Research Article

Fuzzy Γ -Hyperideals in Γ -Hypersemirings by Using Triangular Norms

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The concept of Γ -semihyperring was introduced by Dehkordi and Davvaz as a generalization of semirings, semihyperring, and Γ -semiring. In this paper, by using the notion of triangular norms, we define the concept of triangular fuzzy sub- Γ -semihyperring as well as triangular fuzzy Γ -hyperideals of a Γ -semihyperring, and we study a few results in this respect.

1. Introduction

In [1], Nobusawa introduced Γ -rings as a generalization of ternary rings. Let M be an additive group whose elements are denoted by $a, b, c, \dots$ and Γ another additive group whose elements are $\gamma, \beta, \alpha, \dots$. Suppose that $a\gamma b$ is defined to be an element of M and that $\gamma a\beta$ is defined to be an element of Γ for every a, b, γ , and β . If the products satisfy the following three conditions: (1) $(a_1 + a_2)\gamma b = a_1\gamma b + a_2\gamma b$, $a(\gamma_1 + \gamma_2)b = a\gamma_1 b + a\gamma_2 b$, $a\gamma(b_1 + b_2) = a\gamma b_1 + a\gamma b_2$; (2) $(a\gamma b)\beta c = a\gamma(b\beta c) = a(\gamma\beta c)$; (3) if $a\gamma b = 0$ for any a and b in M , then $\gamma = 0$; then M is called a Γ -ring in the sense of Nobusawa [1]. Barnes [2] weakened slightly the conditions in the definition of Γ -ring and gave a new definition of a Γ -ring. Let M and Γ be two additive abelian groups. Suppose that there is a mapping from $M \times \Gamma \times M \rightarrow M$ (sending (a, γ, b) into $a\gamma b$ such that (1) $(a_1 + a_2)\gamma b = a_1\gamma b + a_2\gamma b$, $a(\gamma_1 + \gamma_2)b = a\gamma_1 b + a\gamma_2 b$, $a\gamma(b_1 + b_2) = a\gamma b_1 + a\gamma b_2$; (2) $(a\gamma b)\beta c = a\gamma(b\beta c)$; then M is called a Γ -ring in the sense of Barnes [2]. Nowadays, Γ -rings mean the Γ -rings due to Barnes and other Γ -rings are known as Γ_N -rings, that is, gamma rings in the sense of Nobusawa. Barnes [2], Luh [3], and Kyuno [4] studied the structure of Γ -rings and obtained various generalization analogous to corresponding parts in ring theory. The notion of Γ -semirings was introduced by Rao [5] as a generalization of semirings as well as Γ -rings. Subsequently, by introducing

the notion of operator semirings of a Γ -semiring, Dutta and Sardar [6] enriched the theory of Γ -semirings. Algebraic hyperstructures represent a natural extension of classical algebraic structures and they were introduced by the French mathematician Marty [7]. Algebraic hyperstructures are a suitable generalization of classical algebraic structures. In a classical algebraic structure, the composition of two elements is an element, while in an algebraic structure, the composition of two elements is a set. Since then, hundreds of papers and several books have been written on this topic, for example, see [8–10]. In [11, 12], Dehkordi and Davvaz studied the notion of a Γ -semihyperring as a generalization of semiring, semihyperring, and Γ -semiring.

Fuzzy sets are sets whose elements have degrees of membership. Fuzzy sets have been introduced by Zadeh (1965) as an extension of the classical notion of sets [13]. Let X be a set. A fuzzy subset A of X is characterized by a membership function $\mu_A : X \rightarrow [0, 1]$ which associates with each point $x \in X$ its grade or degree of membership $\mu_\mu(x) \in [0, 1]$. Fuzzy sets generalize classical sets since the characteristic functions of classical sets are special cases of the membership functions of fuzzy sets, if the latter only take values 0 or 1. After the introduction of fuzzy sets by Zadeh, reconsideration of the concept of classical mathematics began. In 1971, Rosenfeld [14] introduced fuzzy sets in the context of group theory and formulated

the concept of a fuzzy subgroup of a group. Das characterized fuzzy subgroups by their level of subgroups in [15], since then many notions of fuzzy group theory can be equivalently characterized with the help of notion of level subgroups. The concept of a fuzzy ideal of a ring was introduced by Liu [16]. In 1992, Jun and Lee [17] introduced the notion of fuzzy ideals in Γ -rings and studied a few properties. In [6], Dutta and Sardar studied the structures of fuzzy ideals of Γ -rings. Also, see [18]. The study of fuzzy hyperstructures is an interesting research topic of fuzzy sets. There is a considerable amount of work on the connections between fuzzy sets and hyperstructures. In [19], Davvaz introduced the concept of fuzzy H_γ -ideals of H_γ -rings. Then, this concept was studied in depth in several papers, for example, see [20]. Also, see [21]. In [22, 23], Ersoy and Davvaz investigated some properties of fuzzy Γ -hyperideals of Γ -semihyperring. Now, in this paper, we define the concept of triangular fuzzy sub- Γ -semihyperrings and fuzzy Γ -hyperideals of a Γ -semihyperring by using triangular norms, and we study a few results in this respect.

2. Basic Concepts

Let H be a nonempty set and let $\wp^*(H)$ be the set of all nonempty subsets of H . A hyperoperation on H is a map $\circ : H \times H \rightarrow \wp^*(H)$ and the couple $(H, \circ)$ is called a hypergroupoid. If A and B are nonempty subsets of H , then we denote $A \circ B = \bigcup_{a \in A, b \in B} a \circ b$, $x \circ A = \{x\} \circ A$ and $A \circ x = A \circ \{x\}$. A hypergroupoid $(H, \circ)$ is called a semihypergroup if for all x, y, z of H we have $(x \circ y) \circ z = x \circ (y \circ z)$, which means that $\bigcup_{u \in x \circ y} u \circ z = \bigcup_{v \in y \circ z} x \circ v$. A semihyperring is an algebraic structure $(R, +, \cdot)$ which satisfies the following properties: (1) $(R, +)$ is a commutative semihypergroup; that is, $(x + y) + z = x + (y + z)$ and $x + y = y + x$ for all $x, y, z \in R$; (2) $(R, \cdot)$ is a semihypergroup; (3) the hyperoperation $\cdot$ is distributive with respect to the hyperoperation $+$; that is, $x \cdot (y + z) = x \cdot y + x \cdot z$, $(x + y) \cdot z = x \cdot z + y \cdot z$ for all $x, y, z \in R$; (4) the element $0 \in R$ is an absorbing element; that is $x \cdot 0 = 0 \cdot x = 0$ for all $x \in R$. A semihyperring $(R, +, \cdot)$ is called commutative if and only if $a \cdot b = b \cdot a$ for all $a, b \in R$. Vougiouklis in [24] studied the notion of semihyperrings in a general form; that is, both the sum and product are hyperoperations; also see [25]. A semihyperring $(R, +, \cdot)$ with identity $1_R \in R$ means that $1_R \cdot x = x \cdot 1_R = x$ for all $x \in R$. The concept of Γ -semihyperring is introduced and studied by Dehkordi and Davvaz [11]. Let R be a commutative semihypergroup and Γ be a commutative group. Then, R is called a Γ -semihyperring if there exists a map $R \times \Gamma \times R \rightarrow \wp^*(R)$ (image to be denoted by $a\alpha b$ for $a, b \in R$ and $\alpha \in \Gamma$) and $\wp^*(R)$ is the set of all nonempty subsets of R satisfying the following conditions:

- (1) $a\alpha(b + c) = a\alpha b + a\alpha c$,
- (2) $(a + b)\alpha c = a\alpha c + b\alpha c$,
- (3) $a(\alpha + \beta)b = a\alpha b + a\beta b$,
- (4) $a\alpha(b\beta c) = (a\alpha b)\beta c$,

for all $a, b, c \in R$ and for all $\alpha, \beta \in \Gamma$. One can find many examples of Γ -semihyperrings in [11, 12]. In the above

definition if R is a semigroup, then R is called a multiplicative Γ -semihyperring. A Γ -semihyperring R is called commutative if $x\alpha y = y\alpha x$ for every $x, y \in R$ and $\alpha \in \Gamma$. We say that Γ -semihyperring R with zero if there exists $0 \in R$ such that $a \in a + 0$, $0 \in 0\alpha a$, $0 \in a\alpha 0$ for all $a \in R$ and $\alpha \in \Gamma$. Let A and B be two nonempty subsets of Γ -semihyperring R and $x \in R$. We define $A + B = \{x \mid x \in a + b; a \in A, b \in B\}$ and $A\Gamma B = \{x \mid x \in a\alpha b; a \in A, b \in B, \alpha \in \Gamma\}$. A nonempty subset R_1 of Γ -semihyperring R is called a sub- Γ -semihyperring if it is closed with respect to the multiplication and addition. In other words, a nonempty subset R_1 of Γ -semihyperring R is a sub- Γ -semihypergroup if $R_1 + R_1 \subseteq R_1$ and $R_1\Gamma R_1 \subseteq R_1$.

A right (left) Γ -hyperideal I of a Γ -semihyperring R is an additive sub-semihypergroup $(R, +)$ such that $I\Gamma R \subseteq I$ ($R\Gamma I \subseteq I$). If I is both right and left Γ -hyperideal of R , then we say that I is a two-sided Γ -hyperideal or simply a Γ -hyperideal of R . In [22, 23], Ersoy and Davvaz studied fuzzy Γ -hyperideals of Γ -semihyperrings. We recall the notion of a fuzzy Γ -hyperideal of a Γ -semihyperring. Let R be a Γ -semihyperring and μ be a fuzzy subset of R . Then (1) μ is called a fuzzy left Γ -hyperideal of R if $\min\{\mu(x), \mu(y)\} \leq \inf_{z \in x+y} \{\mu(z)\}$ and $\mu(y) \leq \inf_{z \in x\gamma y} \{\mu(z)\}$ for all $x, y \in R$ and $\gamma \in \Gamma$; (2) μ is called a fuzzy right Γ -hyperideal of R if $\min\{\mu(x), \mu(y)\} \leq \inf_{z \in x+y} \{\mu(z)\}$ and $\mu(x) \leq \inf_{z \in x\gamma y} \{\mu(z)\}$ for all $x, y \in R$ and $\gamma \in \Gamma$; (3) μ is called a fuzzy Γ -hyperideal of R if it is both a fuzzy left Γ -hyperideal and a fuzzy right Γ -hyperideal of R .

The concept of a triangular norm was introduced by Menger [26] in order to generalize the triangular inequality of a metric. The current notion of a t -norm and its dual operation is due to Schweizer and Sklar [27]. By a t -norm T we mean a function $T : [0, 1] \times [0, 1] \rightarrow [0, 1]$ satisfying the following conditions: (T1) $T(x, y) = T(y, x)$; (T2) $T(x, T(y, z)) = T(T(x, y), z)$; (T3) $T(x, y) \leq T(x, z)$ if $y \leq z$; (T4) $T(x, 1) = x$, for all $x, y, z \in [0, 1]$. For every t -norm T , we set $\Delta_T = \{x \in [0, 1], T(x, x) = x\}$. A t -norm on $[0, 1]$ is called a continuous t -norm if T is a continuous function from $[0, 1] \times [0, 1]$ to $[0, 1]$ with respect to the usual topology. Note that the function "Min" is a continuous t -norm. A triangular conorm (t -norm for short) is a binary operation S on the unit interval $[0, 1]$, that is, a function $S : [0, 1] \times [0, 1] \rightarrow [0, 1]$, which for all $x, y, z \in [0, 1]$ satisfies (T1)–(T3) and (S4) $S(x, 0) = x$. From an axiomatic point of view, t -norms and t -conorms differ only with respect to their respective boundary conditions. In fact, the concepts of t -norms and t -conorms are dual in some sense. Anthony and Sherwood [28] redefined a fuzzy subgroup of a group by using the notion of t -norm.

3. T -Fuzzy Sub- Γ -Semihyperrings and T -Fuzzy Γ -Hyperideals

In this section, we define the notion of T -fuzzy sub- Γ -semihyperrings and T -fuzzy Γ -hyperideals of a Γ -semihyperring and we study some of their properties. Let T be a t -norm and μ be a fuzzy subset of a Γ -semihyperring R . Then, we say μ has imaginable property if $\text{Im } \mu \subseteq \Delta_T$.

Definition 1. Let R be a Γ -semihyperring, T be a t -norm, and μ be a fuzzy subset of R . Then, μ is called a T -fuzzy sub- Γ -semihyperring of R if

- (1) $T(\mu(x), \mu(y)) \leq \inf_{z \in x+y} \{\mu(z)\},$
- (2) $T(\mu(x), \mu(y)) \leq \inf_{z \in x\gamma y} \{\mu(z)\},$

for all $x, y \in R$ and for all $\gamma \in \Gamma$.

A T -fuzzy sub- Γ -semihyperring of R is said to be imaginable if it satisfies the imaginable property. Clearly, if R is a Γ -semiring, then μ is a T -fuzzy sub- Γ -semiring of R when

- (1') $T(\mu(x), \mu(y)) \leq \mu(x + y),$
- (2') $T(\mu(x), \mu(y)) \leq \mu(x\gamma y),$

for all $x, y \in R$ and for all $\gamma \in \Gamma$.

Example 2. Suppose that $R = \mathbb{N}$, the set of natural numbers, and Γ is a nonempty subset of R . For any $x, y \in R$ and $\gamma \in \Gamma$, we define $x + y = \{x, y\}$ and $x\gamma y = \{x, \gamma, y\}$. Then, R is a Γ -semihyperring. We define the fuzzy subset μ of R by

$$\mu(x) = \begin{cases} \frac{3}{4} & \text{if } x \in \Gamma \\ \frac{5}{9} & \text{otherwise} \end{cases} \quad (1)$$

and we consider the t -norm $T(r, s) = rs/(2 - (r + s - rs))$, where $r, s \in [0, 1]$. Then, for any $x, y \in R$ and $\gamma \in \Gamma$, we have

$$\begin{aligned} \inf_{z \in x+y} \{\mu(z)\} &= \min \{\mu(x), \mu(y)\} \\ &= \begin{cases} \frac{3}{4} & \text{if } x, y \in \Gamma \\ \frac{5}{9} & \text{otherwise,} \end{cases} \end{aligned} \quad (2)$$

$$\inf_{z \in x\gamma y} \{\mu(z)\} = \min \{\mu(x), \mu(y), \mu(y)\} = \frac{5}{9}.$$

On the other hand, we have the following cases:

- (1) $x, y \in \Gamma,$
- (2) $x \notin \Gamma$ and $y \in \Gamma$ (or, $x \in \Gamma$ and $y \notin \Gamma$),
- (3) $x, y \notin \Gamma.$

Regarding the above cases, we have $T(3/4, 3/4) = 9/17$, $T(3/4, 5/9) = 15/40$, and $T(5/9, 5/9) = 25/81$. Thus, in every case, we obtain

$$\begin{aligned} T(\mu(x), \mu(y)) &\leq \inf_{z \in x+y} \{\mu(z)\}, \\ T(\mu(x), \mu(y)) &\leq \inf_{z \in x\gamma y} \{\mu(z)\}. \end{aligned} \quad (3)$$

Therefore, μ is a T -fuzzy sub- Γ -semihyperring of R .

Lemma 3. Let R be a Γ -semihyperring, T be a t -norm, and μ be a T -fuzzy sub- Γ -semihyperring of S . Then

$$\begin{aligned} T_n(\mu(x_1), \dots, \mu(x_n)) &\leq \inf_{z \in x_1 + \dots + x_n} \{\mu(z)\}, \\ T_n(\mu(x_1), \dots, \mu(x_n)) &\leq \inf_{z \in x_1 \gamma \dots \gamma x_n} \{\mu(z)\}, \end{aligned} \quad (4)$$

for all $x_1, \dots, x_n \in R$ and $\gamma \in \Gamma$, where

$$\begin{aligned} T_n(t_1, \dots, t_n) &= \begin{cases} t_1 & \text{if } n = 1 \\ T(t_1, t_2) & \text{if } n = 2 \\ T(t_i, T_{n-1}(t_1, \dots, t_{i-1}, t_{i+1}, \dots, t_n)) & \text{if } n > 2. \end{cases} \end{aligned} \quad (5)$$

Proof. The proof is straightforward by mathematical induction. $\square$

Lemma 4. Let R be a Γ -semihyperring, T be a t -norm, and μ be a T -fuzzy sub- Γ -semihyperring of S . Let A and B be nonempty subsets of R . Then

$$\begin{aligned} T\left(\inf_{a \in A} \{\mu(a)\}, \inf_{b \in B} \{\mu(b)\}\right) &\leq \inf_{z \in A+B} \{\mu(z)\}, \\ T\left(\inf_{a \in A} \{\mu(a)\}, \inf_{b \in B} \{\mu(b)\}\right) &\leq \inf_{z \in A\gamma B} \{\mu(z)\}, \end{aligned} \quad (6)$$

for all $\gamma \in \Gamma$.

Proof. The proof is straightforward. $\square$

Theorem 5. Let R be a Γ -semihyperring, T be a t -norm, and μ be a fuzzy subset of S with imaginable property and b the maximum of $\text{Im } \mu$. Then, the following two statements are equivalent:

- (1) μ is a T -fuzzy sub- Γ -semihyperring of S ,
- (2) $\mu^{-1}[a, b]$ is a sub- Γ -semihyperring of S whenever $a \in \Delta_T$ and $0 < a \leq b$.

Proof. (1) $\Rightarrow$ (2): Suppose that $a \in \Delta_T$ and $0 < a \leq b$. If $x, y \in \mu^{-1}[a, b]$, then $\inf_{z \in x+y} \{\mu(z)\} \geq T(\mu(x), \mu(y)) \geq T(a, a) = a$, which implies that $x + y \subseteq \mu^{-1}[a, b]$. Similarly, assume that $a \in \Delta_T$ and $0 < a \leq b$. If $x, y \in \mu^{-1}[a, b]$ and $\gamma \in \Gamma$, then $\inf_{z \in x\gamma y} \{\mu(z)\} \geq T(\mu(x), \mu(y)) \geq T(a, a) = a$. Then, we have $x\gamma y \subseteq \mu^{-1}[a, b]$, and so $\mu^{-1}[a, b]$ is a sub- Γ -semihyperring of R .

(2) $\Rightarrow$ (1): Suppose that $x, y \in S$ and $\gamma \in \Gamma$. Since $\text{Im } \mu \subseteq \Delta_T$, both $\mu(x)$ and $\mu(y)$ are in Δ_T . Now, we have

$$\begin{aligned} T(T(\mu(x), \mu(y)), T(\mu(x), \mu(y))) &= T(T(\mu(x), T(\mu(y), \mu(x))), \mu(y)) \\ &= T(T(\mu(x), T(\mu(x), \mu(y))), \mu(y)) \\ &= T(T(\mu(x), \mu(x)), T(\mu(y), \mu(y))) \\ &= T(\mu(x), \mu(y)), \end{aligned} \quad (7)$$

and so $T(\mu(x), \mu(y)) \in \Delta_T$. Assume that $a = T(\mu(x), \mu(y))$. If $a = 0$, then

$$\begin{aligned} T(\mu(x), \mu(y)) &= 0 \leq \inf_{z \in x+y} \{\mu(z)\}, \\ T(\mu(x), \mu(y)) &= 0 \leq \inf_{z \in x\gamma y} \{\mu(z)\}. \end{aligned} \quad (8)$$

Now, let $0 < a = T(\mu(x), \mu(y)) \leq \mu(x) \wedge \mu(y) \leq \mu(x) \leq b$. Hence $x, y \in \mu^{-1}[a, b]$, which implies that $x + y \subseteq \mu^{-1}[a, b]$ and $x\gamma y \subseteq \mu^{-1}[a, b]$. Therefore $T(\mu(x), \mu(y)) \leq \inf_{z \in x+y} \{\mu(z)\}$ and $T(\mu(x), \mu(y)) \leq \inf_{z \in x\gamma y} \{\mu(z)\}$. $\square$

Definition 6. Let R be a Γ -semihyperring, T be a t -norm, and μ be a fuzzy subset of R . Then

- (1) μ is called a T -fuzzy left Γ -hyperideal of R if

$$\begin{aligned} T(\mu(x), \mu(y)) &\leq \inf_{z \in x+y} \{\mu(z)\}, \quad \forall x, y \in R, \\ \mu(y) &\leq \inf_{z \in x\gamma y} \{\mu(z)\}, \quad \forall x, y \in R, \quad \forall \gamma \in \Gamma. \end{aligned} \quad (9)$$

- (2) μ is called a T -fuzzy right Γ -hyperideal of R if

$$\begin{aligned} T(\mu(x), \mu(y)) &\leq \inf_{z \in x+y} \{\mu(z)\}, \quad \forall x, y \in R, \\ \mu(x) &\leq \inf_{z \in x\gamma y} \{\mu(z)\}, \quad \forall x, y \in R, \quad \forall \gamma \in \Gamma. \end{aligned} \quad (10)$$

- (3) μ is called a T -fuzzy Γ -hyperideal of R if it is both a T -fuzzy left Γ -hyperideal and a T -fuzzy right Γ -hyperideal of R .

Theorem 7. Let R be a Γ -semihyperring, T be a t -norm, and μ be a fuzzy subset of S with imaginable property and b the maximum of $\text{Im } \mu$. Then, the following two statements are equivalent:

- (1) μ is a T -fuzzy Γ -hyperideal of R ,
 (2) $\mu^{-1}[a, b]$ is a Γ -hyperideal of R whenever $a \in \Delta_T$ and $0 < a \leq b$.

Proof. The proof is similar to the proof of Theorem 5. $\square$

Let μ be a fuzzy subset of R and $t \in [0, 1]$. The set $U(\mu; t) = \{x \in R \mid \mu(x) \geq t\}$ is called a level subset of μ . So, we obtain the following corollary.

Corollary 8. Let R be a Γ -semihyperring and μ be a fuzzy subset of R . Then

- (1) μ is a Min-fuzzy sub- Γ -semihyperring of R if and only if every nonempty level subset is a sub- Γ -semihyperring of R ;
 (2) μ is a Min-fuzzy Γ -hyperideal of R if and only if every nonempty level subset is a Γ -hyperideal of R .

Corollary 9. Let A be a subset of R . Then

- (1) the characteristic function χ_A is a T -fuzzy sub- Γ -semihyperring of R if and only if A is a sub- Γ -semihyperring of R ;
 (2) the characteristic function χ_A is a T -fuzzy Γ -hyperideal of R if and only if A is a Γ -hyperideal of R .

Theorem 10. Let R be a Γ -semihyperring and K be a sub- Γ -semihyperring of R . Let T be the t -norm defined by $T(a, b) = \max\{0, a + b - 1\}$ and μ be a fuzzy subset of R defined by

$$\mu(x) = \begin{cases} r & \text{if } x \in K \\ s & \text{otherwise,} \end{cases} \quad (11)$$

for all $a, b \in [0, 1]$ and $x \in R$, where $r, s \in [0, 1]$ such that $s < r$. Then, μ is a T -fuzzy sub- Γ -semihyperring of R . In particular, if $r = 1$ and $s = 0$, then μ is imaginable.

Proof. The proof is similar to the proof of Theorem 2.6 in [29]. $\square$

Definition 11. Let R_1 and R_2 be Γ_1 and Γ_2 -semihyperrings, respectively. If there exists a map $\varphi : R_1 \rightarrow R_2$ and a bijection $f : \Gamma_1 \rightarrow \Gamma_2$ such that

$$\begin{aligned} \varphi(x + y) &= \{\varphi(z) \mid z \in x + y\} = \varphi(x) + \varphi(y), \\ \varphi(x\gamma y) &= \{\varphi(z) \mid z \in x\gamma y\} = \varphi(x) f(\gamma) \varphi(y), \end{aligned} \quad (12)$$

for all $x, y \in R_1$ and $\gamma \in \Gamma_1$, then we say (φ, f) is a homomorphism from R_1 to R_2 . Also, if φ is a bijection then (φ, f) is called an isomorphism and R_1 and R_2 are isomorphic.

Proposition 12. Let R_1 and R_2 be Γ_1 and Γ_2 -semihyperrings, respectively. Let (φ, f) be a homomorphism from R_1 to R_2 . If λ is a T -fuzzy sub- Γ -semihyperring of R_2 , then $\varphi^{-1}(\lambda)$ is a T -fuzzy sub- Γ -semihyperring of R_1 too.

Proof. Suppose that $x, y \in R_1$ and $\gamma \in \Gamma_1$. Then, we have

$$\begin{aligned} \inf_{z \in x+y} \{\varphi^{-1}(\lambda)(z)\} &= \inf_{z \in x+y} \{\lambda(\varphi(z))\} \\ &\geq \inf_{\varphi(z) \in \varphi(x+y)} \{\lambda(\varphi(z))\} \\ &\geq \inf_{\varphi(z) \in \varphi(x) + \varphi(y)} \{\lambda(\varphi(z))\} \\ &\geq T(\lambda(\varphi(x)), \lambda(\varphi(y))) \\ &= T(\varphi^{-1}(\lambda)(x), \varphi^{-1}(\lambda)(y)); \\ \inf_{z \in x\gamma y} \{\varphi^{-1}(\lambda)(z)\} &= \inf_{z \in x\gamma y} \{\lambda(\varphi(z))\} \\ &\geq \inf_{\varphi(z) \in \varphi(x\gamma y)} \{\lambda(\varphi(z))\} \\ &\geq \inf_{\varphi(z) \in \varphi(x) f(\gamma) \varphi(y)} \{\lambda(\varphi(z))\} \\ &\geq T(\lambda(\varphi(x)), \lambda(\varphi(y))) \\ &= T(\varphi^{-1}(\lambda)(x), \varphi^{-1}(\lambda)(y)). \end{aligned} \quad (13)$$

Therefore, $\varphi^{-1}(\lambda)$ is a T -fuzzy sub- Γ -semihyperring of R_1 . $\square$

Proposition 13. Let R_1 and R_2 be Γ_1 and Γ_2 -semihyperrings, respectively. Let (φ, f) be a homomorphism from R_1 to R_2 . If λ is a T -fuzzy Γ -hyperideal of R_2 , then $\varphi^{-1}(\lambda)$ is a T -fuzzy Γ -hyperideal of R_1 too.

Proof. The proof is similar to the proof of Proposition 12. $\square$

Let $\{a_i\}_{i \in I}$ and $\{b_j\}_{j \in J}$ be two sets of real numbers in $[0, 1]$. Then, we say T is infinitely distributive if

$$T\left(\sup_{i \in I} \{a_i\}, \sup_{j \in J} \{b_j\}\right) = \sup_{i \in I} \left\{T\left(a_i, \sup_{j \in J} \{b_j\}\right)\right\}. \quad (14)$$

If T is continuous, then T is infinitely distributive [30].

Lemma 14. Let T be a continuous t -norm and $\{\mu_i\}_{i \in I}$ be a family of T -fuzzy sub- Γ -semihyperring of R . Then, $\bigcap_{i \in I} \mu_i$ is a T -fuzzy sub- Γ -semihyperring of R .

Proof. For any $x, y \in R$ and $\gamma \in \Gamma$, we have

$$\begin{aligned} \inf_{z \in x+y} \left\{ \left(\bigcap_{i \in I} \mu_i \right) (z) \right\} &= \inf_{z \in x+y} \left\{ \inf_{i \in I} \{ \mu_i (z) \} \right\} \\ &= \inf_{i \in I} \left\{ \inf_{z \in x+y} \{ \mu_i (z) \} \right\} \\ &\geq \inf_{i \in I} \{ T(\mu_i(x), \mu_i(y)) \} \\ &= T\left(\inf_{i \in I} \{ \mu_i(x) \}, \inf_{i \in I} \{ \mu_i(y) \} \right) \\ &= T\left(\left(\bigcap_{i \in I} \mu_i \right) (x), \left(\bigcap_{i \in I} \mu_i \right) (y) \right); \\ \inf_{z \in x\gamma y} \left\{ \left(\bigcap_{i \in I} \mu_i \right) (z) \right\} &= \inf_{z \in x\gamma y} \left\{ \inf_{i \in I} \{ \mu_i (z) \} \right\} \\ &= \inf_{i \in I} \left\{ \inf_{z \in x\gamma y} \{ \mu_i (z) \} \right\} \\ &\geq \inf_{i \in I} \{ T(\mu_i(x), \mu_i(y)) \} \\ &= T\left(\inf_{i \in I} \{ \mu_i(x) \}, \inf_{i \in I} \{ \mu_i(y) \} \right) \\ &= T\left(\left(\bigcap_{i \in I} \mu_i \right) (x), \left(\bigcap_{i \in I} \mu_i \right) (y) \right). \end{aligned} \quad (15)$$

□

Lemma 15. Let R_1 and R_2 be Γ_1 and Γ_2 -semihyperrings, respectively, and (φ, f) be an onto homomorphism from R_1 to R_2 . Then, for every $t \in [0, 1]$, we have $\varphi(\mu)_t = \bigcap_{t > \epsilon > 0} \varphi(\mu_{t-\epsilon})$.

Proof. The proof is similar to the proof of Lemma 3.5 in [31]. □

Proposition 16. Let R_1 and R_2 be Γ_1 and Γ_2 -semihyperrings, respectively, and let μ be a fuzzy subset of R_1 . Let (φ, f) be an onto homomorphism from R_1 to R_2 . If μ is a Min-fuzzy sub- Γ -semihyperring of R_1 , then $\varphi(\mu)$ is a Min-fuzzy sub- Γ -semihyperring of R_2 too.

Proof. Suppose that μ is a Max-fuzzy sub- Γ -semihyperring of R_1 . By Corollary 8, $\varphi(\mu)$ is a Max-fuzzy sub- Γ -semihyperring

of R_2 if every nonempty level subset $U(\varphi(\mu); t)$ is a sub- Γ -semihyperring of R_2 . Thus, assume that $U(\varphi(\mu); t)$ is any nonempty level subset. If $t = 0$, then $U(\varphi(\mu); t) = R_2$, and if $0 < t \leq 1$, then by Lemma 15, we have $U(\varphi(\mu); t) = \bigcap_{t > \epsilon > 0} \varphi(U(\mu; t - \epsilon))$. By Corollary 8, $U(\mu; t - \epsilon)$ for each $t > \epsilon > 0$ is a sub- Γ -semihyperring of R_1 . Hence, $\varphi(U(\mu; t - \epsilon))$ is a sub- Γ -semihyperring of R_2 . By Lemma 14, $\varphi(U(\mu; t))$ being an intersection of a family of sub- Γ -semihyperrings is also a sub- Γ -semihyperring of R_2 and the proof is completed. □

Definition 17. Let R_1 and R_2 be two Γ -semihyperrings and let μ and λ be fuzzy subsets of R_1 and R_2 , respectively. The product of μ and λ is defined to be the fuzzy subset $\mu \times \lambda$ of $R_1 \times R_2$ with $(\mu \times \lambda)(x, y) = T(\mu(x), \lambda(y))$, for all $(x, y) \in R_1 \times R_2$.

Proposition 18. Let R_1 and R_2 be two Γ -semihyperrings and let μ and λ be fuzzy subsets of R_1 and R_2 , respectively. Then

- (1) if μ and λ are T -fuzzy sub- Γ -semihyperrings of R_1 and R_2 , respectively, then $\mu \times \lambda$ is a T -fuzzy sub- Γ -semihyperring of $R_1 \times R_2$;
- (2) if μ and λ are T -fuzzy Γ -hyperideals of R_1 and R_2 , respectively, then $\mu \times \lambda$ is a T -fuzzy Γ -hyperideal of $R_1 \times R_2$.

Proof. It is straightforward. □

In [12], Dehkordi and Davvaz studied Noetherian and Artinian Γ -semihyperrings in crisp case. A collection $\mathcal{A}$ of subsets of a Γ -semihyperring R satisfies the ascending chain condition (or Acc) if there does not exist a properly ascending infinite chain $A_1 \subset A_2 \subset \dots$ of subsets from $\mathcal{A}$. Recall that a subset $B \in \mathcal{A}$ is a maximal element of $\mathcal{A}$ if there does not exist a subset in $\mathcal{A}$ that properly contains B . Similar to [18], in the following, we obtain some results related to fuzzy sets and Noetherian Γ -semihyperrings.

Proposition 19 (see [12]). Let R be a Γ -semihyperring. Then, the following conditions are equivalent:

- (1) R satisfying the Acc condition on right (left) Γ -hyperideals,
- (2) every nonempty family of right (left) Γ -hyperideals has a maximal element,
- (3) every right (left) Γ -hyperideals is finitely generated.

Definition 20 (see [12]). A Γ -semihyperring R is right (left) Noetherian if the equivalent conditions of the above proposition are satisfied. In the same way, we can define an Artinian Γ -semihyperring. Let I be a Γ -hyperideal of a Γ -semihyperring R and I be a Noetherian Γ -semihyperring. Then, I is called a Noetherian Γ -hyperideal of R .

Example 21 (see [12]). Let $A_n = [n, n_{n+1}]$ for every $n \in \mathbb{Z}$, $R = \bigcup_{n \in \mathbb{Z}} A_n$, and $\Gamma = \mathbb{Z}$. Then, R is a Noetherian Γ -semihyperring with respect to the following hyperoperations:

$$x \oplus y = A_{n+m}, \quad x\gamma y = A_{n\gamma m}, \quad (16)$$

where $x \in A_n$ and $y \in A_m$.

Theorem 22. Let $\{A_k \mid k \in \mathbb{N}\}$ be a family of Γ -hyperideals of a Γ -semihyperring R , where $A_1 \supset A_2 \supset A_3 \cdots$. Let μ be a fuzzy subset of R defined by

$$\mu(x) = \begin{cases} \frac{k}{k+1} & \text{if } x \in A_k \setminus A_{k+1}, \quad k = 0, 1, 2, \dots \\ 1 & \text{if } x \in \bigcap_{k=0}^{\infty} A_k, \end{cases} \quad (17)$$

for all $x \in R$, where A_0 stands for R . Let T be a t -norm with $\text{Im } \mu \subseteq \Delta_T$. Then, μ is a T -fuzzy Γ -hyperideal of R .

Proof. Let $x, y \in R$. Suppose that $x \in A_k \setminus A_{k+1}$ and $y \in A_r \setminus A_{r+1}$ for $k = 0, 1, 2, \dots$ and $r = 0, 1, 2, \dots$. Without loss of generality we may assume that $k \leq r$. Then, obviously $y \in A_k$. Since A_k is a Γ -hyperideal of R , it follows that $x + y \subseteq A_k$ and $x\alpha y \subseteq A_k$ which imply that $\inf_{z \in x+y} \mu(z) \geq k/(k+1) = T(\mu(x), \mu(y))$ and $\inf_{z \in x\alpha y} \mu(z) \geq k/(k+1) = \mu(y)$ for all $\alpha \in \Gamma$. If $x \in \bigcap_{k=0}^{\infty} A_k$ and $y \in \bigcap_{k=0}^{\infty} A_k$, then $x + y \subseteq \bigcap_{k=0}^{\infty} A_k$. Hence, $\inf_{z \in x+y} \mu(z) = 1 = T(\mu(x), \mu(y))$ and $\inf_{z \in x\alpha y} \mu(z) = 1 \geq \mu(y)$. If $x \notin \bigcap_{k=0}^{\infty} A_k$ and $y \in \bigcap_{k=0}^{\infty} A_k$, then there exists $n \in \mathbb{N}$ such that $x \in A_n \setminus A_{n+1}$. It follows that $x + y \subseteq A_n$ which implies that $\inf_{z \in x+y} \mu(z) \geq n/(n+1) = T(\mu(x), \mu(y))$ and $\inf_{z \in x\alpha y} \mu(z) \geq n/(n+1) = \mu(y)$ for all $\alpha \in \Gamma$. Finally, assume that $x \in \bigcap_{k=0}^{\infty} A_k$ and $y \notin \bigcap_{k=0}^{\infty} A_k$, then $y \in A_n \setminus A_{n+1}$ for some $n \in \mathbb{N}$. Therefore, $x + y \subseteq A_n$, and thus $\inf_{z \in x+y} \mu(z) \geq n/(n+1) = T(\mu(x), \mu(y))$ and $\inf_{z \in x\alpha y} \mu(z) \geq n/(n+1) = \mu(y)$, for all $\alpha \in \Gamma$. Hence, μ is a T -fuzzy Γ -hyperideal of R . $\square$

Theorem 23. Let R be a Γ -semiring satisfying descending chain condition, let μ be a fuzzy subset of R , and let T be a t -norm with $\text{Im } \mu \subseteq \Delta_T$. Let μ be a T -fuzzy Γ -hyperideal of R . If a sequence of elements of $\text{Im } \mu$ is strictly increasing, then μ has finite number of values.

Proof. Let $\{t_k\}$ be a strictly increasing sequence of elements of $\text{Im } \mu$. Then $0 \leq t_1 \leq t_2 \leq \dots \leq 1$. Then, $U(\mu; t_r)$ is an ideal of M for all $r = 2, 3, \dots$. Let $x \in U(\mu; t_r)$. Then $\mu(x) \geq t_r \geq t_{r-1}$, and so $x \in U(\mu; t_{r-1})$. Hence $U(\mu; t_r) \subseteq U(\mu; t_{r-1})$. Since $t_{r-1} \in \text{Im } \mu$, there exists $x_{r-1} \in M$ such that $\mu(x_{r-1}) = t_{r-1}$. It follows that $x_{r-1} \in U(\mu; t_{r-1})$ but $x_{r-1} \notin U(\mu; t_r)$. Thus, $U(\mu; t_r) \subset U(\mu; t_{r-1})$ and so we obtain a strictly descending sequence $U(\mu; t_1) \supset U(\mu; t_2) \supset U(\mu; t_3) \supset \dots$ of Γ -hyperideals of R which is not terminating. This contradicts the assumption that M satisfies the descending chain condition. Consequently, μ has finite number of values. $\square$

Theorem 24. Let R be a Γ -semiring, μ be a fuzzy subset of R , and T be a t -norm with $\text{Im } \mu \subseteq \Delta_T$. Then, the following conditions are equivalent:

- (1) R is a Noetherian Γ -semihyperring,
- (2) the set of values of any Γ -hyperideal of R is a well-ordered subset of $[0, 1]$.

Proof. (1) $\Rightarrow$ (2): Let μ be a T -fuzzy Γ -hyperideal of R . Suppose that the set of values of μ is not a well-ordered subset of $[0, 1]$. Then, there exists a strictly decreasing sequence $\{t_k\}$

such that $\mu(x_k) = t_k$. It follows that $U(\mu; t_1) \subset U(\mu; t_2) \subset \dots$ is a strictly ascending chain of Γ -hyperideals of M , where $U(\mu; t_r) = \{x \in M \mid \mu(x) \geq t_r\}$, for every $r = 1, 2, \dots$. This contradicts the assumption that R is a Noetherian Γ -semihyperring.

(2) $\Rightarrow$ (1): Suppose that the condition (2) is satisfied and R is not a Noetherian Γ -hyperring. There exists a strictly ascending chain

$$A_1 \subset A_2 \subset A_3 \subset \dots \quad (18)$$

of Γ -hyperideals of R . Note that $A = \bigcap_{k=0}^{\infty} A_k$ is a Γ -hyperideal of R . Define a fuzzy subset in R by

$$\mu(x) = \begin{cases} \alpha & \text{if } x \notin A, \\ \frac{1}{r} & \text{where } r = \min \{k \in \mathbb{N} \mid x \in A_k\}. \end{cases} \quad (19)$$

We claim that μ is a T -fuzzy Γ -hyperideal of R . Let $x, y \in R$. If $x \in A_k \setminus A_{k-1}$ and $y \in A_k \setminus A_{k-1}$, then $x + y \subseteq A_k$ and $x\alpha y \subseteq A_k$. It follows that $\inf_{z \in x+y} \mu(z) = 1/k = T(\mu(x), \mu(y))$ and $\inf_{z \in x\alpha y} \mu(z) \geq 1/k \geq \mu(y)$, for all $\beta \in \Gamma$. Suppose that $x \in A_k$ and $y \in A_k \setminus A_r$ for all $r < k$. Since A_k is a Γ -hyperideal of R , it follows that $x + y \subseteq A_k$. Hence, $\inf_{z \in x+y} \mu(z) \geq 1/k \geq 1/(k+1) \geq \mu(y)$ and so $\inf_{z \in x+y} \mu(z) \geq \min\{\mu(x), \mu(y)\} \geq T(\mu(x), \mu(y))$. Similarly, for the case $x \in A_k \setminus A_r$ and $y \in A_k$, we have $\inf_{z \in x+y} \mu(z) \geq \min\{\mu(x), \mu(y)\} \geq T(\mu(x), \mu(y))$ and $\inf_{z \in x\alpha y} \mu(z) \geq \mu(y)$, for all $\beta \in \Gamma$. Thus, μ is a T -fuzzy Γ -hyperideal of R . Since the chain is not terminating, μ has a strictly descending sequence of values. This contradicts the assumption that the value set of any ideal is well-ordered. Therefore, R is a Noetherian Γ -semihyperring. $\square$

For a family $\{\mu_\alpha \mid \alpha \in \Lambda\}$ of fuzzy subsets in R , we define the join $\bigvee_{\alpha \in \Lambda} \mu_\alpha$ and the meet $\bigwedge_{\alpha \in \Lambda} \mu_\alpha$ as follows:

$$\begin{aligned} \left(\bigvee_{\alpha \in \Lambda} \mu_\alpha \right)(x) &= \sup \{ \mu_\alpha(x) \mid \alpha \in \Lambda \}, \\ \left(\bigwedge_{\alpha \in \Lambda} \mu_\alpha \right)(x) &= \inf \{ \mu_\alpha(x) \mid \alpha \in \Lambda \}, \end{aligned} \quad (20)$$

for all $x \in R$, where Λ is any index set.

Theorem 25. The family of T -fuzzy Γ -hyperideals in R is a completely distributive lattice with respect to meet “ $\wedge$ ” and join “ $\vee$ ”.

Proof. Since $[0, 1]$ is a completely distributive lattice with respect to the usual ordering in $[0, 1]$, it is sufficient to show that $\bigvee_{\alpha \in \Lambda} \mu_\alpha$ and $\bigwedge_{\alpha \in \Lambda} \mu_\alpha$ are T -fuzzy Γ -hyperideals of R for family $\{\mu_\alpha \mid \alpha \in \Lambda\}$ of T -fuzzy Γ -hyperideals of R .

For any $x, y \in R$, we have

$$\begin{aligned}
 \inf_{z \in x+y} \left(\bigvee_{\alpha \in \Lambda} \mu_{\alpha} \right) (z) &\geq \sup \{ T(\mu_{\alpha}(x), \mu_{\alpha}(y)) \mid \alpha \in \Lambda \} \\
 &\geq T(\sup \{ \mu_{\alpha}(x) \mid \alpha \in \Lambda \}, \\
 &\quad \sup \{ \mu_{\alpha}(y) \mid \alpha \in \Lambda \}) \\
 &= T\left(\left(\bigvee_{\alpha \in \Lambda} \mu_{\alpha} \right) (x), \left(\bigvee_{\alpha \in \Lambda} \mu_{\alpha} \right) (y) \right), \\
 \inf_{z \in x+y} \left(\bigwedge_{\alpha \in \Lambda} \mu_{\alpha} \right) (z) &\geq \inf \{ T(\mu_{\alpha}(x), \mu_{\alpha}(y)) \mid \alpha \in \Lambda \} \\
 &= T\left(\left(\bigwedge_{\alpha \in \Lambda} \mu_{\alpha} \right) (x), \left(\bigwedge_{\alpha \in \Lambda} \mu_{\alpha} \right) (y) \right).
 \end{aligned} \tag{21}$$

Now, let $x, y \in M$ and $\beta \in \Gamma$. Then

$$\begin{aligned}
 \inf_{z \in x\beta y} \left(\bigvee_{\alpha \in \Lambda} \mu_{\alpha} \right) (z) &\geq \sup \{ \mu_{\alpha}(y) \mid \alpha \in \Lambda \} \\
 &= \left(\bigvee_{\alpha \in \Lambda} \mu_{\alpha} \right) (y),
 \end{aligned} \tag{22}$$

$$\begin{aligned}
 \inf_{z \in x\beta y} \left(\bigwedge_{\alpha \in \Lambda} \mu_{\alpha} \right) (z) &\geq \inf \{ \mu_{\alpha}(y) \mid \alpha \in \Lambda \} \\
 &= \left(\bigwedge_{\alpha \in \Lambda} \mu_{\alpha} \right) (y).
 \end{aligned} \tag{23}$$

Hence, $\bigvee_{\alpha \in \Lambda} \mu_{\alpha}$ and $\bigwedge_{\alpha \in \Lambda} \mu_{\alpha}$ are T -fuzzy Γ -hyperideals of R . This completes the proof. $\square$

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

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