Research Article

On a Subclass of Meromorphic Close-to-Convex Functions

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Received 5 July 2014; Accepted 14 August 2014; Published 28 August 2014

Academic Editor: Rosihan M. Ali

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The main purpose of this paper is to introduce and investigate a certain subclass of meromorphic close-to-convex functions. Such results as coefficient inequalities, convolution property, inclusion relationship, distortion property, and radius of meromorphic convexity are derived.

1. Introduction and Preliminaries

Let Σ denote the class of functions f of the form

$$f(z) = \frac{1}{z} + \sum_{n=1}^{\infty} a_n z^n, \quad (1)$$

which are analytic in the punctured open unit disk

$$\mathbb{U}^* := \{z : z \in \mathbb{C}, 0 < |z| < 1\} =: \mathbb{U} \setminus \{0\}. \quad (2)$$

For two functions f and g , analytic in $\mathbb{U}$, we say that the function f is subordinate to g in $\mathbb{U}$ and write

$$f(z) \prec g(z) \quad (z \in \mathbb{U}), \quad (3)$$

if there exists a Schwarz function ω , which is analytic in $\mathbb{U}$ with

$$\omega(0) = 0, \quad |\omega(z)| < 1 \quad (z \in \mathbb{U}), \quad (4)$$

such that

$$f(z) = g(\omega(z)) \quad (z \in \mathbb{U}). \quad (5)$$

Indeed, it is known that

$$\begin{aligned} f(z) \prec g(z) \quad (z \in \mathbb{U}) &\implies f(0) = g(0), \\ f(\mathbb{U}) &\subset g(\mathbb{U}). \end{aligned} \quad (6)$$

Furthermore, if the function g is univalent in $\mathbb{U}$, then we have the following equivalence:

$$\begin{aligned} f(z) \prec g(z) \quad (z \in \mathbb{U}) &\iff f(0) = g(0), \\ f(\mathbb{U}) &\subset g(\mathbb{U}). \end{aligned} \quad (7)$$

Let $f, g \in \Sigma$, where f is given by (1) and g is defined by

$$g(z) = \frac{1}{z} + \sum_{n=1}^{\infty} b_n z^n. \quad (8)$$

Then the Hadamard product (or convolution) $f * g$ of the functions f and g is defined by

$$(f * g)(z) := \frac{1}{z} + \sum_{n=1}^{\infty} a_n b_n z^n =: (g * f)(z). \quad (9)$$

A function $f \in \Sigma$ is said to be in the class $\mathcal{MS}^*$ of meromorphic starlike functions if it satisfies the inequality

$$\Re \left(\frac{zf'(z)}{f(z)} \right) < 0 \quad (z \in \mathbb{U}). \quad (10)$$

A function $f \in \Sigma$ is said to be in the class $\mathcal{MK}$ of meromorphic convex functions if it satisfies the inequality

$$\Re \left(1 + \frac{zf''(z)}{f'(z)} \right) < 0 \quad (z \in \mathbb{U}). \quad (11)$$

Moreover, a function $f \in \Sigma$ is said to be in the class $\mathcal{MC}$ of meromorphic close-to-convex functions if it satisfies the condition

$$\Re \left(\frac{zf'(z)}{g(z)} \right) < 0 \quad (z \in \mathbb{U}; g \in \mathcal{MS}^*). \quad (12)$$

Let

$$\mathfrak{f}(z) = z + a_2 z^2 + a_3 z^3 + \dots \quad (13)$$

be analytic in $\mathbb{U}$. If there exists a function $g \in \mathcal{K}$ such that

$$\left| \frac{(z\mathfrak{f}'(z)/g(z)) - 1}{(z\mathfrak{f}'(z)/g(z)) + (1 - 2\alpha)} \right| < \beta \quad (14)$$

$$(z \in \mathbb{U}; 0 \leq \alpha < 1; 0 < \beta \leq 1),$$

then we say that $\mathfrak{f} \in \mathcal{C}(\alpha, \beta)$, where $\mathcal{K}$ denotes the usual class of convex functions. The function class $\mathcal{C}(\alpha, \beta)$ was introduced and studied recently by Peng [1] (see also Peng and Han [2], Selvaraj [3], Gao and Zhou [4], Kowalczyk and Leś-Bomba [5], and Xu et al. [6]).

Motivated essentially by the above mentioned function class $\mathcal{C}(\alpha, \beta)$, we now introduce and investigate the following class of meromorphic close-to-convex functions.

Definition 1. A function $f \in \Sigma$ is said to be in the class $\mathcal{MC}(\alpha, \beta)$ if it satisfies the inequality

$$\left| \frac{(zf'(z)/g(z)) + 1}{(zf'(z)/g(z)) + (2\alpha - 1)} \right| < \beta \quad (z \in \mathbb{U}; g \in \mathcal{MS}^*), \quad (15)$$

where (and throughout this paper unless otherwise mentioned) the parameters α and β are constrained as follows:

$$0 \leq \alpha < 1, \quad 0 < \beta \leq 1. \quad (16)$$

It is easy to verify that $f \in \mathcal{MC}(\alpha, \beta)$ if and only if

$$-\frac{zf'(z)}{g(z)} < \frac{1 + (1 - 2\alpha)\beta z}{1 - \beta z} \quad (z \in \mathbb{U}). \quad (17)$$

We observe that

$$\Re \left(\frac{1 + (1 - 2\alpha)\beta z}{1 - \beta z} \right) > 0 \quad (z \in \mathbb{U}), \quad (18)$$

and, thus, the function class $\mathcal{MC}(\alpha, \beta)$ is a subclass of meromorphic close-to-convex functions.

Clearly, the class $\mathcal{MC}(\alpha, 1) =: \mathcal{MC}(\alpha)$ is the familiar class of meromorphic close-to-convex functions of order α .

For some recent investigations of meromorphic functions, see, for example, the works of [7–22] and the references cited therein.

To derive our main results, we need the following lemmas.

Lemma 2 (see [23]). Let

$$f_1(z) = 1 + \sum_{n=1}^{\infty} c_n z^n \quad (19)$$

be analytic in $\mathbb{U}$ and let

$$f_2(z) = 1 + \sum_{n=1}^{\infty} d_n z^n \quad (20)$$

be analytic and convex in $\mathbb{U}$. If $f_1 < f_2$, then

$$|c_n| \leq |d_n| \quad (n \in \mathbb{N} := \{1, 2, \dots\}). \quad (21)$$

Lemma 3 (see [24]). Suppose that

$$\mathfrak{h}(z) = \frac{1}{z} + \sum_{n=1}^{\infty} c_n z^n \in \mathcal{MS}^*. \quad (22)$$

Then

$$|c_n| \leq \frac{2}{n+1} \quad (n \in \mathbb{N}). \quad (23)$$

Each of these inequalities is sharp, with the extremal function given by

$$\mathfrak{h}(z) = z^{-1} (1 + z^{n+1})^{2/(n+1)}. \quad (24)$$

Lemma 4 (see [25]). Let $-1 \leq D < C \leq 1$ and $-1 \leq F < E \leq 1$. Then,

$$\frac{1 + Cz}{1 + Dz} < \frac{1 + Ez}{1 + Fz} \quad (25)$$

if and only if

$$|ED - FC| \leq (E - F) - (C - D). \quad (26)$$

Lemma 5 (see [26]). Suppose that the function $g \in \mathcal{MS}^*$. Then

$$\frac{(1-r)^2}{r} \leq |g(z)| \leq \frac{(1+r)^2}{r} \quad (|z| = r; 0 < r < 1). \quad (27)$$

Lemma 6 (see [27]). Suppose that

$$\mathfrak{p}(z) < \frac{1 + (1 - 2\alpha)z}{1 - z} \quad (z \in \mathbb{U}; 0 \leq \alpha < 1). \quad (28)$$

Then,

$$\Re \left(\frac{z\mathfrak{p}'(z)}{\mathfrak{p}(z)} \right) \geq \begin{cases} -\frac{2(1-\alpha)r}{(1+r)[1+(2\alpha-1)r]} & (|z| = r; 0 < r \leq r_0), \\ \left(\sqrt{4\alpha(1-r^2)[1+(1-2\alpha)r^2]} - [1+(1-2\alpha)r^2] \right) \times ((1-\alpha)(1-r^2))^{-1} & \\ -\frac{\alpha}{1-\alpha} & (|z| = r; r_0 < r < 1), \end{cases} \quad (29)$$

where r_0 is the unique root of the equation

$$(2\alpha - 1)r^4 - 2(2\alpha - 1)r^3 - 6\alpha r^2 - 2r + 1 = 0 \quad (30)$$

in the interval $(0, 1)$. The results are sharp.

In the present paper, we aim at proving some coefficient inequalities, convolution property, inclusion relationship, distortion property, and radius of meromorphic convexity of the class $\mathcal{MC}(\alpha, \beta)$.

2. Main Results

We begin by stating the following coefficient inequality of the class $\mathcal{MC}(\alpha, \beta)$.

Theorem 7. Suppose that

$$f(z) = \frac{1}{z} + \sum_{n=1}^{\infty} a_n z^n \in \mathcal{MC}(\alpha, \beta). \quad (31)$$

Then

$$|a_1| \leq 1, \quad (32)$$

$$|a_n| \leq \frac{2(1-\alpha)\beta}{n} \left(1 + \sum_{k=1}^{n-1} \frac{2}{k+1} \right) + \frac{2}{n(n+1)} \quad (33)$$

$(n \in \mathbb{N} \setminus \{1\}).$

Proof. Let $f \in \mathcal{MC}(\alpha, \beta)$ and suppose that

$$p(z) := -\frac{zf'(z)}{g(z)} \quad (z \in \mathbb{U}), \quad (34)$$

where

$$g(z) = \frac{1}{z} + \sum_{n=1}^{\infty} b_n z^n \in \mathcal{MS}^*. \quad (35)$$

It follows that

$$p(z) = 1 + \sum_{n=1}^{\infty} p_n z^n < \frac{1 + (1-2\alpha)\beta z}{1 - \beta z} \quad (z \in \mathbb{U}). \quad (36)$$

In view of Lemma 2, we know that

$$|p_n| \leq 2(1-\alpha)\beta \quad (n \in \mathbb{N}). \quad (37)$$

By substituting the series expressions of functions f , g , and p into (34), we get

$$\begin{aligned} & (1 + p_1 z + p_2 z^2 + \cdots + p_n z^n + p_{n+1} z^{n+1} + \cdots) \\ & \times \left(\frac{1}{z} + b_1 z + b_2 z^2 + \cdots + b_n z^n + \cdots \right) \\ & = \frac{1}{z} - a_1 z - 2a_2 z^2 - \cdots - na_n z^n - \cdots \end{aligned} \quad (38)$$

Since f is univalent in $\mathbb{U}^*$, it is well known that $|a_1| \leq 1$.

On the other hand, we find from (38) that

$$-na_n = b_n + p_1 b_{n-1} + p_2 b_{n-2} + \cdots + p_{n-1} b_1 + p_{n+1} \quad (39)$$

$(n \in \mathbb{N} \setminus \{1\}).$

By noting that $g \in \mathcal{MS}^*$, it follows from Lemma 3 that

$$|b_n| \leq \frac{2}{n+1} \quad (n \in \mathbb{N}). \quad (40)$$

Combining (37), (39), and (40), we have

$$n|a_n| \leq 2(1-\alpha)\beta \left(\frac{2}{n} + \frac{2}{n-1} + \cdots + \frac{2}{3} + 1 + 1 \right) + \frac{2}{n+1} \quad (41)$$

$(n \in \mathbb{N} \setminus \{1\}).$

Thus, the assertion (33) of Theorem 7 follows directly from (41). $\square$

Theorem 8. Let

$$g(z) = \frac{1}{z} + \sum_{n=1}^{\infty} b_n z^n \in \mathcal{MS}^*. \quad (42)$$

If $f \in \Sigma$ satisfies the condition

$$(1+\beta) \sum_{n=1}^{\infty} n|a_n| + (1+|2\alpha-1|\beta) \sum_{n=1}^{\infty} |b_n| \leq 2(1-\alpha)\beta, \quad (43)$$

then $f \in \mathcal{MC}(\alpha, \beta)$.

Proof. To prove $f \in \mathcal{MC}(\alpha, \beta)$, it suffices to show that (15) holds. From (43), we know that

$$\begin{aligned} & \beta \left(2(1-\alpha) - \sum_{n=1}^{\infty} n|a_n| - |2\alpha-1| \sum_{n=1}^{\infty} |b_n| \right) \\ & \geq \sum_{n=1}^{\infty} n|a_n| + \sum_{n=1}^{\infty} |b_n| > 0. \end{aligned} \quad (44)$$

Now, by the maximum modulus principle, we deduce from (1) and (44) that

$$\begin{aligned} & \left| \frac{(zf'(z)/g(z)) + 1}{(zf'(z)/g(z)) + (2\alpha-1)} \right| \\ & = \left| \frac{\sum_{n=1}^{\infty} na_n z^{n+1} + \sum_{n=1}^{\infty} b_n z^{n+1}}{\sum_{n=1}^{\infty} na_n z^{n+1} + (2\alpha-1) \sum_{n=1}^{\infty} b_n z^{n+1} + 2(\alpha-1)} \right| \\ & < \frac{\sum_{n=1}^{\infty} n|a_n| + \sum_{n=1}^{\infty} |b_n|}{2(1-\alpha) - \sum_{n=1}^{\infty} n|a_n| - |2\alpha-1| \sum_{n=1}^{\infty} |b_n|} \leq \beta. \end{aligned} \quad (45)$$

This evidently completes the proof of Theorem 8. $\square$

Example 9. By applying Theorem 8, it is obvious to see that the function

$$f(z) = \frac{1}{z} + z \in \mathcal{MC}(0, 1). \quad (46)$$

Theorem 10. Let $|\xi| = 1$ and $g \in \mathcal{MS}^*$. A function $f \in \mathcal{MC}(\alpha, \beta)$ if and only if

$$f(z) * \left[(1 - \beta\xi) \frac{1 - 2z}{z(1 - z)^2} \right] - g(z) * \left\{ [1 + (1 - 2\alpha)\beta\xi] \frac{z^2 - z + 1}{z(1 - z)} \right\} \neq 0 \quad (47)$$

$(z \in \mathbb{U}^*).$

Proof. A function $f \in \mathcal{MC}(\alpha, \beta)$ if and only if

$$-\frac{zf'(z)}{g(z)} \neq \frac{1 + (1 - 2\alpha)\beta\xi}{1 - \beta\xi} \quad (z \in \mathbb{U}; |\xi| = 1). \quad (48)$$

It is easy to see that condition (48) can be written as

$$zf'(z)(1 - \beta\xi) + g(z)[1 + (1 - 2\alpha)\beta\xi] \neq 0 \quad (49)$$

$(z \in \mathbb{U}^*; |\xi| = 1).$

We observe that

$$-zf'(z) = f(z) * \left(\frac{1}{z} - \frac{z}{(1 - z)^2} \right) = f(z) * \frac{1 - 2z}{z(1 - z)^2},$$

$$g(z) = g(z) * \left(\frac{1}{z} + \frac{z}{1 - z} \right) = g(z) * \frac{z^2 - z + 1}{z(1 - z)}. \quad (50)$$

By substituting (50) into (49), we get the desired assertion (47) of Theorem 10. $\square$

Theorem 11. Let

$$0 \leq \alpha_2 \leq \alpha_1 \leq \frac{1}{2}, \quad 0 < \beta_1 \leq \beta_2 < 1. \quad (51)$$

Then,

$$\mathcal{MC}(\alpha_1, \beta_1) \subset \mathcal{MC}(\alpha_2, \beta_2). \quad (52)$$

Proof. Suppose that $f \in \mathcal{MC}(\alpha_1, \beta_1)$. We easily know that

$$-\frac{zf'(z)}{g(z)} < \frac{1 + (1 - 2\alpha_1)\beta_1 z}{1 - \beta_1 z} \quad (z \in \mathbb{U}). \quad (53)$$

By setting $C = 1 + (1 - 2\alpha_1)\beta_1$, $D = -\beta_1$, $E = 1 + (1 - 2\alpha_2)\beta_2$, and $F = -\beta_2$, it follows from (51) that

$$\begin{aligned} |ED - FC| &= |(1 - 2\alpha_2)\beta_1\beta_2 + (1 - 2\alpha_1)\beta_1\beta_2| \\ &\leq |(1 - 2\alpha_2)\beta_1\beta_2 + (1 - 2\alpha_1)\beta_1^2| \\ &\quad + |(1 - 2\alpha_1)\beta_1^2 + (1 - 2\alpha_1)\beta_1\beta_2| \\ &\leq [(1 - 2\alpha_2)\beta_2 - (1 - 2\alpha_1)\beta_1] + (\beta_2 - \beta_1) \\ &= [(1 - 2\alpha_2)\beta_2 - (-\beta_2)] \\ &\quad - [(1 - 2\alpha_1)\beta_1 - (-\beta_1)] \\ &= (E - F) - (C - D). \end{aligned} \quad (54)$$

In view of Lemma 4, we deduce that

$$-\frac{zf'(z)}{g(z)} < \frac{1 + (1 - 2\alpha_1)\beta_1 z}{1 - \beta_1 z} < \frac{1 + (1 - 2\alpha_2)\beta_2 z}{1 - \beta_2 z} \quad (55)$$

$(z \in \mathbb{U}),$

which implies that $f \in \mathcal{MC}(\alpha_2, \beta_2)$. Thus, the assertion (52) of Theorem 11 holds. $\square$

Theorem 12. Let $f \in \mathcal{MC}(\alpha, \beta)$. Then,

$$\frac{(1 - r)^2 [1 - (1 - 2\alpha)\beta r]}{r^2 (1 + \beta r)} \leq |f'(z)| \leq \frac{(1 + r)^2 [1 + (1 - 2\alpha)\beta r]}{r^2 (1 - \beta r)} \quad (56)$$

$(|z| = r; 0 < r < 1).$

Proof. Let $f \in \mathcal{MC}(\alpha, \beta)$. By definition, we know that

$$-\frac{zf'(z)}{g(z)} < \frac{1 + (1 - 2\alpha)\beta z}{1 - \beta z} \quad (z \in \mathbb{U}). \quad (57)$$

Suppose that the function p is defined by (36). Then, we have

$$\frac{1 - (1 - 2\alpha)\beta r}{1 + \beta r} \leq |p(z)| \leq \frac{1 + (1 - 2\alpha)\beta r}{1 - \beta r}, \quad (58)$$

$(|z| = r; 0 < r < 1).$

Since $g \in \mathcal{MS}^*$, by Lemma 5, we know that

$$\frac{(1 - r)^2}{r} \leq |g(z)| \leq \frac{(1 + r)^2}{r} \quad (|z| = r; 0 < r < 1). \quad (59)$$

Thus, by virtue of (36), (58), and (59), we readily get the assertion (56) of Theorem 12. $\square$

Finally, we derive the radius of meromorphic convexity for the class $\mathcal{MC}(\alpha)$.

Theorem 13. Let $f \in \mathcal{MC}(\alpha)$ with $0 < \alpha < 1$. Then,

- (1) for $r_1 \leq r_0$, f is meromorphic convex in $0 < |z| < r_1$;
- (2) for $r_1 > r_0$, f is meromorphic convex in $0 < |z| < r_2$,

where r_0 is the unique root of the equation

$$(2\alpha - 1)r^4 - 2(2\alpha - 1)r^3 - 6\alpha r^2 - 2r + 1 = 0 \quad (60)$$

in the interval $(0, 1)$ and r_1 and r_2 are the smallest root of the equations

$$\begin{aligned} (2\alpha - 1)r^3 + 3(2\alpha - 1)r^2 + 3r + 1 &= 0, \\ (1 - 2\alpha)r^4 + 2(1 - 2\alpha)r^3 + 3(1 - \alpha)r^2 + 2r + 1 &= 0 \end{aligned} \quad (61)$$

in the interval $(0, 1)$, respectively.

Proof. Let $f \in \mathcal{MC}(\alpha)$ and suppose that

$$q(z) := -\frac{zf'(z)}{g(z)} \quad (z \in \mathbb{U}). \quad (62)$$

Then,

$$q(z) < \frac{1 + (1 - 2\alpha)z}{1 - z} \quad (z \in \mathbb{U}). \quad (63)$$

It follows from (62) that

$$-zf'(z) = q(z)g(z). \quad (64)$$

Differentiating both sides of (64) logarithmically, we get

$$-\left(1 + \frac{zf''(z)}{f'(z)}\right) = \frac{zq'(z)}{q(z)} + \frac{zg'(z)}{g(z)}. \quad (65)$$

Since $g \in \mathcal{MS}^*$, we know that

$$\Re\left(\frac{zg'(z)}{g(z)}\right) \geq -\frac{1+r}{1-r} \quad (|z| = r). \quad (66)$$

Combining (63), (65), (66), and Lemma 6, we obtain

$$\begin{aligned} & -\Re\left(1 + \frac{zf''(z)}{f'(z)}\right) \\ & \geq \begin{cases} -\frac{1+r}{1-r} - \frac{2(1-\alpha)r}{(1+r)[1+(2\alpha-1)r]} \\ =: \tilde{H}_\alpha(r) & (|z| = r; 0 < r \leq r_0), \\ -\frac{1+r}{1-r} + \frac{\sqrt{4\alpha(1-r^2)[1+(1-2\alpha)r^2]} - [1+(1-2\alpha)r^2]}{(1-\alpha)(1-r^2)} \\ \times ((1-\alpha)(1-r^2))^{-1} \\ -\frac{\alpha}{1-\alpha} =: \tilde{F}_\alpha(r) & (|z| = r; r_0 < r < 1), \end{cases} \end{aligned} \quad (67)$$

where r_0 is the unique root of (30) in the interval $(0, 1)$. It follows from (67) that the bound of meromorphic convexity for the class $\mathcal{MC}(\alpha)$ is determined either by the equation

$$-\frac{1+r}{1-r} - \frac{2(1-\alpha)r}{(1+r)[1+(2\alpha-1)r]} = 0, \quad (68)$$

or by the equation

$$\begin{aligned} & -\frac{1+r}{1-r} \\ & + \frac{\sqrt{4\alpha(1-r^2)[1+(1-2\alpha)r^2]} - [1+(1-2\alpha)r^2]}{(1-\alpha)(1-r^2)} \\ & - \frac{\alpha}{1-\alpha} = 0. \end{aligned} \quad (69)$$

We note that (68) and (69) can be rewritten as follows:

$$\begin{aligned} H_\alpha(r) &:= (2\alpha - 1)r^3 + 3(2\alpha - 1)r^2 + 3r + 1 = 0, \\ F_\alpha(r) &:= (1 - 2\alpha)r^4 + 2(1 - 2\alpha)r^3 + 3(1 - \alpha)r^2 + 2r + 1 \\ &= 0. \end{aligned} \quad (70)$$

Let r_1 and r_2 be the smallest root of the equations $H_\alpha(r) = 0$ and $F_\alpha(r) = 0$ in the interval $(0, 1)$, respectively. By observing that $H_\alpha(0) = 1 > 0$, we deduce that $H_\alpha(r) > 0$ for $r < r_1$.

Similarly, we know that $F_\alpha(r) > 0$ for $r < r_2$, since $F_\alpha(0) = 1 > 0$.

We observe that

$$\begin{aligned} \tilde{H}_\alpha(r) \geq 0 &\iff H_\alpha(r) \geq 0, \\ \tilde{F}_\alpha(r) \geq 0 &\iff F_\alpha(r) \geq 0. \end{aligned} \quad (71)$$

Thus, when $r_1 \leq r_0$, f is meromorphic convex in $0 < |z| < r_1$; when $r_1 > r_0$, f is meromorphic convex in $0 < |z| < r_2$.

The proof of Theorem 13 is thus completed. $\square$

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

Acknowledgments

This research was supported by the National Natural Science Foundation under Grant nos. 11301008 and 11226088, the Foundation for Excellent Youth Teachers of Colleges and Universities of Henan Province under Grant no. 2013GGJS-146, and the Natural Science Foundation of Educational Committee of Henan Province under Grant no. 14B110012 of the People's Republic of China. The authors would like to thank the referees for their valuable comments and suggestions which essentially improved the quality of this paper.

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