Research Article
The Modelled Raindrop Size Distribution of Skudai, Peninsular Malaysia, Using Exponential and Lognormal Distributions

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This paper presents the modelled raindrop size parameters in Skudai region of the Johor Bahru, western Malaysia. Presently, there is no model to forecast the characteristics of DSD in Malaysia, and this has an underpinning implication on wet weather pollution predictions. The climate of Skudai exhibits local variability in regional scale. This study established five different parametric expressions describing the rain rate of Skudai; these models are idiosyncratic to the climate of the region. Sophisticated equipment that converts sound to a relevant raindrop diameter is often too expensive and its cost sometimes overrides its attractiveness. In this study, a physical low-cost method was used to record the DSD of the study area. The Kaplan-Meier method was used to test the aptness of the data to exponential and lognormal distributions, which were subsequently used to formulate the parameterisation of the distributions. This research abrogates the concept of exclusive occurrence of convective storm in tropical regions and presented a new insight into their concurrence appearance.

1. Introduction

Rain event is normally an expression of varied composition of raindrops diameters as a function of their volumetric diameters per unit volume of space [1]. Raindrop size distribution (DSD) defines the variation in the composition of different raindrop sizes (diameters) within a storm and could be used as a tool for classifying rain events [2, 3]. The rainfalls in temperate climatic zones are composed of small to average size drops in contrast to tropical zones, which are composed of higher proportions of larger raindrops, typically from short-duration high-intensity storms [4]. In the field of communication, rainfalls are categorised into three categories: the drizzle, showers, and thunderstorms. In communication field, DSD is one of the major sources of error in any DSD model because of its temporal and spatial variation between geoclimatic regions [5]. However, in the field of hydrology, rainfalls are categorised into two categories based on their physical processes: the convective and the stratiform. Majority of the established DSD functions used disdrometer recordings [5–9]. However, disdrometer was found to be biased towards larger raindrops by underestimated smaller drops [10].

Although studies of DSD were carried out in other tropical regions [3, 6, 7, 11–15], to the best of our knowledge, there was no formulation or model representing the characteristics of DSD in Malaysia. The climate of Malaysia is more local than regional [16] and it is distinctive in its characteristics and often cannot be subjected to similarity to other regions. The previous research of DSD using data obtained from Kuala Lumpur by Lam [17] focused on investigating the dependence of the rain attenuation on the DSD and finding the key raindrop diameter for computing specific rain attenuation, rather than establishing the relationships of DSD in the region with defined equation’s fittings and coefficients. This lack of established relations could limit further studies in the region. Therefore, the aim of this study is to establish DSD relations based on the region’s rainfall characteristics that could provide a tool for modelling urban hydrological
process, flood appraisal, and prediction of rain attenuation with ease.

Techniques used to measure raindrop diameter, and its distribution, can broadly be classified into two: the automatic equipment and the manual methods. The absorbent paper method devised by Lowe [18] and documented by Wiesner [19], the flour pellet method developed by Bentley [20] which was modified by Laws and Parsons [21], and the oil immersion method fashioned by Eigil and Moore [22] are examples of the latter category, while disdrometer, an electromechanical feeder that translates the momentum of falling raindrop into electrical recordable pulses developed by Joss and Waldvogel [23], and photography method developed by Jones [24] and advanced into the Optical-Spectropluviometer are examples of the former category. Most researchers use acoustic instrument to measure raindrop diameter and its distribution [5, 14, 25–27]. However, this equipment is known to underestimate small raindrop's diameters [10]. In this research, we used unbiased method to bin each raindrop size appropriately to its size by sieving method.

A raindrop breaks into smaller diameters when it reaches its limiting size of about 5 mm to 8 mm [28–30]. The breaking up of raindrop size after reaching a threshold value of 5 mm suggests that DSD follows the form of an exponential function at higher intensity. Lenard [31] was the first to study the breakup of water drops based on separation of electric charges principle. Raindrop breaks because of the induced aerodynamics of resisting air acting on the centre mass of the drop and other reasons such as collision between drops. A raindrop exhibits a complicated shape; however, it forms an almost perfect sphere at small diameters less than 1.25 mm and is flattened at the bottom due to resisting air pressure forming an oblate spheroid shape at larger diameters [29, 32, 33]. At about 10 mm, the hydrodynamic forces overcome the internal binding forces causing air forces to cause a break of the drop into smaller sizes [12]. Recently, Villermaux and Bossa [30] investigated both the shape of the drops' sizes and their distribution and concluded that the DSD parameters are related to the dynamics of the whole spectrum of sizes observed in rain. They further highlighted that topological withering in raindrops from big to smaller unwavering sizes is attained within a much shorter timescale than the typical collision time between the drops.

2. Materials and Methods

2.1. Study Area. The map of the study area is shown in Figure 1. Skudai is located within west Peninsular Malaysia which boarders Thailand to the north and stretched southward to Singapore. It lies between 6°45' and 1°20'N latitudes and 99°40' and 104°20'E longitudes. Skudai lies within the Intertropical Convergence Zone (ITCZ) which results in elevated temperature and high humidity [34]. Skudai experiences an average of 2000 to 2500 mm rainfall depth annually [35]. The study area is characterised by a monthly regular uniform rain distribution dominated by convective storms [34, 35].

![Figure 1: Location map of the study area.](image-url)
Table 1: Sampled storm profile.

<table>
<thead>
<tr>
<th>Sampling date</th>
<th>Duration (minutes)</th>
<th>Average storm intensity (mm hr(^{-1}))</th>
<th>Number of samples</th>
</tr>
</thead>
<tbody>
<tr>
<td>09/10/12</td>
<td>120</td>
<td>35</td>
<td>2</td>
</tr>
<tr>
<td>01/11/12</td>
<td>60</td>
<td>12</td>
<td>7</td>
</tr>
<tr>
<td>27/6/13</td>
<td>33</td>
<td>12</td>
<td>1</td>
</tr>
<tr>
<td>16/7/13</td>
<td>47</td>
<td>16</td>
<td>1</td>
</tr>
<tr>
<td>18/08/13</td>
<td>22</td>
<td>29</td>
<td>1</td>
</tr>
<tr>
<td>24/08/13</td>
<td>15</td>
<td>8</td>
<td>1</td>
</tr>
<tr>
<td>26/08/13</td>
<td>110</td>
<td>65</td>
<td>1</td>
</tr>
<tr>
<td>03/10/13</td>
<td>56</td>
<td>32</td>
<td>1</td>
</tr>
<tr>
<td>05/10/13</td>
<td>13</td>
<td>21</td>
<td>1</td>
</tr>
<tr>
<td>12/10/13</td>
<td>34</td>
<td>23</td>
<td>1</td>
</tr>
</tbody>
</table>

2.3. Parameterization of Drop Size Distributions. Raindrop distribution can be estimated from exponential equation suggested by Marshall and Palmer [36] of the form shown in

\[ N(D) = N_0 e^{-\lambda D}, \]  

where \( D \) is the drop diameter (mm), \( N_0 \) is a constant (8000 m\(^{-3}\) mm\(^{-1}\)) that corresponds to \( N(D = 0) \), and \( \lambda \) is a parameter (mm\(^{-1}\)) that depends on the rainfall intensity \( I \)

\[ \lambda = c I^{-d}, \]  

where \( c = 4.1 \) and \( d = 0.21 \). It is well established in the literature that the DSD follows gamma distribution [14, 37] which is an improved form of the Marshall and Palmer [36] equation proposed by Ulbrich [38] of the form shown in

\[ N(D) = N_0 D^{\mu} e^{-\lambda D}, \]  

where \( \mu \) is a dimensionless shape coefficient.

The basic difference between (1) proposed by Marshall and Palmer [36] and (3) by Ulbrich [38] is the number of parameters. The lognormal model, shown in the form of (4), has advantages over other functions, because all of its three parameters have physical significance and the parameters have linear relations to the moment of DSD [5]

\[ N(D) = \frac{N_0 D^\mu}{\sigma D \sqrt{2\pi}} \exp \left[ -\frac{(\ln D - \mu)^2}{2\sigma^2} \right], \]  

where \( N(D) \) is the number of densities (m\(^{-3}\) mm\(^{-1}\)), \( N_0 \) is the drops count (m\(^{-3}\)), and \( \mu \) and \( \sigma \) are the logarithmized mean and standard deviation of the drop diameters and could be obtained from (5) and (6), respectively. Consider

\[ \sigma = \sqrt{\ln \left( \frac{1 + \frac{V_r}{m^2}}{m^2} \right)}; \]  

\[ \mu = \ln \left( \frac{m^2}{\sqrt{V_r} + m^2} \right); \]  

\( m \) and \( V_r \) are the mean and variance of nonlogarithmized values in the measured samples obtained from method of moment. The third, fourth, and sixth moments were used for estimating the parameters.

3. Results and Discussion

The quantile-quantile (Q-Q) and probability plots were respectively used to test whether our study data follows the exponential and lognormal distributions. The Kaplan-Meier method was used for the survival analyses. The Q-Q and probability plots for the median drop diameter for each intensity were presented in Figures 3 and 4, respectively. The figures indicated the veracity of our study data to suit the lognormal and exponential distributions. Figure 3 presents the exponential Q-Q plot showing the studied raindrop sizes values on x-axis and their expected values on y-axis, while Figure 4 is the lognormal probability plot which is a plot showing the observed cumulative percentage of raindrop sizes on x-axis and their expected cumulative percentiles on y-axis.

The difference between the two figures is the representation of the values in percentiles in Figure 4 instead of their real values as shown in Figure 3. The closer the scattered points
are to the expected value line the stronger the indication that it follows the given distribution. Therefore, subject to this survival test, the method of moment was used to estimate the real mean and standard deviation of (5) and (6) which were used in defining scale and shape parameters in (4).

The exponential $N_0$ parameter in (1) was estimated from regression analysis of our data after eight iterations and was found to be 7627 counts. The $\lambda$ parameter depends on intensity [3, 14]. The $\lambda$-rain rate relationship of Skudai climatic region was established from our data by individually fitting the intensity with the corresponding $\lambda$ value from (2). This resulted in the following:

$$\lambda = 3.31^{-0.25}. \quad (7)$$

Brodie and Rosewell [1] summarised $N_0$ obtained from different studies. They noted that $N_0$ varies from 1400 counts for thunderstorm to a maximum value of 30000 for drizzle and 7000 for widespread rain. Coutinho and Tomás [3] reported $N_0$ counts higher than 15500 and a least of 3900 counts from their studies which was composed of similar upper and lower rain intensities considered in this study. $N_0$ values can be used to classify storm classes based on their metrological nature. According to Waldvogel [2], $N_0$ less than 2000 signifies convective storm (where the weighted composition of DSD leans towards disposition of larger drops than smaller drops), while $N_0$ in excess of 20000 implies stratiform storm (where the continuum balance between larger and smaller drops swings towards small drops). The storms considered in this study have spatial representation. The storms are composed of rain intensities less than 35 mm h$^{-1}$ with only two of the intensities higher than 70 mm h$^{-1}$. The $N_0$ obtained from this study supported the description of the study area rainfall pattern by Zin et al. [39] and Shamsudin and Dan’azumi [34]. Likewise, the summarised result from Brodie and Rosewell [1] recorded the $c$ coefficient in (2) between 3.0 and 5.8 while the $d$ coefficients range between 0.20 and 0.21 units.

Figure 5 shows the modelled rain rate parameter for the seventeen different rain intensities; the coefficient of determination obtained from the fit was 0.56. $\lambda$ decreases with increasing rain intensity. The $\lambda$-intensity relationship suggests that smaller rain intensities are composed of smaller but higher counts of raindrops, while higher intensities have the least collection of raindrops number but with larger drops diameters.

Figure 6 presents the characteristics of Skudai DSD. The fitted lines conform to (1) and (4), respectively, for exponential and lognormal models with $R^2$ of 0.72 and 0.64, respectively. The exponential and lognormal models in Figure 6 represented the range of data obtained from this study, with the lognormal model tending to underestimate the DSD of smaller intensities at drop diameters smaller than 4 mm, while the exponential model predisposed to the bigger diameters at moderate to higher intensities.

Both the lognormal and the exponential models show consistent trends at drop diameter of less than 3.3 mm.
The result also shows that higher rain intensities are composed of larger proportions of raindrop diameters than lighter intensities. The rain intensity in Skudai is considerably composed of raindrop diameters of less than 4 mm in large part. Taking into cognisance the \( N_0 \) count obtained from this study, the study area could be characterised by combined convective and stratiform widespread uniform rainfall. Similar occurrence of convective storm and stratiform in tropical regions has been reported in geographical regions of western equatorial Pacific and northern Australia [40].

The exponential DSD model obtained from this study is compared with Marshall and Palmer [36] model at intensities of 4 mm h\(^{-1}\) and 25 mm h\(^{-1}\) as shown in Figure 7. The results of the two models are in agreement with maximum divergence at higher raindrop diameter. Thus, the models compare more than 70% at drop diameters of 4 mm or less. This is quite expected as the maximum drop diameter considered by Marshall and Palmer [36] is in the order of 4 mm. But both models approached a common value as the rain diameters approach zero. The model converges at \( N_0 = 7627 \) corresponding to \( N(D=0) \) for all rain intensities considered in this study.

The lognormal parameters of the DSD obtained in (4) were related to the intensity using regression analysis on the data. \( N_D, \sigma, \) and \( \mu \) are known to relate to the intensity [5]. These parameters relate to the intensity of the region as presented in

\[
N_D = 763 I^{0.69}, \quad \sigma = 0.31 \ln(I) - 0.44, \quad (8) \mu = 2.34 - 2.00 \ln(I). \]

These modelled parameters and their relationship with intensity are presented in Figures 8–10.
Figures 8 and 9 indicated that \( N_D \) and \( \sigma \) increase with increasing rain rate while Figure 10 suggests decrease of \( \mu \) with increasing intensity. Timothy et al. [5] suggested that the three modelled lognormal parameters are not sufficient to describe the rain rate based on their data. The result of this study, however, indicated that the \( \sigma \) and \( \mu \) relationships in Figures 8 and 9 are sufficient to describe rain rate especially at intensity of less than 40 mm h\(^{-1}\). But the \( N_D \) relationship with intensity presented in Figure 8 suggests that the \( N_D \) depends not only on the rain rate but also on other climatic parameters like the rain type.

A differentiation between the convective and stratiform storms is very valuable in the tropics and in mid-scoipes in the warm season of other geographies, as condensation peaks during the latent heat liberation in troposphere zones of stratiform precipitation. Therefore, a combined model of the exponential and the lognormal distributions could describe tropical storm of both convective and stratiform storms in more appropriate manner than using a single model.

4. Conclusions

Five different parametric expressions describing the rain rate of Skudai were established from this study. The exponential and the lognormal models were used to describe the DSD of the study area. The parameters of these models were empirically instituted from the experimental result using regression analysis on the data. The modelled rain rate and the drop count per unit volume of rain obtained from this study infer that the study area experiences uniform precipitation. This research also demonstrated that the convective storm in the tropical region of Skudai occurred concurrently with stratiform storm.

The advantage of using more than one model to predict storm behaviour has been put forward in this study. The results of the two models are in agreement, with a maximum divergence at a higher raindrop diameter. The lognormal model tends to underestimate the DSD of lighter rain intensities at drop diameters smaller than 4 mm, while the exponential model was predisposed to the bigger diameters at moderate to severe rain intensities. The \( \lambda \)-intensity relationship suggested that lighter rain intensities were composed of smaller but higher counts of raindrops, while higher intensities have the least collection of the raindrop’s numbers but with larger drop’s diameters. The result from this study indicated that the \( \sigma \) and \( \mu \) relationships from the lognormal model are sufficient to describe rain rate, especially at intensities of less than 40 mm h\(^{-1}\). But the \( N_D \) relationship with intensity suggested that the drop counts depend not only upon the rain rate but also on other climatic parameters.

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

References


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