Spectral Transforms Calculation through Decision Diagrams

RADOMIR S. STANKOVIĆ^a and BOGDAN J. FALKOWSKI^{b,*}

^a*Braće Taskovića 17/29, Niš 18000, Minor Yugoslavia;* ^b*Nanyang Technological University, School of Electrical and Electronic Engineering, Block SI, Nanyang Avenue, Singapore, Singapore 639798*

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In this paper, calculation of spectral transforms through Decision diagrams (DDs) and relationship of this method with FFT-like algorithms is discussed. It is shown that in DDs methods the basic operations in FFT-like algorithms are performed not on vectors but instead on parts of DDs as a data structure. Such a data structure represents the input signals, the intermediate results obtained during the calculation as well as the final output results. It should be noticed that, unlike FFT-like algorithms, DDs methods permit to take advantages from both, the properties of the transform matrices and the particular properties of the processed signals.

Keywords: Spectral transforms; Kronecker transforms; Complex Hadamard transforms; Fast Fourier transform; Decision diagrams

DISCRETE FUNCTIONS

Let G be a finite Abelian group of order g decomposable in the form

$$G = \times_{i=1}^n G_i, \quad g = \prod_{i=1}^n g_i. \quad (1)$$

Denote by P a field that may be a finite (Galois) field $GF(p)$, the fields R or C of real and complex numbers.

A discrete function is defined as a mapping $f: G \rightarrow P$. For a given G and P , the set $P(G)$ of such functions endorse the structure of a vector (Hilbert) space. From Eq. (1), a function $f \in P(G)$ can be considered as an n -variable function $f(x_1, \dots, x_n)$, $x_i \in G_i$.

Switching functions are a particular example for $G = C_2^n$, $C_2 = (\{0, 1\}, \oplus)$ and $P = GF(2)$. In this case $\oplus$ is the addition modulo 2, denoted as EXOR.

DECISION DIAGRAMS

Decision diagrams (DDs) are a data structure convenient for representation of discrete functions [5,19,28,34]. DDs are derived by the reduction of the corresponding Decision trees (DTs). The reduction is performed by sharing the isomorphic subtrees and deleting the redundant

information in the DT using the suitably defined reduction rules [28].

Various DDs are defined for representation of different classes of discrete functions [19,28]. Probably the most widely known are the Binary decision diagrams (BDDs) [1,4], for representation of switching functions. For a given f , the BDD is derived by the reduction of the corresponding Binary decision tree (BDT) [28].

BDT for switching functions of a given number of variables can be derived by the recursive application of the Shannon decomposition rule $f = \bar{x}_i f_0 \oplus x_i f_1$, where $f_0 = f(x_i = 0)$ and $f_1 = f(x_i = 1)$ to all the variables in f .

Example 1 Figure 1 shows BDT for $f(x_1, x_2, x_3)$ given by the truth-vector $\mathbf{F} = [0, 1, 0, 0, 1, 1, 0, 1]^T$. This function will be used in further examples illustrating the considerations in this paper.

In a BDT for f , some isomorphic subtrees may appear. They correspond to some equal subvectors or the constant subvectors of orders 2^k , $k \leq n - 1$, in $\mathbf{F}$. Thanks to that, BDT can be reduced into a BDD for f . In the first case, the subtrees may be shared and the redundant information deleted from the BDT. In the second case, the subtree reduces to a single node whose all outgoing edges point to the same constant node. Thus, it can be deleted and its input edge connected directly to the corresponding constant node. The impact of the deleted nodes can be

*Corresponding author.

FIGURE 1 Shannon decision tree.

represented by the cross points defined as points where an edge crosses a level in the BDD. In a BDD, a level consists of nodes corresponding to the same variable. As example, two cross points are shown in Fig. 2.

Example 2 In $\mathbf{F}$ for f in Example 1, there are two equal subvectors of order 2 and two constant vectors of order 2. Figure 2 shows BDD for f derived by using these properties of f .

In the Shannon decomposition rule, no restrictions are posed on the values that f can take. Thus, $\oplus$ may be replaced by the ordinary addition in $\mathcal{C}$, and the BDDs may be generalized into the Multi-Terminal Binary DDs (MTBDDs) used to represent the complex-valued functions on $\mathcal{C}_2^n$ [7,8]. Extension of the concept to DDs for functions on groups of the form (1) is straightforward. It is done by using the non-terminal nodes at the i -th level with g_i outgoing edges. Such DDs are denoted as Multi-terminal DDs (MTDDs).

SPECTRAL TRANSFORMS

A set Q of g linearly independent functions on G into P form a basis in $P(G)$. Thus, any $f \in P(G)$ can be represented as a linear combination of basic functions from Q . We denote by $\mathbf{Q}(n)$ a $(g \times g)$ matrix whose columns are elements of Q .

DEFINITION 1 For $f \in P(G)$ represented by a vector $\mathbf{F} = [f(0), \dots, f(g-1)]^T$, the Q -spectrum, represented by a vector, $\mathbf{Q}_f = [Q_f(0), \dots, Q_f(g-1)]^T$ is defined by

$$\mathbf{Q}_f = \mathbf{Q}(n)\mathbf{F}. \quad (2)$$

Kronecker Transforms

In practical applications some special attention has been paid to spectral transforms defined by the Kronecker product representable transforms matrices $\mathbf{Q}$, since for such transforms efficient calculation algorithms can be

FIGURE 2 Binary decision diagram.

easily derived. These transforms may be denoted as the Kronecker transforms.

DEFINITION 2 Kronecker transforms for $f \in P(G)$ are defined by the transform matrices

$$\mathbf{K}(n) = \bigotimes_{i=1}^n \mathbf{K}_i(1), \quad (3)$$

where $\mathbf{K}_i(1)$ is an arbitrary $(g_i \times g_i)$ non-singular matrix with elements in P .

The Kronecker spectrum K_f of a given $f \in P(G)$ is defined by Eq. (2) if $\mathbf{Q}(n) = \mathbf{K}(n)$.

If all the submatrices $\mathbf{K}_i(1) = [k_{r,s}]$, $r, s \in \{0, \dots, g_i - 1\}$ are equal to each other, then $\mathbf{K}(n)$ permits the recurrence relation

$$\mathbf{K}(n) = \begin{bmatrix} k_{0,0}\mathbf{K}(n-1) & \dots & k_{0,g-1}\mathbf{K}(n-1) \\ \vdots & \ddots & \vdots \\ k_{g-1,0}\mathbf{K}(n-1) & \dots & k_{g-1,g-1}\mathbf{K}(n-1) \end{bmatrix}. \quad (4)$$

Definition 2 includes some spectral transforms that are already being used in engineering practices [21–23,33].

1. If $P = GF(2)$, and $\mathbf{K}_i = \mathbf{R}(1)$, $\forall i$, where

$$\mathbf{R}(1) = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix},$$

the Reed–Muller transform is defined.

2. If $P = \mathbb{R}$, and $\mathbf{K}_i = \mathbf{A}(1)$, $\forall i$, where

$$\mathbf{A}(1) = \begin{bmatrix} 1 & 0 \\ -1 & 1 \end{bmatrix},$$

the arithmetic transform is defined.

3. If $P = \mathbb{R}$, and $\mathbf{K}_1 = \mathbf{W}(1)$, $\forall i$, where

$$\mathbf{W}(1) = \frac{1}{2} \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix},$$

the discrete Walsh transform is defined.

4. By permitting permutation of columns [29] in $\mathbf{R}(1)$, $\mathbf{A}(1)$ and $\mathbf{W}(1)$, the multi-polarity Reed–Muller, arithmetic and Walsh transforms are defined. These transforms are discussed in detail in [9,10,14,23].

CHT

Complex Hadamard Transforms (CHTs) [15,17,18,26,27], are a broad family of spectral transforms that include several, already used, spectral transforms. At the same time, CHTs belong to the Kronecker transforms and share their useful properties, as for example, the recurrence relation (4) and fast algorithms derived from it [16,17,26].

DEFINITION 3 If $P = C$, and the constituent submatrices $\mathbf{K}_i(1)$ in Eq. (3) are defined by

$$\mathbf{K}_i = j^{\mathbf{A}},$$

where $j = \sqrt{-1}$, and $\mathbf{A}$ is an (2×2) non-singular matrix over $GF(4)$, then the CHT are defined [15].

Example 3 If $\mathbf{A}$ is taken as the modulo 2 multiplication matrix multiplied by 2, i.e. if

$$\mathbf{A} = \begin{bmatrix} 0 & 0 \\ 0 & 2 \end{bmatrix},$$

then the corresponding CHT is the Walsh transform.

Example 4 If

$$\mathbf{A} = \begin{bmatrix} 0 & 0 \\ 1 & 3 \end{bmatrix},$$

the Fresnel transform associated to the Walsh transform is defined [2].

Further generalizations of CHTs are possible if different matrices $\mathbf{A}$ are chosen for any $\mathbf{K}_i$ in Eq. (3), or if higher dimensions for $\mathbf{A}$ are allowed.

Example 5 If $\mathbf{A}$ is taken as the modulo 4 multiplication matrix multiplied by 4, i.e. if

$$\mathbf{A} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 3 & 2 & 1 \\ 0 & 2 & 0 & 2 \\ 0 & 1 & 2 & 3 \end{bmatrix},$$

the complex Walsh transform matrix

$$\mathbf{CW}(1) = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -i & -1 & i \\ 1 & -1 & 1 & -1 \\ 1 & i & -1 & -i \end{bmatrix},$$

is defined.

In what follows, the presentation will be based on the CHT defined in the following example [16].

Example 6 If

$$\mathbf{A} = \begin{bmatrix} 0 & 1 \\ 3 & 2 \end{bmatrix},$$

then

$$\mathbf{CHT}(1) = \begin{bmatrix} 1 & i \\ -i & -1 \end{bmatrix}.$$

Figure 3 shows CHT for $n = 3$ determined by

$$\mathbf{CHT}(3) = \mathbf{CHT}(1) \otimes \mathbf{CHT}(1) \otimes \mathbf{CHT}(1). \quad (5)$$

FFT-LIKE ALGORITHMS

Fast Fourier Transform (FFT) is an algorithm for efficient calculation in terms of space and time of the Discrete Fourier Transform (DFT), see for example, Refs. [3,21,22,40,41].

Extension of FFT to other Kronecker transforms is straightforward, thanks to the so-called Good–Thomas [20] factorization for transform matrices.

DEFINITION 4 Each Kronecker product representable matrix $\mathbf{K}(n)$ defined by Eq. (3) can be represented as

$$\mathbf{K}(n) = \prod_{i=1}^n \mathbf{C}_{K_i}(n),$$

where

$$\mathbf{C}_{K_i}(n) = \bigotimes_{j=1}^n \mathbf{C}_{K_i}^j, \quad \mathbf{C}_{K_i}^j = \begin{cases} \mathbf{K}_i, & i = j, \\ \mathbf{I}_{g_i}, & i \neq j, \end{cases}$$

where $\mathbf{I}_{g_i}$ is the identity matrix of order g_i .

Each of the matrices $\mathbf{C}_{K_i}$ defines the corresponding step in the FFT-like algorithm for $\mathbf{K}(n)$. This statement will be explained in the following example.

$$\mathbf{CHT}(3) = \begin{bmatrix} 1 & i & i & 1 & i & 1 & -1 & -i \\ -i & -1 & 1 & -i & 1 & -i & i & 1 \\ -i & 1 & -1 & -i & 1 & i & -i & 1 \\ -1 & i & i & 1 & -i & 1 & -1 & i \\ -i & 1 & 1 & i & -1 & -i & -i & 1 \\ -1 & i & -i & -1 & i & 1 & -1 & i \\ -1 & -i & i & -1 & i & -1 & 1 & i \\ i & 1 & 1 & -i & 1 & -i & -i & -1 \end{bmatrix}$$

FIGURE 3 CHT for $n = 3$ and $\mathbf{A}$ defined by Eq. (1).

FIGURE 4 FFT-like algorithm.

Example 7 The **CHT**(3) matrix in Fig. 3 can be factorized as follows.

$$\mathbf{CHT}(3) = \mathbf{C}_1 \mathbf{C}_2 \mathbf{C}_3,$$

where

$$\mathbf{C}_1 = \mathbf{CHT}(1) \otimes \mathbf{I}_2 \otimes \mathbf{I}_2, \quad \mathbf{C}_2 = \mathbf{I}_2 \otimes \mathbf{CHT}(1) \otimes \mathbf{I}_2,$$

$$\mathbf{C}_3 = \mathbf{I}_2 \otimes \mathbf{I}_2 \otimes \mathbf{CHT}(1).$$

Figure 4 shows the FFT-like algorithm designed from this factorization. Each step corresponds to a matrix $\mathbf{C}_i$, $i = 1, 2, 3$, and performs calculations shown by the non-zero elements in it.

Major disadvantage of FFT-like algorithms for spectral transform calculations is their exponential complexity in both space and time. For functions on an Abelian group of the form (1), the time complexity of FFT-like algorithms is $O(ng)$. The space complexity approximates to $O(g)$ provided all the calculations are done in-place. Such implementations are possible, since the input in the i -th step is the output from the $(i - 1)$ -th step of the algorithm and the values calculated in the previous steps are not required.

FFT in Multi-processors Environment

From Fig. 4, it is obvious that the calculation of the CHT-spectrum consists of the repeated application of the same basic operation determined by **CHT**(1) performed in each step over a different subset of data. In a multiprocessor environment, it is most convenient if each processor performs this basic operation over the neighboring data. Therefore, in Fig. 4, the input data for each step should be reordered. Figure 5 shows such a realization for **CHT**(3).

DD BASED CALCULATIONS

Calculations in Figs. 4 and 5 differ in the ordering of the elements generated after each step of the algorithm. The

FIGURE 5 FFT-like algorithm with calculations in the multiprocessor environment.

ordering corresponds to the way in which the input data for each step are reordered.

If the operations over vectors are provided, the calculations may be further reorganized. In that case, the calculation procedure may be organized to correspond to the structure of DT representing f .

Example 8 Figure 6 shows the calculation of **CHT**(3) for f in Example 1 through BDT in Fig. 1. Calculations are performed over the subvectors to which the outgoing edges of non-terminal nodes point to.

Similarly, as in Figs. 4 and 5, the calculations over equal subvectors are repeated in Fig. 6. Also, the constant vector whose elements are all equal to zero, is processed in the same way as any other subvector in $\mathbf{F}$ for f . In the BDD for f , such information is removed by the reduction rules. Therefore, by providing vector operations, further simplifications in the calculation procedure in Fig. 6 may be reached if it is based on the BDD instead of BDT for f . That is possible, since these rules do not reduce the information content of a BDT and the contribution of the deleted nodes to the spectrum of f is taken into account through the cross points in the BDD.

Example 9 The CHT-spectrum for f in Example 1 can be calculated if at each node and the cross point in the BDD for f in Fig. 2 the operations defined by **CHT**(1) are performed. In the matrix notation, the calculation procedure is described as follows.

FIGURE 6 FFT-like algorithm with calculations over vectors.

First, the node and cross points at the level corresponding to x_3 are processed, which produces:

$$\mathbf{CHT}_{c_{3,0}} = \begin{bmatrix} 1 \cdot 0 + i \cdot 0 \\ -i \cdot 0 - 1 \cdot 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix},$$

$$\mathbf{CHT}_{S_{3,0}} = \begin{bmatrix} 1 \cdot 0 + i \cdot 1 \\ -i \cdot 0 - 1 \cdot 1 \end{bmatrix} = \begin{bmatrix} i \\ -1 \end{bmatrix},$$

$$\mathbf{CHT}_{c_{3,1}} = \begin{bmatrix} 1 \cdot 1 + i \cdot 1 \\ -i \cdot 1 - 1 \cdot 1 \end{bmatrix} = \begin{bmatrix} 1+i \\ -1-i \end{bmatrix}.$$

Performing **CHT(1)** at the nodes at the level corresponding to x_2 , i.e. over the subvectors where points the outgoing edges of these nodes, we get

$$\mathbf{CHT}_{S_{2,0}} = \begin{bmatrix} 1 \cdot \begin{bmatrix} i \\ -1 \end{bmatrix} + i \cdot \begin{bmatrix} 0 \\ 0 \end{bmatrix} \\ -i \cdot \begin{bmatrix} i \\ -1 \end{bmatrix} - 1 \cdot \begin{bmatrix} 0 \\ 0 \end{bmatrix} \end{bmatrix} = [i \quad -1 \quad 1 \quad i]^T,$$

$$\mathbf{CHT}_{S_{2,1}} = \begin{bmatrix} 1 \cdot \begin{bmatrix} 1+i \\ -1-i \end{bmatrix} + i \cdot \begin{bmatrix} i \\ -1 \end{bmatrix} \\ -i \cdot \begin{bmatrix} 1+i \\ -1-i \end{bmatrix} - 1 \cdot \begin{bmatrix} i \\ -1 \end{bmatrix} \end{bmatrix} = [i \quad -1-2i \quad 1-2i \quad i]^T.$$

Performing **CHT(1)** at the root node we get the **CHT**-spectrum of f as follows

$$\mathbf{CHT}_{S_{1,0}} = \begin{bmatrix} 1 \cdot \begin{bmatrix} i \\ -1 \\ 1 \\ i \end{bmatrix} + i \cdot \begin{bmatrix} i \\ -1-2i \\ 1-2i \\ i \end{bmatrix} \\ -i \cdot \begin{bmatrix} i \\ -1 \\ 1 \\ i \end{bmatrix} - 1 \cdot \begin{bmatrix} i \\ -1-2i \\ 1-2i \\ i \end{bmatrix} \end{bmatrix} = \begin{bmatrix} -1+i \\ 1-i \\ 3+i \\ -1+i \\ 1-i \\ 1+3i \\ -1+i \\ 1-i \end{bmatrix}.$$

The calculation procedure described in the above example can be represented through DD as shown in Fig. 7. It is suitable for coding in a programming language.

From the above discussed example, the following Calculation procedure for the Kronecker transforms (CPKT) can be formulated:

Calculation procedure CPKT

1. Given a function f by its DD. Descend DD level by level by starting from the constant nodes.
2. Perform at each non-terminal node and the cross point at the i -th level the operations determined by the submatrix $\mathbf{K}_i(1)$ over the subvectors to which point the outgoing edges of the nodes and cross points.

FIGURE 7 Generation of the MTBDD for CHT-spectrum.

TABLE I Complexity of f and CPU-times

f	i	o	s	w	t (msec)	
					FFT	DDs
5xp1	7	10	88	25	62	30
rd53	5	3	23	6	5	5
rd84	8	4	59	11	157	30
sao2	10	4	154	33	2309	170
9sym	9	1	33	6	564	20

EXPERIMENTAL RESULTS

Similar procedure applies also to the more general multi-polarity CHT in which matrices $\mathbf{K}_i(1)$ have two types corresponding to the polarity bit 0 and 1 accordingly [5].

Table I shows complexity of BDDs for some benchmark function and CPU-times for calculation of CHT in Example 4. The number of inputs (i), outputs (o), size (s) and width (w) of DDs are shown. Calculation time is given in milliseconds. Table II shows complexity of the produced MTBDDs for CHT spectrum of these functions. In this case, the number of non-terminal nodes (ntn) and constant nodes (cn) are given, since in CHTDDs the constant nodes are complex-valued. This table shows that for some functions CHTDDs provide more compact representations than BDDs. As for any other DD, there are the opposite examples. Calculations are performed on a 133 MHz Pentium PC with a 32 MB RAM.

FFT-LIKE ALGORITHMS VERSUS DD METHODS

Basic characteristics of FFT-like algorithms are the following.

1. FFT-like algorithms are based upon the decomposition of the transform matrix $\mathbf{K}(n)$ into the product of n sparse matrices $\mathbf{K}(n) = \mathbf{C}_{K_1} \dots \mathbf{C}_{K_n}$. Each $\mathbf{C}_{K_i}$ describes a step in the algorithm. Decomposition is possible, thanks to the recursive structure of $\mathbf{K}(n)$ originating in the assumed decomposition for G and the way of definition of the basis represented by columns of $\mathbf{K}^{-1}$.
2. The calculations are performed over the vector representations of the input data $\mathbf{F}$ and the intermediate results $\mathbf{F}_i$, $i = 1, \dots, n-1$, after each step, producing finally $\mathbf{S}_f = \mathbf{F}_n$.

TABLE II Complexity of CHTDDs.

f	ntn	cn	s	w
5xp1	127	128	256	255
rd53	15	6	21	5
rd84	36	9	45	8
sao2	965	906	1871	454
9sym	43	6	49	8

3. Although permit parallelization within steps, FFT-like algorithms are essentially sequential algorithms, since the output of the i -th step is the input for the $(i+1)$ -st step.
4. For a given $\mathbf{K}(n)$, and the fixed decomposition of it, the calculations of spectra for different signals are performed in the identical way, although $\mathbf{F}$ may have some particular properties, as for example, many zero elements, many identical or constant subvectors, etc. Thus, FFT-like algorithms do not exploit peculiar properties which $\mathbf{F}$ may have.
5. Complexity of calculations approximates to $O(g)$ in terms of space, providing that calculations in-place are provided, and O/ng in terms of time.

Identical calculations without exploiting possible properties of f and the exponential calculation complexity may be considered as disadvantages of FFT-like algorithms limiting their applications when processing of signals defined in a large number of points is required.

Basic characteristics of DDs methods are the following.

1. Recursive structure of DDs for functions on group G originates in the assumed decomposition of G . Since the same decomposition is assumed in FFT-like algorithms, the recursive structure of DTs matches the recursive structure of the decomposable transform matrices $\mathbf{K}$.
2. Reduction of a DT representing a given f into the corresponding DD for f is possible if f has some properties mentioned in property 4 of FFT-like algorithms, which are not exploited in FFT-like algorithms. Thus, instead of vectors, DDs are used as a structure for data representation in calculation of spectral transforms, some advantages may be taken from such properties of f permitting reduction of complexity of calculations.
3. In DDs, constant subvectors in $\mathbf{F}$ are represented by a single constant node. Equal subvectors are represented by the isomorphic subtrees, which are shared and the redundant subtrees are removed from the DD. Therefore, in DDs methods, the processing of constant subvectors is greatly simplified, and there is no repeated processing of identical subvectors in $\mathbf{F}$.
4. Besides properties of f , in DDs methods, we may also use peculiar properties of $\mathbf{K}(n)$ to improve the efficiency of computation, in the same way as it is done in non-constant geometry FFT-like algorithms. In these algorithms, $\mathbf{C}_{K_i} \neq \mathbf{C}_{K_j}$ for $i \neq j$. The properties of $\mathbf{C}_{K_j}$ may be used to reduce the number of calculations in the j -th step. For example, unlike the FFT for the Walsh transform, the number of calculations in steps of the non-constant geometry FFT for the Haar transform decreases when i increases. The same property was used in [35] to improve efficiency of calculation of the Haar transform through DDs.

From this consideration, the following remark is possible.

Remark 1 In FFT-like algorithms, efficiency of calculations is achieved by taking advantage of the properties of transform matrices decomposable into the product of sparse matrices. These matrices are further expressed in terms of some basic transform matrices, see for example, Refs. [40,41].

In DDs methods, the advantages are taken from both the properties of the transform matrices in the same way as in FFT-like algorithms, and the properties of the processed functions.

Therefore, for a given G and fixed decomposition of G , FFT-like algorithm for any f on G has the same complexity in terms of both space and time. However, in DDs methods, complexity of calculation strongly depends on the complexity of the DD for f . Thanks to that, DDs permit efficient processing when G is large and extend the application of spectral techniques to areas where, for the complexity, the use of FFT-like algorithms is impossible.

GENERALIZATIONS

An n -variable multiple-output switching function f_q with q outputs can be represented by an integer-valued function $f_q(z)$ determined by adding the outputs of f_q multiplied by the weighting coefficients 2^i , $i = 0, \dots, q - 1$. Thanks to that, f_q can be represented by a MTBDD and its spectrum for a spectral transform can be calculated by using the procedure CPKT.

If f_q is represented by a shared BDD (SBDDs) [28] the calculation of a linear spectral transform may be performed over SBDD. The resulting spectrum for $f_q(z)$ is determined by adding the outputs at the root nodes of SBDD multiplied by 2^i .

Several variants of BDDs bases calculation procedures for spectral transform and their extensions of the DDs based calculation methods to other transforms for switching functions are considered in several papers by different authors [5–7,11–13,19,34,35].

The method is also adapted to the so-called Edge-valued binary DDs (EVBDDs) [24] and their generalizations Factored edge-valued binary DDs (FEVBDDs) [39].

Generalizations to spectral transforms and differential operators for multiple-valued function and for functions on finite, not necessarily Abelian, groups into different fields are also provided [25,30,36–38].

CLOSING REMARKS

In this paper, an efficient symbolic algorithm for the calculation of spectral transforms on finite Abelian groups has been presented. Relationships between

standard FFT-type of fast transforms calculations and DDs based calculations have been explained.

As an example, in FFT type calculations through DDs, the Kronecker product based transform called the Complex Hadamard transform was used. Such a transform operates on complex numbers and the advantages of its CHTDDs over standard BDDs for some standard benchmark functions have been shown.

Presented results can be extended to compute other spectral transform on finite Abelian groups. Extensions to computation of the Fourier transform on non-Abelian groups and Generalized Fourier transforms on these groups with generalizations defined through FFT, and complacent Gibbs derivatives on non-Abelian groups are given in [31,32].

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Authors' Biographies

Radomir S. Stanković received BE degree in Electronic Engineering from Faculty of Electronics, University of Niš, in 1976, and MSc, and PhD degrees in Applied Mathematics from Faculty of Electrical Engineering, University of Belgrade, in 1984, and 1986, respectively. He was with High School of Electrotechnic, Niš, from 1976 to 1987. From 1987 to date he is with Faculty of Electronics, Niš. Presently, he is a Professor teaching Logic Design. His research interests include switching theory and multiple-valued logic, signal processing and spectral techniques. He served as the co-editor and editor of two editorials and the author of couple of monographs in spectral techniques.

Bogdan J. Falkowski received the MSEE degree from the Technical University of Warsaw, Poland, and the PhD degree from Portland State University, Oregon, USA. His industrial experience includes research and development positions at several companies from 1978 to 1986. He then joined the Electrical Engineering Department at Portland State University. Currently, he is an Associate professor with the School of Electrical and Electronic Engineering, Nanyang Technological University, Singapore, which he joined in 1992. His research interests include VLSI systems and design, synthesis and optimization of switching circuits, multiple-valued systems, design of algorithms, design automation, digital signal and image processing. He has published 3 book chapters and over 10 articles in the refereed journals and conferences. Dr Falkowski is a Senior Member of IEEE and a Member of IEEE Computer Society and IEEE Circuits and Systems Society. He is a member of Eta Kappa Nu and Pi Beta Upsilon.