Research Article

A New Extended Soft Intersection Set to \((M, N)\)-SI Implicative Fitters of BL-Algebras

Jianming Zhan, 1 Qi Liu, 1 and Hee Sik Kim 2

1 Department of Mathematics, Hubei Minzu University, Enshi, Hubei 445000, China
2 Department of Mathematics, Hanyang University, Seoul 133-791, Republic of Korea

Correspondence should be addressed to Hee Sik Kim; heekim@hanyang.ac.kr
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Molodtsov’s soft set theory provides a general mathematical framework for dealing with uncertainty. The concepts of \((M, N)\)-SI implicative (Boolean) filters of BL-algebras are introduced. Some good examples are explored. The relationships between \((M, N)\)-SI filters and \((M, N)\)-SI implicative filters are discussed. Some properties of \((M, N)\)-SI implicative (Boolean) filters are investigated. In particular, we show that \((M, N)\)-SI implicative filters and \((M, N)\)-SI Boolean filters are equivalent.

1. Introduction

We know that dealing with uncertainties is a major problem in many areas such as economics, engineering, medical sciences, and information science. These kinds of problems cannot be dealt with by classical methods because some classical methods have inherent difficulties. To overcome them, Molodtsov [1] introduced the concept of a soft set as a new mathematical tool for dealing with uncertainties that is free from the difficulties that have troubled the usual theoretical approaches. Since then, especially soft set operations have undergone tremendous studies; for examples, see [2–5]. At the same time, soft set theory has been applied to algebraic structures, such as [6–8]. We also note that soft set theory emphasizes balanced coverage of both theory and practice. Nowadays, it has promoted a breath of the discipline of information sciences, decision support systems, knowledge systems, decision-making, and so on; see [9–13].

BL-algebras, which have been introduced by Hájek [14] as algebraic structures of basic logic, arise naturally in the analysis of the proof theory of propositional fuzzy logic. Turunen [15] proposed the concepts of implicative filters and Boolean filters in BL-algebras. Liu et al. [16, 17] applied fuzzy set theory to BL-algebras. After that, some researchers have further investigated some properties of BL-algebras. Further, Ma et al. investigated some kinds of generalized fuzzy filters BL-algebras and obtained some important results; see [18, 19]. Zhang et al. [20, 21] described the relations between pseudo-BL, pseudo-effect algebras, and BCC-algebras, respectively. The other related results can be found in [22, 23].


In this paper, we introduce the concept of \((M, N)\)-soft intersection implicative filters of BL-algebras. Some related properties are investigated. In particular, we show that \((M, N)\)-SI implicative filters and \((M, N)\)-SI Boolean filters are equivalent.

2. Preliminaries

Recall that an algebra \(L = (L, \leq, \wedge, \vee, \odot, \rightarrow, 0, 1)\) is a BL-algebra [14] if it is a bounded lattice such that the following conditions are satisfied:

(i) \((L, \odot, 1)\) is a commutative monoid,

(ii) \(\odot\) and \(\rightarrow\) form an adjoin pair, that is, \(z \leq x \rightarrow y\) if and only if \(x \odot z \leq y\) for all \(x, y, z \in L\),
(iii) \( x \land y = x \odot (x \rightarrow y) \),
(iv) \( (x \rightarrow y) \lor (y \rightarrow x) = 1 \).

In what follows, \( L \) is a BL-algebra unless otherwise is specified.

In any BL-algebra \( L \), the following statements are true (see [14, 15]):

(a\textsubscript{1}) \( x \leq y \Leftrightarrow x \rightarrow y = 1 \),
(a\textsubscript{2}) \( x \rightarrow (y \rightarrow z) = (x \odot y) \rightarrow (x \rightarrow z) \),
(a\textsubscript{3}) \( x \odot y \leq x \land y \),
(a\textsubscript{4}) \( x \rightarrow y \leq (z \rightarrow x) \rightarrow (z \rightarrow y), x \rightarrow y \leq (y \rightarrow z) \rightarrow (x \rightarrow z) \),
(a\textsubscript{5}) \( x \rightarrow x' = x'' \rightarrow x \),
(a\textsubscript{6}) \( x \lor x' = 1 \Rightarrow x \land x' = 0 \),
(a\textsubscript{7}) \( (x \rightarrow y) \odot (y \rightarrow z) \leq x \rightarrow z \),
(a\textsubscript{8}) \( x \leq y \Rightarrow y \rightarrow z \leq z \rightarrow y \),
(a\textsubscript{9}) \( x \leq y \Rightarrow z \rightarrow x \leq z \rightarrow y \),
(a\textsubscript{10}) \( x \lor y = ((x \rightarrow y) \rightarrow y) \land ((y \rightarrow x) \rightarrow x) \),

where \( x' = x \rightarrow 0 \).

A nonempty subset \( A \) of \( L \) is called a filter of \( L \) if it satisfies the following conditions: (II) \( 1 \in A \), (I\textsubscript{2}) for all \( x \in A \), for all \( y \in L \), \( x \rightarrow y \in A \Rightarrow y \in A \).

It is easy to check that a nonempty subset \( A \) of \( L \) is a filter of \( L \) if and only if it satisfies (I\textsubscript{3}) for all \( x, y, z \in L \).

(14) for all \( x, y \in A \), for all \( x \in A \), for all \( x \rightarrow y \in A \Rightarrow y \in A \).

Now, we call a nonempty subset \( A \) of \( L \) an implicative filter if it satisfies (II) and (I\textsubscript{5}) \( x \rightarrow (z' \rightarrow y) \in A, y \rightarrow z \in A \Rightarrow x \rightarrow z \in A \).

A nonempty subset \( A \) of \( L \) is said to be a Boolean filter of \( L \) if it satisfies \( x \land x' \in A, x \rightarrow y \in A \Rightarrow y \in A \). (see [15–18]).

From now on, we let \( L \) be an BL-algebra, \( U \) an initial universe, \( E \) a set of parameters, \( P(U) \) the power set of \( U \), and \( A, B, C \subseteq E \). We let \( \emptyset \subseteq M \subseteq N \subseteq U \).

**Definition 1** (see [1]). A soft set \( f_A \) over \( U \) is a set defined by \( f_A : E \rightarrow P(U) \) such that \( f_A(x) = \emptyset \) if \( x \notin A \). Here \( f_A \) is also called an approximate function. A soft set over \( U \) can be represented by the set of ordered pairs \( f_A = \{(x, f_A(x)) \mid x \in E, f_A(x) \in P(U)\} \). It is clear to see that a soft set \( f_A \) is a parameterized family of subsets of \( U \). Note that the set of all soft sets over \( U \) will be denoted by \( S(U) \).

**Definition 2** (see [9]). Let \( f_A, f_B \in S(U) \).

(1) \( f_A \) is said to be a soft subset of \( f_B \) and denoted by \( f_A \subseteq f_B \) if \( f_A(x) \subseteq f_B(x) \), for all \( x \in E \). \( f_A \) and \( f_B \) are said to be soft equally, denoted by \( f_A = f_B \), if \( f_A \subseteq f_B \) and \( f_B \subseteq f_A \).

(2) The union of \( f_A \) and \( f_B \), denoted by \( f_A \cup f_B \), is defined as \( f_A \cup f_B = f_{A \cup B} \), where \( f_{A \cup B}(x) = f_A(x) \cup f_B(x) \), for all \( x \in E \).

(3) The intersection of \( f_A \) and \( f_B \), denoted by \( f_A \cap f_B \), is defined as \( f_A \cap f_B = f_{A \cap B} \), where \( f_{A \cap B}(x) = f_A(x) \cap f_B(x) \), for all \( x \in E \).

**Definition 3** (see [26]). (1) A soft set \( f_L \) over \( U \) is called an SI-filter of \( L \) over \( U \) if it satisfies

\( (S_1) f_L(x) \subseteq f_L(1) \) for any \( x \in L \),

\( (S_2) f_L(x \rightarrow y) \cap f_L(x) \subseteq f_L(y) \) for all \( x, y \in L \).

(2) A soft set \( f_L \) over \( U \) is called an SI-implicative filter of \( L \) over \( U \) if it satisfies \((S_1)\) and

\( (S_3) f_L(x \rightarrow (z' \rightarrow y)) \cap f_L(y \rightarrow z) \subseteq f_L(x \rightarrow z) \), for all \( x, y, z \in L \).

In [27], Ma and Kim introduced the concept of \((M, N)-SI\) filters in BL-algebras.

**Definition 4** (see [27]). A soft set \( f_S \) over \( U \) is called an \((M, N)-soft\) intersection filter (briefly, \((M, N)-SI\) filter) of \( L \) over \( U \) if it satisfies

\( (SI_1) f_S(x) \cap N \subseteq f_L(I \cup M) \) for all \( x \in L \),

\( (SI_2) f_S(x \rightarrow y) \cap f_L(x) \cap N \subseteq f_L(y \cup M) \) for all \( x, y \in L \).

Define an ordered relation \( \preceq^{(M, N)} \) on \( S(U) \) as follows. For any \( f_L, f_M \in S(U), \emptyset \subseteq M \subseteq N \subseteq U \), we define

\( f_L \preceq^{(M, N)} f_M \Leftrightarrow f_L \subseteq f_M \cap N \subseteq f_L \cup M \)

And we define a relation \( \preceq \) as follows:

\( f_L \preceq f_M \Leftrightarrow f_L \preceq^{(M, N)} f_M \).

**Definition 5** (see [27]). A soft set \( f_S \) over \( U \) is called an \((M, N)-soft\) intersection filter (briefly, \((M, N)-SI\) filter) of \( L \) over \( U \) if it satisfies

\( (SI_1') f_S(x) \subseteq f_L(1) \) for all \( x \in L \),

\( (SI_2') f_S(x \rightarrow y) \cap f_L(x) \subseteq f_L(y) \) for all \( x, y \in L \).

**3. \((M, N)-SI\) Implicative (Boolean) Filters**

In this section, we investigate some characterizations of \((M, N)-SI\) implicative filters of BL-algebras. Finally, we prove that a soft set in BL-algebras is an \((M, N)-SI\) implicative filter if and only if it is an \((M, N)-SI\) Boolean filter.

**Definition 6**. A soft set \( f_L \) over \( U \) is called an \((M, N)-soft\) intersection implicative filter (briefly, \((M, N)-SI\) implicative filter) of \( L \) over \( U \) if it satisfies \((SI_1)\) and \((SI_3)\)

\( f_L(x \rightarrow (z' \rightarrow y)) \cap f_L(y \rightarrow z) \subseteq f_L(x \rightarrow z) \cup M \). for all \( x, y, z \in L \).

**Remark 7**. If \( f_L \) is an \((M, N)-SI\) implicative filter of \( L \) over \( U \), then \( f_L \) is an \((\emptyset, U)-SI\) implicative filter of \( L \). Hence every \( SI\)-implicative filter of \( L \) is an \((M, N)-SI\) implicative filter of \( L \), but the converse need not be true in general. See the following example.

**Example 8**. Assume that \( U = D_2 = \{(x, y) \mid x^2 = y^2 = e, xy = yx\} = \{e, x, y, xy\} \), dihedral group, is the universe set.
Let \( L = \{0, a, b, 1\} \), where \( 0 < a < b < 1 \). Then we define 
\[ x \wedge y = \min\{x, y\}, \quad x \vee y = \max\{x, y\} \]
and \( \odot \) and \( \rightarrow \) as follows:
\[
\begin{array}{c|cccc}
\odot & 0 & a & b & 1 \\
\hline
0 & 0 & 0 & 0 & 0 \\
a & 0 & a & a & a \\
b & 0 & a & b & b \\
1 & 0 & 1 & 1 & 1 \\
\end{array}
\quad \begin{array}{c|cccc}
\rightarrow & 0 & a & b & 1 \\
\hline
0 & 0 & 0 & 0 & 0 \\
a & 0 & 1 & 1 & 1 \\
b & 0 & 1 & 1 & 1 \\
1 & 0 & a & b & 1 \\
\end{array}
\tag{1}
\]

Then \( (\land, \lor, \odot, \rightarrow, 1) \) is a BL-algebra.

Let \( M = \{e, y\} \) and \( N = \{e, x, y\} \).

Define a soft set \( f_L \) over \( U \) by \( f_L(1) = \{e, x\}, f_L(a) = f_L(b) = \{e, x, y\}, \) and \( f_L(0) = \{e, y\} \). Then one easily check that \( f_L \) is an \((M, N)-SI\) implicat filter of \( L \) over \( U \), but it is not an SI implicat filter of \( L \) over \( U \) since \( f_L(1) = \{e, x\} \not\supseteq f_L(a) \).

By means of \( \overline{\circ}(M, N) \), we can obtain the following equivalent concept.

**Definition 9.** A soft set \( f_L \) over \( U \) is called an \((M, N)-SI\) implicat filter of \( L \) over \( U \) if it satisfies \((SI4)\) \( f_L(x \lor x' \rightarrow 1) = (M, N) f_L(1) \) for all \( x \in L \).

From the above definitions, we have the following.

**Proposition 10.** Every \((M, N)-SI\) implicat filter of \( L \) over \( U \) is an \((M, N)-SI\) filter, but the converse may not be true as shown in the following example.

**Example 11.** Define \( x \odot y = \min\{x, y\} \) and 
\[
x \rightarrow y = \begin{cases} 
1, & \text{if } x \leq y, \\
y, & \text{if } x > y.
\end{cases}
\tag{2}
\]

Then \( L = (\{0, 1\}, \land, \lor, \odot, \rightarrow, 0, 1) \) is a BL-algebra.

Let \( U = L, M = \{0, 0.5, 0.75\} \), and \( N = \{0, 0.5, 0.75, 1\} \).

Define a soft set \( f_L \) over \( U \) by 
\[
f_L(x) = \begin{cases} 
\{0, 0.5\}, & \text{if } x \in \left[0, \frac{1}{2}\right], \\
\{0.5, 1\}, & \text{if } x \in \left[\frac{1}{2}, 1\right].
\end{cases}
\tag{3}
\]

Then one can easily check that \( f_L \) is an \((M, N)-SI\) filter of \( L \) over \( U \), but it is not an \((M, N)-SI\) implicat filter of \( L \) over \( U \). Since \( f_L(1) \cap f_L(1) \cap N = f_L(1) \cap N = \{0, 1\} \cap \{0, 0.5\} = \{0, 1\} \) and \( f_L(\frac{3}{2}) \cap \{0, 0.5, 0.75, 1\} = \{0, 1\} \), this implies that \( f_L(1) \cap f_L(1) \cap N \not\supseteq f_L(x \rightarrow z) \cup M \).

**Lemma 12** (see [27]). If a soft set \( f_L \) over \( U \) is an \((M, N)-SI\) filter of \( L \), then for any \( x, y, z \in L \), we have
\[
\begin{align*}
(1) & \quad x \leq y \Rightarrow f_L(x) \bar{\circ}(M, N) f_L(y), \\
(2) & \quad f_L(x \rightarrow y) = f_L(1) \cap f_L(x) \bar{\circ}(M, N) f_L(y), \\
(3) & \quad f_L(x \odot y) = f_L(x) \cap f_L(y) = f_L(x \wedge y), \\
(4) & \quad f_L(0) = (M, N) f_L(x) \cap f_L(x'), \\
(5) & \quad f_L(x \rightarrow y) \cap f_L(y \rightarrow z) \bar{\circ}(M, N) f_L(x \rightarrow z), \\
(6) & \quad f_L(x) \cap f_L(y) \bar{\circ}(M, N) f_L(y \rightarrow z), \\
(7) & \quad f_L(x \rightarrow y) \bar{\circ}(M, N) f_L(y \rightarrow z) \cap f_L(x \rightarrow z), \\
(8) & \quad f_L(x \rightarrow y) \bar{\circ}(M, N) (f_L(z \rightarrow x) \cap f_L(z \rightarrow y)).
\end{align*}
\]

**Theorem 13.** Let \( f_L \) be an \((M, N)-SI\) filter of \( L \) over \( U \), then the following are equivalent:
\[
\begin{align*}
(1) & \quad f_L \text{ is an } (M, N)-SI \text{ implicat filter of } L, \\
(2) & \quad f_L(x \rightarrow z) \bar{\circ}(M, N) f_L(x \rightarrow (z' \rightarrow z)) \text{ for all } x, y, z \in L, \\
(3) & \quad f_L(x \rightarrow z) = (M, N) f_L(x \rightarrow (z' \rightarrow z)) \text{ for all } x, y, z \in L, \\
(4) & \quad f_L(x \rightarrow z) = (M, N) f_L (y \rightarrow (x \rightarrow (z' \rightarrow z))) \cap f_L(y), \text{ for all } x, y, z \in L.
\end{align*}
\]

**Proof.** (1) \( \Rightarrow \) (2) Assume that \( f_L \) is an \((M, N)-SI\) filter of \( L \) over \( U \). Putting \( y = z \) in \((SI_3)\), then
\[
\begin{align*}
f_L(x \rightarrow z) & \cup M \\
& \supseteq \left(f_L(x \rightarrow z) \cup M\right) \cap (f_L(z \rightarrow z) \cap N) \\
& = \left(f_L(x \rightarrow z) \cap f_L(z \rightarrow z) \cap N\right) \cup M \\
& \supseteq f_L(x \rightarrow z) \cap (f_L(1) \cap N) \cap M \\
& \supseteq f_L(x \rightarrow z) \cap N;
\end{align*}
\]
that is, \( f_L(x \rightarrow z) \bar{\circ}(M, N) f_L(x \rightarrow (z' \rightarrow z)) \). Thus, (2) holds.

(2) \( \Rightarrow \) (3) By \((a_1)\) and \((a_2)\), \( x \rightarrow z \leq z' \rightarrow x \rightarrow z \); then it follows from Lemma 12 (1) that \( f_L(x \rightarrow z) \bar{\circ}(M, N) f_L(x \rightarrow (z' \rightarrow z)) \). Thus, (3) holds.

(3) \( \Rightarrow \) (4) Assume that (4) holds. By Lemma 12 (5), we have \( f_L(x \odot z' \rightarrow y) \cap f_L(y \rightarrow z) \bar{\circ}(M, N) f_L(x \odot z' \rightarrow z) \). By \((a_2)\), \( f_L(x \rightarrow (z' \rightarrow y)) \cap f_L(y \rightarrow z) \bar{\circ}(M, N) f_L(x \rightarrow (z' \rightarrow z)) \).

(4) \( \Rightarrow \) (1) Putting \( y = 1 \) in (4), we have
\[
f_L(x \rightarrow z) \bar{\circ}(M, N) f_L(x \rightarrow (z' \rightarrow z)).
\]

Hence
\[
f_L(z \rightarrow z) \bar{\circ}(M, N) f_L(x \rightarrow (z' \rightarrow y)) \cap f_L(y \rightarrow z).
\]

Thus, \((SI_4)\) holds. This shows that \( f_L \) is an \((M, N)-SI\) implicat filter of \( L \) over \( U \).

Now, we introduce the concept of \((M, N)-SI\) Boolean filters of BL-algebras.

**Definition 14.** Let \( f_L \) be an \((M, N)-SI\) filter of \( L \) over \( U \), then \( f_L \) is called an \((M, N)-SI\) Boolean filter of \( L \) over \( U \) if it satisfies
\[
(SI_4) \quad f_L(x \lor x') = (M, N) f_L(1) \text{ for all } x \in L.
\]
Theorem 15. A soft set $f_L$ over $U$ is an $(M, N)$-SI implicative filter of $L$ if and only if it is an $(M, N)$-SI Boolean filter.

Proof. Assume that $f_L$ over $U$ is an $(M, N)$-SI Boolean filter of $L$ over $U$. Then

$$f_L(x \rightarrow z)$$

$$\equiv_{(M,N)} f_L((z \lor z') \rightarrow (x \rightarrow z)) \cap f_L((z \lor z') \rightarrow (x \rightarrow z))$$

$$= f_L((x \rightarrow x') \rightarrow x')$$

By (a10) and (a1), we have

$$((z \lor z') \rightarrow (x \rightarrow z)) = (z \rightarrow (x \rightarrow z)) \land (z' \rightarrow (x \rightarrow z))$$

Therefore, it follows from Theorem 13 that $f_L$ is an $(M, N)$-SI implicative filter of $L$ over $U$.

Conversely, assume that $f_L$ is an $(M, N)$-SI implicative filter of $L$ over $U$. By Theorem 13, we have

$$f_L((x \rightarrow z) = f_L((x \rightarrow x') \rightarrow (x' \rightarrow x)) = f_L((x \rightarrow x') \rightarrow (x' \rightarrow x'))$$

$$= f_L((x \rightarrow x') \rightarrow (x \rightarrow x)) = f_L(1).$$

By Lemma 12, we have

$$f_L(x \lor x')$$

$$= f_L((x \rightarrow z) = f_L((x \rightarrow x') \rightarrow (x' \rightarrow x)) = f_L((x \rightarrow x') \rightarrow (x \rightarrow x))$$

$$= f_L(1).$$

Hence $f_L$ is an $(M, N)$-SI Boolean filter of $L$ over $U$.

Remark 16. Every $(M, N)$-SI implicative filter and $(M, N)$-SI Boolean filter in BL-algebras are equivalent.

Next, we give some characterizations of $(M, N)$-SI implicative (Boolean) filters in BL-algebras.

Theorem 17. Let $f_L$ be an $(M, N)$-SI filter of $L$ over $U$, then the following are equivalent:

1. $f_L$ is an $(M, N)$-SI implicative (Boolean) filter,
2. $f_L(x) = f_L((x' \rightarrow x))$, for all $x \in L$,
3. $f_L((x \rightarrow y) \rightarrow x) \equiv_{(M,N)} f_L((x \rightarrow y) \rightarrow x)$, for all $x, y \in L$,
4. $f_L((x \rightarrow y) \rightarrow x) \equiv_{(M,N)} f_L((x \rightarrow y) \rightarrow x)$, for all $x, y \in L$,
5. $f_L((x \rightarrow y) \rightarrow x) \equiv_{(M,N)} f_L((x \rightarrow y) \rightarrow x)$, for all $x, y, z \in L$.

Proof. (1) $\Rightarrow$ (2). Assume that $f_L$ is an $(M, N)$-SI implicative (Boolean) filter of $L$ over $U$. By Theorem 13, we have

$$f_L(x) = f_L((x' \rightarrow x) \equiv_{(M,N)} f_L((x' \rightarrow x)).$$

Thus, (2) holds.

(2) $\Rightarrow$ (3). By (a1), (a2), and (a6), we have $x' \leq x \rightarrow y$ and so $(x \rightarrow y) \leq x \rightarrow x$. By Lemma 12, we have $f_L((x \rightarrow y) \rightarrow x) \equiv_{(M,N)} f_L(x' \rightarrow x)$. Combining (2), we have $f_L((x \rightarrow y) \rightarrow x) \equiv_{(M,N)} f_L(x' \rightarrow x)$. Thus, (3) holds.

(3) $\Rightarrow$ (4). Since $x \leq (x \rightarrow y) \rightarrow x$, then by Lemma 12, we have $f_L((x \rightarrow y) \rightarrow x) \equiv_{(M,N)} f_L((x \rightarrow y) \rightarrow x)$. Combining (3), we have $f_L(x) \equiv_{(M,N)} f_L((x \rightarrow y) \rightarrow x)$. Finally, we investigate extension properties of $(M, N)$-SI implicative filters of BL-algebras.

Theorem 18 (extension property). Let $f_L$ and $g_L$ be two $(M, N)$-SI filters of $L$ over $U$ such that $f_L(1) = g_L(1)$ and $f_L(x) = g_L(x)$ for all $x \in L$. If $f_L$ is an $(M, N)$-SI implicative (Boolean) filter of $L$, then so is $g_L$.

Proof. Assuming that $f_L$ is an $(M, N)$-SI implicative (Boolean) filter of $L$ over $U$, then $f_L(x \lor x') = f_L(1)$ for all $x \in L$. By hypothesis, $g_L(x \lor x') = g_L(1) = f_L(1)$. By (SI'), we have $g_L(1) = g_L(x \lor x')$. Thus, $g_L(x \lor x') = g_L(1)$. Hence $g_L$ is an $(M, N)$-SI implicative (Boolean) filter of $L$.

4. Conclusions

In this paper, we introduce the concepts of $(M, N)$-SI implicative filters and $(M, N)$-SI Boolean filters of BL-algebras. Then we show that every $(M, N)$-SI Boolean filter is equivalent to $(M, N)$-SI implicative filters. In particular, some equivalent conditions for $(M, N)$-SI Boolean filters are obtained. We hope it can lay a foundation for providing a new soft algebraic tool in many uncertainties problems.
To extend this work, one can apply this theory to other fields, such as algebras, topology, and other mathematical branches. To promote this work, we can further investigate $(M, N)$-Sp prime (semiprime) Boolean filters of $BL$-algebras. Maybe one can apply this idea to decision-making, data analysis, and knowledge based systems.

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

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