical and data support for the design of hydrostatic bearing and the evaluation of spindle stiffness.

Data Availability

The data used to support the findings of this study are included within the article.

Conflicts of Interest

The authors declare that there are no conflicts of interest regarding the publication of this paper.

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