Research Article

Interval-Valued Optimization Problems Involving (α, ρ) -Right Upper-Dini-Derivative Functions

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We consider an interval-valued multiobjective problem. Some necessary and sufficient optimality conditions for weak efficient solutions are established under new generalized convexities with the tool-right upper-Dini-derivative, which is an extension of directional derivative. Also some duality results are proved for Wolfe and Mond-Weir duals.

1. Introduction

Recently, Yuan and Liu [1] considered some new generalized convexity concepts using right upper-Dini-derivative, which is an extension of directional derivative. Thus some optimality and duality results are established for a nondifferentiable multiobjective programming problem. For various approaches relative to generalized convexity, we refer to [2–9].

In many real-life situations data suffer from inexactness. The interval-valued optimization problems are closely related to optimization problems with inexact data. Recently, Wu [10–12] derived optimality conditions and duality results for a multiobjective programming problem with interval-valued objective functions. See also [13] and their references.

In this paper we consider an interval multiobjective optimization problem. Some new optimality conditions and duality results are stated under new generalized convexities with the tool-right upper-Dini-derivative. The paper is organized as follows. In Section 2 some definitions, notations, and some basic arithmetic of interval calculus are given. In Section 3, we state necessary optimality conditions and in Section 4 we present sufficient optimality conditions. The duality results are stated in Sections 5 and 6. The last section gives some conclusions.

2. Notations and Preliminaries

Let $\mathbb{R}^n$ be the n -dimensional Euclidean space and let $\mathbb{R}_+^n$ be its nonnegative orthant. For $x = (x_1, \dots, x_n) \in \mathbb{R}^n$ and $y = (y_1, \dots, y_n) \in \mathbb{R}^n$ we consider the following conventions:

$$\begin{aligned} x \leq y & \text{ iff } x_i \leq y_i, \quad i = \overline{1, n} \quad (\text{i.e., } y_i - x_i \in \mathbb{R}_+, i = \overline{1, n}); \\ x \leq y & \text{ iff } x \leq y, \quad x \neq y; \\ x < y & \text{ iff } x_i < y_i, \quad i = \overline{1, n}. \end{aligned} \quad (1)$$

Let $X \subset \mathbb{R}^n$ be an arcwise connected set in Avriel and Zang [14] and Bhatia and Mehra [15] and φ a real-valued function defined on X . Let $x_1, x_2 \in X$, and H_{x_1, x_2} be the arc connecting x_1 and x_2 in X .

Definition 1. The right derivative (or right differential) of φ with respect to H at $t = 0$ is defined as

$$\varphi' (H_{x_1, x_2} (0^+)) = \lim_{t \rightarrow 0^+} \frac{\varphi (H_{x_1, x_2} (t)) - \varphi (x_1)}{t}. \quad (2)$$

Yuan and Liu [1] give some new generalized convexity with the upper-Dini-derivative concept.

Definition 2. The right upper-Dini-derivative relative to H is defined by

$$(d\varphi)^+(H_{x_1, x_2}(0^+)) = \limsup_{t \rightarrow 0^+} \frac{\varphi(H_{x_1, x_2}(t)) - \varphi(x_1)}{t}. \quad (3)$$

Definition 3. X is locally arcwise connected (LAC) at $\bar{x}$ if for any $x \in X$ and $x \neq \bar{x}$ there exists a positive number $a(x, \bar{x})$, with $0 < a(x, \bar{x}) \leq 1$ and a continuous arc $H_{\bar{x}, x}$ s.t. $H_{\bar{x}, x}(t) \in X$ for any $t \in (0, a(x, \bar{x}))$. The set X is LAC if X is LAC at any $x \in X$.

Definition 4 (see [1]). Let $X \subset \mathbb{R}^n$ be a LAC set and let $\varphi : X \rightarrow \mathbb{R}$ be a real function defined on X . The function φ is said to be (α, ρ) -right upper-Dini-derivative locally arcwise connected with respect to H at u , if there exist real functions $\alpha : X \times X \rightarrow \mathbb{R}$ and $\rho : X \times X \rightarrow \mathbb{R}$ such that

$$\varphi(x) - \varphi(u) \geq \alpha(x, u) (d\varphi)^+(H_{u, x}(0^+)) + \rho(x, u), \quad (4)$$

$$\forall x \in X.$$

If φ is (α, ρ) -right upper-Dini-derivative locally arcwise connected (with respect to H) at u for any $u \in X$, then φ is called (α, ρ) -right upper-Dini-derivative locally arcwise connected (with respect to H) on X .

Definition 5 (see [1]). A k -dimensional vector-valued function $f : X \rightarrow \mathbb{R}^k$ is called (α, ρ) -right upper-Dini-derivative arcwise connected (with respect to H) at u , if the i th component of f is (α_i, ρ_i) -right upper-Dini-derivative arcwise connected (with respect to H) at u for $i \in K$, where $\alpha = (\alpha_1, \dots, \alpha_k)^T$ and $\rho = (\rho_1, \dots, \rho_k)^T$. If f is (α, ρ) -right upper-Dini-derivative arcwise connected (with respect to H) at any $u \in X$, then f is called (α, ρ) -right upper-Dini-derivative arcwise connected (with respect to H) on X .

Definition 6 (see [3]). A k -dimensional vector-valued function $f : X \rightarrow \mathbb{R}^k$ is called convex-like (with respect to η) on X if for all $x, u \in X$ and any $t \in [0, 1]$, there exists $z \in X$ such that $f(z) \leq tf(x) + (1-t)f(u)$.

Definition 7 (see [1]). A k -dimensional vector-valued function $f : X \rightarrow \mathbb{R}^k$ is called ρ -generalized (strong) pseudoright upper-Dini-derivative arcwise connected (with respect to H) at u , if there exists vector-valued function ρ such that $f(x) < (\leq) f(u) \Rightarrow (d\varphi)^+(H_{u, x}(0^+)) < \rho(x, u)$, for $x \in X$; f is called ρ -generalized (weak) quasi-right upper-Dini-derivative arcwise connected (with respect to H) at u , if there exists vector-valued function ρ such that $f(x) \leq (<) f(u) \Rightarrow (d\varphi)^+(H_{u, x}(0^+)) \leq \rho(x, u)$, for $x \in X$, where $\rho = (\rho_1, \dots, \rho_k)^T$.

Lemma 8 (see [16]). Let $S \subset \mathbb{R}^n$ be a nonempty set and let $\Psi : S \rightarrow \mathbb{R}^m$ be a convex-like vector-valued function on S . Then either $\Psi(x) < 0$ has a solution $x \in S$, or there exists $\lambda \in \mathbb{R}_+^m$ such that the system $\lambda^T \Psi(x) \geq 0$ holds for all $x \in S$, but both are never true at the same time.

Let $\text{CBI}(\mathbb{R})$ be the class of all closed and bounded intervals in $\mathbb{R}$. Thus if $a = [a^L, a^U] \in \text{CBI}(\mathbb{R})$, we have

$$a = [a^L, a^U] = \{x \in \mathbb{R} : a^L \leq x \leq a^U\}, \quad (5)$$

where a^L and a^U mean lower and upper bounds of a . If $a^L = a^U$, then $a = [a, a]$ is a real number. Also, let $b = [b^L, b^U]$. Then, by definition we have

$$\begin{aligned} a + b &= [a^L + b^L, a^U + b^U], \\ a - b &= [a^L - b^U, a^U - b^L]. \end{aligned} \quad (6)$$

For a real number α , we have

$$\alpha a = \begin{cases} [\alpha a^L, \alpha a^U], & \text{if } \alpha > 0, \\ [\alpha a^U, \alpha a^L], & \text{if } \alpha < 0, \\ [0, 0] = 0, & \text{if } \alpha = 0. \end{cases} \quad (7)$$

Using [17, 18], we consider some preliminary results about interval arithmetic calculus.

Definition 9. Let $a = [a^L, a^U]$ and $b = [b^L, b^U] \in \text{CBI}(\mathbb{R})$. We say that a is less than b and write $a < b$ if $a_i < b_i, \forall i = \overline{1, n}$.

Definition 10. Let $a, b \in \text{CBI}(\mathbb{R}^n)$. We say that a is less than or equal to b and write $a \leq b$ if $a^L \leq b^L$ and $a^U \leq b^U$.

Let X be a nonempty subset of $\mathbb{R}^n$. A function $\varphi : X \rightarrow \text{CBI}(\mathbb{R})$ is called an interval-valued function. In this case,

$$\varphi(x) = [\varphi^L(x), \varphi^U(x)], \quad (8)$$

with $\varphi^L, \varphi^U : X \rightarrow \mathbb{R}, \varphi^L(x) \leq \varphi^U(x), \forall x \in X$.

We consider the following multiobjective interval-valued optimization problem:

$$\begin{aligned} \text{cmin } f(x) &= (f_1(x), \dots, f_n(x)) \\ \text{subject to } g(x) &\leq 0 \\ x &\in X, \end{aligned} \quad (\text{IVP})$$

with $g = (g_1, \dots, g_m)^T, f^L = (f_1^L, \dots, f_k^L)^T, f^U = (f_1^U, \dots, f_k^U)^T$, where $f_i^L, f_i^U : \mathbb{R}^n \rightarrow \mathbb{R}, f_i(x) = [f_i^L(x), f_i^U(x)]$ for $i = \overline{1, k}$ and $g_j : \mathbb{R}^n \rightarrow \mathbb{R}, j = \overline{1, m}$. Let $X_0 = \{x \in X : g(x) \leq 0\}$ be the set of all feasible points of (IVP). We put $K = \{1, \dots, k\}$.

Definition 11. Let $\bar{x} \in X$. We say that $\bar{x}$ is a weak efficient solution of (IVP) if there exists no $\tilde{x} \in X$ such that $f(\tilde{x}) < f(\bar{x})$.

3. Necessary Optimality Conditions

In this section, we establish Fritz John and Karush-Kuhn-Tucker necessary optimality conditions for problem (IVP).

Theorem 12 (Fritz John necessary condition). Assume that x^* is an efficient solution for (IVP). If $(df^L)^+(H_{x^*,x}(0^+))$, $(df^U)^+(H_{x^*,x}(0^+))$, and $(dg_{J(x^*)})^+(H_{x^*,x}(0^+))$ are convex-like on X with respect to the variable x and g_j is upper semicontinuous at x^* for $j \in J \setminus J(x^*)$, then there exist $\xi^L, \xi^U \in \mathbb{R}_+^k$, $\mu \in \mathbb{R}_+^m$, $(\xi^L, \xi^U, \mu) \neq 0$ such that

$$(\xi^L)^T (df^L)^+(H_{x^*,x}(0^+)) + (\xi^U)^T (df^U)^+(H_{x^*,x}(0^+)) + \mu^T (dg)^+(H_{x^*,x}(0^+)) \geq 0, \quad \forall x \in X; \quad (9)$$

$$\mu^T g(x^*) = 0. \quad (10)$$

Proof. We prove firstly that the following system of inequalities

$$(df^\alpha)^+(H_{x^*,x}(0^+)) < 0, \quad s \in \{L, U\}; \quad (11)$$

$$(dg_{J(x^*)})^+(H_{x^*,x}(0^+)) < 0$$

has no solution for $x \in X$. We proceed by contradicting. If there exists $x' \in X$, a solution of this system, we get that for each $i = \overline{1, n}$ there exist $\delta_i^L > 0$ and $\delta_i^U > 0$ such that for $s \in \{L, U\}$

$$f_i^s(H_{x^*,x'}(t)) < f_i^s(x^*), \quad \text{for } t \in (0, \delta_i^s), \quad (12)$$

and for each $j \in J(x^*)$ there exists $\delta_j > 0$ such that

$$g_j(H_{x^*,x'}(t)) < g_j(x^*) = 0, \quad \text{for } t \in (0, \delta_j). \quad (13)$$

Since g_j , for $j \notin J(x^*)$, is semicontinuous at x^* , then $g_j(H_{x^*,x'}(t))$ is semicontinuous at $t = 0$. Finally we get

$$f_i^s(H_{x^*,\bar{x}}(t)) < f_i^s(x^*), \quad \text{for } i \in K, s \in \{L, U\}; \quad (14)$$

$$g_j(H_{x^*,x'}(t)) < 0, \quad \text{for } j \in M,$$

for any $t \in (0, \delta_0)$, where $\delta_0 = \min\{\min_i \delta_i^L, \min_i \delta_i^U, \min_j \delta_j\} > 0$. These inequalities contradict that x^* is a weak efficient solution for (IVP). Hence the systems (9) and (10) have no solution for $x \in X$. Now we can apply Lemma 8. Since $(df^L)^+$, $(df^U)^+$, and $(dg_{J(x^*)})^+(H_{x^*,x}(0^+))$ are convex-like on X , we obtain that (9) and (10) hold and the proof is complete. $\square$

Theorem 13 (Karush-Kuhn-Tucker necessary optimality condition). Let $(df^L)^+(H_{x^*,x}(0^+))$, $(df^U)^+(H_{x^*,x}(0^+))$, and $(dg_{J(x^*)})^+(H_{x^*,x}(0^+))$ be convex-like on X with respect to the variables x and g_j , for $j \notin J(x^*)$, be upper semicontinuous at x^* . If there exists $x' \in X_0$ such that

$$(dg_{J(x^*)})^+(H_{x^*,x'}(0^+)) < 0, \quad (15)$$

and x^* is a weak efficient solution for the problem (IVP), then there exist $\xi^\alpha = (\xi_1^\alpha, \dots, \xi_k^\alpha)^T \in \mathbb{R}_+^k$, $\alpha \in \{L, U\}$, and $\mu \in \mathbb{R}_+^m$ satisfying (9), (10), and $(\xi^L, \xi^U) \neq 0$.

Proof. We suppose $(\xi^L, \xi^U) = 0$. Then, by (9) we obtain

$$\mu^T (dg)^+(H_{x^*,x}(0^+)) \geq 0, \quad \forall x \in X. \quad (16)$$

Since $\mu_j = 0$ for $j \in J \setminus J(x^*)$, by (10), and there exists $j \in J(x^*)$ such that $\mu_j > 0$, by (15), it results $\mu^T (dg)^+(H_{x^*,x'}(0^+)) < 0$, which contradicts (15) and theorem is proved. $\square$

4. Sufficient Optimality Conditions

In this section we give some Karush-Kuhn-Tucker type sufficient optimality conditions under generalized convexity with upper-Dini-derivative concept.

Theorem 14. Let x^* be a feasible solution of (IVP). Assume that there exist ξ^{L^*}, ξ^{U^*} , $(\xi^{L^*}, \xi^{U^*}) \neq 0$ and $\mu^* \geq 0$, $i \in J(x^*)$, such that f_i^s ($i \in K, s \in \{L, U\}$) and g_j ($j \in J$) are (α, ρ_i^s) and (α, ρ_j') -right upper-Dini-derivative locally arcwise connected at x^* with respect to H , respectively. Also one assumes $\alpha(x, x^*) \geq 0$ and

$$(\xi^{L^*})^T \rho^L(x, x^*) + (\xi^{U^*})^T \rho^U(x, x^*) + \mu^{*T} \rho_{J(x^*)}' \geq 0 \quad (17)$$

hold for all $x \in X_0$ feasible. Then x^* is a weak efficient solution for (IVP).

Proof. We suppose to the contrary that x^* is not a weak efficient solution for (IVP). Then there exists $x' \in D$ such that $f(x') < f(x^*)$. Now, since $(\xi^{L^*}, \xi^{U^*}) \geq 0$, $\mu_{J(x^*)}^* \geq 0$, and $g_{J(x^*)}(x') \leq 0 = g_{J(x^*)}(x^*)$, we get

$$(\xi^{L^*})^T (f^L(x') - f^L(x^*)) + (\xi^{U^*})^T (f^U(x') - f^U(x^*)) + \mu_{J(x^*)}^{*T} (g_{J(x^*)}(x') - g_{J(x^*)}(x^*)) < 0. \quad (18)$$

Since f_i^s ($i \in K, s \in \{L, U\}$) and g_j ($j \in J$) are (α, ρ_i^s) and (α, ρ_j') -right upper-Dini-derivative locally arcwise connected at x^* , by (18) we obtain

$$\alpha(x', x^*) \left((\xi^{L^*})^T (df^L)^+(H_{x^*,x'}(0^+)) + (\xi^{U^*})^T (df^U)^+(H_{x^*,x'}(0^+)) + \mu_{J(x^*)}^{*T} (dg_{J(x^*)})^+(H_{x^*,x'}(0^+)) + (\xi^{L^*})^T \rho^L(\bar{x}, x') + (\xi^{U^*})^T \rho^U(\bar{x}, x') \right) < 0. \quad (19)$$

Now, by $\alpha(\bar{x}, x') \geq 0$ and (17) we get a contradiction. Thus the theorem is proved. $\square$

The next sufficient optimality condition is given in the case of generalized pseudo- and quasi-right upper-Dini-derivative arcwise connected type, where the proof is on the line of the above theorem.

Theorem 15. Let x^* be a feasible solution of (IVP). Assume that there exist $\xi^{L*}, \xi^{U*}, (\xi^{L*}, \xi^{U*}) \geq 0$ and $\mu^* \geq 0, i \in J(x^*)$, such that f^s ($s \in \{L, U\}$) is generalized pseudoright upper-Dini-derivative locally arcwise connected at x^* with respect to H and g is ρ' -generalized quasi-right upper-Dini-derivative locally arcwise connected at x^* with respect to H , respectively. Also one assumes that there exist $\xi^{L*}, \xi^{U*}, (\xi^{L*}, \xi^{U*}) \geq 0$ and $\mu^* \geq 0$ such that $(\xi^{L*})^T \rho^L(x, x^*) + (\xi^{U*})^T \rho^U(x, x^*) + \mu^{*T} \rho'(x, x^*) \geq 0$ for any $x \in X_0$. Then x^* is a weak efficient solution for (IVP).

5. Wolfe Duality

Relative to (IVP) we consider the following Wolfe type dual problem:

$$\begin{aligned} \min \quad & \Psi(y, \xi^L, \xi^U, \mu) = f(y) + \mu^T g(y) e \\ \text{subject to} \quad & (\xi^{L*})^T (df^L)^+(H_{y,x}(0^+)) \\ & + (\xi^{U*})^T (df^U)^+(H_{y,x}(0^+)) \\ & + \mu^T (dg)^+(H_{y,x}(0^+)) \geq 0, \quad \forall x \in X_0 \\ & (\xi^L, \xi^U) \geq 0, \quad \mu \geq 0, \quad y \in X, \end{aligned} \quad (\text{IWVD})$$

where $f(y) = [f^L(y), f^U(y)]$ and $e = (1, 1, \dots, 1)$ such that $[e^T \xi^L, e^T \xi^U] = [1, 1] = 1$. Let F_D denote the set of all feasible solutions of (IWVD) and let $\text{pr}_X F_D$ be the projection of the set F_D on X . Now we present weak, strong, and strict converse duality theorems relative to (IVP) and (IWVD). The proofs, which will be skipped here, follow the classic lines of multiobjective optimization [4, 7] and interval optimization problems [11].

Theorem 16 (weak duality). Let x and (y, ξ^L, ξ^U, μ) be a feasible solution for (IVP) and (IWVD), respectively. One supposes that f^s ($s \in \{L, U\}$) and g are (α, ρ^s) and (α, ρ') -right upper-Dini-derivative arcwise connected at y on $X_0 \cup \text{pr}_X F_D$ with respect to H , respectively. Moreover, one assumes $\alpha(x, y) \geq 0$ and $(\xi^L)^T \rho^L + (\xi^U)^T \rho^U + \mu^T \rho' \geq 0$, for all $x \in X_0$. Then the following cannot hold: $f(x) < \Psi(y, \xi^L, \xi^U, \mu)$.

Theorem 17 (strong duality). Let x^* be a weak efficient solution of (IVP), at which the assumptions of Karush-Kuhn-Tucker necessary optimality conditions are satisfied. Then there exist $(\xi^{L*}, \xi^{U*}) \geq 0$ and $\mu^* \in \mathbb{R}^m$ such that $(x^*, \xi^{L*}, \xi^{U*}, \mu^*) \in F_D$ and the objective values of (IVP) and (IWVD) are equal. Further, if the hypotheses of weak duality theorem hold for all feasible solutions (y, ξ^L, ξ^U, μ) for (IWVD), then $(x^*, \xi^{L*}, \xi^{U*}, \mu^*)$ is an efficient solution of (IWVD).

Theorem 18 (converse duality). Let $(y^*, \xi^{L*}, \xi^{U*}, \mu^*)$ be a weak efficient solution of (IWVD). One assumes that f^s ($s \in \{L, U\}$) and g are (α, ρ^s) and (α, ρ') -right upper Dini derivative

arcwise connected at y on $X_0 \cup \text{pr}_X F_D$ with respect to H , respectively. If $\alpha(x, y^*) \geq 0$ and $(\xi^{L*})^T \rho^L(x, y^*) + (\xi^{U*})^T \rho^U(x, y^*) + (\mu^*)^T \rho'(x, y^*) \geq 0$, for all $x \in X_0$, then y^* is a weak efficient solution of (IVP).

6. Mond-Weir Duality

In this section we consider the following interval multiobjective dual problem, which is Mond-Weir dual type [6]:

$$\begin{aligned} \max \quad & f(y) \\ \text{subject to} \quad & (\xi^L)^T (df^L)^+(H_{y,x}(0^+)) \\ & + (\xi^U)^T (df^U)^+(H_{y,x}(0^+)) \\ & + \mu^T (dg)^+(H_{y,x}(0^+)) \geq 0, \quad \forall x \in X_0 \\ & \mu^T g(y) \geq 0 \\ & \xi^L, \xi^U \in \mathbb{R}^n, \quad (\xi^L, \xi^U) \geq 0, \quad \mu \geq 0, \end{aligned} \quad (\text{IMWVD})$$

where $e = (1, \dots, 1) \in \mathbb{R}^k$, with $[e^T \xi^L, e^T \xi^U] = [1, 1] = 1$. Let F_D denote the set of all feasible solutions of (IMWVD) and $\text{pr}_X F_D$ as the projection of the set F_D on X .

As in Section 5, we can establish some duality results. Here, we simply state them in the next theorems.

Theorem 19 (weak duality). Let x and (y, ξ^L, ξ^U, μ) be a feasible solution for (IVP) and (IMWVD), respectively. Assume that f^s ($s \in \{L, U\}$) and g are (α, ρ^s) and (α, ρ') -right upper-Dini-derivative arcwise connected at y on $X_0 \cup \text{pr}_X F_D$ with respect to H , respectively. Also one supposes that $\alpha(x, y) \geq 0$ and $(\xi^L)^T \rho^L(x, y) + (\xi^U)^T \rho^U(x, y) + \mu^T \rho'(x, y) \geq 0$, for all $x \in X_0$. Then the following cannot hold: $f(x) \leq f(y)$.

Theorem 20 (strong duality). Let x^* be a weak efficient solution of the interval multiobjective programming problem (IVP), at which the assumptions of Karush-Kuhn-Tucker necessary optimality conditions are satisfied. Then there exist $\xi^{L*}, \xi^{U*} \in \mathbb{R}^k$, $(\xi^{L*}, \xi^{U*}) \geq 0$, and $\mu^* \in \mathbb{R}^m$, $\mu^* \geq 0$, such that $(x^*, \xi^{L*}, \xi^{U*}, \mu^*)$ is a feasible solution for (IMWVD) and the objective values of (IVP) and (IMWVD) are equal. Moreover, if the weak duality result between (IVP) and (IMWVD) holds, then $(x^*, \xi^{L*}, \xi^{U*}, \mu^*)$ is a weak efficient solution for (IMWVD).

Theorem 21 (converse duality). Let $(y^*, \xi^{L*}, \xi^{U*}, \mu^*)$ be a weak efficient solution of (IMWVD). Suppose that f^s ($s \in \{L, U\}$) and g are (α, ρ^s) and (α, ρ') -right upper-Dini-derivative arcwise connected at y on $X_0 \cup \text{pr}_X F_D$ with respect to H , respectively. Further, if $\alpha(x, y^*) \geq 0$ and $(\xi^{L*})^T \rho^L(x, y^*) + (\xi^{U*})^T \rho^U(x, y^*) + (\mu^*)^T \rho'(x, y^*) \geq 0$, for all $x \in X_0$, then y^* is a weak efficient solution of (IVP).

7. Conclusions

In this paper we considered an interval multiobjective optimization problem. Some new optimality conditions and duality results were studied under the generalized convexity considered by Yuan and Liu [1]. Necessary optimality conditions and sufficient optimality conditions were derived. Duality results were established. These results can be extended to a class of univex generalized convexity of Mishra type [4] with the tool-right upper-Dini-derivative. Further it is possible to establish duality results relative to a mixed dual of Xu type [9].

Conflict of Interests

The author declares that there is no conflict of interests regarding the publication of this paper.

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