Research Article

Flexible Queuing Model for Number of Active Users in Cognitive Radio Network Environment

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1. Introduction

Cognitive Radios (CR) seem to be the prominent area of research in future communication systems including 5G which is based on Massive MIMO and heterogeneous networks [1, 2]. A reliable communication model for cognitive radio is the requirement of the time. With the development of the idea, its implication in almost all the existing communication setups, including 4G and mobile communication, and the momentum it has gained, it is definite to continue its research importance in future as well [3, 4]. Recent works on smart grid for future energy requirements also incorporate the idea of cognitive radio for communication in different layers of networks; i.e., the network of home appliances, the community, and then the mega scale networks up to metropolitan level are supposed to be exploiting the idea of cognitive radio for communications [5, 6].

Surprisingly, a reliable channel model is still missing for cognitive radio. Specifically if we get the idea of cooperative users with a fusion centre operating in community in parallel with the primary networks and service providers, then, how much resources will be available to cognitive radio network is a basic question. This will also link with the level of reliability that a cognitive radio network may offer to its clients.

There have been lot of research in CR, however, that focus on three major areas. The first one is the channel sensing [7, 8] while second one is the access of spectrum holes or channels [9] and the third one is optimization of resources to utilize the available channels for maximum throughput [10].

Now in case of the reliable communication on cognitive radio network, the question will be how many users are present at some given instant [11] and how many channels are available to accommodate these users [12]? Answer to the questions lie in probability theory. The available channels to cognitive radio network will be based on the parameters such as the overall available/free channels in a primary base station operating in the region and of course on the sensing capability of the cognitive radio network under consideration.

The second question related to the above discussed three major areas of CRN will have certain channels available and will be offering service to its clients; then, how many clients can be accommodated at a given instant is a question whose answer lies in queuing theory.
A queuing model for number of channels occupied by the users in primary radio network is relatively less complicated and has been derived similar as in [13]. However, that is for infinite resources, which has been modified for the limited available resources; i.e., maximum available channels are assumed to be finite. That seems logical and is being employed in existing primary radio networks.

In third phase the idea has been extended to the cognitive radio networks with flexible upper bound. Since the upper limit for available channels in cognitive radio networks is flexible hence, accordingly, a Flexible Queuing Model is being proposed in this paper. The model has been implemented in simulations and the results for diversified scenarios have been presented.

Rest of the paper has been organized as follows: Section 2 presents mathematical framework and proposed model for existing networks with fixed upper limit for number of users. In Section 3 proposed Flexible Queuing Model for cognitive radio networks is discussed. Section 4 presents the whole system model. Simulation results are given in Section 5 and the conclusion is included in Section 6.

In the document $p_j$ represents probability of $j$ number of active users, while $n$ and $k$ are random variables representing, respectively, the available number of channels and number of active users for cognitive radio network.

### 2. Mathematical Framework and Proposed Model for Existing Networks

In case of existing cellular networks, the BS dedicates a fraction of available communication channels to accommodate the smooth hand over (HO). The HO is predictable based on the physical location of Mobile user; i.e., the handover probability will be on higher side near the cell boundaries. Yet there could be other causes for handover such as Quality of Service (QoS) and Conjunction; however, they are secondary reasons in comparison.

In case of cognitive radios the available number of channels and hence the active users are random, depending on the sensing capabilities and occupying the available frequency spots for the spectrum. Hence the Flexible Queuing Model was the ultimate requirement to address this issue. Since the model was not available in the existing literature, therefore this model has been proposed to deal with two different scenarios which may be considered as an extension of M/M/1.

**Scenario-I.** This case is considered in current Section 2 where the length of queue has been taken as finite. Yet this length is kept fixed. Accordingly the mathematical model has been developed and results have been derived for finite number of users ($K$ in the manuscript).

**Scenario-II.** The major contribution in our work is the environment in which the finite number $K$ is taken as a random. Accordingly the phrase “Flexible Queuing Model” has been introduced in the title. All the derivations have been carried out and results have been generated for a stochastic upper limit as in Section 3.

In case of existing cellular networks, the number of available channels for a BS is considered to be deterministic and fixed. Let this number be $N$. Moreover let the maximum allowable number of active users be $K$. Therefore, the extra channels, i.e., $h_{extra} = N - K$, will be reserved for handovers. The ratio of maximum allowable active users to the available channels reflects the worst situation that appears in peak hours when the entire $K$ allowable channels will be occupied. As a matter of routine the number of active users will be random occupying a value from the set $\{0, 1, \ldots, K\}$. Assuming $k$ number of active users at an instant when another user appears, the network will move to $k + 1$ state and conversely; if an active user leaves the network, the system will move to $k - 1$ state. Ultimately the number of active users will follow a queuing system model as illustrated in the Figure 1.

In the figure $\alpha$ is the rate at which a new user appears and the process moves forward from state $k$ to $k + 1$ and $0 \leq k \leq K - 1$ and $\beta$ is the rate at which process moves backward from state $k$ to $k - 1$ when an active user leaves the system with the condition $1 \leq k \leq K$.

Accordingly the probability of finding the process in state $k$ at some given instant will be

$$p_k = \zeta^k p_0, \quad 0 \leq k \leq K$$

where $\zeta = \alpha/\beta$ and $p_0$ is the probability of finding the process in state $k$ at some given instant. The detailed derivation may follow through the queuing model presented in [13].

In order to compute $p_0$, i.e., the probability of finding the process in state $0$ at some given instant, we may apply the normalization i.e., $\sum_{k=0}^{\infty} p_k = 1$

That is, $(1 + \zeta + \zeta^2 + \ldots + \zeta^K) p_0 = 1$.

Now the series, $(1 + \zeta + \zeta^2 + \ldots + \zeta^K) = (1 + \zeta + \zeta^2 + \ldots) - (\zeta^{K+1} + \zeta^{K+2} + \ldots)$, which is the difference of two infinite series hence will be given as

$$p_0 \sum_{k=1}^{K} \zeta^k = p_0 \sum_{k=0}^{\infty} \zeta^k - p_0 \zeta^{K+1} \sum_{k=0}^{\infty} \zeta^k$$

$$p_0 - p_0 \zeta^{K+1} \sum_{k=0}^{\infty} \zeta^k = p_0 \left(1 - \zeta^{K+1}\right) \sum_{k=0}^{\infty} \zeta^k = 1$$

It will converge provided $\zeta < 1$ and hence

$$p_0 \left(1 - \zeta^{K+1}\right) \frac{1}{1 - \zeta} = 1$$

This gives

$$p_0 = \frac{1 - \zeta}{1 - \zeta^{K+1}}$$
Thus
\[ p_j = \zeta^j \frac{1 - \zeta}{1 - \zeta^{K+1}} \] (5)

with the condition that \( \zeta < 1 \) and hence \( \alpha < \beta \). Therefore in this case the forward transition rate \( \alpha \) which is the rate at which some new channel is needed must be less than the reverse transition rate \( \beta \), which is the rate at which the channel is released by the user and is added back in the queue of available resources, which will be the ultimate condition for global balance.

These transition probabilities and queuing model are valid for the existing cellular networks. Below is a little bit complicated model for cognitive radio network. This model will be applicable to the cognitive radio networks committed to provide specific level of reliability to the users.

3. Proposed Flexible Queuing Model for Cognitive Radio Networks

In above section we have proposed a queuing model for cellular networks. In this case a state is an indicator of the active number of users at some given instant. The advantage in case of above derivation was that the maximum allowable active users and number of channels, i.e., \( K \) and \( N \), were known to us and HO occurs with relatively less probability that was only \( p_j \). However, in case of CRN the situation will be little bit more complicated; i.e., the available number of channels in this case will be a random variable \( n \) which may take a value from the set \( \{0, 1, 2, \ldots, N\} \) with respective probabilities changing with outcome. Evaluation of these probabilities may not be simple, specifically when there is the existence of multiple CRNs operating in parallel on cooperative basis. Yet having \( n \) available channels the CRN will be in position to accommodate \( k \) users which will be decided based on the level of reliability, i.e.,
\[ h_{ratio} = \frac{n - k}{h_{active}} = \frac{n - k}{k} \implies k = n \frac{n}{h_{ratio} + 1} \] (6)

Ratio in (6) is decided by the CRN fusion centre. Accordingly for \( k \) active users, \( m = h_{ratio} k \), where \( m = k \leq n \) will be upper bounded, where \( n \) is the total number of channels available to CRN after sensing and it will be random as well.

\[ k + m = n \implies k + kh_{ratio} = n \implies k = n \frac{n}{1 + h_{ratio}} \] (7)

This \( k \) represents the maximum allowable users. Obviously \( k \) will also be a random variable in this case. Depending on this set up, the queuing system, as developed in case of above cellular model, will need to be flexible. The updated Figure 2 is given below.

The transition rates \( \alpha_c \) and \( \beta_c \) in this scenario will be greater than their corresponding counterparts in previous section, i.e., \( \alpha_c > \alpha \) and \( \beta_c > \beta \), which indicates more vibrant environment in this case.

\[ \text{Figure 2: State model for channel occupation and release by active users in cognitive radio network.} \]

**Theorem 1.** In given scenario, the state probabilities \( p_j \) and condition for convergence are derived as below.

**Proof.** Given fixed \( k \), i.e., \( k = K \), the state probabilities \( p_j \) will become
\[ p_j = \lambda^j \frac{1 - \lambda}{1 - \lambda^{K+1}}, \quad 0 \leq j \leq K \] (8)

The scenario of expression (8) is similar to that discussed in equation (5). However present scenario is flexible and \( \lambda = \alpha_c/\beta_c \). Similarly the condition for convergence will be \( \beta_c > \alpha_c \). Since \( k \) is random, the above probability in this case will become
\[ p_j = \sum_{k=0}^{K} \lambda^j \frac{1 - \lambda}{1 - \lambda^{K+1}} P(k = k) \] (9)

where \( P(k = k) \) will be derived from \( P(n) \) available channels for CRN which is based on the sensing strategies, the environment temperature, and number of active CRNs in given area. The expression in (9) is proposed with the help of probability theory and specifically total probability theorem.

Rearranging expression (9) we get
\[ p_j = \lambda^j (1 - \lambda) \sum_{k=0}^{K} \frac{P(k = k)}{1 - \lambda^{K+1}} = \lambda^j (1 - \lambda) \left( \frac{P(k = 0)}{1 - \lambda} + \frac{P(k = 1)}{1 - \lambda^2} + \ldots + \frac{P(k = K)}{1 - \lambda^{K+1}} \right) \] (10)

4. Whole System Model

Following flowchart in Figure 3 describes the whole system model including the architecture of primary and cognitive radio user environment.

5. Simulations

In order to verify the performance of proposed models, simulations are carried out in MATLAB. For this purpose different cases, under different situations, are considered.

**Case 1.** In Case 1, the probability of number of active users for existing cellular networks is determined. For this purpose, the number of available channels and maximum number of
active users for base station are taken as 20 and 15, respectively \((N = 20, K = 15)\). Random variable \(k\) representing number of active users will occupy values from the set \(\{0, 1, \ldots, 15\}\). Required probabilities are determined using expression (5) for different values of \(\zeta\), i.e., 0.75, 0.85, and 0.95. The corresponding probabilities are shown in Figure 4.

**Case 2.** This case deals with probability of active users for cognitive radio network. For this purpose let \(N = \) number of active users for primary BS=100.

\[
N' = \text{number of vacant channels for cognitive users} = N/2 = 50
\]

\[
M_{\text{CRN}} = \text{number of cognitive networks} = 5
\]

The probability of active users for CRN is evaluated using Poisson random variable distribution in expression (10) and the resulting expression becomes

\[
p_j = \lambda^j (1 - \lambda) e^{-\lambda} \sum_{k=-n}^{+n} 1 - \frac{1}{\lambda^{k+1}} \frac{\alpha^k}{k!}
\]

The resultant probabilities for \(\lambda = 0.75, 0.85, 0.95\) are shown in Figure 5.

**Case 3.** For Case 3 simulations are performed under the same condition as Case 2 except the variance of \(\sigma\) which is taken to be as 5 and the simulation results are shown in Figure 6.
Case 4. Case 4 is same as Case 2 except that the probability of active users for CRN is evaluated using uniform random variable distribution in expression (10) and the resulting expression becomes

$$p_j = \lambda^j (1 - \lambda) \left( \frac{1}{N} + \sum_{k=\alpha}^{\lambda \alpha + \eta} \frac{1}{1 - \lambda^{k+1}} \right)$$

(12)

The resultant probabilities for $\lambda = 0.75, 0.85, 0.95$ are shown in Figure 7.

Case 5. Case 5 presents simulation for the comparison between numerical results for Uniform and Poisson distributions using different values of $\lambda$. These results are shown in Figure 8.

Above simulation results show that an exponential-like decay in probabilities is observed in almost all the plots by increasing the number of users. This is quite in line with the expectation, i.e., considering the expression (9), and $\lambda$ is kept constant which is less than 1. Hence $\lambda^{K+1}$ will be negligible in comparison with 1 and

$$p_j \approx \lambda^j (1 - \lambda) \sum_{k=0}^{K} \frac{P(k=k)}{1 - \lambda^{k+1}} \approx \lambda^j (1 - \lambda) = \lambda^j$$

(13)

where $\sum_{k=0}^{K} P(k=k) \approx 1$.

Hence by increasing $j$, $p_j$ decreases exponentially as is evident from plots.

In case we take the value of $\lambda$ closer to unity, the PDF becomes almost linear i.e., a straight line. It may be observed that the area under the curve which can be easily computed
in case of straight line is approximately unity that is the normalization of PDF.

6. Conclusion

Methods of finding probabilities of number of active users for existing cellular networks and for cognitive radio networks are presented. For existing cellular networks, the number of channels for base station and maximum allowable number of active users are considered to be fixed. The state of the system at any time is defined by the number of active users at that time. The probability of the system to be in a particular state is evaluated on the basis of the rate $\alpha$ at which a new user appears in the system and the rate $\beta$ at which an active user leaves the system. Then flexible queuing state model addresses the complicated scenario for cognitive radio networks where the available channels are actually the spare channels of primary BS. Hence both the number of channels for cognitive network and the maximum number of active cognitive users are not deterministic. The probability of the cognitive network system to be in a particular state is evaluated on the basis of the rate $\alpha_c$ at which a new cognitive user may appear in the system, the rate $\beta_c$ at which an active cognitive user may leave the system and pmf of number of available channels.

The work is important to deal with handover problem for number of active users in existing cellular networks and in cognitive radio networks on the basis of their probabilities.

Data Availability

The data used to support the findings of this study are included within the article.

Conflicts of Interest

All the authors declare that there are no conflicts of interest.

References


