Research Article

Research on Active Control of Rotating Motion and Vibration of Flexible Manipulator

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In this study, PZT (piezoelectric) actuators and PD control (PDs' command line tool) method is selected to control the vibration of the flexible manipulator. The dynamic equations of the flexible manipulator system are established based on Lagrange principle. The control strategy of PZT actuators and joint control torque are designed. It is investigated by a Lyapunov approach that a combined scheme of PD feedback and command voltages applies to segmented PZT actuators. By comparison, only PD feedback control is also considered to control the flexible manipulator. The numerical simulations prove that the method of the designed PZT actuators' control strategy and PD control is effective to compress the vibration of the flexible manipulator.

1. Introduction

Flexible components are widely used in modern engineering with the rapid development of science and technology, and the resulting vibration control problems have attracted widespread attention. For example, in the manufacturing industry, a flexible system is formed when a robotic arm is running at high speed, and its positioning accuracy is restricted by the small deformation elastic vibration of the robotic arm [1]. The flexible manipulator is a typical rigid-flexible coupling dynamics system. The research on the active control of the flexible manipulator is developed in parallel with its modeling theory. Various control methods in the field of automatic control are used in the flexible manipulator. Active control research, such as PD control, optimal control, variable structure control, and adaptive control, as well as robust control, intelligent control methods, etc. [2–4]. Shafei et al. [5, 6] research the study of a general formulation and numerical solution for the dynamic load-carrying capacity of a mechanical manipulator with elastic links. They obtain the dynamic equations of motion for the system by means of the Gibbs–Appell formulation

and by applying the assumed modes method. The elastic characteristics of the members have been modeled based on the Timoshenko beam theory and its associated mode shapes. And they propose the approach that generates the optimal trajectory of the flexible mobile manipulators in point-to-point motion based on an open-loop optimal control. Considering the application of smart materials such as piezoelectric ceramics, magnetorheological fluids, and shape memory alloys in rigid-flexible coupling dynamic systems, people have also studied smart structural systems for active control of flexible manipulators [7–10].

In this study, a comprehensive design of the manipulator joint torque and piezoelectric actuator control law is carried out. The control method adopts the PD control method. To ensure that the system reaches the predetermined position within a limited time, the piezoelectric actuator is used to ensure the suppression of the elastic vibration of the flexible mechanical arm. First, it describes the application of a piezoelectric smart material in the vibration control of a single-rod flexible manipulator. The Lyapunov stability theory is used to derive suitable PD feedback control and PZT actuator control. The control law consists of two parts.

The first part is the PD feedback control of the joint angle to control the rotation of the entire manipulator. Although the open-loop control system has simple structure and low cost, it has low control accuracy and is vulnerable to external interference. Once the output error occurs, it cannot be compensated. Therefore, PD feedback control is selected in this paper. The second part reduces the vibration of the manipulator by applying voltage to the actuator. The PZT actuator is fixed on the surface of the flexible mechanical arm, and the PZT actuator is controlled by an external voltage. This material (the material is lead zirconate titanate) can generate a shear force to prevent the deformation of the structure, which can effectively control the movement of the manipulator and well suppress the vibration of the flexible manipulator. Finally, the effectiveness of the given control method is verified by numerical simulation.

2. Principle of Active Vibration Control of the Flexible Manipulator Arm with the Piezoelectric Actuator

The PZT piezoelectric actuator can be used to control the vibration of the flexible mechanical arm. The basic principle is that piezoelectric materials are used as sensors and actuators for controlled structures. The sensors sense structural deformation caused by vibration and convert them into electrical signals. A certain control law is used to generate a control signal, which is amplified and then applied to the actuator, and the active control of the mechanical structure vibration is realized through the actuator [11].

The control principle of a flexible manipulator with a PZT piezoelectric actuator is shown in Figure 1. When the drive motor acts on the flexible manipulator to produce a large range of motion, it will cause the flexible manipulator to vibrate. At this time, the bending vibration signal is obtained by the resistance strain sensor pasted on it, filtered and amplified by the dynamic strain gauge, and then inputted to the industrial computer through data collection and conversion. The control algorithm is implemented on the industrial computer, and the control signal is converted into a voltage signal output by D/A to drive the PZT piezoelectric actuator to generate the corresponding bending moment to offset the vibration of the flexible manipulator.

3. Dynamic Equation of the Flexible Manipulator with the Piezoelectric Actuator

Take a single-rod flexible mechanical arm as an example. As shown in Figure 2, one end of the single-rod flexible mechanical arm is fixedly connected to the central rigid body. Suppose x_0oy_0 is the fixed coordinate system and oxy is the reference coordinate system fixedly connected to the flexible manipulator. The joint control moment is applied at the central rigid body to make the flexible manipulator rotate in the horizontal plane. Control the rotation of the flexible arm. L is the length of the flexible manipulator, h is the section height, b is the section width, θ is the joint rotation angle of the flexible manipulator, and m_L is the additional mass at the end of the flexible manipulator.
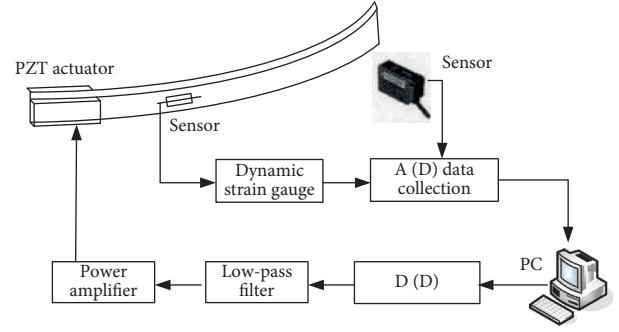

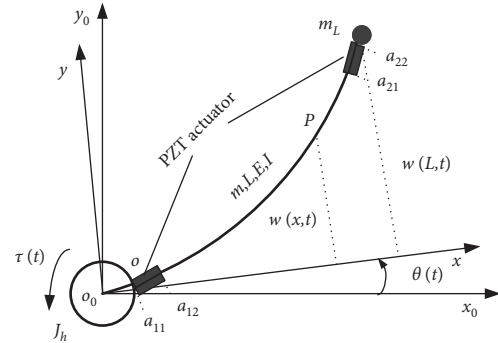Figure 1: Vibration control principle of the PZT actuator.

Figure 2: Model of the flexible manipulator with PZT actuators.

Four pieces of PZT piezoelectric actuators of the same specification are pasted on the surface of the robotic arm, which are used to control the elastic vibration generated during a large range of motion. The positions relative to the robotic arm are a_{i1} and a_{i2}, $i = 1, 2$. In the dynamic modeling of the robotic arm, the following assumptions are made:

(1) The flexible mechanical arm is Euler–Bernoulli beam, and the axial deformation of the beam is ignored

(2) The thickness of each piezoelectric actuator is unchanged, and the width is consistent with the width of the beam

(3) Ignore the influence of gravity and the central rigid body

Let p be any point on the flexible manipulator, and its coordinate is (p_x, p_y). The deformation at this point is $w(x, t)$. According to the continuum vibration theory, $w(x, t) = \phi(x)q(t)$ can be set. The kinetic energy of the robotic arm system considering the piezoelectric actuator is

$$T = \frac{1}{2} \int_0^L \rho_1 A_1 \left(\dot{p}_x^2 + \dot{p}_y^2 \right) dx + \frac{1}{2} \sum_{i=1}^n \int_{a_{i1}}^{a_{i2}} \rho_2 A_2 \left(\dot{p}_x^2 + \dot{p}_y^2 \right) dx$$

$$+ \frac{1}{2} m_L \left(\dot{p}_x^2 + \dot{p}_y^2 \right)|_{x=L},$$

(1)

where ρ_1 and A_1 are the density and cross-sectional area of the robotic arm, ρ_2 and A_2 are the density and cross-sectional area of the PZT piezoelectric actuator, and $\dot{p}_x$ and $\dot{p}_y$

are the components of the velocity at point P in the x and y directions, respectively.

The total potential energy considering the elastic deformation of the mechanical arm and the deformation of the PZT actuator is

$$V = \frac{1}{2}\int_0^L E_1 I_1 \left(w''(x,t)\right)^2 dx + \frac{1}{2}\sum_{i=1}^n \int_{a_{i1}}^{a_{i2}} E_2 I_2 \left(w''(x,t)\right)^2 dx, \tag{2}$$

where E_1 and E_2 are the elastic modulus of the mechanical arm and the piezoelectric actuator, respectively, and I_1 and I_2 are the section moments of inertia of the mechanical arm and the piezoelectric actuator, respectively.

The virtual work caused by the PZT actuator and the joint input torque $u(t)$ is

$$\delta W_{\theta q} = u(t)\delta\theta + \sum_{i=1}^n \int_{a_{i1}}^{a_{i2}} \tau_i(x,t)bh\left(\delta\theta + \phi'(x)\delta q\right)dx, \tag{3}$$

where $\tau_i(x,t)$ is the distributed shear stress generated by the PZT actuator. The virtual work of the system generated by the PZT actuator can be simplified into the following form [5]:

$$\delta W_{\theta q} = u(t)\delta\theta + \sum_{i=1}^n cU_i(t)\delta\theta + \sum_{i=1}^n cU_i(t)\tilde\varphi'(i)\delta\dot q, \tag{4}$$

where $cU_i(t)$ represents the bending moment produced by the ith PZT Actuator, c is a constant with a positive value, $U_i(t)$ is the control voltage applied to the ith PZT Actuator, and $\tilde\varphi\prime(i) = \phi\prime(L) - \phi(0)$. From equation (4), we get the generalized force Q_θ about θ, and the generalized force about q is Q_q, and $Q_q = u(t) + \sum_{i=1}^n cU_i(t)$ and $Q_\theta = \sum_{i=1}^n cU_i(t)$.

Applying the Lagrange principle, the dynamic equation of the entire system is

$$\underbrace{\begin{bmatrix} m_{\theta\theta} & m_{\theta q} \\ m_{\theta q}^T & m_{qq} \end{bmatrix}}_{M(q)} \begin{bmatrix} \ddot\theta \\ \ddot q \end{bmatrix} + \underbrace{\begin{bmatrix} \dot q^T m_{qq} q & \dot\theta q^T m_{qq} \\ -\dot\theta q^T m_{qq} & 0 \end{bmatrix}}_{C(\dot\theta, q, \dot q)} \begin{bmatrix} \dot\theta \\ \dot q \end{bmatrix} + \begin{bmatrix} 0 \\ K_q q \end{bmatrix} = \begin{bmatrix} Q_\theta \\ Q_q \end{bmatrix}, \tag{5}$$

where $m_{\theta\theta} = (1/3)\rho_1 A_1 + (1/3)\sum_{i=1}^n \rho_2 A_2 (a_{i2}^3 - a_{i1}^3) + q^T m_{qq} q + (1/2)m_L L^2$, $m_{\theta q} = \int_0^L \rho_1 A_1 x\phi(x)dx + \sum_{i=1}^n \int_{a_{i1}}^{a_{i2}} \rho_2 A_2 x\phi(x)dx$, $m_{qq} = \int_0^L \rho_1 A_1 \phi^2(x)dx + \sum_{i=1}^n \int_{a_{i1}}^{a_{i2}} \rho_2 A_2 \phi^2(x)dx + (1/2)m_L\phi^2(L)$, and $K_q = \int_0^L E_b I_b (w''(x,t))^2 dx + \sum_{i=1}^n \int_{a_{i1}}^{a_{i2}} E_p I_p (w''(x,t))^2 dx$.

4. Active Control of Rotation and Vibration of the Flexible Manipulator

For the flexible manipulator system with PZT piezoelectric actuator shown in formula (5), it is necessary to design a reasonable input torque $u(t)$ and PZT actuator control voltage $U_i(t)$ in order to realize the joint control of its rotation and vibration of the manipulator. First, we design

the input torque $u(t)$ in the following form, which considers the joint torque control item (PD control is used here) and the control voltage $U_i(t)$ part of the piezoelectric actuator:

$$u(t) = -k_p\Delta\theta - k_v\Delta\dot\theta - \sum_{i=1}^n cU_i(t), \tag{6}$$

where k_p is the proportional gain, k_v is the integral gain, $\Delta\theta$ and $\Delta\dot\theta$ are the joint angle and angular velocity errors, respectively, and c is the piezoelectric actuator control constant.

Applying a suitable voltage $U_i(t)$ on the piezoelectric actuator,

$$\begin{aligned} U_i(t) &= \tilde\varphi'(i)\dot q \\ &= \dot w'(a_{i2}, t) - \dot w\prime(a_{i1}, t). \end{aligned} \tag{7}$$

For the angular velocities $\dot w\prime(a_{i2}, t)$ and $\dot w\prime(a_{i1}, t)$ in the control voltage expression of formula (7), it is difficult to measure them in practical applications if they are used as feedback quantities, while the linear velocities $\dot w(a_{i2}, t)$ and $\dot w(a_{i1}, t)$ are easy to obtain; that is, by comparing the accelerometers, we integrate the acceleration amount to get the linear velocity. Therefore, a linear velocity feedback control method is used here, and the control voltage is redesigned as follows:

$$\begin{aligned} U_i(t) &= -k_i\tilde\varphi(i)\dot q \\ &= -k_i\left(\dot w(a_{i2}, t) - \dot w(a_{i1}, t)\right). \end{aligned} \tag{8}$$

Thereinto, $\dot w(a_{i2}, t)$ and $\dot w(a_{i1}, t)$ represent the linear velocity at points a_{i2} and a_{i1}, respectively.

5. Numerical Simulation

First, we only consider the simulation analysis of the vibration control of the flexible manipulator system controlled by PD. In contrast, we consider the flexible robotic arm system, as shown in Figure 3, where a pair of PZT piezoelectric actuators are attached to the root and tail surfaces, each of which is $0.1L$ ($L = 1$ m) in length. Suppose the PZT actuator located at the root of the flexible manipulator arm has a control voltage of $U_1(t)$, and the actuator at the root has a control voltage of $U_2(t)$. The material and structural parameters of the flexible manipulator system are shown in Table 1.

Suppose the ideal position of the flexible manipulator joint is $\theta = 0.1$, the initial condition is $y = \begin{bmatrix} 0 & 0 & 0 & 0 \end{bmatrix}^T$, and the controller parameters are $k_p = k_v = 1$ and $k_1 = k_2 = 1$.

5.1. Simulation Analysis of Vibration Control of the Flexible Manipulator System considering Only PD Control. Considering the situation of PD control, the ideal position of the flexible manipulator joint is $\theta = 0.1$, the initial condition is $y = \begin{bmatrix} 0 & 0 & 0 & 0 \end{bmatrix}^T$, and the controller parameters are $k_p = 100$ (Nm/rad) and $k_v = 100$ (Nms/rad).

The simulation results are shown in Figures 4–7. Figure 4 shows the joint angular displacement curve of the flexible manipulator arm. At $t = 5$ s, the joint angle of the flexible
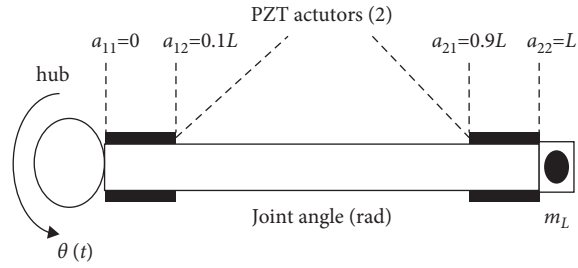
Figure 3: The flexible manipulator system with four PZT actuators.

Table 1: System parameters of the flexible manipulator.

Flexible manipulator	PZT actuator
Material: aluminum	Material: lead zirconate titanate
Elastic modulus: $E_b = 7.6 \times 10^9$ Nm	Elastic modulus: $E_p = 6.3 \times 10^{10}$ Nm
Length: $L = 1$ m	Length: $a_2 - a_1 = 0.1$ m
Thickness: $h_b = 4 \times 10^{-3}$	Thickness: $h_p = 0.75 \times 10^{-3}$
Width: $b = 0.05$ m	Width: $b = 0.05$ m
Density: $\rho_b = 2840$ kg/m^3	Density: $\rho_p = 7600$ kg/m^3
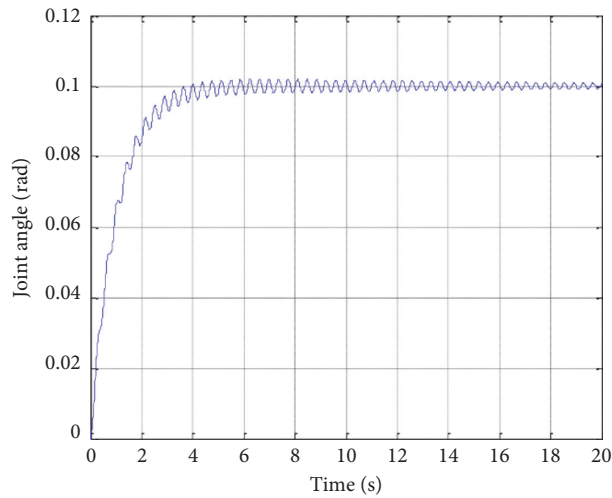
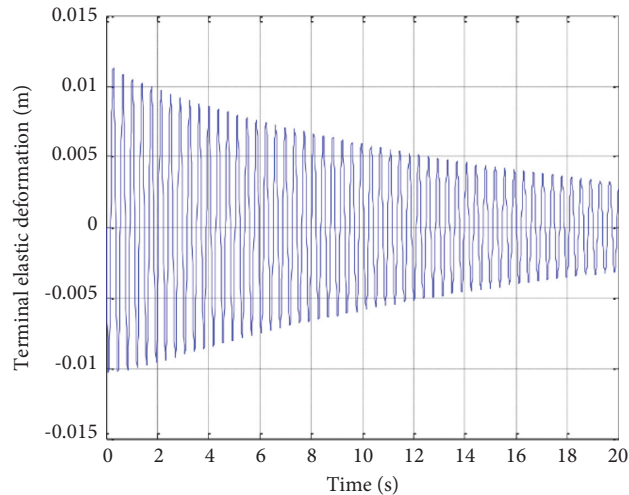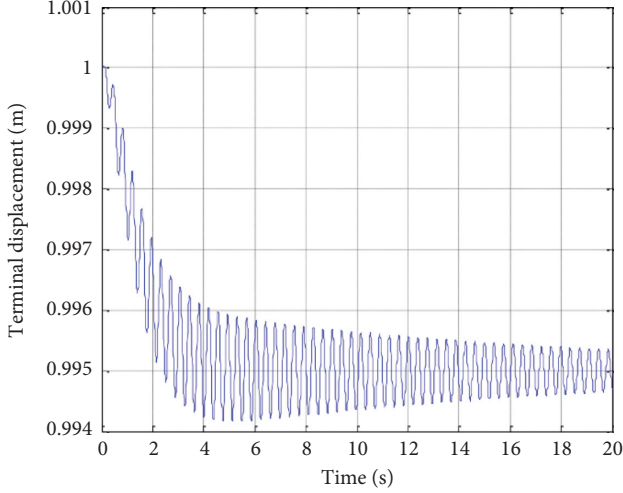Figure 4: Joint angular displacement of the flexible manipulator with PD control.
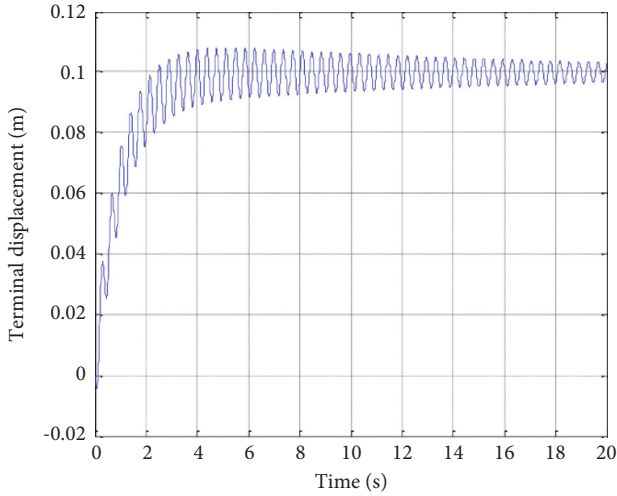

Figure 5: Tip deflection of the flexible manipulator with PD control.

FIGURE 6: Tip displacement p_x with PD control.

FIGURE 7: Tip displacement p_y with PD control.

manipulator arm reaches the specified position, but during the movement of 0–5 s, the joint has maintained a large vibration. After 5 s, the joint stays at $\theta = 0.1$ and carried out a small vibration here, showing a decay trend. Figure 5 shows the end vibration of the flexible manipulator. It can be seen that the end vibration has been vibrating continuously during the 20 s of the simulation, and the amplitude of the vibration is very large. The maximum amplitude is ±0.01 m, but it increases with time. Due to the effect of damping, the vibration is constantly attenuated. Figures 6 and 7 show the displacement of the end of the flexible manipulator in the x and y directions. It can be seen that their vibration conditions are not well controlled; especially, when the target position is about to be reached, the amplitude becomes significantly larger. During $t = 20$ s simulation time, the end of the flexible robotic arm will continue to vibrate.

Therefore, only PD controls the vibration of the flexible manipulator system. Although the movement of the flexible manipulator arm can also reach the ideal position, it takes a long time to reach the specified position. More importantly,

the movement is always accompanied by large values. Vibration is contrary to the requirements of high response, positioning accuracy, and vibration suppression of the flexible manipulator system. Therefore, while using PD control, other methods of vibration control must be supplemented to enable the flexible manipulator to achieve an ideal vibration control effect, in order to comprehensively study the PD and PZT actuators to control the vibration of the flexible manipulator.

5.2. Simulation Analysis of Vibration Control of the Flexible Manipulator System Based on PD + PZT Actuator Control. It can be seen from the foregoing that, for a flexible manipulator system, a single PD control is difficult to effectively control its vibration; especially, when working in a space environment, vibration will have a great impact on the work of the flexible manipulator. Therefore, this section will focus on the application of PZT actuators in the vibration control of flexible manipulators. The vibration control of the flexible manipulator system includes the control of its joints and distributed piezoelectric actuators, designing a reasonable system control law and then establishing the modal shape function of the flexible manipulator.

5.2.1. Controller Design. Similarly, for equation (8), in order to ensure $\dot{V} \leq 0$, reasonable input torques $u(t)$ and $U_i(t)$ need to be designed. First, we design the input torque $u(t)$, including PD control and PZT actuator control, and the expression is as follows:

$$u(t) == -k_p \Delta\theta - k_v \dot{\theta} - \sum_{i=1}^{n} cU_i(t), \qquad (9)$$

where k_p is the proportional gain, k_v is the integral gain, and the last term on the right side of equation (9) is to offset the influence of the PZT actuator control on the rigid body motion of the flexible manipulator.

Substituting equation (9) into (8), we can obtain

$$\dot{V} = -k_v \dot{\theta}^2 + \sum_{i=1}^{n} cU_i(t)(\tilde{\varphi}'(i)\dot{q}), \qquad (10)$$

where $\phi'(i)\dot{q}$ is the angular velocity difference of the ith PZT actuator, denoted as

$$\tilde{\phi}'(i)\dot{q} = \dot{w}\prime(a_{i2}, t) - \dot{w}'(a_{i1}, t). \qquad (11)$$

In order to ensure that $\dot{V}$ is less than zero, a suitable voltage $U_i(t)$ must be applied to the PZT actuator such that

$$\sum_{i=1}^{n} cU_i(t)\left(\tilde{\varphi}'(i)\dot{q}\right) \leq 0. \qquad (12)$$

Obviously, the expression of $U_i(t)$ can be designed as

$$\begin{aligned} U_i(t) &= -k_i \tilde{\varphi}'(i)\dot{q} \\ &= -k_i\left(\dot{w}'(a_{i2}, t) - \dot{w}'(a_{i1}, t)\right). \end{aligned} \qquad (13)$$

Substituting the above formula into formula (10), then

$$\dot{V} = -k_v \dot{\theta}^2 - \sum_{i=1}^{n} c k_i \left(\widetilde{\phi}'(i)\dot{q} \right)^2 \le 0. \tag{14}$$

Therefore, the above control law can ensure the stability of the system.

5.2.2. Simulation Results. In order to verify the above control, consider the flexible manipulator system shown in Figure 3. Similarly, paste a pair of PZT piezoelectric actuators on the root and tail surfaces, each with a length of $0.1L$ ($L = 1$ m). The control voltage of the actuator at the root of the flexible manipulator is $U_1(t)$, and the control voltage of the actuator at the root is $U_2(t)$. The system parameters of the flexible manipulator are the same as before.

As the flexible manipulator system is considered, vibration will inevitably occur during the movement. Therefore, in addition to complete the ideal rigid body movement, it is necessary to ensure that the vibration of the flexible manipulator can be suppressed as much as possible. The ideal position of the flexible manipulator joint is $\theta = 0.1$, the initial condition of the arm joint is $y = \begin{bmatrix} 0 & 0 & 0 & 0 \end{bmatrix}^T$, and the parameters of the controller are controlled at $k_p = 100$ (Nm/rad), $k_v = 100$ (Nm/rad), and $k_1 = k_2 = 5000$ (Vs/rad).

The simulation results of the flexible manipulator system are shown in Figures 8–13. Figures 8 and 9 show that the joint angular displacement and end deformation of the flexible manipulator can be well controlled. It can be seen from Figure 8 that, under the control of the PZT actuator, the joint did not vibrate significantly when it reached the designated position. After reaching the designated position, that is, after $t = 5$ s, the joint of the flexible manipulator remained stable at the joint angle $\theta = 0.1$ and the residual vibration of the flexible mechanical arm is effectively suppressed. It can be seen from Figure 9 that the end of the flexible manipulator is damped and vibrated in time $t < 10$ s. Due to the added control law, after 10 s, it maintains a stable state and no longer vibrates. Figure 10 shows the input torque $u(t)$ of the flexible manipulator system. Figure 11 shows the input piezoelectric control voltages $V_1(t)$ and $V_2(t)$, respectively. Figures 12 and 13 show the displacement of the end of the flexible manipulator in the x and y directions. It can be seen that, at $t = 8$ s, the vibration of the end of the flexible manipulator has approached zero, indicating a good control effect.

The previous simulation studies have shown that the flexible manipulator system controlled by PD and PZT actuators and the flexible manipulator system controlled only by PD can realize the joint movement of the flexible manipulator in terms of large-scale motion and vibration control. Suppress residual vibration, but the difference between the two situations is also obvious, mainly in the following two aspects:

(1) Although the joint angles under the control of the two controllers reach the designated position at about $t = 5$ s, the flexible manipulator system controlled only by PD has been accompanied by small
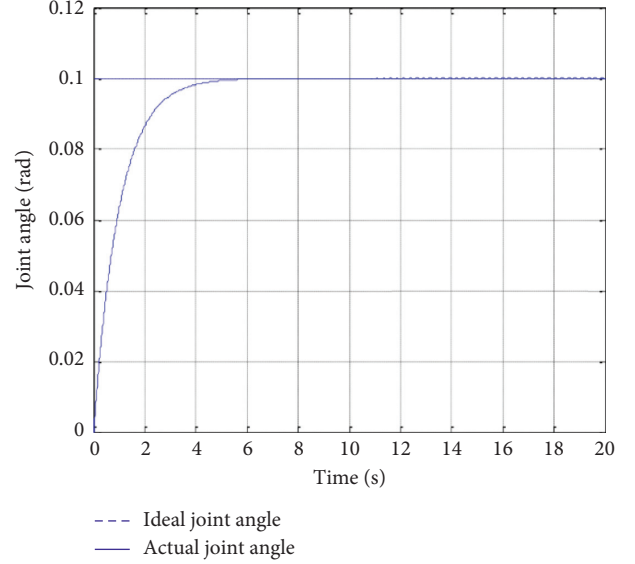

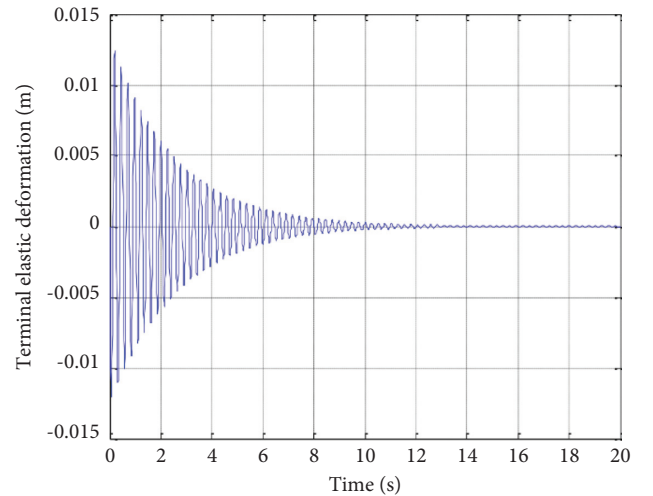FIGURE 8: Joint angular displacement of the flexible manipulator.

FIGURE 9: Tip deflection of the flexible manipulator.

vibrations. After reaching the designated position, it also vibrates up and down at $\theta = 0.1$.

(2) Under the two control methods, the response time of the end deformation of the flexible manipulator is significantly different. Under the simultaneous control of PD and PZT, when the end of the flexible manipulator is at $t = 10$ s, all residual vibrations can be suppressed, but when only controlled by PD. During the 20 s simulation time, the end of the flexible manipulator has been vibrating. Due to the influence of damping factors, the vibration has been decayed continuously.

In a comprehensive comparison of the two cases, it can be clearly seen that it is difficult to obtain a satisfactory control effect if only the PD control is used to suppress the vibration of the flexible manipulator. On the contrary, using PD control and PZT actuator control at the same time can
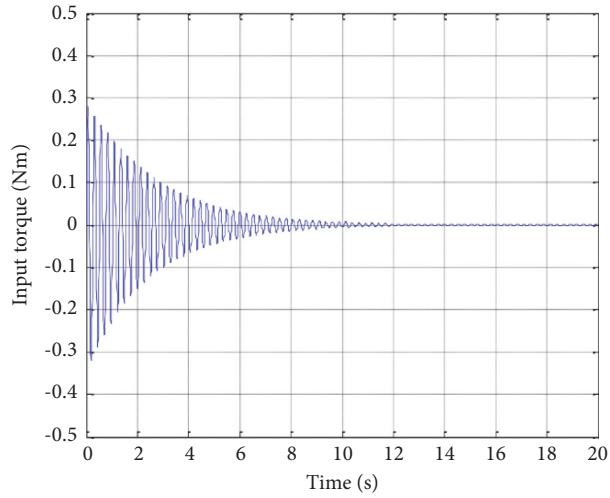
FIGURE 10: Torque input of the flexible manipulator.
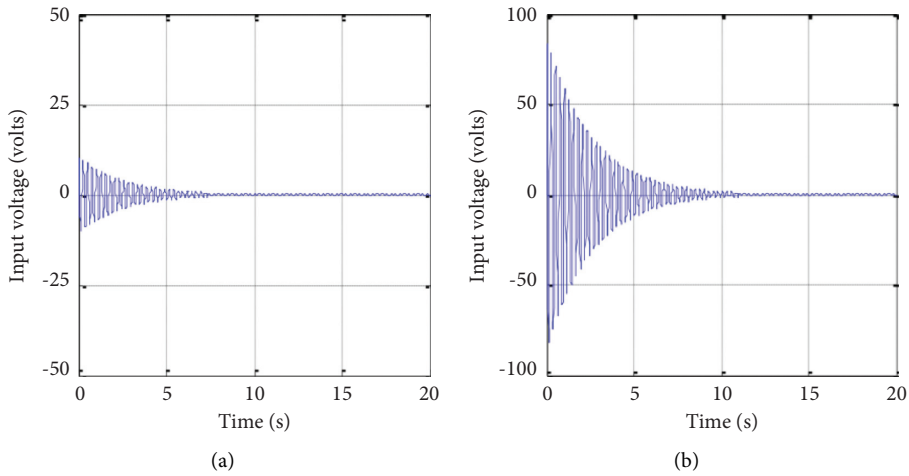

(a)

(b)

FIGURE 11: Control voltages $V_1(t)$ and $V_2(t)$. (a) Voltage $V_1(t)$ applied to the first pair of piezoelectric actuators. (b) Voltage $V_2(t)$ acting on the first pair of piezoelectric actuators.
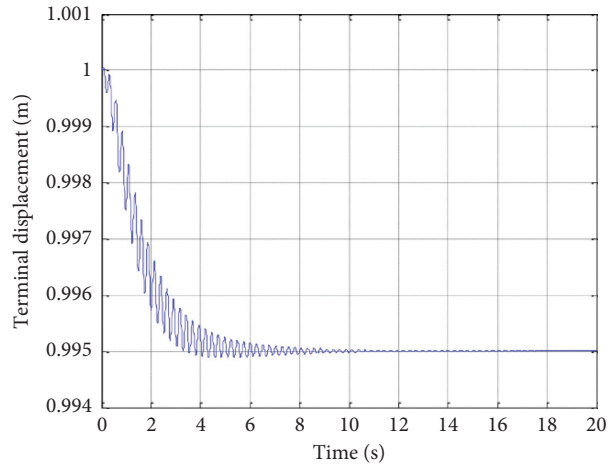

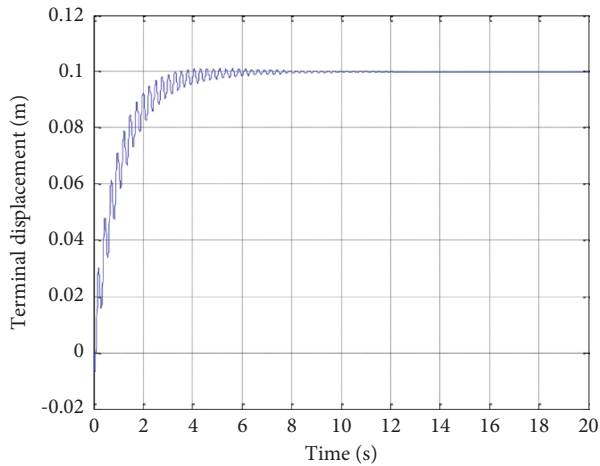FIGURE 12: Tip displacement p_x with combined PD and PZT control.

Figure 13: Tip displacement p_y with combined PD and PZT control.

effectively suppress the vibration of the flexible manipulator, which also proves that the above control law is very effective for the vibration control of the flexible manipulator system.

6. Conclusion and Prospect

PD control combined with PZT piezoelectric control has been studied to control the motion of the flexible manipulator and suppress vibration. Among them, the PZT control law uses linear velocity feedback to make its control algorithm easy to implement. Finally, simulations verify the effectiveness of alternative control methods in the motion and vibration suppression of the flexible manipulator.

This study has completed the exploration of the single flexible links. Zheng-Hua Luo and Korayem have carried out research on multiple flexible links. In the following research, we can refer to the experience and combine this paper to study active control of flexible manipulators with multiple flexible links [12–14].

Data Availability

All data generated or analyzed during this study are included within the article.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

Acknowledgments

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