Research Article

Countably QC-Approximating Posets

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Received 17 May 2014; Accepted 21 July 2014; Published 5 August 2014

Academic Editor: Jianming Zhan

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As a generalization of countably C -approximating posets, the concept of countably QC-approximating posets is introduced. With the countably QC-approximating property, some characterizations of generalized completely distributive lattices and generalized countably approximating posets are given. The main results are as follows: (1) a complete lattice is generalized completely distributive if and only if it is countably QC-approximating and weakly generalized countably approximating; (2) a poset L having countably directed joins is generalized countably approximating if and only if the lattice $\sigma_c(L)^{op}$ of all σ -Scott-closed subsets of L is weakly generalized countably approximating.

1. Introduction

The notion of continuous lattices as a model for the semantics of programming languages was introduced by Scott in [1]. Later, a more general notion of continuous directed complete partially ordered sets (i.e., continuous dcpos or domains) was introduced and extensively studied (see [2–4]). Lawson in [4] gave a remarkable characterization that a dcpo L is continuous if and only if the lattice $\sigma(L)^{op}$ of all Scott-closed subsets of L is completely distributive. Gierz et al. in [5] introduced quasicontinuous domains, the most successful generalizations of continuous domains, and proved that quasicontinuous domains equipped with the Scott topology are precisely the spectra of hypercontinuous distributive lattices. Venugopalan in [6] introduced generalized completely distributive (GCD) lattices and Xu in his Ph.D. thesis [7] proved that GCD lattices are precisely the dual of hypercontinuous lattices. Ho and Zhao in [8] introduced the concept of C -continuous lattices. And they showed that for any poset L , $\sigma(L)^{op}$ is a C -continuous lattice and that L is continuous if and only if $\sigma(L)^{op}$ is continuous.

On the other hand, Lee in [9] introduced the concept of countably approximating lattices, a generalization of continuous lattices, and showed that this new larger class has many properties in common with continuous lattices. In [10], Han et al. further generalized the concept of

countably approximating lattices to the concept of countably approximating posets and characterized countably approximating posets via the σ -Scott topology. Yang and Liu in [11] introduced the concept of generalized countably approximating posets, a generalization of countably approximating posets. Making use of the ideas of [8, 10], Mao and Xu in [12] introduced the concept of countably C -approximating posets and showed that the lattice of all σ -Scott-closed subsets of a poset is a countably C -approximating lattice and that a complete lattice is completely distributive if and only if it is countably approximating and countably C -approximating.

In this paper, we generalize the concept of countably C -approximating posets to the concept of countably QC-approximating posets. With the countably QC-approximating property, we present some characterizations of GCD lattices and generalized countably approximating posets.

2. Preliminaries

We quickly recall some basic notions and results (see, e.g., [3, 8] or [11]). Let $(L, \leq)$ be a poset. Then L with the dual order is also a poset and denoted by L^{op} . A *principal ideal* (resp., *principal filter*) is a set of the form $\downarrow x = \{y \in L \mid y \leq x\}$ (resp., $\uparrow x = \{y \in L \mid x \leq y\}$). For $X \subseteq L$, we write $\downarrow X = \{y \in L \mid \exists x \in X, y \leq x\}$, $\uparrow X = \{y \in L \mid \exists x \in$

FIGURE 1: A complete lattice with countably directed sets having maximal elements.

$X, x \leq y\}$. A subset X is a(n) *lower set* (resp., *upper set*) if $X = \downarrow X$ (resp., $X = \uparrow X$). The supremum of X is denoted by $\vee X$ or $\sup X$. A subset D of L is *directed* if every finite subset of D has an upper bound in D . A subset D of L is *countably directed* if every countable subset of D has an upper bound in D . Clearly every (countably) directed set is nonempty, and every countably directed set is directed but not vice versa. A poset L is a *directed complete partially ordered set* (dcpo, in short) if every directed subset of L has a supremum. A poset is said to have *countably directed joins* if every countably directed subset has a supremum.

Remark 1. It is clear that if D is countably directed and itself is countable, then D has a maximal element. By this observation, we see that every countable poset must have countably directed joins and thus a poset having countably directed joins need not be a dcpo.

The following definitions give various induced relations by the order of a poset.

Definition 2 (see [3]). Let L be a poset and $x, y \in L$. We say that x is *way-below* y or x *approximates* y , written $x \ll y$ if whenever D is a directed set that has a supremum $\sup D \geq y$, then there is some $d \in D$ with $x \leq d$. For each $x \in L$, we write $\downarrow x = \{y \in L \mid y \ll x\}$. A poset is said to be *continuous* if every element is the directed supremum of elements that approximate it. A continuous poset which is also a complete lattice is called a *continuous lattice*.

Definition 3 (see [10]). Let L be a poset and $x, y \in L$. We say that x is *countably way-below* y , written $x \ll_c y$ if for any countably directed subset D of L with $\sup D \geq y$, there is some $d \in D$ with $x \leq d$. For each $x \in L$, we write $\downarrow_c x = \{y \in L \mid y \ll_c x\}$ and $\uparrow_c x = \{y \in L \mid x \ll_c y\}$. A poset L having countably directed joins is called a *countably approximating poset* if for each $x \in L$, the set $\downarrow_c x$ is countably directed and $x = \vee \downarrow_c x$. A countably approximating poset which is also a complete lattice is called a *countably approximating lattice*.

In a poset L , it is clear that $x \ll_c y$ implies that $x \leq y$. Since every countably directed set is directed, we have that $x \ll y$ implies $x \ll_c y$ for all $x, y \in L$. In other words, $\downarrow y \subseteq \downarrow_c y$ for each $y \in L$. However, the following example shows that the reverse implication need not be true.

Example 4. Let L be the unit interval $[0, 1]$. For all $x, y \in [0, 1]$, it is easy to check that $x \ll_c y \Leftrightarrow x \leq y$ and that $x \ll y \Leftrightarrow x = 0 = y$ or $x < y$.

By Remark 1, it is clear that every countable poset is a countably approximating poset.

Proposition 5. Let L be a poset and S a countable subset of L such that $\vee S$ exists. If $s \ll_c x$ for all $s \in S$, then $\vee S \ll_c x$.

Proof. Straightforward. $\square$

By Proposition 5, in a complete lattice L , the set $\downarrow_c x$ is automatically countably directed for each $x \in L$. So, a complete lattice L is countably approximating if and only if for each $x \in L$, $x = \vee \downarrow_c x$. Thus every continuous lattice is a countably approximating lattice.

Proposition 6. Let L be a poset. If every countably directed subset of L has a maximal element, then L is a countably approximating poset.

Proof. Straightforward by Definition 3. $\square$

Example 7. Let L be the complete lattice formed by uncountably many incomparable unit intervals $[0, 1]$ with all the 0's being pasted as a $\perp$ and all the 1's being pasted as a $\top$ (See Figure 1). Then it is easy to check that the resulting complete lattice satisfies the condition in Proposition 6 and thus is a countably approximating lattice.

Proposition 8. Let L be a poset. If every countably directed subset of L is countable, then L is a countably approximating poset.

Proof. It is straightforward by Remark 1 and Proposition 6. $\square$

Example 9. If $\mathbb{N}$ with its usual order is augmented with uncountably many incomparable upper bounds, then it is easy to check that the resulting poset satisfies the condition in Proposition 8 and thus is a countably approximating poset.

For a set X , we use $\mathcal{P}(X)$ to denote the power set of X and $\mathcal{P}_{\text{fin}}(X)$ to denote the set of all nonempty finite subsets of X . For a poset L , define a preorder $\leq$ (sometimes called *Smyth preorder*) on $\mathcal{P}(L) \setminus \{\emptyset\}$ by $G \leq H$ if and only if $\uparrow H \subseteq \uparrow G$ for all $G, H \subseteq L$. That is, $G \leq H$ if and only if for each

$y \in H$ there is an element $x \in G$ with $x \leq y$. We say that a nonempty family $\mathcal{F}$ of subsets of L is (countably) *directed* if it is (countably) directed in the Smyth preorder. More precisely, $\mathcal{F}$ is directed if for all $F_1, F_2 \in \mathcal{F}$, there exists $F \in \mathcal{F}$ such that $F_1, F_2 \leq F$; that is, $F \subseteq \uparrow F_1 \cap \uparrow F_2$.

Generalizing the relation $\ll_c$ on points of L to the nonempty subsets of L , one obtains the concept of weakly generalized countably approximating posets.

Definition 10. Let L be a poset having countably directed joins. A binary relation $\ll_c$ on $\mathcal{P}(L) \setminus \{\emptyset\}$ is defined as follows. $A \ll_c B$ if and only if for any countably directed set $D \subseteq L$, $\bigvee D \in \uparrow B$ implies $D \cap \uparrow A \neq \emptyset$. We write $F \ll_c x$ for $F \ll_c \{x\}$ and $y \ll_c H$ for $\{y\} \ll_c H$. If for each $x \in L$, $\uparrow x = \bigcap \{ \uparrow F \mid F \in \omega(x) \}$, where $\omega(x) = \{F \mid F \in \mathcal{P}_{\text{fin}}(L) \text{ and } F \ll_c x\}$, then L is called a *weakly generalized countably approximating poset*. A weakly generalized countably approximating poset which is also a complete lattice is called a *weakly generalized countably approximating lattice*.

A weakly generalized countably approximating poset (lattice) L with the condition that for each $x \in L$, $\omega(x)$ is countably directed is called a *generalized countably approximating poset (lattice)* in [11].

As a generalization of completely distributive lattice, the following concept of GCD lattices was introduced in [6].

Definition 11 (see [6]). Let L be a poset. A binary relation $\triangleleft$ on $\mathcal{P}(L)$ is defined as follows. $A \triangleleft B$ if and only if whenever S is a subset of L for which $\bigvee S$ exists, $\bigvee S \in \uparrow B$ implies $S \cap \uparrow A \neq \emptyset$. A complete lattice L is called a *generalized completely distributive lattice* or shortly a *GCD lattice*, if and only if for all $x \in L$, $\uparrow x = \bigcap \{ \uparrow F \mid F \in \mathcal{P}_{\text{fin}}(L) \text{ and } F \triangleleft x \}$.

Definition 12 (see [3]). A subset U of a poset L is *Scott-open* if $\uparrow U = U$ and for any directed set $D \subseteq L$, $\sup D \in U$ implies $U \cap D \neq \emptyset$. All the Scott-open sets of L form a topology, called the *Scott topology* and denoted by $\sigma(L)$. The complement of a Scott-open set is called a *Scott-closed set*. The collection of all Scott-closed sets of L is denoted by $\sigma(L)^{op}$. The topology on L generated by $\{L \setminus \downarrow x \mid x \in L\}$ as a subbase is called the *upper topology* and denoted by $\nu(L)$.

Replacing directed sets with countably directed sets in Definition 12, we can get the concept of σ -Scott-open sets.

Definition 13 (see [10]). Let L be a poset. A subset U of L is called *σ -Scott-open* if $\uparrow U = U$ and for any countably directed set $D \subseteq L$, $\sup D \in U$ implies $U \cap D \neq \emptyset$. All the σ -Scott-open sets of L form a topology, called the *σ -Scott topology* and denoted by $\sigma_c(L)$. The complement of a σ -Scott-open set is called a *σ -Scott-closed set*. The collection of all σ -Scott-closed sets of L is denoted by $\sigma_c(L)^{op}$.

Remark 14 (see [10], Remark 2.1). (1) For a poset L , the σ -Scott topology $\sigma_c(L)$ is closed under countably intersections and the Scott topology $\sigma(L)$ is coarser than $\sigma_c(L)$; that is, $\sigma(L) \subseteq \sigma_c(L)$.

(2) A subset of a poset is σ -Scott-closed if and only if it is a lower set and closed under countably directed joins.

To study the order structure of the lattice of all σ -Scott-closed subsets for a poset, Mao and Xu in [12] introduced the concept of countably C -approximating posets.

Definition 15 (see [12]). Let L be a poset and $x, y \in L$. We say that x is σ -beneath y , denoted by $x <_\sigma y$, if for any nonempty σ -Scott-closed set $F \subseteq L$ for which $\bigvee F$ exists, $\bigvee F \geq y$ always implies that $x \in F$. Poset L is said to be *countably C -approximating* if for each $x \in L$, $x = \bigvee \downarrow^<_\sigma x$, where $\downarrow^<_\sigma x = \{y \in L \mid y <_\sigma x\}$. A complete lattice which is also countably C -approximating is called a *countably C -approximating lattice*.

Lemma 16 (see [12]). For a poset L , the lattice $\sigma_c(L)^{op}$ is countably C -approximating.

Proof. Let L be a poset and $\mathcal{C} \in \sigma_c(\sigma_c(L)^{op})^{op}$. It is straightforward to check that $\bigvee_{\sigma_c(L)^{op}} \mathcal{C} = \bigcup \mathcal{C}$. For each $F \in \sigma_c(L)^{op}$, we have that $F = \bigvee_{\sigma_c(L)^{op}} \{\downarrow x \mid x \in F\}$. Suppose $\mathcal{C} \in \sigma_c(\sigma_c(L)^{op})^{op}$ with $\bigvee_{\sigma_c(L)^{op}} \mathcal{C} \supseteq F$. Then for each $x \in F$, since $\bigvee_{\sigma_c(L)^{op}} \mathcal{C} = \bigcup \mathcal{C} \supseteq F$, there exists $A \in \mathcal{C}$ such that $x \in A$. Noticing that $A \in \sigma_c(L)^{op}$ is a lower set, we have $\downarrow x \subseteq A \in \mathcal{C}$. It follows from $\mathcal{C} \in \sigma_c(\sigma_c(L)^{op})^{op}$ being a lower set that $\downarrow x \in \mathcal{C}$. Thus by Definition 15, $\downarrow x <_\sigma F$ holds in $\sigma_c(L)^{op}$. Hence, $F = \bigvee_{\sigma_c(L)^{op}} \{\downarrow x \mid x \in F\} \subseteq \bigvee \downarrow^<_\sigma F \subseteq F$. So, $F = \bigvee \downarrow^<_\sigma F$ and by the arbitrariness of $F \in \sigma_c(L)^{op}$, we conclude that $\sigma_c(L)^{op}$ is countably C -approximating. $\square$

3. Countably QC-Approximating Posets

In this section, we introduce the concept of countably QC-approximating posets. Firstly, we generalize the relation $<_\sigma$ on points of a poset L to the nonempty subsets of L .

Definition 17. For a poset L , the σ -beneath relation $<_\sigma$ on nonempty subsets of L is defined as follows: $A <_\sigma B$ if and only if whenever S is a nonempty σ -Scott-closed subset of L for which $\bigvee S$ exists, $\bigvee S \in \uparrow B$ implies $S \cap \uparrow A \neq \emptyset$. We write $F <_\sigma x$ for $F <_\sigma \{x\}$. Set $c(x) = \{F \mid F \in \mathcal{P}_{\text{fin}}(L) \text{ and } F <_\sigma x\}$.

The next proposition is basic and the proof is omitted.

Proposition 18. Let L be a poset. Then

- (i) $\forall G, H \subseteq L, G <_\sigma H \Rightarrow G \leq H$;
- (ii) $\forall G, H \subseteq L, G <_\sigma H \Leftrightarrow \forall h \in H, G <_\sigma h$;
- (iii) $\forall E, F, G, H \subseteq L, E \leq G <_\sigma H \leq F \Rightarrow E <_\sigma F$;
- (iv) $\forall x, y \in L, \{x\} <_\sigma \{y\} \Leftrightarrow x <_\sigma y$.

With the relation $<_\sigma$, we have the concept of countably QC-approximating posets.

Definition 19. A poset L is said to be *countably quasi- C -approximating*, shortly *countably QC-approximating*, if for all

$x \in L$, $\uparrow x = \cap\{\uparrow F \mid F \in c(x)\}$. A countably QC-approximating poset which is also a complete lattice is called a *countably QC-approximating lattice*.

Proposition 20. *Countably C-approximating posets are countably QC-approximating.*

Proof. Let L be a countably C-approximating poset. Then for all $x \in L$,

$$\begin{aligned} \uparrow x &\subseteq \cap\{\uparrow F \mid F \in c(x)\} \\ &= \cap\{\uparrow y \mid y <_{\sigma} x\} \cap \{\uparrow F' \mid F' \in c(x)\} \\ &\subseteq \cap\{\uparrow y \mid y <_{\sigma} x\} = \uparrow x. \end{aligned} \quad (1)$$

Thus $\cap\{\uparrow F \mid F \in c(x)\} = \uparrow x$. By Definition 19, L is countably QC-approximating. $\square$

By Lemma 16 and Proposition 20, we immediately have the following.

Corollary 21. *For any poset L , the lattice $\sigma_c(L)^{op}$ is countably QC-approximating.*

In the sequel, we explore relationships between countably QC-approximating lattices and GCD lattices.

Proposition 22. *Every GCD lattice is weakly generalized countably approximating.*

Proof. Let L be a GCD lattice. For all $x \in L$ and $F \in \mathcal{P}_{fin}(L)$, $F \triangleleft x$ implies $F \ll_c x$. Then $\uparrow x \subseteq \cap\{\uparrow F \mid F \in \omega(x)\} \subseteq \cap\{\uparrow F \mid F \in \mathcal{P}_{fin}(L) \text{ and } F \triangleleft x\} = \uparrow x$. So $\uparrow x = \cap\{\uparrow F \mid F \in \omega(x)\}$. By Definition 10, L is weakly generalized countably approximating. $\square$

Proposition 23. *Every GCD lattice is countably QC-approximating.*

Proof. Let L be a GCD lattice. For each $x \in L$ and $F \in \mathcal{P}_{fin}(L)$, $F \triangleleft x$ implies $F \ll_{\sigma} x$. Then $\uparrow x \subseteq \cap\{\uparrow F \mid F \in c(x)\} \subseteq \cap\{\uparrow F \mid F \in \mathcal{P}_{fin}(L) \text{ and } F \triangleleft x\} = \uparrow x$. Thus $\uparrow x = \cap\{\uparrow F \mid F \in c(x)\}$. By Definition 19, L is countably QC-approximating. $\square$

The following theorem characterizes GCD lattices.

Theorem 24. *Let L be a complete lattice. Then the following statements are equivalent:*

- (1) L is a GCD lattice;
- (2) L is countably QC-approximating and weakly generalized countably approximating.

Proof. (1) $\Rightarrow$ (2): follows from Propositions 22 and 23.

(2) $\Rightarrow$ (1): suppose that L is countably QC-approximating and weakly generalized countably approximating. Then for each $x \in L$, by the weakly generalized countably approximating property of L , we have $\uparrow x = \cap\{\uparrow F \mid F \in \omega(x)\}$.

Now for each $F \in \omega(x)$, we show that $\uparrow F = \cap\{\uparrow F' \mid F' \in \mathcal{P}_{fin}(L) \text{ and } F' \ll_{\sigma} F\}$. To this end, it suffices to show that $\cap\{\uparrow F' \mid F' \in \mathcal{P}_{fin}(L) \text{ and } F' \ll_{\sigma} F\} \subseteq \uparrow F$. Suppose $t \in \cap\{\uparrow F' \mid F' \in \mathcal{P}_{fin}(L) \text{ and } F' \ll_{\sigma} F\}$ and $t \notin \uparrow F$. Then for any $y_F \in F$, $t \notin y_F$. By the countably QC-approximating property of L , there exists $F_{y_F} \in c(y_F)$ such that $F_{y_F} \ll_{\sigma} y_F$ and $t \notin \uparrow F_{y_F}$. Let $\bar{F} = \bigcup_{y_F \in F} F_{y_F}$. Then $\bar{F}$ is still finite and $\bar{F} \ll_{\sigma} F$. It is clear that $t \notin \bar{F}$, contradicting to that $t \in \cap\{\uparrow F' \mid F' \in \mathcal{P}_{fin}(L) \text{ and } F' \ll_{\sigma} F\}$. Thus $\uparrow x = \cap\{\uparrow F \mid F \in \omega(x)\} = \cap\{\uparrow F' \mid F' \in \mathcal{P}_{fin}(L), \exists F \in \mathcal{P}_{fin}(L) \text{ such that } F' \ll_{\sigma} F \ll_c x\}$.

Suppose $F' \ll_{\sigma} F \ll_c x$, we will show that $F' \triangleleft x$. For any $A \subseteq L$ with $\forall A \geq x$, let $G = \{\downarrow E \mid E \text{ is a countable subset of } A\}$. Then G is a countably directed set and $\forall G = \forall A \in \uparrow x$. Since $F \ll_c x$, there exists a countable subset $E \subseteq A$ such that $\forall E = \downarrow E \in \uparrow F$. By Remark 14 (1), $\downarrow E$ is σ -Scott-closed. It follows from $F' \ll_{\sigma} F$ that $\downarrow E \cap \uparrow F' \neq \emptyset$. This implies $A \cap \uparrow F' \neq \emptyset$, showing that $F' \triangleleft x$. Thus, $\uparrow x \subseteq \cap\{\uparrow W \mid W \in \mathcal{P}_{fin}(L), W \triangleleft x\} \subseteq \cap\{\uparrow F' \mid F' \in \mathcal{P}_{fin}(L), \exists F \in \mathcal{P}_{fin}(L), F' \ll_{\sigma} F \ll_c x\} = \uparrow x$. So, $\uparrow x = \cap\{\uparrow W \mid W \in \mathcal{P}_{fin}(L) \text{ and } W \triangleleft x\}$. Therefore, L is a GCD lattice. $\square$

Recall that a poset L is called a *hypercontinuous poset* (see [13]) if for all $x \in L$, the set $\{y \in L \mid y <_{\nu(L)} x\}$ is directed and $x = \sup\{y \in L \mid y <_{\nu(L)} x\}$, where $y <_{\nu(L)} x \Leftrightarrow x \in \text{int}_{\nu(L)} \uparrow y$. A hypercontinuous poset which is also a complete lattice is called a *hypercontinuous lattice*.

Lemma 25 (see [7], Theorem 4.1.4). *Let L be a complete lattice. Then L is a GCD lattice if and only if L^{op} is a hypercontinuous lattice.*

It is easy to see that for a finite lattice L , both L and L^{op} are continuous, and $\nu(L) = \sigma(L)$. It follows from ([14], Theorem 2.1) that L and L^{op} are hypercontinuous lattices; hence by Lemma 25, L^{op} and L are GCD lattices. By this observation, we see that every finite lattice is a countably QC-approximating lattice. So, countably QC-approximating lattices need not be distributive.

It is known from Proposition 4.1 in [12] that any countably C-approximating lattice is distributive. So, countably QC-approximating lattices need not be countably C-approximating.

Lemma 26 (see [11], Theorem 3.4). *Let L be a poset having countably directed joins. Then L is generalized countably approximating if and only if the lattice $\sigma_c(L)$ is hypercontinuous.*

So, in view of Lemma 25, a poset having countably directed joins is generalized countably approximating if and only if the lattice $\sigma_c(L)^{op}$ is a GCD lattice. The following theorem gives comprehensive characterizations of generalized countably approximating posets.

Theorem 27. *Let L be a poset having countably directed joins. Then the following statements are equivalent:*

- (i) L is a generalized countably approximating poset;

- (ii) $\sigma_c(L)$ is a hypercontinuous lattice;
- (iii) $\sigma_c(L)^{op}$ is a GCD lattice;
- (iv) $\sigma_c(L)^{op}$ is a weakly generalized countably approximating lattice.

Proof. (i) $\Leftrightarrow$ (ii) by Lemma 26.

(ii) $\Leftrightarrow$ (iii) by Lemma 25.

(iii) $\Leftrightarrow$ (iv) follows from Theorem 24 and Corollary 21. $\square$

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

Acknowledgments

The authors are grateful to the anonymous reviewers for their valuable comments and helpful suggestions. This work is supported by NSF of china (11101212 and 61103018).

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