Research Article

Theoretical and Experimental Study on the Transmission Loss of a Side Outlet Muffler

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Vibration and noise reduction methods are important research directions in many industries. Mufflers are widely used in pipelines to reduce noise and vibration transmission. A theoretical analysis is conducted to determine the noise reduction mechanism of a muffler with a side outlet, and the results are verified with experiments. The quadrupole parameters of the muffler are obtained using transfer matrix analysis, and sensitivity analysis of the quadrupole parameters is conducted. The influence of the side outlet location on the transmission loss of the muffler is investigated experimentally using an impedance tube. The key factors affecting the transmission loss are analyzed. The theoretical calculation results of the transmission loss of the side outlet muffler are in good agreement with the experimental results.

1. Introduction

Vibration and noise reduction technology represent important research directions in many industries. The transmission of noise can be significantly reduced by installing noise reduction devices. Mufflers are widely used in pipeline systems to reduce the transmission of noise and vibration [1–4].

At present, the design of mufflers in many enterprises depends largely on the experience of engineers [5, 6]. The performance of mufflers has been determined using experiments, but this method relies on the experience of engineers and is inefficient. There is an urgent need to investigate mufflers used in pipelines.

Traditional expansion chamber mufflers are widely used in pipeline systems. The muffling mechanism of these mufflers has been investigated in the literature [7–12]. If low-frequency noise has to be eliminated, the expansion chamber of this type of muffler has to be long, requiring a large muffler. Traditional expansion chamber mufflers suffer from periodic transmission loss and do not eliminate noise at the passing frequency [13–15].

In this study, a muffler with a side outlet is investigated. The quadrupole parameters of the muffler are obtained using the transfer matrix analysis method [16]. The sensitivity of the quadrupole parameters of the muffler is analyzed, and the influence of the side outlet position on the transmission loss of the muffler is investigated. The transmission loss of the muffler is measured using an impedance tube. The system parameters of the muffler are obtained using the two-load method and changing the boundary conditions at the end of the impedance tube. Most studies on muffler noise used simulations and experiments to determine the transmission loss characteristics of side outlet mufflers. Therefore, a theoretical analysis of the transmission loss mechanism of the side outlet muffler is conducted, and the results are verified using experiments.

2. Theoretical Analysis of the Transmission Loss of the Side Outlet Muffler

Theoretical analysis is used to obtain the quadrupole parameters of the muffler using the transfer matrix analysis method, and the calculation formula of the transmission
loss of the muffler attributed to the side outlet position is derived.

2.1. Basic Assumptions. The following assumptions and simplifications are required to establish the wave equation of the sound wave propagation [17, 18]:

(1) The medium is an ideal fluid without viscous properties, and sound waves propagate in the medium without energy loss
(2) Sound propagation is an adiabatic process, and there is no heat exchange with the outside world
(3) The sound waves propagate in the medium with a small amplitude, and the acoustic field parameters in the medium are first-order small quantities, which is described by a linear wave equation

2.2. Method for Calculating the Transmission Loss. The sound pressure and vibration velocity of the muffler inlet and outlet are calculated to determine the transmission loss of the muffler using the quadrupole transfer matrix method [1]:

\[
\begin{bmatrix}
  p_1 \\
  v_1
\end{bmatrix} =
\begin{bmatrix}
  A & B \\
  C & D
\end{bmatrix}
\begin{bmatrix}
  p_2 \\
  v_2
\end{bmatrix},
\]

where \( p_1 \) and \( p_2 \) are the sound pressure of the muffler inlet and outlet, respectively, and \( v_1 \) and \( v_2 \) are the vibration velocity of the muffler inlet and outlet, respectively. The parameters in equation (1) are defined as follows [1]:

\[
\begin{align*}
A &= (p_1/p_2) \bigg|_{v_1=0, v_2=b'} \\
B &= (v_1/v_2) \bigg|_{p_1=0, v_1=b'} \\
C &= (v_1/p_2) \bigg|_{v_1=0, v_2=b'} \\
D &= (v_1/v_2) \bigg|_{p_1=0, v_1=b'}
\end{align*}
\]

If the inlet and outlet temperatures of the muffler are the same and the cross-sectional areas of the inlet and outlet are the same, the transmission loss obtained by the quadrupole transfer matrix method is as follows [1]:

\[
\text{TL} = 20 \log\left(\frac{1}{2} A + B \left(\frac{S_1}{c}\right) + C \left(\frac{c}{S_1}\right) + D \right),
\]

where \( S_1 \) denotes the cross-sectional area of the inlet and outlet and \( c \) denotes the sound velocity in the air.

2.3. Transmission Loss of Expansion Chamber Mufflers. An expansion chamber muffler consists of an intake pipe, an outlet pipe, and a chamber. The intake pipe and the outlet pipe are located on both sides of the chamber and on the same axis, as shown in Figure 1.

For the original expansion chamber muffler, the quadrupole transfer matrix is given by [19]

\[
[T] = \begin{bmatrix}
\cos(kl_1) & j\left(c/S\right)\sin(kl_1) \\
\left(j/c\right)\sin(kl_1) & \cos(kl_1)
\end{bmatrix}
\]

By substituting equation (4) into equation (3), the transmission loss of the traditional expansion chamber muffler is obtained as follows [19]:

\[
\text{TL}_1 = 20 \log \left\{ \left(\frac{1}{2} \cos(kl_1) + \frac{S_1}{S} \sin(kl_1) + \frac{S}{S_1} \sin(kl_1) + \cos(kl_1) \right) \right\}
\]

A simplification of equation (5) results in [20]

\[
\text{TL}_1 = 10 \log \left\{ 1 + \frac{1}{4} \left( m - \frac{1}{m} \right)^2 \sin^2(kl_1) \right\},
\]

where \( m = S/S_1 \) is the expansion ratio. The transmission loss of the traditional expansion chamber muffler is a function of the frequency, the expansion chamber length, and the expansion ratio. The transmission loss of the traditional expansion chamber muffler has periodic passage frequency, at which the transmission loss is zero. In this study, a side outlet muffler is investigated to improve the acoustic performance of the traditional expansion chamber muffler and make better use of the space.

2.4. Transmission Loss of the Side Outlet Muffler. The side outlet muffler also consists of an intake pipe, an outlet pipe, and a chamber, but the direction of the outlet and the intake axis is perpendicular, as shown in Figure 2.

The transfer matrix of the side outlet muffler can be divided into two types of elements, i.e., a straight tube element with an equal cross section and an element with a change in the cross section. The transfer matrix of the side outlet muffler is the product of the transfer matrix of the two basic units [21]:

\[
[T] = \begin{bmatrix}
\cos(kl_1) & j\left(c/S\right)\sin(kl_1) \\
\left(j/c\right)\sin(kl_1) & \cos(kl_1)
\end{bmatrix}
\]

where \( l_1 = l - l_2 \); by simplifying equation (7), we obtain the following equation:

\[
[T] = \begin{bmatrix}
\cos(kl_1) - \sin(kl_1)\tan(kl_2) & j\left(c/S\right)\sin(kl_1) \\
\left(j/c\right)\sin(kl_1) + \cos(kl_1)\tan(kl_2) & \cos(kl_1)
\end{bmatrix}
\]

\[\text{Figure 1: Diagram of an expansion chamber muffler.}\]
The transmission loss of the side outlet muffler is obtained by substituting equation (8) into equation (3):

\[
TL_2 = 20 \log \left\{ \frac{1}{2} \left( \cos(kl_1) - \sin(kl_1)\tan(kl_2) + \frac{1}{m}\sin(kl_1) \right) \right\}.
\]

The transmission loss of the side outlet muffler is a function of the frequency, length of the expansion chamber, the exit position, and the expansion ratio. In the next section, we describe the parameters of the side outlet muffler.

3. Key Parameter Analysis of the Side Outlet Muffler

It has been shown that the transmission loss of the side outlet muffler is a function of the frequency, the expansion cavity length, the outlet position, and the expansion ratio. The sensitivity of the quadrupole parameters of the muffler is analyzed theoretically, and the influence of the side outlet position on the transmission loss of the muffler is determined. The mechanism of noise reduction in the side outlet muffler is ascertained.

3.1. Influence of the Quadrupole Parameters on the Transmission Loss

It is evident from equation (3) that the quadrupole parameters are required for the calculation of the transmission loss of the muffler. In general, the expansion ratio \( m > 1 \); therefore, in equation (5) and equation (9), the coefficient \((1/m) < 1\) of the second term (quadrupole parameter B) in the equation for calculating the transmission loss. The larger the expansion ratio is, the smaller the coefficient of the second term is; therefore, this item is a small amount with little influence. In contrast, the coefficient \( m > 1 \) of the third term (quadrupole parameter C) in the equation for calculating the transmission loss is an important item with relatively large influence, and the larger the expansion ratio is, the larger the coefficient of the third term is. We analyze the influence of all parameters on the transmission loss.

Table 1 shows the parameters and values of the side outlet muffler. The influence of the quadrupole parameters on the transmission loss of the side outlet muffler is determined with numerical calculations using MATLAB software. The effect of the first term of the quadrupole parameters is determined by comparing the results obtained for \( A = 0 \) with those obtained from equation (9). As is shown in Figure 3, there are some effects at the passage, but they have little influence on the overall transmission loss. As can be seen from Figure 4, the second quadrupole parameter has almost no effect on the transmission loss of the muffler, which is consistent with the above analysis. The third quadrupole parameter has a significant effect on the transmission loss of the muffler (Figure 5), which is consistent with the above analysis. The fourth quadrupole parameter has only a small effect on the passing frequency but has little influence on the overall transmission loss. As can be seen from Figure 4, the second quadrupole parameter has almost no effect on the overall transmission loss. As can be seen from Figure 4, the second quadrupole parameter has almost no effect on the transmission loss of the muffler, which is consistent with the above analysis. The third quadrupole parameter has a significant effect on the transmission loss of the muffler (Figure 5), which is consistent with the above analysis. The fourth quadrupole parameter has only a small effect on the passing frequency but has little influence on the overall transmission loss (Figure 6), which is similar to the results of the first quadrupole parameter.

3.2. Influence of the Outlet Position on the Transmission Loss of the Muffler

The theoretical analysis in Section 3.1 showed that the third quadrupole parameter \( C \) had a significant influence on the transmission loss of the muffler. Therefore, it is necessary to conduct an in-depth analysis.

We assume that \( \xi = \sin(kl_1) + \cos(kl_1)\tan(kl_2) \), and by transforming the trigonometric functions, we obtain the following equation:

\[
\xi = \sin(kl_1)(\cos(kl_2) + \sin(kl_2)\tan(kl_2)).
\]

Equation (10) shows that the result is affected by the length of the chamber \( l \) and the distance \( l_2 \) from the outlet axis to the right side of the muffler. The transmission loss of the muffler can be reduced by adjusting these two parameters.

We investigate the functional properties of \( \xi \) and assume that \( \varphi = \sin(kl_1) \) and \( \psi = \cos(kl_1) + \sin(kl_1)\tan(kl_2) \); the characteristics of these two functions are analyzed separately, and the absolute values of these two functions are used for the convenience of analysis.

As shown in Figure 7, as the independent variable \( kl_1 \) increases, the function value \( \varphi \) reaches a passing frequency at which the transmission loss is zero at regular intervals. Figure 8 shows that as the independent variable \( kl_2 \) increases, the function value \( \psi \) reaches a maximum at regular intervals, and the transfer loss at the maximum value is theoretically infinite. When function \( \varphi \) is zero, it satisfies \( kl_1 = \pm n_1 \pi \), and when function \( \psi \) reaches the maximum value, it satisfies \( kl_2 = \pm (2n_2 + 1/2)\pi \).

Since \( \xi = \varphi\psi \), we can match the two functions by choosing appropriate values of \( l \) and \( l_2 \) to eliminate the passing frequency of the function value \( \xi \) so that when function \( \varphi \) is zero, the function \( \psi \) is at the maximum value,
i.e., \((l/n_1) = (2l_2/2n_2 + 1)\). By simplification, we obtain the following equation:

\[
\frac{l_2}{T} = \frac{2n_2 + 1}{2n_1}. \tag{11}
\]

Table 2 shows the results of \(l_2/l\) (equation (11)) when \(n_1\) and \(n_2\) are different values.

It is observed that \(l_2/l = 1/2\) occurs for odd numbers column of \(n_1\) and \(l_2/l = 1/4\) occurs for double odd series column of \(n_1\). We use the model data in Table 1 to determine the transmission loss of the side outlet muffler by changing the distance \(l_2\) between the outlet axis and the right side. The transmission loss is compared with that of the traditional expansion chamber muffler.

As shown in Figure 9, when \(l_2/l = 1/2\), i.e., the axis of the side exit is 1/2 of the total length of the boundary, the odd number times of the passing frequency of ½ the wavelength can be eliminated. In Figure 10, when \(l_2/l = 1/4\), i.e., the axis of the side exit is ¼ of the total length of the boundary, the odd times of the passing frequency of the wavelength can be eliminated.

In previous studies [22–24], most of the research studies on the transmission loss characteristics of mufflers used simulation and experimental methods (references are cited here). Some phenomena can be found, but the internal mechanism is difficult to reveal accurately. In this paper, the theoretical analysis method based on the period matching of trigonometric function is used to reveal the noise eliminating
mechanism of side outlet muffler. The results of this paper are helpful for the relevant scholars to better understand the structure of muffler and carry out relevant design.

4. Experimental Verification and Analysis of the Side Outlet Muffler

The transmission loss of the muffler is determined using an impedance tube in an experiment. The theoretical calculation results and the experimental results of the transmission loss of the side outlet muffler are compared, and the key factors are analyzed.


Four methods are commonly used to measure the transmission loss of a muffler, i.e., the transfer function method, the acoustic wave decomposition method, the two-sound source method, and the two-load method. The first two measurement methods require that complete noise elimination is achieved at the end of the system. In the experiment, the end of the muffler pipe is usually filled with sound-absorbing material. Because the absorption coefficient of the sound-absorbing material in the low-frequency section is not high, the muffler cannot achieve complete noise elimination, resulting in a large error. The last two methods do not require sound-absorbing material but require two measurements. The difference between the two methods is that the former obtains the system parameters based on the characteristics of the sound source generated by the two measurements, whereas the latter obtains the system parameters based on changes in the acoustic impedance conditions at the end of the muffler. The two-load method is easy to implement and widely used and is, therefore, used in...
4.2. Experimental Equipment for Determining the Transmission Loss of the Muffler. In this study, the B&K 4206-T impedance tube is used to determine the transmission loss of the muffler, as shown in Figure 12. The system consists of three parts. The first part is the signal generation system, which consists of a power amplifier and a loudspeaker, which is part of the impedance tube. The second part (signal acquisition system) consists of four ¼ inch B&K microphones, a data acquisition system, and a signal acquisition and processing software (B&K pulse) system. The third part (muffler and pipeline system) consists of a connecting pipe, the muffler, and the end device.

4.3. Comparative Analysis of the Experimental and Theoretical Results. Two side outlet mufflers are designed using the parameters listed in Table 1. The distances between the outlet axis and the right side are 45 mm and 90 mm, respectively, corresponding to $l_2/l_1 = 1/4$ and $l_2/l_1 = 1/2$. Three-dimensional printing technology is used to fabricate the two mufflers. The impedance tube test equipment is used to measure the transmission losses of the two mufflers, as shown in Figure 13. The wall thickness of the muffler is 10 mm to avoid the influence of sound radiation and vibration of the muffler wall.

Figure 14 shows the theoretical and experimental results of the transmission loss of the muffler for a distance of 33.58 mm between the outlet axis and the right side. The theoretical calculation results are in good agreement with the experimental results. The first peak of the transmission loss of the muffler is observed at 1900 Hz in the experiment.
coinciding with the theoretical frequency. The transmission loss corresponding to the resonance frequency obtained in the experiment is 34.4 dB, which is less than the 62.7 dB obtained from the theoretical analysis.

Figure 15 shows the theoretical and experimental results of the transmission loss for a distance of 67 mm between the outlet axis and the right side. The theoretical calculation results are consistent with the experimental results. The first peak value of the transmission loss of the muffler is observed at 950 Hz in the experiment, coinciding with the theoretical frequency. The transmission loss corresponding to the resonance frequency obtained in the experiment is 30.2 dB, which is less than the 62.4 dB obtained from the theoretical analysis.

The experimental value is lower than the theoretical value. The reasons are as follows. The theoretical calculation is based on ideal boundary conditions. The incident sound pressure is absorbed entirely by the end outlet and reflected sound pressure and other noises occur in the experiment. Although the theoretical results are slightly different from the experimental results, the overall agreement is good, providing reliable information for determining the influence mechanism of the transmission loss of complex mufflers in future studies.

5. Conclusions

A theoretical analysis was conducted on the transmission loss mechanism of a side outlet muffler, and the quadrupole parameters of the muffler were obtained using the transfer matrix analysis method. A sensitivity analysis of the quadrupole parameters of the muffler was performed, and the influence of the side outlet position on the transmission loss of the muffler was determined. The theoretical and experimental results of the transmission loss of the side outlet muffler were compared.

The following conclusions were drawn:

1) The transmission loss of the side outlet muffler is a function of the frequency, the length of the expansion chamber, the exit port position, and the expansion ratio. An analysis of the parameters of the transfer matrix indicated that the second quadrupole parameter had the least influence on the transmission loss, and the third quadrupole parameter had the largest influence on the transmission loss.

2) Theoretical analysis of the third quadrupole parameter showed that when $l_2/l = 1/2$, the odd number times passing frequency of $\frac{1}{2}$ of the wavelength could be eliminated; when $l_2/l = 1/4$, the odd number times passing frequency of the wavelength could be eliminated.
(3) The theoretical calculation results of the transmission loss of the side outlet muffler were in good agreement with the experimental results, indicating that the proposed method is suitable for determining the influence mechanism of the transmission loss of complex mufflers in future studies.

Data Availability

The data used to support the findings of this study are available from the corresponding author upon request.

Conflicts of Interest

The authors declare no conflicts of interest.

Authors’ Contributions

Liang Zhang did put forward the overall idea, contributed to the theoretical and experimental analysis, and wrote the manuscript. He-Mu Shi contributed to experimental analysis. Xiao-Hui Zeng and Zhuo Zhuang contributed to theoretical analysis.

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