Research Article

On Fuzzy Positive Implicative Filters in BE-Algebras

Sun Shin Ahn¹ and Jeong Soon Han²

¹ Department of Mathematics Education, Dongguk University, Seoul 100-715, Republic of Korea

² Department of Applied Mathematics, Hanyang University, Ansan 426-791, Republic of Korea

Correspondence should be addressed to Jeong Soon Han; han@hanyang.ac.kr

Received 28 March 2014; Accepted 9 May 2014; Published 25 May 2014

Academic Editor: Hee S. Kim

Copyright © 2014 S. S. Ahn and J. S. Han. This is an open access article distributed under the Creative Commons Attribution License, which permits unrestricted use, distribution, and reproduction in any medium, provided the original work is properly cited.

We study several degrees in defining a fuzzy positive implicative filter, which is a generalization of a fuzzy filter in BE-algebras.

1. Introduction

In [1], H. S. Kim and Y. H. Kim introduced the notion of a BE-algebra. Ahn and So [2, 3] introduced the notion of ideals in BE-algebras. Ahn et al. [4] fuzzified the concept of BE-algebras and investigated some of their properties. Jun and Ahn [5] provided several degrees in defining a fuzzy implicative filter.

In this paper, we study several degrees in defining a fuzzy positive implicative filter, which is a generalization of a fuzzy filter in BE-algebras.

2. Preliminaries

We recall some definitions and results discussed in [1–3].

An algebra $(X; *, 1)$ of type $(2, 0)$ is called a BE-algebra if

$$(BE1) \quad x * x = 1 \text{ for all } x \in X,$$

$$(BE2) \quad x * 1 = 1 \text{ for all } x \in X,$$

$$(BE3) \quad 1 * x = x \text{ for all } x \in X,$$

$$(BE4) \quad x * (y * z) = y * (x * z), \text{ for all } x, y, z \in X \text{ (exchange)}.$$

We introduce a relation “ $\leq$ ” on a BE-algebra X by $x \leq y$, if and only if $x * y = 1$. A nonempty subset S of a BE-algebra X is said to be a *subalgebra* of X , if it is closed under the operation “ $*$.” By noticing that $x * x = 1$, for all $x \in X$, it is clear that $1 \in S$. A BE-algebra $(X; *, 1)$ is said to be self-distributive, if $x * (y * z) = (x * y) * (x * z)$, for all $x, y, z \in X$.

Definition 1 (see [1]). Let $(X; *, 1)$ be a BE-algebra and let F be a nonempty subset of X . Then, F is called a *filter* of X if

$$(F1) \quad 1 \in F,$$

$$(F2) \quad x * y \in F \text{ and } x \in F \text{ imply } y \in F,$$

for all $x, y, z \in X$.

A nonempty subset F of a BE-algebra X is called an *implicative filter* of X if it satisfies (F1) and

$$(F3) \quad x * (y * z) \in F \text{ and } x * y \in F \text{ imply } x * z \in F, \text{ for all } x, y, z \in X.$$

Example 2 (see [1]). Let $X := \{1, a, b, c, d, 0\}$ be a BE-algebra with the following table:

$*$	1	a	b	c	d	0
1	1	a	b	c	d	0
a	1	1	a	c	c	d
b	1	1	1	c	c	c
c	1	a	b	1	a	b
d	1	1	a	1	1	a
0	1	1	1	1	1	1

(1)

Then $F_1 := \{1, a, b\}$ is a filter of X , but $F_2 := \{1, a\}$ is not a filter of X , since $a * b \in F_2$ and $a \in F_2$, but $b \notin F_2$.

Proposition 3. Let $(X; *, 1)$ be a BE-algebra and let F be a filter of X . If $x \leq y$ and $x \in F$, for any $y \in X$, then $y \in F$.

Proposition 4. Let $(X; *, 1)$ be a self-distributive BE-algebra. Then, the following hold, for any $x, y, z \in X$:

- (i) if $x \leq y$, then $z * x \leq z * y$ and $y * z \leq x * z$;
- (ii) $y * z \leq (z * x) * (y * x)$;
- (iii) $y * z \leq (x * y) * (x * z)$.

A BE-algebra $(X; *, 1)$ is said to be transitive if it satisfies Proposition 4 (iii).

Definition 5 (see [5]). A fuzzy subset μ of a BE-algebra X is called a fuzzy filter of X , if it satisfies, for all $x, y \in X$,

- (d1) $\mu(1) \geq \mu(x)$,
- (d2) $\mu(x) \geq \min\{\mu(y * x), \mu(y)\}$.

A fuzzy subset μ of a BE-algebra X is called a fuzzy implicative filter of X if it satisfies (d1) and

- (d3) $\mu(x * z) \geq \min\{\mu(x * (y * z)), \mu(x * y)\}$, for all $x, y, z \in X$.

Definition 6 (see [5]). Let F be a nonempty subset of a BE-algebra X which is not necessarily a filter of X . One says that a subset G of X is an enlarged filter of X related to F , if it satisfies the following:

- (1) F is a subset of G ,
- (2) $1 \in G$,
- (3) $(\forall x, y \in X)(\forall x \in F)(x * y \in F \Rightarrow y \in G)$.

3. Fuzzy Positive Implicative Filters of BE-Algebras with Degrees in $(0, 1]$

X denotes a BE-algebra unless specified otherwise.

Definition 7. A nonempty subset F of X is called a positive implicative filter of a BE-algebra X if it satisfies (F1) and

- (F4) $x * ((y * z) * y) \in F$ and $x \in F$ imply $y \in F$, for all $x, y, z \in X$.

Note that every positive implicative filter of a BE-algebra X is a filter of X .

Example 8. (1) Let $X := \{1, a, b, c, d\}$ be a self-distributive BE-algebra ([1]) with the following table:

$*$	1	a	b	c	d
1	1	a	b	c	d
a	1	1	b	c	d
b	1	a	1	c	c
c	1	1	b	1	b
d	1	1	1	1	1

Then $\{1, b\}$ is an implicative filter of X but not a positive implicative filter of X , since $b * ((a * d) * a) = 1, b \in \{1, b\}$ and $a \notin \{1, b\}$.

(2) Consider a BE-algebra $X := \{1, a, b, c, d, 0\}$ as in Example 2. Then $\{1\}$ is a filter of X but not an implicative filter

of X , since $d * (a * 0) = 1 \in \{1\}, d * a = 1 \in \{1\}$, and $d * 0 = a \notin \{1\}$. Also, it is not a positive implicative filter of X , since $1 * ((a * b) * a) = 1 \in \{1\}, 1 \in \{1\}$ and $a \notin \{1\}$.

(3) Let $X := \{1, a, b, c\}$ be a self-distributive BE-algebra ([5]) with the following table:

$*$	1	a	b	c
1	1	a	b	c
a	1	1	b	c
b	1	a	1	c
c	1	1	b	1

Then $\{1, b\}$ is an implicative filter of X and $\{1, a, b\}$ is a positive implicative filter of X .

Definition 9. A fuzzy subset μ of a BE-algebra X is called a fuzzy positive implicative filter of X , if it satisfies (d1) and

- (d4) $\mu(y) \geq \min\{\mu(x * ((y * z) * y)), \mu(x)\}$, for all $x, y \in X$.

Definition 10. Let F be a nonempty subset of a BE-algebra X which is not necessarily a positive implicative filter of X . One says that a subset G of X is an enlarged positive implicative filter of X related to F , if it satisfies the following:

- (1) F is a subset of G ,
- (2) $1 \in G$,
- (3) $(\forall x, y, z \in X)(\forall x \in F)(x * ((y * z) * y) \in F \Rightarrow y \in G)$.

Obviously, every positive implicative filter is an enlarged positive implicative filter of X related to itself. Note that there exists an enlarged positive implicative filter of X related to any nonempty subset F of X .

Example 11. Consider a BE-algebra $X = \{1, a, b, c, d\}$ which is given in Example 8 (1). Note that $F := \{1, b\}$ is not a positive implicative filter. Then, $G := \{1, a, b, c\}$ is an enlarged positive implicative filter of X related to F and G is not a positive implicative filter of X , since $b * ((d * d) * d) = c \in G, b \in G$ and $d \notin G$.

Proposition 12. Let F be a nonempty subset of a BE-algebra X . Every enlarged positive implicative filter of X related to F is an enlarged filter of X related to F .

Proof. Let G be an enlarged positive implicative filter of X related to F . By putting $z := 1$ in Definition 10 (3), we have

$$(\forall x, y \in X) \quad (x * ((y * 1) * y) = x * y \in F, \quad (4)$$

$$x \in F \Rightarrow y \in G).$$

Hence, G is an enlarged filter of X related to F . $\square$

The converse of Proposition 12 is not true in general as seen in the following example.

Example 13. Let $X := \{1, a, b, c\}$ be a transitive BE-algebra ([2]) with the following table:

$*$	1	a	b	c
1	1	a	b	c
a	1	1	a	a
b	1	1	1	a
c	1	1	a	1

(5)

Let $F := \{1\}$ and $G := \{1, c\}$. Then G is an enlarged filter of F but it is not an enlarged positive implicative filter of F , since $1 * ((a * c) * a) = 1 \in F$, $1 \in F$ and $a \notin G$.

In what follows let λ and κ be members of $(0, 1]$, and let n and k denote a natural number and a real number, respectively, such that $k < n$, unless otherwise specified.

Definition 14 (see [5]). A fuzzy subset μ of a BE-algebra X is called a fuzzy filter of X with degree (λ, κ) , if it satisfies the following:

- (e1) $(\forall x \in X)(\mu(1) \geq \lambda\mu(x))$,
 (e2) $(\forall x, y \in X)(\mu(x) \geq \kappa \min\{\mu(y * x), \mu(y)\})$.

A fuzzy subset μ of a BE-algebra X is called a fuzzy implicative filter of X with degree (λ, κ) , if it satisfies (e1) and

- (e3) $(\forall x, y, z \in X)(\mu(x * z) \geq \kappa \min\{\mu(x * (y * z)), \mu(x * y)\})$.

Definition 15. A fuzzy subset μ of a BE-algebra X is called a fuzzy positive implicative filter of X with degree (λ, κ) , if it satisfies (e1) and

- (e4) $(\forall x, y, z \in X)(\mu(y) \geq \kappa \min\{\mu(x * ((y * z) * y)), \mu(x)\})$.

Proposition 16 (see [5]). Every fuzzy filter of a BE-algebra X with degree (λ, κ) satisfies the following assertions:

- (i) $(\forall x, y \in X)(\mu(x * y) \geq \lambda\kappa\mu(y))$;
 (ii) $(\forall x, y \in X)(y \leq x \Rightarrow \mu(x) \geq \lambda\kappa\mu(y))$;
 (iii) $(\forall x, y, z \in X)(x \leq y * z \Rightarrow \mu(z) \geq \min\{\kappa\mu(y), \lambda\kappa^2\mu(x)\})$.

Note that if $\lambda \neq \kappa$, then a fuzzy positive implicative filter with degree (λ, κ) may not be a fuzzy positive implicative filter with degree (κ, λ) and vice versa. Obviously, every fuzzy positive implicative filter is a fuzzy positive implicative filter with degree (λ, κ) , but the converse may not be true.

Example 17. Consider a self-distributive BE-algebra $X = \{1, a, b, c\}$ which is given in Example 8 (3). Define a fuzzy subset $\mu : X \rightarrow [0, 1]$ by

$$\mu = \begin{pmatrix} 1 & a & b & c \\ 0.7 & 0.4 & 0.8 & 0.4 \end{pmatrix}. \quad (6)$$

Then, μ is a fuzzy implicative filter of X with degree $(2/5, 3/5)$ and a fuzzy filter of X with degree $(2/5, 3/5)$, but it is neither

a fuzzy filter of X nor a fuzzy positive implicative filter of X with degree $(2/5, 3/5)$ since

$$\begin{aligned} \mu(1) &= 0.7 \not\geq 0.8 = \mu(b), \\ \mu(a) &= 0.4 \not\geq 0.42 = \frac{3}{5} \times 0.7 \\ &= \frac{3}{5} \times \mu(1) \\ &= \frac{3}{5} \times \min\{\mu(1 * ((a * c) * a)) = \mu(1), \mu(1)\}. \end{aligned} \quad (7)$$

Example 18. Let $X := \{1, a, b, c\}$ be a BE-algebra ([2]) with the following table:

$*$	1	a	b	c
1	1	a	b	c
a	1	1	b	c
b	1	1	1	1
c	1	a	c	1

(8)

Define a fuzzy subset $\mu : X \rightarrow [0, 1]$ by

$$\mu = \begin{pmatrix} 1 & a & b & c \\ 0.7 & 0.8 & 0.4 & 0.4 \end{pmatrix}. \quad (9)$$

Then, μ is a fuzzy positive implicative filter of X with degree $(3/5, 2/5)$. But it is neither a fuzzy filter of X nor a fuzzy positive implicative filter of X with degree $(2/5, 3/5)$ since

$$\begin{aligned} \mu(1) &= 0.7 \not\geq 0.8 = \mu(a), \\ \mu(c) &= 0.4 \not\geq 0.42 = \frac{3}{5} \times 0.7 \\ &= \frac{3}{5} \times \mu(1) \\ &= \frac{3}{5} \times \min\{\mu(a * ((c * b) * c)) = \mu(1), \mu(a)\}. \end{aligned} \quad (10)$$

Also, it is not a fuzzy implicative filter of X with degree $(2/5, 3/5)$ since

$$\begin{aligned} \mu(c * b) &= \mu(c) = 0.4 \not\geq 0.42 = \frac{3}{5} \times 0.7 \\ &= \frac{3}{5} \times \mu(1) \\ &= \frac{3}{5} \times \min\{\mu(c * (c * b)) \\ &= \mu(1), \mu(c * c) = \mu(1)\}. \end{aligned} \quad (11)$$

Proposition 19. If μ is a fuzzy positive implicative filter of a BE-algebra X with degree (λ, κ) , then μ is a fuzzy filter of X with degree (λ, κ) .

Proof. By putting $z := y$ in (e4), we have

$$\begin{aligned} \mu(y) &\geq \kappa \min\{\mu(x * ((y * y) * y)), \mu(x)\} \\ &= \kappa \min\{\mu(x * y), \mu(x)\} \end{aligned} \quad (12)$$

for any $x, y \in X$. Thus, μ is a fuzzy filter of X with degree (λ, κ) . $\square$

The converse of Proposition 19 is not true in general (see Example 17).

Note that a fuzzy filter with degree (λ, κ) is a fuzzy filter if and only if $(\lambda, \kappa) = (1, 1)$.

Proposition 20. Let μ be a fuzzy positive implicative filter of a BE-algebra X with degree (λ, κ) . Then, the following holds:

$$(\forall x, y \in X) \quad (\mu(x) \geq \kappa \lambda \mu((x * y) * x)). \quad (13)$$

Proof. Assume that μ is a fuzzy positive implicative filter of a BE-algebra X with degree (λ, κ) and let $x, y \in X$. Using (e4) and (e1), we have

$$\begin{aligned} \mu(x) &\geq \kappa \min \{ \mu(1 * ((x * y) * x)), \mu(1) \} \\ &= \kappa \min \{ \mu((x * y) * x), \lambda \mu((x * y) * x) \} \\ &= \kappa \lambda \mu((x * y) * x). \end{aligned} \quad (14)$$

This completes the proof. $\square$

Proposition 21. Let μ be a fuzzy filter of a BE-algebra X with degree (λ, κ) satisfying

$$(\forall x, y \in X) \quad (\mu(x) \geq \mu((x * y) * x)). \quad (15)$$

Then, μ is a positive implicative filter of X with degree (λ, κ) .

Proof. Let $x, y, z \in X$. Using (e2), we have

$$\begin{aligned} \mu(y) &\geq \mu((y * z) * y) \\ &\geq \kappa \min \{ \mu(x * ((y * z) * y)), \mu(x) \}. \end{aligned} \quad (16)$$

Thus, μ is a positive implicative filter of a BE-algebra X with degree (λ, κ) .

Corollary 22. Let μ be a fuzzy filter of X . Then, μ is a fuzzy positive implicative filter of X , if and only if

$$(\forall x, y \in X) \quad (\mu(x) \geq \mu((x * y) * x)). \quad (17)$$

Proof. It follows from Propositions 20 and 21. $\square$

Proposition 23. Every fuzzy positive implicative filter of a BE-algebra X with degree (λ, κ) satisfies the following assertions:

- (i) $(\forall x, y \in X) (\mu(x * y) \geq \lambda \kappa \mu(y))$;
- (ii) $(\forall x, y \in X) (x \leq y \Rightarrow \mu(y) \geq \lambda \kappa \mu(x))$.

Proof. It follows from Propositions 16 and 19. $\square$

Corollary 24. Let μ be a fuzzy positive implicative filter of a BE-algebra X with degree (λ, κ) . If $\lambda = \kappa$, then

- (i) $(\forall x, y \in X) (\mu(x * y) \geq \lambda^2 \mu(y))$,
- (ii) $(\forall x, y \in X) (x \leq y \Rightarrow \mu(y) \geq \lambda^2 \mu(x))$.

Proposition 25. Let X be a self-distributive BE-algebra X . Let μ be a fuzzy positive implicative filter of X with degree (λ, κ) . Then,

$$(\forall x, y \in X) \quad (\mu((y * x) * x) \geq \lambda^2 \kappa^3 \mu(y)). \quad (18)$$

Proof. Let $x, y \in X$. Let μ be a fuzzy positive implicative filter of X with degree (λ, κ) . By Proposition 19, μ is a fuzzy filter of X with degree (λ, κ) . Since $x \leq (y * x) * x$, using Proposition 4 (i), we have $((y * x) * x) * y \leq x * y$. Hence,

$$\begin{aligned} (x * y) * y &\leq (y * x) * ((x * y) * x) \\ &= (x * y) * ((y * x) * x) \\ &\leq (((y * x) * x) * y) * ((y * x) * x). \end{aligned} \quad (19)$$

Using Propositions 16 and 23, we have

$$\begin{aligned} \mu((y * x) * x) &\geq \min \{ \kappa \mu(((y * x) * x) * y), \lambda \kappa^2 \mu((x * y) * y) \} \\ &\geq \min \{ \kappa^2 \lambda \mu(y), (\lambda \kappa^2) \kappa \lambda \mu(y) \} \\ &= \kappa^2 \lambda \min \{ \mu(y), \lambda \kappa \mu(y) \} \\ &= \kappa^2 \lambda (\lambda \kappa) \mu(y) \\ &= \lambda^2 \kappa^3 \mu(y). \end{aligned} \quad (20)$$

This completes the proof. $\square$

Definition 26. ([6]) Let X be a BE-algebra. X is said to be commutative if the following identity holds:

$$(C) \quad (x * y) * y = (y * x) * x; \text{ that is, } x \vee y = y \vee x, \text{ where } x \vee y = (y * x) * x, \text{ for all } x, y \in X.$$

Theorem 27. Let X be a commutative self-distributive BE-algebra. Every fuzzy positive implicative filter of X with degree (λ, κ) is a fuzzy implicative filter of X with degree $(\lambda, \kappa^3 \lambda^2)$.

Proof. Let μ be a fuzzy positive implicative filter of X with degree (λ, κ) . By Proposition 19, μ is a fuzzy filter of X with degree (λ, κ) . Using (BE4) and Proposition 4 (iii), we obtain $(x * (y * z)) * ((x * y) * (x * (x * z))) = 1$, for any $x, y, z \in X$. Hence, by Proposition 16 (iii), we have $\mu(x * (x * z)) \geq \min \{ \kappa \mu(x * (y * z)), \lambda \kappa^2 \mu(x * y) \}$. On the other hand, using (BE4) and (C), we obtain

$$\begin{aligned} ((x * z) * z) * (x * z) &= x * (((x * z) * z) * z) \\ &= x * ((z * (x * z)) * (x * z)) \\ &= x * (1 * (x * z)) \\ &= x * (x * z). \end{aligned} \quad (21)$$

Using Proposition 20, we have

$$\begin{aligned}
 \mu(x * z) &\geq \kappa \lambda \mu(((x * z) * z) * (x * z)) \\
 &= \kappa \lambda \mu(x * (x * z)) \\
 &\geq \kappa \lambda \min \{ \kappa \mu(x * (y * z)), \lambda \kappa^2 \mu(x * y) \} \quad (22) \\
 &= \kappa^2 \lambda \min \{ \mu(x * (y * z)), \kappa \lambda \mu(x * y) \} \\
 &\geq \kappa^3 \lambda^2 \min \{ \mu(x * (y * z)), \mu(x * y) \}.
 \end{aligned}$$

This completes the proof. $\square$

Denote by $\mathcal{F}_{PI}(X)$ the set of all positive implicative filters of a BE-algebra X . Note that a fuzzy subset μ of a BE-algebra X is a fuzzy positive implicative filter of X , if and only if

$$(\forall t \in [0, 1]) \quad (U(\mu; t) \in \mathcal{F}_{PI}(X) \cup \{\emptyset\}). \quad (23)$$

But we know that, for any fuzzy subset μ of a BE-algebra X , there exist $\lambda, \kappa \in (0, 1)$ and $t \in [0, 1]$ such that

- (1) μ is a fuzzy positive implicative filter of X with degree (λ, κ) ,
- (2) $U(\mu; t) \notin \mathcal{F}_{PI}(X) \cup \{\emptyset\}$.

Example 28. Consider a BE-algebra $X = \{1, a, b, c, d\}$ which is given in Example 8 (1). Define a fuzzy subset $\mu : X \rightarrow [0, 1]$ by

$$\mu = \begin{pmatrix} 1 & a & b & c & d \\ 0.4 & 0.3 & 0.5 & 0.3 & 0.3 \end{pmatrix}. \quad (24)$$

If $t \in (0.4, 0.6]$, then $U(\mu; t) = \{1, b\}$ is not a positive implicative filter of X , since $b * ((a * d) * a) = 1, b \in \{1, b\}$, and $a \notin \{1, b\}$. But μ is a fuzzy positive implicative filter of X with degree $(0.4, 0.6)$.

Theorem 29. Let μ be a fuzzy subset of a BE-algebra X . For any $t \in [0, 1]$ with $t \leq \max\{\lambda, \kappa\}$, if $U(\mu; t)$ is an enlarged positive implicative filter of X related to $U(\mu; t/\max\{\lambda, \kappa\})$, then μ is a fuzzy positive implicative filter of X with degree (λ, κ) .

Proof. Assume that $\mu(1) < t \leq \lambda \mu(x)$, for some $x \in X$ and $t \in (0, \lambda]$. Then $\mu(x) \geq t/\lambda \geq t/\max\{\lambda, \kappa\}$ and so $x \in U(\mu; t/\max\{\lambda, \kappa\})$; that is, $U(\mu; t/\max\{\lambda, \kappa\}) \neq \emptyset$. Since $U(\mu; t)$ is an enlarged filter of X related to $U(\mu; t/\max\{\lambda, \kappa\})$, we have $1 \in U(\mu; t)$; that is, $\mu(1) \geq t$. This is a contradiction, and thus $\mu(1) \geq \lambda \mu(x)$, for all $x \in X$.

Now suppose that there exist $a, b, c \in X$ such that $\mu(b) < \kappa \min\{\mu(a * ((b * c) * b)), \mu(a)\}$. If we take $t := \kappa \min\{\mu(a * ((b * c) * b)), \mu(a)\}$, then $t \in (0, \kappa] \subseteq (0, \max\{\lambda, \kappa\}]$. Hence, $a * ((b * c) * b) \in U(\mu; t/\kappa) \subseteq U(\mu; t/\max\{\lambda, \kappa\})$ and $a \in U(\mu; t/\kappa) \subseteq U(\mu; t/\max\{\lambda, \kappa\})$. It follows from Definition 10(3) that $b \in U(\mu; t)$ so that $\mu(b) \geq t$, which is impossible. Therefore,

$$\mu(y) \geq \kappa \min \{ \mu(x * ((y * z) * y)), \mu(x) \} \quad (25)$$

for all $x, y, z \in X$. Thus, μ is a fuzzy positive implicative filter of X with degree (λ, κ) . $\square$

Corollary 30. Let μ be a fuzzy subset of a BE-algebra X . For any $t \in [0, 1]$ with $t \leq k/n$, if $U(\mu; t)$ is an enlarged positive implicative filter of X related to $U(\mu; (n/k)t)$, then μ is a fuzzy positive implicative filter of X with degree $(k/n, k/n)$.

Theorem 31. Let $t \in [0, 1]$ be such that $U(\mu; t) (\neq \emptyset)$ is not necessarily a positive implicative filter of a BE-algebra X . If μ is a fuzzy positive implicative filter of X with degree (λ, κ) , then $U(\mu; t \min\{\lambda, \kappa\})$ is an enlarged positive implicative filter of X related to $U(\mu; t)$.

Proof. Since $t \min\{\lambda, \kappa\} \leq t$, we have $U(\mu; t) \subseteq U(\mu; t \min\{\lambda, \kappa\})$. Since $U(\mu; t) \neq \emptyset$, there exists $x \in U(\mu; t)$ and so $\mu(x) \geq t$. By (e1), we obtain $\mu(1) \geq \lambda \mu(x) \geq \lambda t \geq t \min\{\lambda, \kappa\}$. Therefore, $1 \in U(\mu; t \min\{\lambda, \kappa\})$.

Let $x, y, z \in X$ be such that $x * ((y * z) * y) \in U(\mu; t)$ and $x \in U(\mu; t)$. Then $\mu(x * ((y * z) * y)) \geq t$ and $\mu(x) \geq t$. It follows from (e4) that

$$\begin{aligned}
 \mu(y) &\geq \kappa \min \{ \mu(x * ((y * z) * y)), \mu(x) \} \\
 &\geq \kappa t \geq t \min \{ \lambda, \kappa \};
 \end{aligned} \quad (26)$$

so that $y \in U(\mu; t \min\{\lambda, \kappa\})$. Thus, $U(\mu; t \min\{\lambda, \kappa\})$ is an enlarged positive implicative filter of X related to $U(\mu; t)$. $\square$

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

Acknowledgment

The authors are grateful to the referee for their valuable suggestions and help.

References

- [1] H. S. Kim and Y. H. Kim, "On BE-algebras," *Scientiae Mathematicae Japonicae*, vol. 66, no. 1, pp. 113–116, 2007.
- [2] S. S. Ahn and K. S. So, "On ideals and upper sets in BE-algebras," *Scientiae Mathematicae Japonicae*, vol. 68, no. 2, pp. 279–285, 2008.
- [3] S. S. Ahn and K. S. So, "On generalized upper sets in BE-algebras," *Bulletin of the Korean Mathematical Society*, vol. 46, no. 2, pp. 281–287, 2009.
- [4] S. S. Ahn, Y. H. Kim, and K. S. So, "Fuzzy BE-algebras," *Journal of Applied Mathematics & Informatics*, vol. 29, no. 3-4, pp. 1049–1057, 2011.
- [5] Y. B. Jun and S. S. Ahn, "Fuzzy implicative filters with degrees in the interval $(0, 1]$," *Journal of Computational Analysis and Applications*, vol. 15, pp. 1456–1466, 2013.
- [6] S. S. Ahn, Y. H. Kim, and J. M. Ko, "Filters in commutative BE-algebras," *Korean Mathematical Society. Communications*, vol. 27, no. 2, pp. 233–242, 2012.