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Research Article

Simplified Method for Consolidation Settlement Calculation of Combined Composite Foundation

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Treating weak saturated soil foundation by drainage powder jetting pile foundation can not only increase the bearing capacity of the foundation but also accelerate the drainage and consolidation process of the foundation. Having been commonly used in engineering, the plum blossom pile layout scheme is based on the foundation axisymmetric consolidation model (the powder jetting pile is the model center and the drainage board is located at the outer boundary of the model). It adopts reasonable boundary conditions and foundation seepage conditions and the method of pile-soil composite modulus and obtains the expression of the average excess pore water pressure of the composite foundation of the instantaneous loading of the drainage powder jetting pile under the simplified model. Therefore, the average consolidation degree of the foundation is acquired. The expression of the average consolidation degree can comprehensively reflect factors such as pile-soil modulus ratio, displacement ratio, drain spacing ratio, and those which affect the consolidation process of the foundation. The obtained analytical solution is of certain practical application significance for the consolidation settlement calculation of such engineering.

1. Introduction

In recent years, with the rapid development of engineering construction, the foundation treatment technology in China has also presented a rapid development era and has brought considerable economic benefits. However, engineering practice always ahead of theoretical research is a common problem facing the engineering field. For instance, drainage consolidation method and cement soil mixing method are the two methods most widely used in foundation treatment technology in China, but engineering practice clearly indicates that both have serious defects. The drainage consolidation method is mainly used in constructing the first high-speed railway in China. However, after two years of operation, the postconstruction settlement has greatly exceeded the standard [1, 2]. The quality of the deep construction is difficult to achieve and the treatment cost is extremely high when the cement soil mixing pile is used to treat deep soft soil. On the contrary, the use of a single cement soil pile to treat soft soil foundation is not enough to meet the needs of the current construction cycle. For the purpose of meeting the bearing capacity and deformation requirements during the construction and use period of the project, the combined composite foundation treatment technology (the combined treatment technology of vertical drainage body and cement soil mixing pile) has been widely used [3–5]. There are still many shortcomings in terms of theoretical calculation although this new combined composite foundation treatment technology can satisfy the requirements of foundation bearing capacity and accelerate the consolidation process of foundation soil.

The combined composite foundation differs from the conventional single solidified composite foundation, and they also have commonalities [6–8]. The combined composite foundation increases the vertical drainage channel of the composite foundation and improves the seepage path of the foundation water, compared with the traditional cement soil mixing pile composite foundation. Compared with the vertical drainage foundation, the existence of cement soil mixing pile can play the pile effect and make this kind of foundation have obvious characteristics of composite foundation. Lorenzo and Bergado [9] and Lorenzo and Bergado [10] reckoned that the excess pore water pressure of cement soil piles could not be assumed to be zero at any time during the consolidation process of cement soil pile composite foundation. On such basis, they established the governing equation of cement soil pile composite foundation. Zhang et al. [11] set up a consolidation differential equation for composite foundations under the assumption that the cement soil pile is an undrained pile regardless of the deformation of the cement soil pile. According to the initial and boundary conditions of consolidation process, the governing equation of foundation consolidation is solved. Yang and Li [3] deemed the composite foundation as a homogeneous composite material and established the consolidation equation of the undrained end-bearing pile composite foundation factoring the interaction between pile and soil.

Lu et al. [12] derived the consolidation control equation of the undrained pile composite foundation based on the undrained boundary conditions, according to the axisymmetric consolidation model and assuming that the pile is an undrained pile. It is theoretically proved that there is only vertical seepage in the undrained pile composite foundation and no radial seepage. The solution of Terzaghi's one-dimensional consolidation theory can be obtained by the degradation of the analytical solution.

Xing et al. [13] deduced the consolidation of cement soil pile composite foundation under the rigid foundation by considering the consolidation of cement soil piles and the relationship between the pore water displacement and the volume reduction of the unit body in the consolidation process of cement soil pile composite foundation. Consolidation analysis of the cement soil pile and the surrounding soil is carried out under the assumption of the strain of the pile and the soil. The consolidation degree of the foundation is solved by the method of weighted average of consolidation degree, and the feasibility of the analytical solution is verified by finite element analysis and examples. Chen et al. [14] studied the consolidation problem of the composite foundation of drainage powder jetting piles. Through the simplified analysis of the foundation model, the drainage pile is centered on the drainage board, and the powder injection pile is located at the outer boundary of the affected area unit. The dissipation law of the excess pore water pressure in the foundation is infiltrated from the periphery of the affected area to the center of the drainage board. Assuming that the powder jetting pile is impervious to water, the consolidation of the foundation will be the consolidation of the soil in the affected area of the entire unit. The consolidation analytical solution of the drainage powder pile foundation is obtained by the theory of axisymmetric consolidation. Ye et al. [15] investigated the consolidation of combined composite foundation of cement soil piles combined with plastic drainage boards. The pore water is modeled as being infiltrated from the center to the periphery. The model assumes that the pile is a permeable pile, but does not consider the well resistance of the vertical drainage body. To sum up, the foundation treatment method in China has developed from the traditional single foundation treatment method to the combined composite foundation treatment method [16, 17]. Because of the complex foundation model of composite foundation, the research on consolidation settlement characteristics is difficult, and the theoretical results are far behind practice [18, 19]. Therefore, it is of great significance to carry out the simplified study of consolidation settlement theory of composite foundation for improving the guiding role of engineering practice.

2. Establishment of Consolidation Control Equation

2.1. Calculation Model. Based on the research results of Buddhima et al. [20], the vertical drainage body is simplified in this chapter and the discontinuous sand pile on the boundary is simplified into a continuous circular drainage wall. In terms of use in the project, the powder jetting pile is mostly arranged with plum blossom piles. As shown in Figure 1, the contribution of one drainage board to the powder jetting pile at the shaded part is one-third of the cross-sectional area of the drainage board. A powder jetting pile is located at the boundary of the drainage interface of the drainage board, and the contribution of the powder jetting pile to the affected area of the drainage board is one-third of the cross-sectional area of the powder jetting pile. Therefore, the effect of the powder jetting pile in the affected area is still the cross-sectional area of a powder jetting pile. Within the scope of the affected area, the composite foundation model is simplified into a foundation model of a powder jetting pile, a pile surrounding soil, and a drainage board.

In order to be able to determine the consolidation analytical solution of the combined composite foundation of this type, the affected area of the hexagon is simplified into a circular influence area (as shown in Figure 2). The drainage board is located at the outer boundary of the affected area of the cement soil pile. The area of a drainage board is small compared to the area of the entire affected area when it is equivalent to the drainage wall, which is negligible, so the area of the soil in the affected area can be approximated to the area that the cement soil pile has been removed. The pore water in the composite foundation soil penetrates from the center to the periphery. It is assumed that the modulus variation in the affected zone is simplified to the pile-soil composite modulus in the consolidation analysis of this type of foundation, and the vertical seepage of the soil around the pile is considered in the affected area. The well resistance effect of the drainage board can be effectively considered according to the relationship between the amount of water discharged from the soil in the affected area and the amount

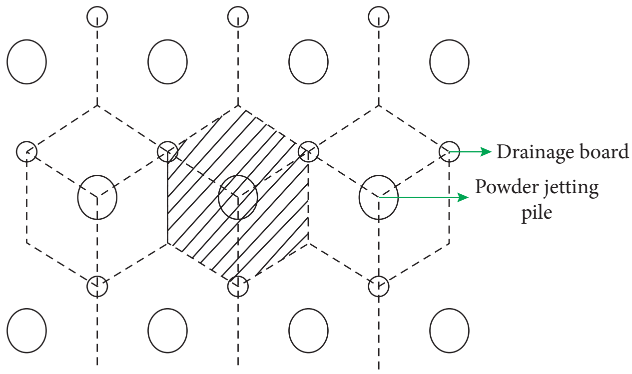

Figure 1: Drainage powder jetting pile floor plan.

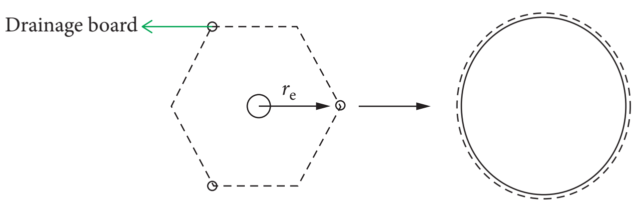

Figure 2: Simplified analysis model for the drainage system.

of water discharged from the drainage board in the affected area.

The model in the area affected by single pile is often used as the calculation model in the consolidation analysis of composite foundation. In this section, by referring to the research method of the consolidation model of composite foundation, the powder jetting pile-drainage board is divided into the same affected area for analysis. The upper boundary of the model is used for drainage, and the lower boundary is the impervious boundary. The pile reinforcement depth is H_0.

2.2. Establishment of Consolidation Control Equation. Axisymmetric consolidation equation:

$$\frac{1}{r}\frac{\partial}{\partial r}\left[\frac{k_h}{\gamma_w} r \frac{\partial u}{\partial r}\right] + \frac{k_v}{\gamma_w}\frac{\partial^2 \overline{u}}{\partial z^2} = -\frac{\partial \varepsilon_v}{\partial t}. \quad (1)$$

The relationship between the strain assumption of the drainage wall foundation, etc. and the radial average pore pressure of the drainage wall foundation:

$$\frac{\partial \varepsilon_v}{\partial t} = -\frac{1}{E_{sp}}\frac{\partial \overline{u}}{\partial t}. \quad (2)$$

In equations (1) and (2), $E_{\mathrm{sp}} = mE_p + (1-m)E_s$, $\overline{u}$ is the average excess pore water pressure of the drainage wall foundation at any depth; E_s is the compression modulus of the foundation soil, E_p is the compression modulus of the powder jetting pile, and E_{sp} is the pile-soil composite compression modulus; m is the displacement ratio of the powder jetting pile; ε_p is the vertical strain of the foundation; k_h is the radial permeability coefficient of the foundation soil; k_v is the vertical permeability coefficient of the foundation soil; γ_w is the unit weight of water.

Boundary conditions:

$$\begin{aligned} &r = 0\colon \frac{\partial u}{\partial r} = 0,\\ &r = r_e\colon u = \overline{u}_c,\\ &z = 0\colon u = 0,\\ &z = H_0\colon \frac{\partial u}{\partial z} = 0. \end{aligned} \quad (3)$$

It is assumed that the flow is equal, so

$$\left[2\pi r\mathrm{d}z \frac{k_h}{\gamma_w}\frac{\partial u}{\partial r}\right]\Big|_{r=r_e} = \pi r_c^2 \mathrm{d}z \frac{k_c}{\gamma_w}\frac{\partial^2 \overline{u}_c}{\partial z^2}. \quad (4)$$

By combining the above formulas, the consolidation control equation for the combined composite foundation of this type can be obtained:

$$B\frac{\partial^4 \overline{u}}{\partial z^4} + A\frac{\partial^3 \overline{u}}{\partial z^2 \partial t} + C\frac{\partial^2 \overline{u}}{\partial z^2} + \frac{\partial \overline{u}}{\partial t} = 0, \quad (5)$$

wherein

$$\begin{aligned} &A = -m'\frac{r_e^2 k_c}{8k_h};\ B = m'\frac{r_e^2 k_c}{8k_h}\frac{E_{\mathrm{sp}} k_v}{\gamma_w};\\ &C = -\left(m'\frac{E_{\mathrm{sp}} k_c}{\gamma_w} + \frac{E_{\mathrm{sp}} k_v}{\gamma_w}\right). \end{aligned} \quad (6)$$

The volumetric strain of the composite foundation at the initial moment is zero since the external load acting on the foundation is instantaneously applied, and the external load is entirely borne by the pore water in the foundation, namely,

$$t = 0,\ \overline{u}(z,0) = \sigma_0. \quad (7)$$

3. Solution of Average Pore Pressure of Composite Foundation

By using the separation variable method, let the solution of equation (5) be

$$\overline{u} = Z(z)T(t). \quad (8)$$

Substituting equation (8) into (5) gives that

$$-\frac{BZ^{(4)} + CZ''}{AZ'' + Z} = \frac{T'}{T} = -\beta, \quad (9)$$

wherein β is a number greater than zero.

Two homogeneous differential equations can be obtained from equation (9):

$$T' + \beta T = 0, \quad (10)$$

$$BZ^{(4)} + (C - \beta A)Z'' - \beta Z = 0. \quad (11)$$

The characteristic equation of equation (11) is

$$B\kappa^4 + (C - \beta A)\kappa^2 - \beta = 0. \tag{12}$$

The solution of the above formula can be expressed as follows:

$$\kappa^2 = \frac{(-C + \beta A) \pm \sqrt{(-C + \beta A)^2 + 4B\beta}}{2B}. \tag{13}$$

It can be known from the theory of ordinary differential equations that the general solution of equation (12) is

$$\overline{u} = \sum_{m=1}^{\infty} A_m \begin{bmatrix} a_m \sin(\lambda_m z) + b_m \cos(\lambda_m z) + \\ c_m \sinh(\mu_m z) + d_m \cosh(\mu_m z) \end{bmatrix} e^{-\beta_m t}, \tag{14}$$

wherein

$$\lambda_m = \sqrt{\frac{-(-C + \beta_m A) + \sqrt{(-C + \beta_m A)^2 + 4B\beta_m}}{2B}}, \tag{15}$$

$$\mu_m = \sqrt{\frac{-(-C + \beta_m A) + \sqrt{(-C + \beta_m A)^2 + 4B\beta_m}}{2B}}. \tag{16}$$

The matrix equations for a_m, b_m, c_m, and d_m can be obtained from the boundary conditions, and is given that

$$b_m = c_m = d_m = 0, \tag{17}$$

$$a_m \lambda_m \cos(\lambda_m H_0) = 0. \tag{18}$$

From equation (17), we know that $a_m \neq 0$ and $\lambda_m \neq 0$, so the above equation (18) can be obtained:

$$\lambda_m = \frac{M}{H_0}, \tag{19}$$

wherein $M = ((2m-1)/2)\pi$, $(m = 1, 2, 3, \ldots)$.

It gives that

$$\beta_m = \frac{B(M/H_0)^4 - C(M/H_0)^2}{1 - A(M/H_0)^2}. \tag{20}$$

Substituting A, B, and C expressions into the above formula give that

$$\beta_m = \frac{E_{sp}k_v}{\gamma_w}\left(\frac{M}{H_0}\right)^2 + \frac{E_{sp}k_h}{\gamma_w}\frac{8}{d_e^2(1/4 + D)}. \tag{21}$$

In short, the average excess pore water pressure at any depth of the composite foundation can be obtained:

$$\overline{u} = \sum_{m=1}^{\infty} A_m a_m \sin\left(\frac{M}{H_0} z\right) e^{-\beta_m t}. \tag{22}$$

Then, by substituting formula (7) into the above formula, the expression of the average excess pore pressure in the composite foundation can be obtained as follows:

$$\sum_{m=1}^{\infty} (A_m a_m) \sin\left(\frac{M}{H_0} z\right) = \sigma_0. \tag{23}$$

By using the orthogonality of the trigonometric function on both sides of the above formula, we can obtain

$$\overline{u} = \frac{2\sigma_0}{M} \sum_{m=1}^{\infty} e^{-\beta_m t} \sin\left(\frac{M}{H_0} z\right). \tag{24}$$

4. Solution of Average Consolidation of Composite Foundation

Under the load, the ground consolidation degree calculated by foundation deformation is used. The ratio of the consolidation settlement to the final settlement over time t becomes the consolidation degree, i.e.,

$$U = \frac{s_t}{s_\infty}, \tag{25}$$

wherein s_t is the amount of settlement at the time t of the foundation and s_∞ is the total settlement of the foundation, which can be calculated by referring to the stratified sum method.

Calculate the average consolidation degree under the instantaneous load of the foundation using the average pore pressure of the foundation, then substitute equation (24) into the following formula:

$$U_p(t) = \frac{\int_0^{H_0} (\sigma - \overline{u}) dz}{\int_0^{H_0} \sigma_0 dz}, \tag{26}$$

$$U_p(t) = 1 - \frac{\int_0^{H_0} (2/H_0) \sum_{m=1}^{\infty} e^{-\beta_m t} \sin(M/Hz) dz}{\int_0^{H_0} \sigma_0 dz}. \tag{27}$$

By using the principle of orthogonality of the trigonometric function, the expression of the average consolidation degree of the consolidated layer of the composite foundation can be obtained, namely,

$$U_p(t) = 1 - \sum_{m=1}^{\infty} \frac{2}{M^2} e^{-\beta_m t}, \tag{28}$$

wherein $D' = 8G'/M^2$; $G' = (k_h/k_c)(H_0/d_c)^2$ is a well-resistance factor; $M = ((2m-1)/2)\pi$, $m = 1, 2, 3, \ldots\ldots, d_c$ is the diameter of the vertical drainage body; d_e is the diameter of the affected area. $(E_{sp}k_v/\gamma_w)(M/H_0)^2$ in β_m reflects the vertical consolidation of the foundation. $((E_{sp}k_h)/\gamma_w)(8/d_e^2(1/4 + D'))$ reflects the radial consolidation of the foundation.

Equation (28) also shows the dissipation of pore water pressure and the process of effective stress growth, which is the solution to the average consolidation degree of the foundation under the simplified model. This formula can factor the role of cement soil piles or the well resistance of the vertical drainage body.

5. Analysis of Consolidation Properties of Composite Foundation

Analytical solution for the average consolidation degree of combined composite foundation under transient loading is obtained in this paper, based on the cement soil pile as the center and the axisymmetric consolidation model of the drainage board at the outer boundary of the affected area and on the axisymmetric consolidation equation of the soil. The analytical solution can comprehensively reflect factors affecting the consolidation rate of the combined composite foundation of the type. In order to reflect the influence of various factors on the consolidation rate of the foundation, the consolidation factor β_m is first made dimensionless. Let

$$\beta_m t = \tau_m T_h, \tag{29}$$

wherein T_h is a time factor, which is a dimensionless number; let

$$T_h = \frac{c_h t}{4r_e^2}. \tag{30}$$

Let c_h of F be the horizontal consolidation coefficient of the soil, $c_h = E_s k_h/\gamma_w$, so

$$\tau_m = \frac{\beta_m t}{T_h} = \left(m\frac{E_p}{E_s} + 1 - m\right)\left[\frac{k_v}{k_h}\left(\frac{d_e M}{H_0}\right)^2 + \frac{32}{(1+4D')}\right]. \tag{31}$$

Among them, $a = E_p/E_s$ is the pile-soil stress ratio.

As shown in Figure 3, when the ratio of the radius of the affected area of the composite foundation to the radius of the sand pile in the affected area is constant (drain spacing ratio), at the same moment, the greater the ratio of the radius of the affected area and the composite foundation of the composite pile to the drainage board radius, the smaller the average consolidation degree of the composite foundation.

Figure 4 reflects the influence of the drainage board permeability coefficient on the consolidation of the foundation. As shown in Figure 4, as the drainage coefficient of the drainage board decreases, the consolidation rate of the composite foundation gradually becomes slower. It is because that the drainage board acts as the main vertical drainage channel of the weak soil foundation when the composite foundation increases with the external load and the average consolidation degree of the foundation increases. When the permeability coefficient is reduced, the well-resistance effect of the pore water flowing out of the vertical drainage body in the soil in the affected area is more significant. Therefore, it takes a longer period that the water in the vertical drainage body flows out of its pores. Furthermore, a sand cushion layer should be laid on the top surface of the composite foundation in the composite foundation drainage design, to ensure that the water flow discharged by each drainage board (vertical drainage body) forms a connected drainage surface on the surface of the composite foundation.

As shown in Figure 5, under the instantaneous loading, the displacement ratio of the cement soil pile changes, affecting the consolidation rate of the composite pile composite foundation. As shown in Figure 5, the consolidation speed of the composite foundation of the composite pile is accelerated as the displacement ratio of the cement soil pile increases. Increasing the diameter of the cement soil piles is equivalent to reducing the radial drainage distance of the pore water in the foundation soil when the spacing of the cement soil piles is constant. Therefore, increasing the area displacement ratio of the cement soil piles can speed up the consolidation rate of the combined pile composite foundation.

The influence of the pile-soil modulus ratio on the consolidation rate of composite piles under transient loading is reflected in Figure 6. With the increase of the cement soil pile-soil modulus ratio in composite foundation, the

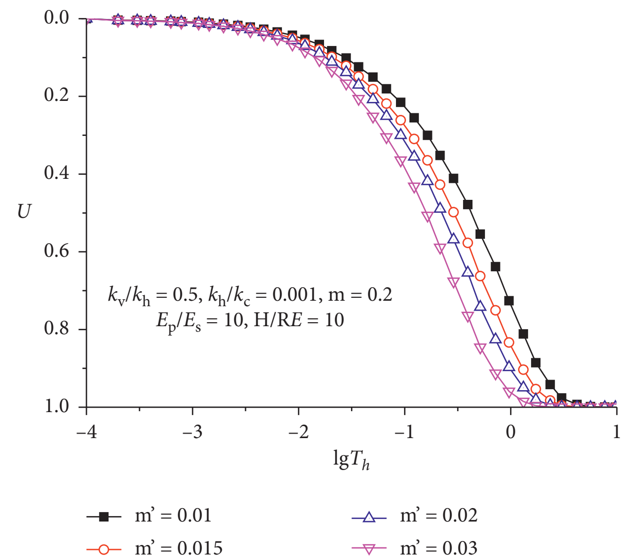

FIGURE 3: Influence of displacement ratio of drainage board on foundation consolidation rate.

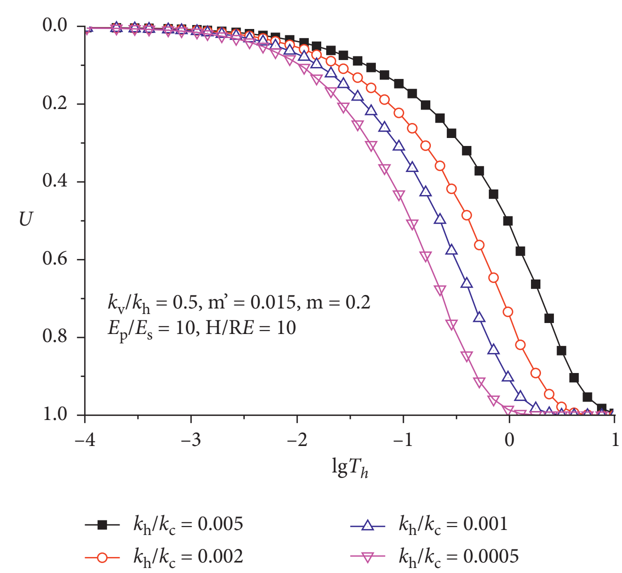

FIGURE 4: Influence of permeability coefficient of drainage board on foundation consolidation rate.

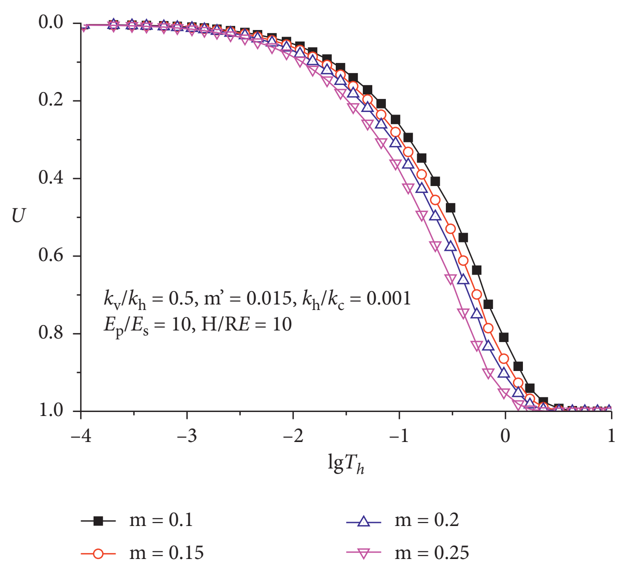

Figure 5: Influence of displacement ratio of cement soil pile on foundation consolidation rate.

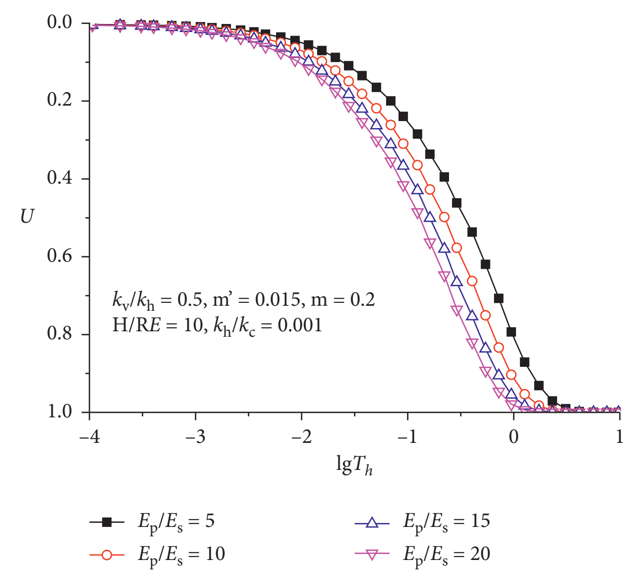

Figure 6: Influence of pile-soil modulus ratio on consolidation rate of foundation.

consolidation speed of the composite pile composite foundation accelerates. The effective stress in the soil around the undrained piles will gradually increase with the consolidation of the composite foundation of composite piles and the gradual dissipation of pore water pressure in the soil around the undrained piles, when the compression modulus of cement soil piles becomes larger. It can be seen from the equal strain relationship that the stress on the cement soil pile is increasing. The pile-soil together bears the upper load in the composite foundation design, but the engineering cannot simply rely on increasing the compression modulus of the cement soil pile to accelerate the consolidation rate of the composite pile composite foundation. The cement soil pile and the surrounding soil in the whole foundation will not be consistent in the vertical strain, and the isostrain relationship will not be established when the pile-soil modulus ratio is large and the composite foundation is under load. Therefore, the selection of the compression modulus of the cement soil pile should be controlled according to the actual conditions of the project.

The comparison between the solution and the Chen Lei solution is reflected in Figure 7. It can be known from the figure that the solution of the average consolidation degree of the foundation at the same time is basically identical under different conditions. There are similarities between the consolidation factor and the foundation consolidation factor in Chen Lei's solution in the average consolidation degree of the composite foundation obtained in this paper. The difference lies in the radial consolidation of the foundation, mainly due to the difference in the calculation model. The same point lies in the vertical consolidation process of the foundation and the effect of the powder jetting pile on the consolidation rate of the combined composite foundation can be extracted into the form of $(1 + m \cdot (E_p/(E_s - 1)))$. This formula can reflect the displacement ratio of the powder jet pile and the influence of the compression modulus of the powder jet pile on the consolidation rate of the foundation.

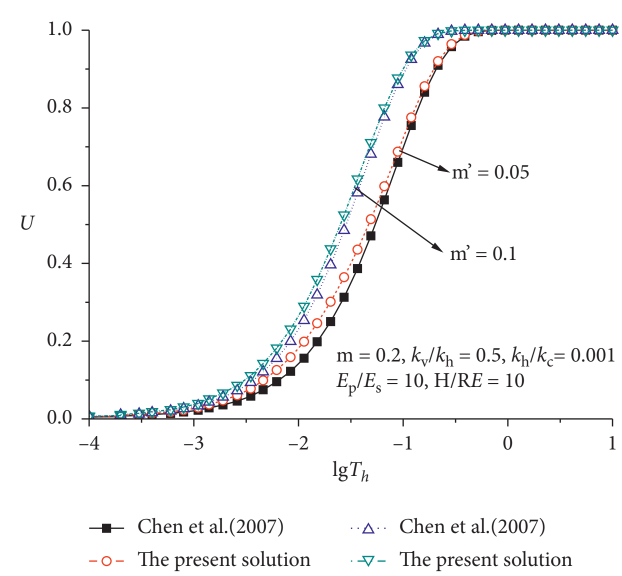

Figure 7: Comparative analysis of consolidation rate of combined composite foundation.

Therefore, it can be seen through the above analysis that the consolidation analytical solution of the drainage powder jetting pile under the simplified conditions can reflect the consolidation process of the foundation to a certain extent, which has certain practical significance for guiding engineering practice.

6. Conclusion

The simplified consolidation analytical solution of the combined composite foundation is deduced by simplifying the consolidation model of the composite foundation of drainage powder jetting piles, based on the axisymmetric consolidation. The factors affecting the consolidation rate of the foundation are analyzed and the following conclusions are reached:

(1) Increasing the cement soil pile displacement ratio and compression modulus can accelerate the consolidation process of the composite foundation, which is consistent with other research results.

(2) Reducing the spacing of the piles can accelerate the consolidation rate of the foundation.

(3) The influence on the consolidation rate of the foundation is slowed down by reducing the permeability coefficient of the drainage board, and its influence on the consolidation rate of the foundation is greater than other factors. Therefore, the well resistance of the drainage board cannot be ignored.

(4) There is certain practical significance of the consolidation analytical solution of the composite foundation of drainage powder jetting piles in this paper for guiding engineering practice.

Data Availability

The data used to support the findings of this study are available from the corresponding author upon request.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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