Research Article

Stress-Strain Relationship of Ca(OH)\(_2\)-Activated Hwangtoh Concrete

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This study examined the stress-strain behavior of 10 calcium hydroxide (Ca(OH)\(_2\))-activated Hwangtoh concrete mixes. The volumetric ratio of the coarse aggregate (\(V_{\text{agg}}\)) and the water-to-binder (W/B) ratio were selected as the main test variables. Two W/B ratios (25% and 40%) were used and the value of \(V_{\text{agg}}\) varied between 0% and 40.0%, and 0% and 46.5% for W/B ratios of 25% and 40%, respectively. The test results demonstrated that the slope of the ascending branch of the stress-strain curve of Ca(OH)\(_2\)-activated Hwangtoh concrete was smaller, and it displayed a steeper drop in stress in the descending branch, compared with those of ordinary Portland cement (OPC) concrete with the same compressive strength. This trend was more pronounced with the increase in the W/B ratio and decrease in \(V_{\text{agg}}\). Based on the experimental observations, a simple and rational stress-strain model was established mathematically. Furthermore, the modulus of elasticity and strain at peak stress of the Ca(OH)\(_2\)-activated Hwangtoh concrete were formulated as a function of its compressive strength and \(V_{\text{agg}}\). The proposed stress-strain model predicted the actual behavior accurately, whereas the previous models formulated using OPC concrete data were limited in their applicability to Ca(OH)\(_2\)-activated Hwangtoh concrete.

1. Introduction

With the increasing importance of leadership in energy and environmental design (LEED) certifications for buildings and reducing greenhouse gas (GHG) emissions, many concrete industry players are strongly focusing on minimizing the use of ordinary Portland cement (OPC). Although OPC has played a prominent role in building and infrastructure development as the basic component of concrete and mortar, the production of a ton of OPC is commonly associated with the following environmental issues [1–4]: (1) CO\(_2\) emissions of 0.8–0.9 ton, which is approximately 7% of the total GHG emissions into the earth’s atmosphere; (2) very high energy consumption including 90–100 kWh of electric power and 3–4 GJ of thermal energy owing to the plasticity temperature exceeding 1300°C; (3) natural resource depletion including 180–190 kg of bituminous coal and 20–30 kg of crude oil. For these reasons, the development of concrete with nil or minimal cement has attracted significant attention since the late 1980s.

Hwangtoh has gained more interest recently as a source material for low-cement concrete, especially in Korea and China, because it is known to be eco-friendly, with high absorption capacity and self-purifying characteristics, as well as offering health benefits by virtue of emitting far infrared radiation [5–7]. Hwangtoh is primarily clay formed by the weathering of rocks and composed of more than 70% inorganic substances, some organic material, water, and air. Yang et al. [6, 7] established that the workability and compressive strength development in calcium hydroxide (Ca(OH)\(_2\))-activated Hwangtoh mortars are comparable to those of OPC mortars with the same mixing proportions. However, more extensive use of Hwangtoh concrete in structural engineering applications warrants precise evaluations of its inherent characteristics including mechanical properties, inelastic deformation behavior, and durability. In
Table 1: Chemical composition of Hwangtoh (% by mass).

<table>
<thead>
<tr>
<th>SiO₂</th>
<th>Al₂O₃</th>
<th>Fe₂O₃</th>
<th>CaO</th>
<th>MgO</th>
<th>K₂O</th>
<th>Na₂O</th>
<th>TiO₂</th>
<th>MnO</th>
<th>LOI*</th>
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<tbody>
<tr>
<td>52.50</td>
<td>32.90</td>
<td>4.31</td>
<td>0.40</td>
<td>4.37</td>
<td>1.50</td>
<td>2.00</td>
<td>0.69</td>
<td>0.24</td>
<td>1.09</td>
</tr>
</tbody>
</table>

* Loss on ignition.

Table 2: Details of Ca(OH)₂-activated Hwangtoh concrete mix proportions and summary of test results.

<table>
<thead>
<tr>
<th>Group</th>
<th>Specimen*</th>
<th>W/B (%)</th>
<th>S/a (%)</th>
<th>Vagr (%)</th>
<th>W unit</th>
<th>B</th>
<th>S</th>
<th>G</th>
<th>S_f</th>
<th>w_c (kg/m³)</th>
<th>f_c² (MPa)</th>
<th>E_c (MPa)</th>
<th>𝜀_0</th>
<th>𝜀_0.5</th>
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</thead>
<tbody>
<tr>
<td>I</td>
<td>25–100</td>
<td>100</td>
<td>0.0</td>
<td>1469</td>
<td>0</td>
<td>2419</td>
<td>30.8</td>
<td>17468</td>
<td>0.0033</td>
<td>0.0038</td>
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<td></td>
<td>25–75</td>
<td>75</td>
<td>14.3</td>
<td>1102</td>
<td>372</td>
<td>2424</td>
<td>33.1</td>
<td>18563</td>
<td>0.0031</td>
<td>0.0042</td>
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<td></td>
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<tr>
<td>II</td>
<td>25–60</td>
<td>25</td>
<td>60</td>
<td>22.9</td>
<td>180</td>
<td>720</td>
<td>595</td>
<td>10.8</td>
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<td>19873</td>
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<td>25–45</td>
<td>45</td>
<td>31.4</td>
<td>661</td>
<td>818</td>
<td>2429</td>
<td>35.0</td>
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<td>25–30</td>
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<td>441</td>
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<td></td>
<td>40–30</td>
<td>30</td>
<td>46.5</td>
<td>513</td>
<td>1210</td>
<td>2335</td>
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<td>0.0024</td>
<td>0.0066</td>
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</table>

W/B: water-to-binder ratio by weight, S/a: fine aggregate-to-total aggregate ratio by volume, Vagr: volumetric ratio of coarse aggregate, W: water, B: binder, S: fine aggregate, G: coarse aggregate, S_f: polycarboxylate-based, high-range water-reducing admixture, w_c: dry density of concrete, f_c²: compressive strength, E_c: modulus of elasticity, 𝜀_0: strain at peak stress, and 𝜀_0.5: strain corresponding to 0.5f_c² after the peak stress.

* In specimen names, the first and second parts refer to the W/B ratio and S/a ratio, respectively, for example, 25–75 refers to Ca(OH)₂-activated Hwangtoh concrete mix with W/B and S/a ratios of 25% and 75%, respectively.

particular, the compressive stress-strain relations of concrete incorporating a new binder should definitely be evaluated for use in the rational analysis and design of structural elements because the cohesion between pastes and aggregate particles significantly influences the extent of increase in strain due to the applied load [8]. The stress-strain relations are typically understood through various experiments and formulated using simple equations. However, no such experimental observations on the stress-strain relations of Ca(OH)₂-activated Hwangtoh concrete are available in existing literature.

The objective of the present study is to propose a simple and rational model for obtaining the nonlinear stress-strain curves for Ca(OH)₂-activated Hwangtoh concrete under compression. To formulate the key factor determining the slopes of the ascending and descending branches of the curve, 10 concrete mixes were prepared with a range of water-to-binder (W/B) ratios by weight and fine aggregate-to-total aggregate ratios (S/a) by volume. The developed model was then compared with existing models [9–11] that were empirically derived from the test data on OPC concrete. Furthermore, the modulus of elasticity and strain at peak stress of Ca(OH)₂-activated Hwangtoh concrete were compared with those of OPC concrete collected from a variety of specimens [12] with compressive strength in similar ranges.

2. Experimental Program

2.1. Materials. Hwangtoh acquired high pozzolanic reactivity as a result of its calcination at a temperature of 850°C using the Hoffman method. The calcinated Hwangtoh, which was used as a source material, was activated by 7.5% Ca(OH)₂. This Hwangtoh contained less calcium oxide (CaO) but was rich in both silicon oxide (SiO₂) and aluminium oxide (Al₂O₃) as inferred from Table 1. This indicates that the chemical composition of Hwangtoh is very similar to that of fly ash and/or metakaolin [13]. All dry powdered alkali activators were preblended with the source materials in dry form.

The measurement of the physical properties of materials revealed the specific gravity and specific surface area of Hwangtoh to be 2.8 and 3200 cm²/g, respectively. The specific gravity, fineness modulus, and water absorption of the sand (used as the fine aggregate) were 2.42%, 2.51%, and 1.7%, respectively, and those of the crushed granite (used as the coarse aggregate) were 2.24% and 21.2%.

2.2. Specimen Mix Characteristics. In general, with increase in the compressive strength of concrete, the initial slope of its stress-strain curve increases, whereas the descending branch of the curves after peak stress has a more rapidly decreasing slope [14]. Moreover, concrete displays higher modulus of elasticity (E_c) and smaller strain (ε_0) values at peak stress than do mortars and pastes, indicating that the volumetric ratio of the coarse aggregates (Vagr) directly affects the stress-strain curve of concrete[15, 16]. Considering these critical factors, the 10 cementless concrete mixes using Ca(OH)₂-activated Hwangtoh binder were prepared with a range of W/B and S/a ratios, as listed in Table 2. The selected W/B ratios were 25% and 40%, and within each of these ratios, the
S/\(a\) ratio of the mixes for each \(W/B\) ratio varied from 30\% to 100\% in intervals of 15\%. The concrete specimen with an \(S/a\) ratio of 100\% indicates a mortar without coarse aggregates. As a result, \(V_{\text{agg}}\) ranged between 0\% and 40.0\%, and 0\% and 46.5\% for \(W/B\) ratios of 25\% and 40\%, respectively. The unit water content was fixed at 180 kg/m\(^3\) for all concrete mixes. To achieve workability of the mixes for casting, a polycarboxylate-based, high-range water-reducing agent was added with binder ratio by weight of 1.5\% and 1.2\% for concrete mixes with \(W/B\) ratios of 25\% and 40\%, respectively.

2.3. Testing. All concrete specimens were mixed using a pan mixer of 0.35 m\(^3\) capacity and equipped with rubber wiper blades. The initial slump of fresh concrete was measured in accordance with the ASTM C143 provision [17]. Immediately after casting, all specimens used for plotting the stress-strain curve were cured at a constant temperature and relative humidity of \(23 \pm 2\)\(^\circ\)C and \(70 \pm 5\)\%, respectively, until testing at an age of 28 days. All steel moulds were removed after 1 day. The stress-strain curve was recorded using 100 \(\times\) 200 mm cylindrical concrete specimens equipped with a compressometer comprising linear variable differential transducers (LVDTs) of 50 mm capacity on both sides, as shown in Figure 1. The compressometer was combined with a convenient unbounded device for measuring transverse strain. Electrical resistance strain gages with gage length of 75 mm were also attached to the specimens. Prior to testing, both ends of each cylinder were leveled using a grinding machine. Concentric load was applied using a universal testing machine of 500 kN capacity, equipped with a closed-loop servocontrol system. Each specimen was preloaded to 20\% of its compressive strength after which its position was adjusted based on strain gage and LVDT readings to achieve a concentric axial load. A spherical hinge was also positioned between the testing machine head and the specimens to minimize eccentricity in case of a large deformation. To obtain a complete stress-strain curve, a low strain rate of \(2.0 \times 10^{-4}\) per min was employed. Testing was continued until the final crushing of the concrete. All test data were captured by a data logger and stored automatically. Using the measured stress-strain curve, the \(E_c\) was calculated at 40\% peak stress in accordance with the ASTM C469 provision [17].

3. Test Results and Discussion

3.1. General Behavior and Crack Propagation. Longitudinal cracking started at 50\%–60\% of the peak stress within the mid-height section of the specimen. The stress at which cracks developed was independent of \(V_{\text{agg}}\). As the applied load increased, the initial crack propagated toward the top and bottom surfaces of the specimens. As a result, the failure of specimens after peak stress was governed by the time when the longitudinal crack joined the top and bottom surfaces of the specimen. The width of the longitudinal cracks increased sharply at 80\%–85\% of the peak stress, which was accompanied by a rapid increase in the longitudinal and transverse strains, as shown in Figure 2. The rate of increase in those strains increased as \(V_{\text{agg}}\) decreased. With rapid increase in transverse strain, the volumetric change of the specimens started to shift from compression to tension. This threshold point was found to occur at approximately 80\% and 95\% of the peak stress for concrete mixes with \(V_{\text{agg}}\) of 0\% and 40\%, respectively. This implies that the rate of increase in transverse strain due to the opening of longitudinal cracks decreases as \(V_{\text{agg}}\) increases. After the peak stress, failure tended to occur along the longitudinal cracks, which commonly formed in the pastes between the aggregate particles. The drop in stress subsequent to peak stress was more rapid in concrete mixes with \(V_{\text{agg}} = 0\%\) than those...
The stress-strain curves obtained for the concrete mixes are plotted in Figure 4. The shape of all curves was a second-degree parabola with its vertex at the peak stress point. However, the slopes of the ascending and descending branches of the curves heavily depended on the $V_{agg}$ values. Similar to observations on OPC concrete [9–11], the curves were almost linear up to approximately one-half of the peak stress point, showing that their initial slope increased as $f'_c$ and/or $V_{agg}$ increased. Normal-weight aggregates possess elastic modulus values that are 1.5–5.0 times higher than those of pastes [14], causing the initial slope of the curves to rise with increase in $V_{agg}$. Additionally, the slope of the descending branch was steeper and less steep with increasing $f'_c$ and $V_{agg}$, respectively, though a slightly higher value of $f'_c$ was obtained for the concrete mix with a higher $V_{agg}$ value but the same W/B ratio. Overall, the descending branch of the curve for concrete mixes with $V_{agg} = 0\%$ (i.e., specimens 25–100 and 40–100) was very short compared to those for mixes with coarse aggregates. These trends were more pronounced for a W/B ratio of 25% than for 40%, leading to the inference that the mechanical interlocking of the coarse aggregate along the cracks relieves the brittle fracture (as indicated by the descending branch) and improves the compressive strength of the concrete.

3.4. Modulus of Elasticity ($E_c$). The relationship of $f'_c$ and $E_c$ of the Ca(OH)$_2$-activated Hwangtoh concrete is shown in Figure 5. The figure also shows the test data for normal-weight OPC concrete with $f'_c$ not exceeding 50 MPa, as well as predictions made by Carreira and Chu’s model [9]. In
Figure 4: Stress-strain curves measured for Ca(OH)$_2$-activated Hwangtoh concrete.

Figure 5: Modulus of elasticity ($E_c$) as a function of compressive strength ($f'_c$).

Figure 6: Regression analysis for modulus of elasticity ($E_c$).

The unit weight ($\omega_c$) of normal-weight concrete was assumed to be 2300 kg/m$^3$. The $E_c$ of the concrete was commonly lower than the best-fit curves determined for OPC concrete and predictions of Carreira and Chu's model [9]. The development of bond microcracks at the interfaces between pastes and aggregate particles accelerates the nonlinearity of the stress-strain curves [14], possibly causing the cohesion between them to be lower in Ca(OH)$_2$-activated Hwangtoh binder than OPC.

In general, the $E_c$ of concrete can be expressed as a function of $f'_c$ and $\omega_c$ [14]. However, it was difficult to determine the effect of $\omega_c$ on the $E_c$ of the tested Ca(OH)$_2$-activated Hwangtoh concrete because the variation in the $\omega_c$ values (in the test) was very minimal at the same $W/B$ ratios, ranging between 2419 and 2431 kg/m$^3$, 2321 and 2335 kg/m$^3$ for $W/B$ ratios of 25% and 40%, respectively. Moreover, $E_c$ was affected by $V_{agg}$ (showing a slightly lower value in specimens with an $S/a$ ratio of 60% than those with an $S/a$ ratio of 45% at similar $f'_c$) owing to the considerably higher modulus of elasticity in aggregates than in pastes when individually subjected to load. In addition, an increase in the continuous cracks joining the bond microcracks that developed at the interface between the pastes and aggregate particles leads to a faster rate of increase in strain [8]. To formulate a simple equation for $E_c$, influential parameters such as $f'_c$ and $V_{agg}$ were repeatedly combined and adjusted through a trial-and-error approach until a relatively high correlation coefficient ($R^2$) was obtained. Based on a regression analysis of the test results, the $E_c$ of Ca(OH)$_2$-activated Hwangtoh concrete with $f'_c < 40$ MPa can be empirically expressed as follows (see Figure 6):

$$E_c = 3010\left(f'_c\right)^{0.5} \left(1 + \frac{V_{agg}}{100}\right)^{0.75} \text{ (MPa)}.$$  

3.5. Strain at Peak Stress ($\varepsilon_0$). MacGregor and Wight [14] established that $\varepsilon_0$ of concrete increases with increase in $f'_c$. This trend was also verified for the test case from the data listed in Table 2. As a result, the existent equations for $\varepsilon_0$ have mostly been empirically developed only as a function of
f_c. However, Ca(OH)_2-activated Hwangtoh concrete demonstrated higher values of \( \varepsilon_0 \) than did OPC concrete with the same \( f'_c \), as shown in Figure 7. These \( \varepsilon_0 \) values also increased slightly as \( V_{agg} \) decreased because the increase in paste volume reduced the \( E_c \) and accelerated the nonlinearity of the stress-strain curves, especially after the development of longitudinal cracks. However, none of the predicted equations [9–11] consider the effect of \( V_{agg} \) on the strain although the former is evidently the secondary factor causing the increasing strain rate. Based on a regression analysis of the test results, the \( \varepsilon_0 \) of Ca(OH)_2-activated Hwangtoh concrete can be written as follows (see Figure 7):

\[
\varepsilon_0 = 0.08 \left( \frac{f'_c}{E_c} \right)^{0.5} \left( 1 + \frac{V_{agg}}{100} \right)^{-0.5}.
\] (2)

### 4. Mathematical Equations for Stress-Strain Relations

Figure 4 confirms that the compressive stress-strain curve of Ca(OH)_2-activated Hwangtoh concrete is parabolic and hump-shaped with its vertex at the peak stress point. This physically means that the tangential modulus of elasticity has the maximum value at the origin, gradually decreases to zero at the peak stress point, and becomes negative through the descending branch of the curve. In this study, the same assumption and the following nonlinear equation [9, 11, 12] were applied for generating a complete curve:

\[
Y = \frac{\beta_1 + 1}{X} X^{\beta_1 + 1} + \beta_1,
\] (3)

where \( Y = f'_c/E_c \) is the normalized stress, \( X = \varepsilon_c/\varepsilon_0 \) is the normalized strain, and \( f'_c \) is the concrete stress corresponding to strain \( \varepsilon_c \). Equation (3) gives the following physical meanings: (1) \( Y = 0 \) when \( X = 0 \), representing the origin point; (2) \( Y = 1 \) when \( X = 1 \), representing the peak stress point; (3) \( df_c/de_c = 0 \) when \( X = 1 \) or at the peak stress point. Hence, the factor \( \beta_1 \) in (3) is a key parameter for determining the slopes of the ascending and descending branches of the curve with its values commonly differing in each branch. To determine the slope of the ascending branch, \( E_c \) was selected as the reference parameter. Because \( E_c \) is defined as a secant modulus joining the origin and 40% of the peak stress, substituting it in (3) results in the following equation for determining \( \beta_1 \) for the ascending branch:

\[
0.4(X_a)^{\beta_1 + 1} + (0.4 - X_a) \beta_1 - X_a = 0 \quad \text{for} \quad \varepsilon_c \leq \varepsilon_0, \tag{4}
\]

where \( X_a = 0.4 f'_c/E_c \varepsilon_0 \).

In contrast, some reviews [12, 18] concluded that the secant modulus joining the origin and 50% of the peak stress can be regarded as an adequate reference point to derive the slope of the descending branch. Hence, the present study formulated an equation for determining \( \beta_1 \) for the descending branch using the secant modulus at 0.5 \( f'_c \) as the reference point, as follows:

\[
(X_d)^{\beta_1+1} + (1 - 2X_d) \beta_1 - 2X_d = 0 \quad \text{for} \quad \varepsilon_c > \varepsilon_0, \tag{5}
\]

where \( X_d = \varepsilon_{0.5}/\varepsilon_0 \) and \( \varepsilon_{0.5} \) is the strain corresponding to 0.5 \( f'_c \) after peak stress. The values of \( \beta_1 \) in the nonlinear equations (4) and (5) can be calculated through numerical analysis for the given material properties of \( E_c, \varepsilon_0 \), and \( \varepsilon_{0.5} \).

#### 4.1. Strain at 50% of Peak Stress in Descending Branch (\( \varepsilon_{0.5} \))

The slope of the descending branch of the curve depends on \( f'_c \) and \( V_{agg} \), as inferred from Table 1. A regression analysis was conducted using these parameters (as shown in Figure 8) similar to the procedure employed for \( \varepsilon_0 \) to propose a simple equation for \( \varepsilon_{0.5} \), yielding the following equation:

\[
\varepsilon_{0.5} = -0.17 \left( f'_c/E_c \right)^{0.01} \left( 1 + \frac{V_{agg}}{100} \right)^{-0.03} + 0.166. \tag{6}
\]

#### 4.2. Key Parameter (\( \beta_1 \))

An analytical parametric study was conducted to formulate \( \beta_1 \), which determines the slopes
of the ascending and descending branches. For given \( f'_c \) and \( V_{agg} \), the material properties of \( E_c, \gamma_c, \) and \( f_{0.2} \) were calculated using (1), (2), and (6), respectively. Subsequently, the two non-linear equations ((4) and (5)) were solved using the Newton-Raphson method. Considering the practical strength development in Ca(OH)\(_2\)-activated Hwangtoh concrete, the variables \( f'_c \) and \( V_{agg} \) in the analytical parametric study were selected to range between 10 and 50 MPa and 0% and 60%, respectively. Finally, the analytically obtained results were statistically optimized to drive the following \( \beta_1 \) equations for the ascending branch (Figure 9(a)) and the descending branch (Figure 9(b)):

\[
\beta_1 = 0.33 \exp \left[ 0.65 \left( \frac{f'_c}{f_0} \right)^{0.5} \left( 1 + \frac{V_{agg}}{100} \right)^{0.2} \right] \text{ for } \varepsilon_c \leq \varepsilon_0 
\]

(7)

\[
\beta_1 = 0.43 \exp \left[ 2.1 \left( \frac{f'_c}{f_0} \right)^{0.5} \left( 1 + \frac{V_{agg}}{100} \right)^{-2.5} \right] \text{ for } \varepsilon_c > \varepsilon_0, 
\]

(8)

where \( f_0 \) (= 10 MPa) is the reference value for the compressive strength of concrete. A higher value of \( \beta_1 \) in (3) produces larger slopes of the ascending and descending branches. Hence, an increase in the value of \( V_{agg} \) in (7) and (8) results in a higher value of \( E_c \) and less steep slope of the descending branch, as observed in Figure 4.

In summary, the compressive stress-strain relations for unconfined Ca(OH)\(_2\)-activated Hwangtoh concrete can be generalized as follows:

\[
f_c = \left( \frac{\beta_1 + 1}{\beta_1} \right) \left( \frac{\varepsilon_c}{\varepsilon_0} \right)^{\beta_1 + 1} f'_c, 
\]

(9)

where \( \varepsilon_0 \) and \( \beta_1 \) are obtained using (2) and (7) or (8), respectively. The stress-strain relations proposed for the concrete need to be reexamined for strengths of more than 40 MPa because these equations have been derived using the limited test data pertaining to the present study.

5. Comparisons of Model and Test Results

Figure 10 shows the typical comparisons of the predicted and measured curves for different \( W/B \) ratios and \( V_{agg} \) values. The figure also displays the curves pertaining to previous models [9–11] that were empirically formulated using OPC test data. Table 3 lists the mean (\( \gamma_{c,m} \)) and standard deviation (\( \gamma_{c,s} \)) of the normalized root-mean-square errors (NRMSE) calculated for each stress-strain curve. In general, the previous models commonly overestimate the slope of the ascending branch and underestimate the value of \( \varepsilon_0 \). This trend is more pronounced with the increase in \( f'_c \) and decrease in \( V_{agg} \). Furthermore, the equations proposed by [9] and Wee et al. [11] overestimate the stress in the descending branch, particularly for concrete with low \( V_{agg} \). Consequently, the previous models yield high values for \( \gamma_{c,m} \)(>0.238) and \( \gamma_{c,s} \)(>0.050) and are limited in their applicability to Ca(OH)\(_2\)-activated Hwangtoh concrete, and the error associated with them gradually increases as \( V_{agg} \) decreases.

In contrast to the previous models for OPC concrete, the predictions obtained from the proposed model for Ca(OH)\(_2\)-activated Hwangtoh concrete are in better agreement with

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<tbody>
<tr>
<td>( \gamma_{c,m} )</td>
<td>0.238</td>
<td>0.254</td>
<td>0.259</td>
<td>0.091</td>
</tr>
<tr>
<td>( \gamma_{c,s} )</td>
<td>0.075</td>
<td>0.072</td>
<td>0.050</td>
<td>0.039</td>
</tr>
</tbody>
</table>

*\( \gamma_{c,m} \) and \( \gamma_{c,s} \) refer to the mean and standard deviation, respectively, of the NRMSE calculated for each specimen using the following equation:

\[
\text{NRMSE} = \frac{1}{(f'_c)_{\text{m}}} \sum \left[ (f'_c)_{\text{Exp}} - (f'_c)_{\text{Pre}} \right]^{1/2},
\]

where \( (f'_c)_{\text{m}} \) is the mean of the measured stress, \( (f'_c)_{\text{Exp}} \) and \( (f'_c)_{\text{Pre}} \) are experimental and predicted stresses, respectively, and \( n \) is the number of measured points.

Figure 9: Formulation of \( \beta_1 \) through numerical analysis.

Table 3: Statistical values of NRMSEs calculated by comparing stress-strain curves of various models to experimental data.
test results, showing relatively higher accuracy for both ascending and descending branches, judging from the considerably lower NRMSE values (0.091 and 0.039 for $\gamma_{e,m}$ and $\gamma_{e,s}$, resp.). These results are consistent with the actual responses: in particular, they accurately reflect the effect of $V_{agg}$ on $e_0$ and the slopes of the ascending and descending branches. Hence, the proposed model is useful for accurately evaluating the compressive stress-strain behavior of low- and medium-strength Ca(OH)$_2$-activated Hwangtoh concrete.

6. Conclusions

The present study examined the stress-strain behavior of Ca(OH)$_2$-activated Hwangtoh concrete with different $W/B$ and $V_{agg}$ values. The $E_c$ and $e_0$ values of the specimens were compared to those of OPC concrete compiled by Yang et al. Based on the experimental data, a simple equation was established to reasonably predict the stress-strain relations of Ca(OH)$_2$-activated Hwangtoh concrete under compression,
though further verification is required for strengths of more than 40 MPa because of the limited data used for empirical fitting. From the experimental and comparative observations, the following conclusions may be drawn.

(1) For the same $W/B$, the $f'_c$ of Ca(OH)$_2$-activated Hwangtoh concrete was commonly lower than that of OPC concrete; however, the test concrete gained strength development comparable to that of structural concrete at a $W/B$ less than 30%.

(2) With decrease in $V_{agg}$, the slope of the ascending branch of the stress-strain curve and $f'_c$ decreased, whereas $\varepsilon_0$ and the declining slope of the descending branch of the curve increased. This observation was more pronounced as the $W/B$ ratio increased.

(3) Compared to OPC concrete with the same $f'_c$, Ca(OH)$_2$-activated Hwangtoh concrete had lower $E_c$ and larger $\varepsilon_0$, showing greater discrepancy as $V_{agg}$ decreased. Hence, the $E_c$ and $\varepsilon_0$ of Ca(OH)$_2$-activated Hwangtoh concrete could be formulated as a function of $f'_c$ and $V_{agg}$.

(4) Previous models formulated using OPC concrete data revealed limitations in their applicability to Ca(OH)$_2$-activated Hwangtoh concrete, whereas the proposed stress-strain model predicted the actual behavior quite accurately, as evidenced by the much lower NRMSE values.

Conflict of Interests

The authors declare that they have no conflict of interests regarding the publication of this paper.

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