Closed-Loop and Robust Control of Quantum Systems

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Review Article

For most practical quantum control systems, it is important and difficult to attain robustness and reliability due to unavoidable uncertainties in the system dynamics or models. Three kinds of typical approaches (e.g., closed-loop learning control, feedback control, and robust control) have been proved to be effective to solve these problems. This work presents a self-contained survey on the closed-loop and robust control of quantum systems, as well as a brief introduction to a selection of basic theories and methods in this research area, to provide interested readers with a general idea for further studies.

Closed-Loop Learning Control.

Closed-loop learning control has achieved great successes in controlling the laboratory quantum phenomena [5, 6], where the optimal open-loop control strategy is hard to design directly due to the incomplete knowledge of the system models or unexpected uncertainties.

Feedback Control.

When a control is added to a quantum system, adjusting the control parameters according to instantaneous state of the system can make quantum control more pertinent and more effective, which will improve the control result with robustness and reliability.

Robust Control.

In realistic environment, the quantum system is unavoidable to be subject to disturbances, uncertainties, and incomplete knowledge. These factors can all be viewed as uncertainties in the control field, in the Hamiltonian system,
in the field-coupling coefficient (e.g., the dipole moment),
and so forth and might affect the control results [16]. In order
to achieve robustness in control method and to develop new
insights into complicated quantum plants (such as quantum
networks), it is desirable to apply classical robust control
thory into quantum domain. Various kinds of robust control
approaches [17–23] have been proposed, especially in the
communities of control science.

All of these three kinds of approaches aim at optimizing
the control performances for quantum systems that have
no perfect models or are subject to uncertainties. But they
are different from each other regarding specific motivations,
methods, and applications. A brief comparison between these
three kinds of approaches is shown in Table 1, and, in the
following sections, we give a self-contained survey on these
promising research areas and provide interested readers with
a general idea for further studies.

The remaining of the paper is organized as follows.
In Section 2, the closed-loop learning control problems of
quantum systems are defined with the concept of quantum
control landscape, and three kinds of closed-loop learning
control methods (e.g., gradient-based, GA, and RL methods)
are reviewed. Section 3 introduced several feedback control
approaches including Lyapunov control, measurement-based
feedback control, and coherent-feedback control. Then such
quantum robust control approaches as $H^\infty$ control, sliding
mode control, quantum risk-sensitive control, and quantum
ensemble control are surveyed in Section 4. Conclusions
and the authors’ perspective on the future challenges in the
associated fields are summarized in Section 5.

2. Closed-Loop Learning Control

Learning control is an effective control method that can
learn from experience and optimize the system performance
by searching for the best control strategy in an iterative
way. When applied to the control of quantum systems, as
presented in [5], the closed-loop learning control procedure
generally involves three elements: (i) a trial laser control
input design, (ii) the laboratory generation of the control that
is applied to the sample and subsequently observed for its
impact, and (iii) a learning algorithm that considers the prior
experiments and suggests the form of the next control input.
It is clear that, for each trial of control, it is an open-loop
control, while the control performance will be sent back to the
learning algorithm to direct the optimization for the optimal
control strategy.

The control objective is usually formulated as an optimal
control problem by converting the problem into a problem of
optimizing a functional of such control parameters as the
quantum states, control inputs, control time, and so on. In order
to systematically study the relationship between the
time-dependent controls and the associated values of the
objective functional, a notion of quantum control landscape
[24–25] is defined and related theories are also well developed.
In this section, we will survey the area of closed-loop learning
control from the point of view of quantum control landscape
and introduce several practical and promising learning meth-
ods to explore the quantum control landscape, which includes
the gradient-based methods, stochastic searching methods
(e.g., genetic algorithm), and reinforcement learning meth-
ods.

2.1. Quantum Control Landscape: A Unified View for Closed-
Loop Learning Control. In recent years, quantum control
landscapes [25] have attracted more and more attention in
the research field of quantum control. A control landscape
is defined as the map between the time-dependent control
Hamiltonian and associated values of the control perfor-
ance functional. For example, as shown in Figure 1, the
performance function $f(u)$ is defined as the functional of the
control strategy $u = u_i, i = 1, 2, \ldots, M$, where $M$ is a positive
integer that indicates the number of the control variables
($M = 2$ for the case shown in Figure 1).

Quantum control aims to manipulate the dynamics of
system evolution on the atomic and molecular scales, and
most quantum control problems can be formulated as the
maximization of an objective performance function. From
a unified point of view, the closed-loop learning control is
the approach of exploring a quantum control landscape to
find the optimal control strategy where the objective function
reaches its maximum or minimum. For the past decades,
various algorithms have been proposed to explore the control
landscapes for both theoretical studies and applications [24–
29]. Most traditional learning methods can also be adopted to
analyze or explore different kinds of control landscapes. In the
next subsection, we survey these existing successful methods
and classify them into three categories, that is, gradient-based
methods, stochastic searching methods, and reinforcement
learning methods.

2.2. Typical Learning Control Methods to Explore
Quantum Control Landscape

2.2.1. Gradient-Based Methods. Gradient-based methods are
one of the most important kinds of learning and optimization
control methods for quantum systems [30, 31]. A well-
developed gradient-based method called D-MORPH search
algorithm is introduced in [32].

For most gradient-base methods, for example, we can
introduce a time-like variable $s$ to characterize different
control strategies $u^{(s)}(t)$. A gradient flow in the control space
can be defined as

$$
\frac{du^{(s)}(t)}{ds} = -\nabla J(u^{(s)}(t))
= -\left(\frac{\partial J}{\partial u_1(t)}, \frac{\partial J}{\partial u_2(t)}, \ldots, \frac{\partial J}{\partial u_M(t)}\right),
$$

(1)

where $\nabla J(u^{(s)}(t))$ is the gradient of $J$ with respect to the
control strategy $u^{(s)}(t)$.
**Table 1: Closed-loop and robust control approaches for quantum systems.**

<table>
<thead>
<tr>
<th>Control Type</th>
<th>Motivations</th>
<th>Typical Methods</th>
<th>Applications</th>
</tr>
</thead>
<tbody>
<tr>
<td>Closed-loop learning control</td>
<td>Direct the control results and procedures in an iteratively learning way</td>
<td>(1) Gradient-based methods</td>
<td>Controlling laboratory quantum phenomena with incomplete knowledge or unexpected uncertainties, for example, optimal laser control design.</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(2) Stochastic searching (GA)</td>
<td>Quantum state transition control, entanglement control, design of quantum gates, and so forth.</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(3) RL methods</td>
<td></td>
</tr>
<tr>
<td>Feedback control</td>
<td>Adjust control parameters according to instantaneous feedback information</td>
<td>(1) Lyapunov control</td>
<td>Control design for quantum systems that are fragile and are subject to various kinds of uncertainties.</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(2) Measurement-based control</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>(3) Coherent-feedback control</td>
<td></td>
</tr>
<tr>
<td>Robust control</td>
<td>Design control to achieve the best objective functional under the possible worst uncertainties</td>
<td>(1) $H_\infty$ control</td>
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<tr>
<td></td>
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<td>(2) Sliding mode control</td>
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<td>(3) Risk-sensitive control</td>
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<td>(4) Quantum ensemble control</td>
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</table>

Choosing an arbitrary control strategy $u^0(t), t \in [0, T]$, we can find the optimal one using gradient flow by solving the following initial value problem:

\[
\frac{du^{(i)}}{ds} = -\nabla J(u^{(i)}(t)), \quad u^{(0)}(t) = u^0(t).
\]

(2)

According to (2), generally, we can approach the optimal control strategy by a forward Euler method over the $s$-domain; that is,

\[
u(s + \Delta s, t) = \nu(s, t) - \Delta s \nabla J(u^{(i)}(t)).
\]

(3)

It is clear that, for a quantum control problem, the gradient-based methods are most likely effective provided that (i) we can get the gradient easily and (ii) there are no traps on the control landscape (otherwise, the learning process may get into the traps and cannot reach the maxima). Fortunately, as argued in [5, 33], it is surprising that, under certain conditions, most of the control landscapes are trap free, and it is easy to find the optimal solution for controlling most of the quantum phenomena. But for more complex tasks, we cannot guarantee the previous conditions or the gradient is hard to acquire, and hence other global searching methods for the closed-loop learning control are necessary.

2.2.2. Stochastic Searching Methods. Most of the stochastic searching methods are global searching methods and can step over traps of local maxima. One of the most widely used methods is genetic algorithm (GA), which has also achieved great success in the closed-loop learning control of laboratory quantum systems.

In the early 1990s, Judson and Rabitz [34] use a GA learning procedure to direct the production of pulses with a laboratory measurement device. Thereafter, GA methods have been widely applied to various quantum control problems. For example, in [35], an evolutionary algorithm is applied to femtosecond pulse shaping in optimal control experiments. Tsubouchi and Momose [36] use the GA algorithm to optimize the pulse shape for rovibrational wave-packet manipulation. Atabek et al. [37] use evolutionary algorithms for the optimal laser control of molecular orientation. The control and optimization prospects in the frequency domain are also studied theoretically using GA and shaping Fourier-limited pulses [38].

For more details about stochastic learning control methods like GA, for the laboratory closed-loop learning control, please refer to [6], where these methods are discussed within the concept of experimental adaptive feedback control (AFC).

2.2.3. Reinforcement Learning Methods. Reinforcement learning (RL) [39] is an active area of machine learning and has been extensively applied to traditional control problems ranging from operations research to robotics [40, 41]. Compared with other learning methods, RL is a learning technique based on trial and error and is a more general learning approach that can learn from experience and show great adaptability through an iterative way. That is to say, RL involves approximating solutions to stochastic optimal control problems under the condition of incomplete knowledge of the system, where the feedback for the closed loop is an imprecise value called reward or penalty. So RL methods are also suitable for control design of quantum systems [18] where it is difficult to get a good model or the searching problems are too complex to solve with computational efficiency.
On the other hand, the quantum characteristics also have inspired new algorithms for traditional RL methods. Dong et al. [42] proposed a new learning paradigm called quantum reinforcement learning (QRL) which fuses quantum computation with RL. In their study, the states \( \{ s_i \} \) or actions \( \{ a_i \} \) in traditional RL are denoted as the corresponding orthogonal quantum states and are called the eigen states \( \{ |s_i\rangle \} \) of eigen actions \( \{ |a_i\rangle \} \) in QRL. Here the action \( a_i \) represents the possible operation (or control) that can accomplish the state transition between two states. The state \( s_i \) can represent \( |\phi_i\rangle \) and the action \( a_i \) represents the control function that can drive \( |\phi_i\rangle \) to \( |\phi_{i+1}\rangle \). This kind of representation with a parallel updating mechanism can speed up the learning process and improve the learning performance as well. Compared with the traditional RL, the QRL algorithm learns faster, its convergence range is much larger, and the learning rate is much easier to tune. QRL has been successfully applied for incoherent control of quantum systems [18]. Other quantum-inspired RL methods have also been studied for traditional control problems [43, 44].

Due to the strong learning and adaptive abilities, RL-based methods are promising for the exploration of quantum control landscapes and can be used for those quantum control problems where gradient-based or GA methods cannot work well. Thus the gradient-based methods (local search methods), GA methods (stochastic global search methods), and RL methods (global search methods but sometimes use the direction of gradient-like rewards) constitute three typical and different searching methods to explore the quantum control landscapes.

3. Feedback Control of Quantum Systems

Quantum feedback control includes two central steps, that is, getting the information from the system and adding the control to the quantum system. The information of the quantum system can be obtained by two ways, precisely calculating the evolution of the system or fetching it from the quantum systems by some methods such as measurement. The former method is limited since a quantum system may have many unexpected affections during its evolution, while for the latter method, the measurement on a quantum system will unavoidably influence the states of the measured quantum system, making the situation more complex when applying feedback to quantum systems.

For a system with predictable state in the system, one can easily design the control field according to the instantaneous state of the system, and quantum Lyapunov control theory is such a kind of quantum control methods which obtain the message by exact simulation of the system. Actually, quantum Lyapunov control theory is only a feedback design of open-loop control theory, based on the artificial simulation of the system; thus it cannot be used in the case of unknown initial states or in the presence of unpredictable disturbance to the system.

There are two strategies for feedback control of quantum systems, measurement-based feedback, and coherent-feedback quantum control. The former strategy is to measure some quantum observable or signals of the system and to use and process the measurement results in a controller to produce a classical control signal that drives a suitable actuator, such as a laser beam or a potential well, which exerts direct influence on the quantum system to be controlled. The latter strategy is to use another quantum system without measurement, a full quantum controller, and connect it with the quantum system to be controlled in a feedback loop.

In the next three subsections, we will give a detailed survey of the above mentioned Lyapunov, measurement-based, and coherent quantum feedback control theories.

3.1. Feedback-Designed Open-Loop Quantum Control: Lyapunov Control

Quantum Lyapunov control uses feedback design to construct control fields but applies the fields into quantum systems in an open-loop way. It has been proposed as a good candidate for quantum state transfer [45, 46] and provides us with a simple way to design control fields without measurement and feedback ([47–57]).

Lyapunov function of quantum states is the central concept in quantum Lyapunov control theory; a function \( V \) is specified to design time-varying control fields. The system with state \( \rho \) converges to the target state given by \( \rho \) which monotonically decreases (or increases) to its minimum (or maximum), which is an observation \( P \) of the dynamics that are closely related to some property of target states,

\[
V (\rho) = \text{Tr} (P \rho),
\]

and \( V \) just stands for the distance between the present state and the target state. Then let the derivation of the Lyapunov function \( \dot{V} < 0 \), which leads to the evolution of the system to the target state.

Assume that a closed quantum system with the free Hamiltonian \( H_0 \) and the time-dependent control Hamiltonian \( H_c (t) \) can be described by the following Liouville equation

\[
\frac{d\rho(t)}{dt} = -i[H_0 + H_c (t), \rho(t)].
\]

Then the time derivative of the Lyapunov function can be calculated to design the control fields. By requiring

\[
\dot{V} = \text{Tr} (-iP [H_0 + H_c (t), \rho(t)]) < 0,
\]

one can work out the requirement of the parameter of the control Hamiltonian \( H_c (t) \). Since the previous requirement may not completely determine the parameter, one can also find some further constraints to improve the control efficiency [57]. In essence, Lyapunov control uses the information of the system by simulation of the system, and it is a kind of feedback design control strategies.

This theory can be easily extended to the open quantum systems, such as the systems determined by

\[
\frac{d\rho(t)}{dt} = -i[H_0 + H_c (t), \rho(t)] + \mathcal{L} (\rho(t)),
\]

to study the control in open quantum systems.
In recent years, quantum Lyapunov control has been used to transfer quantum states [45, 46], to drive the states of open quantum system into the decoherence free subspaces [53], and to control the states in the decoherence free subspaces [54]. Also, Lyapunov control can be used to control the entanglement of the quantum systems [48, 52].

3.2. Measurement-Based Feedback Control. Measurement-based feedback (MFC) uses the measurement results to produce a classical control signal to drive a suitable actuator which exerts direct influence on the quantum system to be controlled [4]. During the MFC process, one can perform measurement on the system to get some information of the system state and then design the control law based on the estimation of the state. The system to be controlled is a quantum system, while the controller may be quantum, classical, or a quantum-classical hybrid. Different from the classical feedback control process, which can obtain the information of the system easily without disturbing it, the collapse of quantum state under the measurement process makes the problem of quantum systems rather complex.

Markovian Quantum Feedback. The general theory of quantum-limited feedback for continuously monitored systems was presented by Wiseman and Milburn, based on quantum measurement theory and on Hamiltonian system bath interactions [11, 58]. They considered the instantaneous feedback of some measured photocurrent (homodyne detection) onto the dynamics of a quantum system, and the master equation for the resulting evolution was Markovian; that is, the measurement record is immediately fed back into the system to alter the system dynamics and may then be forgotten, while any time delay is ignored and a memoryless controller is assumed. Hence, the equation describing the resulting evolution is a Markovian master equation. This kind of feedback has already been used to reduce laser noise below the shot-noise level [59] and also has been used in many aspects of physical problems, such as the control of quantum qubits [60] and quantum entanglement [61–64]. In case the feedback delay cannot be ignored, the feedback Hamiltonian must include a delay parameter. Time delay effect of the measurement was investigated in [65].

Bayesian Feedback Method. Doherty and Jacobs presented a formulation of feedback in quantum systems in which the best estimates of the dynamical variables are obtained continuously from the measurement record and fed back to control the system [7, 12]. They considered some arbitrary functional of the entire history of the measurement results that can be used to alter the system evolution. The resulting equation dynamics of the system are then non-Markovian. However, the dynamics of the system and controller remain Markovian, and this is completely analogous to the situation in classical control theory. In Bayesian quantum feedback, the control process is also divided into two steps involving state estimation and feedback control. Due to the fact that, in linear systems, the estimation process may be modeled by its classical analogue, Kalman filtration and classical linear quadratic Gaussian (LQG) control theory may be applied to quantum feedback by estimation. For Bayesian quantum feedback, it uses a more general form of control Hamiltonian with more information from the measurement.

It has been compared in [14] the Bayesian and Markovian feedback quantum controls, where it was proved that Bayesian feedback is never inferior, and since it uses more information, it is usually superior to Markovian feedback. However, it would be far more difficult to implement than Markovian feedback and it loses its superiority when obvious simplifying approximations are made. Thus, it is not clear which form of feedback would be better in the face of inevitable experimental imperfections. Bayesian feedback has also been used in many aspects of systems, such as the preparation of quantum states [66], and quantum error correction [67, 68].

Weak Measurement and Nondemolition Measurement. Making as little influence as possible during the measurement process is important to minimize disturbance to the system to be controlled. Weak measurement makes it possible to modify the evolution continuously via Hamiltonian feedback, where the Hamiltonian feedback applied to the system depends on the measurement record [69], and it can also be modeled by a stochastic master equation by introducing an ancilla system weakly coupled to the system of interest. Weak measurements and local feedback can be used to control the generation of entanglement between two qubits [70]. Motivated by the proposal of Braunçzyk et al. [71], experimentally exploring the use of weak measurement [72] in feedback control on a photonic polarization qubit is given in [73], as well as in the control of nonlinear quantum systems [74]. Quantum nondemolition measurement preserves the integrity of the system and the value of the measured observable, which is best thought of as the ideal quantum projective measurement. Nevertheless, nondemolition does not mean that the state of the system has no wave collapse, and it is extremely difficult to carry out experimentally [75].

Although measurement-based feedback control is effective in many quantum control systems, its drawbacks are also evident. Firstly, measuring a quantum system almost inevitably disturbs it. Even a nondemolition measurement that leaves the system in the state in which it was measured still typically alters the states of the system prior to the measurement [76, 77]. After fluorescence determines whether the ion is in its ground state or excited state, the initial quantum coherence between those states is irrevocably lost. Secondly, the information from the measurement is stochastic because a result of the measurement of the system jumps to one state or another probabilistically. Although the ability to apply coherent operations conditioned on the results of measurements allows the controller to compensate for the probabilistic nature of their results, the introduction of stochastic effects significantly complicates the control process. Furthermore, the measurement-based feedback is limited by its information processing speed that has to be kept up with the evolution of the system dynamics, and it cannot be used in most solid state systems whose time scales range from picoseconds to nanoseconds.
3.3. Coherent-Feedback Control. Coherent-feedback quantum control uses another quantum system as a full quantum controller and connects it with the quantum system to be controlled in a feedback loop; that is, the feedback controller itself is a quantum system, and the control operations consist of unitary transformations. This is greatly different from Markovian and Bayesian quantum feedback controls where the feedback information from measurement results is classical information and the feedback controller is a classical controller.

Since this control uses full quantum information of the system, it can perform a number of tasks that controllers using a classical information feedback loop cannot [15]. Compared with the measurement-based feedback control, coherent-feedback control does not involve measurement, avoiding the introduction of excess measurement noise, while the controller and the system plant can be both quantum systems and are coherently connected. By coherent-feedback control, one can use coherent feedback to guide a quantum system from an unknown initial state to a desired final state without destroying the initial state. In addition, a controller can use a quantum feedback loop to drive a quantum system to a target state that is entangled with another quantum system, while entanglement is a local quantum phenomenon that cannot be created by controllers using classical feedback loops.

The very successful noise-reducing controllers, the $H^\infty$ and the linear quadratic Gaussian (LQG) controllers, have natural coherence control analogues [17, 78, 79]. By basic principles of linear quantum stochastic control theory, it has been presented that optimal and robust design of quantum coherent-feedback loops can be accomplished using sophisticated methods of system engineering [17], and an experimental implementation of coherent-feedback quantum control with optical resonators as the dynamical systems and laser beams as the coherent disturbance and feedback signals has been presented [80]. The experiments of coherent-feedback control in optical field squeezing are proposed in [81], and it was also applied to many other interesting problems, such as cooling quantum oscillator [82], spontaneous switching suppression [83], multipartite quantum entanglement generation [84], and producing optical quantum gates in a four-wave mixing process [85].

Traditional coherence feedback control was established for the Markovian environment. Recently, the non-Markovian coherence feedback control was presented [86]. However, in coherence feedback control, the controller itself will cause quantum decoherence to the controlled system even though it coherently entangles with the system [87]; thus, whether the coherent feedback is better than the open-loop control for quantum control systems needs to be investigated in depth [88].

4. Robust Control

A general formalism of quantum robust optimal control problem was given in [16], which pointed out that to design a control field that achieves the best objective functional under possible worst uncertainties is in essence a minimax problem. Reference [16] also provided a method to calculate the worst possible disturbance to the control process and to design a corresponding robust optimal control field. Another noticeable early attempt to apply robust control theory in quantum field is [89], where the small gain theorem was extended to analyze the stability of quantum feedback networks. Later, different robust control tools were systematically introduced into the quantum domain, which formulated the early development of quantum robust control.

4.1. $H^\infty$ Control of Quantum Systems. For several typical classes of noncommutative linear stochastic systems with many interesting examples in quantum technology, $H^\infty$ control theory was introduced to obtain robust controllers and developed for diverse situations and requirements.

Take the class of linear noncommutative stochastic systems in [17] for example, which encompasses some quantum and classical systems:

$$\begin{align*}
\dot{x}(t) &= Ax(t)dt + B\omega(t), \quad x(0) = x_0, \\
\dot{y}(t) &= Cx(t)dt + D\omega(t),
\end{align*}$$

where $A, B, C,$ and $D$ are, respectively, real $\mathbb{R}^{n\times m}$, $\mathbb{R}^{m\times n}$, $\mathbb{R}^{n\times p}$, and $\mathbb{R}^{p\times m}$ matrices with $n$, $m$, and $p$, all positive integers and $x(t) = [x_1(t), \ldots, x_n(t)]^T$ is a vector of self-adjoint possibly noncommutative system variables, whose initial value $x_0$ consists of operators satisfying the commutation relations

$$[x_j(0), x_k(0)] = 2i\Theta_{jk}, \quad j, k = 1, \ldots, n, \quad (9)$$

where $[A, B] = AB - BA$ is the commutation operator, $\Theta_{jk}$ are components of the real antisymmetric matrix $\Theta$, and $i$ is the imaginary unit. $x_0$ is also assumed to be Gaussian with density operator $\rho$. The vector quantity $\omega$ describes the input signals and is assumed to have the decomposition

$$\dot{\omega}(t) = \beta_\omega(t)dt + d\tilde{\omega}(t), \quad (10)$$

where $\beta_\omega(t)$ is a self-adjoint, adapted process (see [90, 91]). The noise part of $\omega(t)$ is $d\tilde{\omega}(t)$, a vector of self-adjoint quantum noises with Itô table

$$d\tilde{\omega}(t)d\tilde{\omega}^T(t) = F_\omega dt, \quad (11)$$

where $F_\omega$ is a nonnegative Hermitian matrix (see [91, 92]). For more detailed description and assumptions and the physical realizability of this class of systems, one can refer to [17].

The $H^\infty$ controller synthesis problem for the class of systems described by $(8)\sim(11)$ was first formulated and solved in [17]. Furthermore, this quantum $H^\infty$ control problem was extended to a time-varying version, and the corresponding solution was obtained by a dynamic game approach in [93]. For the same plant, the finite horizon dynamic game theory approach was applied in [94], and the solving process was proved equivalent to solving a corresponding deterministic continuous-time problem with imperfect state information.
The finite horizon $H_{\infty}$ control problem in [94] was then extended to the case of delayed measurements in [95].

To simplify the deduction process and obtain more profound results, a more special class of linear quantum systems was considered in [79], which proposed a robust controller designing method probably more easy to implement experimentally.

4.2. Sliding Mode Control of Quantum Systems. Sliding mode control (SMC) approach is a useful robust control tool in classical control theory and industrial applications, especially for nonlinear systems. Since many quantum systems evolve with nonlinear equations, SMC is therefore supposed to be capable of controlling some quantum phenomena [96, 97].

Reference [98] applied the SMC control method into quantum systems. Similar to the classical theory, quantum sliding mode is a system state where the system has some desirable features, such as robustness to a class of uncertainties, and features brought by eigenstates, features brought by invariant state subspaces. Once the sliding mode is selected, one needs to design control laws that can drive the system onto its sliding mode and keep the system on it, which were designed in detail by combining unitary control and periodic projective measurements in [98].

In [99], a sliding mode design method for two-level quantum systems with bounded uncertainties was proposed. The uncertainties were assumed to take the form of perturbations in the Hamiltonian, and the controller design method used the Lyapunov methodology and periodic projective measurements. These results were extended in [100], where the effect of uncertainties in driving the system state back to the sliding mode domain from outside was considered, and the measurement periods were modified when considering uncertainties described as perturbations in the free Hamiltonian. In [101], a sampled-data design approach for decoherence control of a single qubit with operator errors was proposed using a sliding mode domain concept as the required control performance.

Though sliding mode control approach was introduced into quantum systems, the appropriate combination of the essential characters of these two focuses is still worth digging. Furthermore, one may consider extending sliding mode control to open quantum systems and applying other branches of classical nonlinear control theories into the quantum domain.

4.3. Quantum Risk-Sensitive Control. As a modification of the common integral form of criterion, or the so-called a risk-neutral criterion, a risk-sensitive criterion takes the form of an exponential function, which results in the close connections between robust control and risk-sensitive control [102–104]. For example, risk-sensitive control is anticipated to be useful in designing robust controllers [105]. Reference [106] formulated a risk-sensitive optimal control problem for quantum systems, obtained a solution using dynamic programming, and briefly discussed the robustness properties of the risk-sensitive controllers. Reference [107] considered a risk-sensitive optimal control problem for continuously monitored open quantum systems within the framework of quantum Langevin equations and solved the problem with quantum stochastic calculus and dynamic programming. Reference [105] collected related research and systematically illustrated a quantum risk-sensitive control problem and the corresponding dynamic-programming solution. At the end of [105], the author proposed several developing directions for quantum risk-sensitive control, which include theoretical development, practical applications in quantum field, and the exploration of robustness properties.

Filtering aims to extract information from noisy signals and is inherently connected with robust control, which therefore forms robust estimation. Guaranteed-cost filtering and risk-sensitive filtering are two branches of robust estimation, which are quite promising to be extended into quantum theory. Reference [108] obtained a quantum version of the guaranteed-cost filter and showed its unique robustness character compared with optimal Kalman filter and risk-sensitive observer. Reference [23] studied a quantum risk-sensitive estimation problem and analyzed robustness properties of the filter under a discrete approximation model of the aimed quantum system. More systematic work within the associated topics remains to be done.

4.4. Quantum Ensemble Control. Ensemble control means controlling a continuum of dynamical systems with different values of parameters characterizing the system dynamics by using the same control signal. Ensemble control derives from the manipulation of an ensemble of nuclear spins in nuclear magnetic resonance (NMR) spectroscopy and imaging (MRI), where one often needs to develop external excitations that can simultaneously steer the ensemble of systems with variations in their internal parameters from a fiducial state to a target state [109]. Here we view the difference in parameters as system uncertainties. Hence, ensemble control forms a new systematic branch of robust control.

A fundamental question in quantum ensemble control is controllability, which determines whether the control function that transfers the system from initial states to desired target states can exist. Reference [110] introduced the notion of simultaneous controllability; that is, all individuals in the system are simultaneously controllable, and generalized controllability criteria for decomposable systems. References [111, 112] formally proposed the definition of ensemble controllability for quantum systems described by Bloch equations depending continuously on a finite number of scalar parameters and with a finite number of control inputs and analyzed ensemble controllability and optimal control of linear time-invariant systems. Ensemble controllability concerns finding open-loop controls to compensate for the dispersion in element parameters. Reference [113] cast the design of control pulses as an optimal ensemble control problem and introduced a multidimensional pseudospectrum-based solution, whose convergence was shown in [114]. Reference [115] studied the controllability of an ensemble of general finite-dimensional time-varying linear systems and gave necessary and sufficient conditions, which is in connection with singular values of the operator characterizing the system dynamics.
Reference [116] introduced a universal numerical method based on the singular value decomposition to approximate optimal ensemble control problems.

Furthermore, since ensemble is originally a notion in quantum statistics, one may anticipate introducing tools and methods in quantum statistical mechanics into quantum ensemble control to give new in sights and approaches.

5. Conclusions and Discussions

Manipulating system dynamics at the quantum scale is full of challenges for both theoretical and laboratory researchers. Closed-loop and robust control approaches are of most importance to deal with uncertainties and incomplete knowledge about the system dynamics or unexpected disturbances. To conclude this paper, we briefly discuss some open problems and promising research directions as follows.

Learning skills is very important for the control design of quantum systems where no good solutions can be easily obtained from a specific model. Although the closed-loop learning control approach for controlling quantum phenomena has been well developed since the early 1990s, more effective learning theories and algorithms need to be further explored. The experts from different fields such as quantum physics, chemical physics, control theory, computer science, and artificial intelligence need to cooperate on this exciting research area.

Feedback control is one of the most important control strategies for traditional control problems. Almost all the practical industrial control systems use feedback controllers such as PID controllers. In the feedback control approach, the deviations between the measured variable and a set point are fed back to the controller to generate appropriate control actions. When we apply feedback control methods for the quantum control systems, two problems are unavoidable, that is, the problems of measurement and time scale. Quantum state measurement is difficult and much more complex than its counterpart of traditional control systems. Time scale is another nontrivial issue for the control of quantum systems since the feedback signals are always lagging. The feedback control design needs to incorporate the time delay of the feedback signal and satisfy the time scale of the controlled quantum systems.

For practical applications, robustness is an important aspect for the design of controllers, especially for quantum systems that are subject to various kinds of uncertainties and are more fragile. The existing results mainly focus on certain kinds of quantum systems with specific models, and experiments temporarily fall behind the development of theory. In the future, more general and systematic approaches of robust control need to be developed for more general kinds of uncertainties which exist in practical applications.

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