Research Article

Certain Subclasses of Analytic Functions with Complex Order

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Two new subclasses of analytic functions of complex order are introduced. Apart from establishing coefficient bounds for these classes, we establish inclusion relationships involving $(n-\delta)$ neighborhoods of analytic functions with negative coefficients belonging to these subclasses.

1. Introduction

Let $\mathcal{A}(n)$ denote the class of functions of the form

$$f(z) = z - \sum_{k=n+1}^{\infty} a_k z^k \quad (a_k \geq 0, n \in \mathbb{N} := \{1, 2, 3, \dots\}), \quad (1)$$

which are analytic and univalent in the open disc

$$\Delta = \{z \in \mathbb{C} : |z| < 1\}. \quad (2)$$

A function $f(z) \in \mathcal{A}(n)$ is star-like of complex order b , denoted as $f(z) \in S^*(b)$ if and only if it satisfies

$$\Re \left\{ 1 + \frac{1}{b} \left(\frac{zf'}{f} - 1 \right) \right\} > 0 \quad (z \in \Delta). \quad (3)$$

A function $f(z) \in \mathcal{A}(n)$ is convex of complex order b , denoted as $f(z) \in C(b)$ if and only if it satisfies

$$\Re \left\{ 1 + \frac{1}{b} \left(\frac{zf''}{f'} \right) \right\} > 0 \quad (z \in \Delta). \quad (4)$$

These classes, $S^*(b)$, and $C(b)$ are introduced and studied by Nasr and Aouf [1] and Wiatrowski [2].

For the two functions f_j ($j = 1, 2$) given by

$$f_j(z) = z + \sum_{k=2}^{\infty} a_{k,j} z^k \quad (j = 1, 2), \quad (5)$$

the Hadamard product or convolution, denoted by $(f_1 * f_2)(z)$, is given by

$$(f_1 * f_2)(z) = z + \sum_{k=2}^{\infty} a_{k,1} a_{k,2} z^k. \quad (6)$$

Given $f(z)$ of the form (1) and $\delta \geq 0$, we define n - δ neighborhood of a function $f \in \mathcal{A}(n)$ as

$$N_{n,\delta}(f) = \left\{ g \in \mathcal{A}(n) \mid \sum_{k=n+1}^{\infty} k |a_k - b_k| \leq \delta \right\}. \quad (7)$$

In particular, for the identity function $e(z) = z$,

$$N_{n,\delta}(e) = \left\{ g \in \mathcal{A}(n) \mid \sum_{k=n+1}^{\infty} k |b_k| \leq \delta \right\}. \quad (8)$$

The concept of Neighborhood $N_{n,\delta}$ of a function f is introduced and studied by Ruscheweyh [3] and extended further by Silverman [4].

For complex numbers $\alpha_1, \alpha_2, \dots, \alpha_q$ and $\beta_1, \beta_2, \dots, \beta_s$ ($\beta_j \in \mathbb{C} \setminus \mathcal{Z}_0^-$; $\mathcal{Z}_0^- = \{0, -1, -2, \dots\}$ for $j = 1, 2, \dots, s$),

we define the generalized hypergeometric function ${}_qF_s(\alpha_1, \alpha_2, \dots, \alpha_q; \beta_1, \beta_2, \dots, \beta_s; z)$ as

$${}_qF_s(\alpha_1, \alpha_2, \dots, \alpha_q; \beta_1, \beta_2, \dots, \beta_s; z) = \sum_{k=0}^{\infty} \frac{(\alpha_1)_k (\alpha_2)_k \cdots (\alpha_q)_k}{(\beta_1)_k (\beta_2)_k \cdots (\beta_s)_k} \frac{z^k}{k!}, \quad (q \leq s+1; q, s \in \mathbb{N}_0 := \mathbb{N} \cup \{0\}; z \in U), \quad (9)$$

where $\mathbb{N}$ denotes the set of all positive integers and $(x)_k$ is the Pochhammer symbol defined in terms of gamma functions as

$$(x)_k = \frac{\Gamma(x+k)}{\Gamma(x)} = \begin{cases} 1 & \text{if } k=0 \\ x(x+1)\cdots(x+k-1) & \text{if } k \in \mathbb{N}. \end{cases} \quad (10)$$

Corresponding to the function $g_{q,s}(\alpha_1, \beta_1; z)$ defined by

$$g_{q,s}(\alpha_1, \beta_1; z) = {}_qF_s(\alpha_1, \alpha_2, \dots, \alpha_q; \beta_1, \beta_2, \dots, \beta_s; z) \quad (11)$$

recently in [5], an operator $\mathcal{D}_{\lambda,\mu}^m(\alpha_1, \beta_1)f(z) : \mathcal{A} \rightarrow \mathcal{A}$ is defined by

$$\begin{aligned} \mathcal{D}_{\lambda,\mu}^0(\alpha_1, \beta_1)f(z) &:= f(z) * g_{q,s}(\alpha_1, \beta_1; z), \\ \mathcal{D}_{\lambda,\mu}^1(\alpha_1, \beta_1)f(z) &:= (1-\lambda+\mu)(f(z) * g_{q,s}(\alpha_1, \beta_1; z)) \\ &\quad + (\lambda-\mu)z(f(z) * g_{q,s}(\alpha_1, \beta_1; z))' \\ &\quad + \lambda\mu z^2(f(z) * g_{q,s}(\alpha_1, \beta_1; z))'', \\ \mathcal{D}_{\lambda,\mu}^m(\alpha_1, \beta_1)f(z) &:= \mathcal{D}_{\lambda,\mu}^1(\mathcal{D}_{\lambda,\mu}^{m-1}(\alpha_1, \beta_1)f(z)), \end{aligned} \quad (12)$$

where $0 \leq \mu \leq \lambda \leq 1$ and $m \in \mathbb{N}_0$. By the above definition, it is easy to note that

$$\begin{aligned} \mathcal{D}_{\lambda,\mu}^m(\alpha_1, \beta_1)f(z) &= z + \sum_{k=2}^{\infty} [1 + (k-1)(\lambda - \mu + k\mu\lambda)]^m \\ &\quad \times \frac{(\alpha_1)_{k-1}(\alpha_2)_{k-1} \cdots (\alpha_q)_{k-1}}{(\beta_1)_{k-1}(\beta_2)_{k-1} \cdots (\beta_s)_{k-1}} \frac{a_k z^k}{(k-1)!}. \end{aligned} \quad (13)$$

Let us take for convenience that

$$\begin{aligned} B_k &= \frac{(\alpha_1)_{k-1}(\alpha_2)_{k-1} \cdots (\alpha_q)_{k-1}}{(\beta_1)_{k-1}(\beta_2)_{k-1} \cdots (\beta_s)_{k-1} (k-1)!}, \\ C_k &= 1 + (k-1)(\lambda - \mu + k\mu\lambda). \end{aligned} \quad (14)$$

Hence, we have

$$\mathcal{D}_{\lambda,\mu}^m(\alpha_1, \beta_1)f(z) = z + \sum_{k=2}^{\infty} C_k^m B_k a_k z^k. \quad (15)$$

For suitable values of $\alpha_{i's}$, $\beta_{j's}$, q , s , m , λ , and μ , we can deduce several operators such as Sălăgean derivative operator [6], Ruscheweyh derivative operator [7], fractional calculus operator [8], Carlson-Shaffer operator [9], Dziok-Srivatsava operator [10], and also the operator introduced by Abubaker and Darus [11].

Definition 1. For $0 \leq \alpha \leq 1$, we let A be the subclass of $\mathcal{A}(n)$ consisting of functions of the form (1) that satisfy

$$\begin{aligned} &\left| \frac{1}{b} \left(z [\mathcal{D}_{\lambda,\mu}^m(\alpha_1, \beta_1)f(z)]' \right. \right. \\ &\quad \times \left((1-\alpha) \mathcal{D}_{\lambda,\mu}^m(\alpha_1, \beta_1)f(z) \right. \\ &\quad \left. \left. + \alpha z [\mathcal{D}_{\lambda,\mu}^m(\alpha_1, \beta_1)f(z)]' \right)^{-1} \right| < \gamma, \end{aligned} \quad (16)$$

where $z \in \Delta$, $b \in \mathbb{C} \setminus \{0\}$, $0 < \gamma \leq 1$, and $\mathcal{D}_{\lambda,\mu}^m(\alpha_1, \beta_1)f(z)$ are as given in (15).

Definition 2. For $0 \leq \alpha \leq 1$ we let B be the subclass of $\mathcal{A}(n)$, consisting of functions of the form (1) that satisfy

$$\begin{aligned} &\left| \frac{1}{b} \left((1-\alpha) \frac{\mathcal{D}_{\lambda,\mu}^m(\alpha_1, \beta_1)f(z)}{z} \right. \right. \\ &\quad \left. \left. + \alpha [\mathcal{D}_{\lambda,\mu}^m(\alpha_1, \beta_1)f(z)]' - 1 \right) \right| < \gamma, \end{aligned} \quad (17)$$

where $z \in \Delta$, $b \in \mathbb{C} \setminus \{0\}$, $0 < \gamma \leq 1$, and $\mathcal{D}_{\lambda,\mu}^m(\alpha_1, \beta_1)f(z)$ are as given in (15).

By specializing the parameters involved in the above definitions, we could arrive at several known as well as new classes. For example, by taking $\lambda = 1$, $\mu = 0$, $q = 2$, $s = 1$, $\alpha_1 = \beta_1$, and $\alpha_2 = 1$ and the above classes reduced to

$$\begin{aligned} A_1 &= \left\{ f \in \mathcal{A}(n) \mid \left| \frac{1}{b} \left(z [\mathbb{D}^m f(z)]' \right. \right. \right. \\ &\quad \times \left((1-\alpha) \mathbb{D}^m f(z) \right. \\ &\quad \left. \left. \left. + \alpha z [\mathbb{D}^m f(z)]' \right)^{-1} \right| < \gamma \right\}, \end{aligned} \quad (18)$$

where $\mathbb{D}^m f(z)$ denote the Sălăgean derivative of order m given by

$$\mathbb{D}^m f(z) = z - \sum_{k=n+1}^{\infty} k^m a_k z^k,$$

$$B_1 = \left\{ f \in \mathcal{A}(n) \mid \left| \frac{1}{b} \left((1-\alpha) \frac{\mathbb{D}^m f(z)}{z} + \alpha [\mathbb{D}^m f(z)]' - 1 \right) \right| < \gamma \right\}. \quad (19)$$

Similarly, on taking $q = 2, s = 1, \alpha_1 = \eta - 1, (\eta > -1), \alpha_2 = 1, \beta_1 = 1$, one gets

$$A_2 = \left\{ f \in \mathcal{A}(n) \mid \left| \frac{1}{b} \left(z [D^m f(z)]' \times ((1-\alpha) D^m f(z) + \alpha z [D^m f(z)]')^{-1} \right) \right| < \gamma \right\}, \quad (20)$$

where $D^m f(z)$ is the operator introduced and studied by Abubaker and Darus [11] given by

$$D^m f(z) = z - \sum_{k=n+1}^{\infty} C_k^m \left(\frac{\eta + k - 1}{k - 1} \right) z^k,$$

$$B_2 = \left\{ f \in \mathcal{A}(n) \mid \left| \frac{1}{b} \left((1-\alpha) \frac{D^m f(z)}{z} + \alpha [D^m f(z)]' - 1 \right) \right| < \gamma \right\}. \quad (21)$$

Further, by taking $m = 0$ in the definition of the classes A and B , we could arrive at $S_n(q, s, \alpha, b, \gamma)$ and $R_n(q, s, \alpha, b, \gamma)$ which were introduced and studied by Murugusundaramoorthy et al. [12].

In this paper, we establish the coefficient inequalities for the classes A and B and several inclusion relationships involving n - δ neighborhoods of analytic univalent functions with negative and missing coefficients belonging to these classes.

2. Coefficient Inequalities

Theorem 3. Let the function $f \in \mathcal{A}(n)$ as given in (1). Then, $f \in A$ if and only if

$$\sum_{k=n+1}^{\infty} \{ [1 + \alpha(k-1)] (\gamma |b| - 1) + k \} C_k^m B_k a_k \leq \gamma |b|. \quad (22)$$

Proof. Let the functions of form (1) belong to the class A . Then, in view of (15) and (16), we get

$$\left| \frac{\sum_{k=n+1}^{\infty} ([1 + \alpha(k-1)] - k) C_k^m B_k a_k z^k}{z - \sum_{k=n+1}^{\infty} [1 + \alpha(k-1)] C_k^m B_k a_k z^k} \right| < \gamma |b|, \quad (23)$$

$$\frac{\sum_{k=n+1}^{\infty} ([1 + \alpha(k-1)] - k) C_k^m B_k a_k r^{k-1}}{1 - \sum_{k=n+1}^{\infty} [1 + \alpha(k-1)] C_k^m B_k a_k r^{k-1}} < \gamma |b|.$$

Letting $r \rightarrow 1^-$ through real values, we get the required assertion (22). Conversely, suppose $f(z)$ satisfies (22), then, in view of (16), consider

$$\begin{aligned} & \left| z [\mathcal{D}_{\lambda, \mu}^m (\alpha_1, \beta_1) f(z)]' \right. \\ & \quad \left. - (1-\alpha) \mathcal{D}_{\lambda, \mu}^m (\alpha_1, \beta_1) f(z) \right. \\ & \quad \left. - \alpha z [\mathcal{D}_{\lambda, \mu}^m (\alpha_1, \beta_1) f(z)]' \right| \\ & \quad - \gamma |b| \left| (1-\alpha) \mathcal{D}_{\lambda, \mu}^m (\alpha_1, \beta_1) f(z) \right. \\ & \quad \left. + \alpha z [\mathcal{D}_{\lambda, \mu}^m (\alpha_1, \beta_1) f(z)]' \right| \\ & = \left| \sum_{k=n+1}^{\infty} ([1 + \alpha(k-1)] - k) C_k^m B_k a_k z^k \right| \\ & \quad - \gamma |b| \left| z - \sum_{k=n+1}^{\infty} [1 + \alpha(k-1)] C_k^m B_k a_k z^k \right| \\ & < \sum_{k=n+1}^{\infty} \{ ([1 + \alpha(k-1)] (\gamma |b| - 1) + k) \} \\ & \quad \times C_k^m B_k a_k - \gamma |b| \\ & \leq 0. \end{aligned} \quad (24)$$

Hence, the result follows. $\square$

Similarly, we prove the following.

Theorem 4. Let the function $f \in \mathcal{A}(n)$ be as defined in (1). Then, $f \in B$ if and only if

$$\sum_{k=n+1}^{\infty} [1 + \alpha(k-1)] C_k^m B_k a_k \leq \gamma |b|. \quad (25)$$

Corollary 5. Let the function $f \in \mathcal{A}(n)$ as given in (1). Then, $f(z) \in A_1$ if and only if

$$\sum_{k=n+1}^{\infty} \{ [1 + \alpha(k-1)] (\gamma |b| - 1) + k \} k^m a_k \leq \gamma |b|. \quad (26)$$

Corollary 6. Let the function $f \in \mathcal{A}(n)$ be as defined in (1). Then, $f \in B_1$ if and only if

$$\sum_{k=n+1}^{\infty} [1 + \alpha(k-1)] k^m a_k \leq \gamma |b|. \quad (27)$$

Corollary 7. Let the function $f \in \mathcal{A}(n)$ as given in (1). Then, $f \in A_2$ if and only if

$$\sum_{k=n+1}^{\infty} \{ [1 + \alpha(k-1)] (\gamma |b| - 1) + k \} C_k^m \times \left(\frac{\eta + k - 1}{k - 1} \right) a_k \leq \gamma |b|. \quad (28)$$

Corollary 8. Let the function $f(z) \in \mathcal{A}(n)$ be as defined in (1). Then, $f \in B_2$ if and only if

$$\sum_{k=n+1}^{\infty} [1 + \alpha(k-1)] C_k^m \binom{\eta+k-1}{k-1} a_k \leq \gamma |b|. \quad (29)$$

3. Inclusion Relationships

Theorem 9. If

$$\delta = \frac{\gamma |b| (1+n)}{[(1+n\alpha)(\gamma |b| - 1) + n + 1] B_{n+1} C_{n+1}^m} \quad (\gamma |b| > 1), \quad (30)$$

then $A \subset N_{n,\delta}(e)$.

Proof. Let $f \in A$. Then, in view of (22), we have

$$C_{n+1}^m B_{n+1} [(1+n\alpha)(\gamma |b| - 1) + n + 1] \sum_{k=n+1}^{\infty} a_k \quad (31)$$

$$\leq \gamma |b|,$$

$$\sum_{k=n+1}^{\infty} a_k \leq \frac{\gamma |b|}{C_{n+1}^m B_{n+1} [(1+n\alpha)(\gamma |b| - 1) + n + 1]}. \quad (32)$$

Consider

$$\begin{aligned} & C_{n+1}^m B_{n+1} \sum_{k=n+1}^{\infty} k a_k \\ & \leq \gamma |b| + (1+n\alpha)(1-\gamma |b|) B_{n+1} C_{n+1}^m \sum_{k=n+1}^{\infty} a_k \\ & \leq \gamma |b| + \frac{(1+n\alpha)(1-\gamma |b|) \gamma |b|}{(1+n\alpha)(\gamma |b| - 1) + n + 1} \\ & \leq \frac{\gamma |b| (n+1)}{(1+n\alpha)(\gamma |b| - 1) + n + 1}. \end{aligned} \quad (33)$$

Hence,

$$\begin{aligned} \sum_{k=n+1}^{\infty} k a_k & \leq \frac{\gamma |b| (n+1)}{[(1+n\alpha)(\gamma |b| - 1) + n + 1] B_{n+1} C_{n+1}^m} \\ & = \delta. \end{aligned} \quad (34)$$

Hence, the result follows. $\square$

In similar manner, we establish the following result.

Theorem 10. If

$$\delta = \frac{\gamma |b| (1+n)}{(1+n\alpha) B_{n+1} C_{n+1}^m}, \quad (35)$$

then $B \subset N_{n,\delta}(e)$.

Corollary 11. If

$$\delta = \frac{\gamma |b| (1+n)}{[(1+n\alpha)(\gamma |b| - 1) + n + 1] (n+1)^m}, \quad (36)$$

then $A_1 \subset N_{n,\delta}(e)$.

Corollary 12. If

$$\delta = \frac{\gamma |b| (1+n)}{(1+n\alpha)(n+1)^m}, \quad (37)$$

then $B_1 \subset N_{n,\delta}(e)$.

Corollary 13. If

$$\delta = \frac{\gamma |b| (1+n)}{[(1+n\alpha)(\gamma |b| - 1) + n + 1] C_{n+1}^m \left(\frac{\eta+n}{n}\right)}, \quad (38)$$

then $A_2 \subset N_{n,\delta}(e)$.

Corollary 14. If

$$\delta = \frac{\gamma |b| (1+n)}{(1+n\alpha) C_{n+1}^m \left(\frac{\eta+n}{n}\right)}, \quad (39)$$

then $B_2 \subset N_{n,\delta}(e)$.

4. Neighborhoods for A^σ and B^σ

In this section, we determine the neighborhood properties of A^σ and B^σ . Here, the classes A^σ consist of functions $f \in \mathcal{A}(n)$ for which there exists a function $g(z) \in A$ such that

$$\left| \frac{f(z)}{g(z)} - 1 \right| < 1 - \sigma \quad (z \in \Delta, 0 \leq \sigma < 1). \quad (40)$$

In the same way, we define B^σ , consisting of functions $f(z) \in \mathcal{A}(n)$ for which there exists another function $g(z) \in B$ such that

$$\left| \frac{f(z)}{g(z)} - 1 \right| < 1 - \sigma \quad (z \in \Delta, 0 \leq \sigma < 1). \quad (41)$$

Theorem 15. If $g \in A$ and

$$\begin{aligned} \sigma &= 1 - \frac{\delta}{n+1} \\ &\times \left\{ [(1+n\alpha)(\gamma |b| - 1) + n + 1] \right. \\ &\times B_{n+1} C_{n+1}^m \\ &\times ((1+n\alpha)(\gamma |b| - 1) + n + 1) \\ &\times B_{n+1} C_{n+1}^m - \gamma |b|^{-1} \} \\ &\quad (\gamma |b| > 1), \end{aligned} \quad (42)$$

then $N_{n,\delta}(g) \subset A^\sigma$.

Proof. Suppose $f \in N_{n,\delta}(g)$, then

$$\sum_{k=n+1}^{\infty} k |a_k - b_k| \leq \delta, \quad (43)$$

$$\sum_{k=n+1}^{\infty} |a_k - b_k| \leq \frac{\delta}{n+1}.$$

Since $g \in A$, we have

$$\sum_{k=n+1}^{\infty} b_k \leq \frac{b |k|}{[(1+n\alpha)(\gamma|b|-1)+n+1] B_{n+1} C_{n+1}^m}. \quad (44)$$

Consider

$$\begin{aligned} & \left| \frac{f(z)}{g(z)} - 1 \right| \\ & < \frac{\sum_{k=n+1}^{\infty} |a_k - b_k|}{1 - \sum_{k=n+1}^{\infty} b_k} \\ & \leq \frac{\delta}{n+1} \left\{ \frac{[(1+n\alpha)(\gamma|b|-1)+n+1] B_{n+1} C_{n+1}^m}{[(1+n\alpha)(\gamma|b|-1)+n+1] B_{n+1} C_{n+1}^m - \gamma|b|} \right\} \\ & = 1 - \sigma. \end{aligned} \quad (45)$$

Therefore, $f \in A^\sigma$ for σ given by (42). $\square$

Theorem 16. If $g \in B$ and

$$\sigma = 1 - \frac{\delta}{n+1} \left\{ \frac{(1+n\alpha) B_{n+1} C_{n+1}^m}{(1+n\alpha) B_{n+1} C_{n+1}^m - \gamma|b|} \right\}, \quad (46)$$

then $N_{n,\delta}(g) \subset B^\sigma$.

Corollary 17. If $g \in A_1$ and

$$\begin{aligned} \sigma = 1 - \frac{\delta}{n+1} & \left\{ [(1+n\alpha)(\gamma|b|-1)+n+1] (n+1)^m \right. \\ & \times [(1+n\alpha)(\gamma|b|-1)+n+1] \\ & \times (n+1)^m - \gamma|b|^{-1} \} \\ & (\gamma|b| > 1), \end{aligned} \quad (47)$$

then $N_{n,\delta}(g) \subset A_1^\sigma$.

Corollary 18. If $g \in B_1$ and

$$\sigma = 1 - \frac{\delta}{n+1} \left\{ \frac{(1+n\alpha)(n+1)^m}{(1+n\alpha)(n+1)^m - \gamma|b|} \right\}, \quad (48)$$

then $N_{n,\delta}(g) \subset B_1^\sigma$.

Corollary 19. If $g \in A_2$ and

$$\begin{aligned} \sigma = 1 - \frac{\delta}{n+1} & \left\{ [(1+n\alpha)(\gamma|b|-1)+n+1] \binom{\eta+n}{n} C_{n+1}^m \right. \\ & \times \left([(1+n\alpha)(\gamma|b|-1)+n+1] \right. \\ & \times \left. \left. \binom{\eta+n}{n} C_{n+1}^m - \gamma|b| \right)^{-1} \right\} \\ & (\gamma|b| > 1), \end{aligned} \quad (49)$$

then $N_{n,\delta}(g) \subset A_2^\sigma$.

Corollary 20. If $g \in B_2$ and

$$\sigma = 1 - \frac{\delta}{n+1} \left\{ \frac{(1+n\alpha) \binom{\eta+n}{n} C_{n+1}^m}{(1+n\alpha) \binom{\eta+n}{n} C_{n+1}^m - \gamma|b|} \right\}, \quad (50)$$

then $N_{n,\delta}(g) \subset B_2^\sigma$.

Conflict of Interests

The authors declare that they do not have conflict of interests regarding the publication of this paper.

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