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Research Article

Achievable Rates of Gaussian Interference Channel with Multi-Layer Rate-Splitting and Successive Simple Decoding

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The capacity bound of the Gaussian interference channel (IC) has received extensive research interests in recent years. Since the IC model consists of multiple transmitters and multiple receivers, its exact capacity region is generally unknown. One well-known capacity achieving method in IC is Han-Kobayashi (H-K) scheme, which applies two-layer rate-splitting (RS) and simultaneous decoding (SD) as the pivotal techniques and is proven to achieve the IC capacity region within 1 bit. However, the computational complexity of SD grows exponentially with the number of independent signal layers, which is not affordable in practice. To this end, we propose a scheme which employs multi-layer RS at the transmitters and successive simple decoding (SSD) at the receivers in the two-transmitter and two-receiver IC model and then study the achievable sum capacity of this scheme. Compared with the complicated SD, SSD regards interference as noise and thus has linear complexity. We first analyze the asymptotic achievable sum capacity of IC with equal-power multi-layer RS and SSD, where the number of layers approaches to infinity. Specifically, we derive the closed-form expression of the achievable sum capacity of the proposed scheme in symmetric IC, where the proposed scheme only suffers from a little capacity loss compared with SD. We then present the achievable sum capacity with finite-layer RS and SSD. We also derive the sufficient conditions where employing finite-layer RS may even achieve larger sum capacity than that with infinite-layer RS. Finally, numerical simulations are proposed to validate that multi-layer RS and SSD are not generally weaker than SD with respect to the achievable sum capacity, at least for some certain channel gain conditions of IC.

1. Introduction

Due to the broadcast nature of wireless channel, the interference greatly affects the performance of wireless communication when multiple signal streams are transmitted on the same time/frequency resources. As a general description, the interference channel (IC) model has been proposed to describe the channel statistics where multiple transmitters and multiple receivers share the same physical resources [1]. In recent decades, IC has been regarded as an important building block in cognitive radio [2], multicell network [3, 4], and massive input massive output system [5].

A basic IC model is illustrated in Figure 1, where two transmitters, i.e., Tx-1 and Tx-2, aim to simultaneously transmit their signals to two receivers, i.e., Rx-1 and Rx-2, respectively. Before analyzing the capacity bound and the capacity approaching techniques of IC, we may first recall the other two well-studied multiuser channel models, i.e., the multiple access channel (MAC) model and the broadcast channel (BC) model, where rate-splitting (RS), superposition coding (SC), and successive simple decoding (SSD) are usually required to approach the capacity bounds of MAC and BC. However, different from MAC and BC which either has a single transmitter or a single receiver, IC has at least two independent links, i.e., Tx-1-to-Rx-1 and Tx-2-to-Rx-2 as shown in Figure 1, which may interfere with each other. Each receiver in IC receives multiple signal streams, which constitute an MAC. Symmetrically, each transmitter in IC broadcasts the signal to multiple users which exactly follows a BC. Therefore, the IC model can be regarded as a composition of MAC and BC, and this fact makes the analysis in either MAC or BC not sufficient in the IC.

In the past several decades, the problem of finding the exact capacity region of Gaussian IC has been proven to

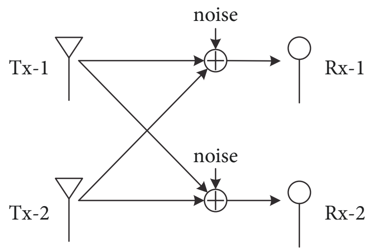


Figure 1: The interference channel model with two transmitters and two receivers.

be pretty hard. The exact capacity region of IC with strong interference is derived in [6]. Meanwhile, [7] analyzes the capacity region of discrete-memoryless IC. Some researchers have focused on the capacity region of degraded IC, e.g., Z-IC [8–12].

Nevertheless, the capacity region of a general IC has not been clearly revealed yet. One best known achievable rate region of a general IC is Han-Kobayashi (H-K) inner bound [13]. The original H-K bound is hard to be analytically described and depicted; therefore, H. Etkin in [14] proposes a simplified H-K scheme, which approaches the IC capacity region within 1 bit. The capacity region achieved by the simplified H-K scheme is termed simple H-K region. To prove the achievability, the simultaneous decoding (SD) of more than one codeword is required, which may lead to exponential computational complexity. Recently, simultaneous nonunique decoding (SND) is proved to be rate optimal when random coding is applied [15]. However, the decoding complexity is still high. Hence, one key question to be studied in the era of IC is

(i) Are there any simple decoding and encoding methods which can be used to achieve the H-K capacity region?

To find the answer, we may look at the capacity approaching techniques in MAC, due to the fact that MAC can be regarded as a degraded version of IC. To approach every rate pair in MAC capacity region, the transmitters employ RS and SC and the receiver employs SSD and successive interference cancellation (SIC) [16]. While SD/SND generally has exponential computational complexity with respect to the number of independent coding layers in the transmission signals, SSD/SIC only requires linear computational complexity. Hence, SSD/SIC are pretty simple compared with SD or SND and have been attractive to the researchers from many fields [17–19]. For example, J. Cao in [19] proves that, with infinite number of rate splits at each transmitter, applying RS and SSD can asymptotically achieve capacity of MAC bound in a distributed manner.

In view of the benefits of RS, two RS-based schemes are proposed in [20, 21], separately, to achieve the H-K inner bound in IC. However, Omar in [22] shows that joint decoding is still required in [20, 21] (instead of SSD) and the receiving complexity of the methods in [20, 21] is actually not reduced. Therefore, whether RS and SSD can achieve the H-K inner bound remains a question. Y. Zhao in [23] studies the maximum achievable rate with SSD in the deterministic model for IC. However, the deterministic model only works in high SNR region. Still, [23] does not answer the aforementioned question. In [24], the authors point out that any finite-layer RS and SSD cannot achieve the corner points of the SD bound of the symmetric Gaussian IC, where the interference is strong but not very strong. Alternatively, a sliding window superposition coding method is proposed in [24] to achieve the simultaneous decoding inner bound where interference cancellation is available at different time slots. However, this method suffers from performance loss since the first and last blocks are not fully loaded with messages. Therefore, with general channel gain settings, the question that whether RS and SSD can be used to achieve the SD achievable rate region is still unsolved.

As conjectured by Omar in [22], multiple-layer RS may be required such that SSD can achieve the bound close to H-K capacity. Following this conjunction, in this paper, we conduct an asymptotic analysis of the achievable rate with multi-layer RS and SSD in IC and aim at answering the question whether infinite-layer RS can achieve the SD inner bound [22, 24]. We note that once multi-layer RS and SSD are able to approach the SD achievable rate region, they can be directly applied in the H-K scheme to achieve the utmost capacity region of IC. We assume that RS is conducted by splitting the transmission power and assigning suitable rate for each split. We start with equal-power RS with infinite number of layers and then find that infinite-layer equal-power RS and SSD cannot approach the SD bound in general. Especially, we derive the closed-form formula of the performance gap between the proposed scheme and the SD bound in symmetric IC. We note that the performance gap is pretty small. Based on the above results, we then analyze the achievable rate with finite-layer RS. Surprisingly, with certain channel gain conditions, we show that employing finite-layer RS and SSD can achieve even better sum capacity than SD. To sum up, the main result of this paper is that employing multiple layers RS and SSD/SIC can nearly approach the SD bound in IC. The result can be further exploited as a guideline in designing capacity approaching technologies in practical scenarios, such as designing good inter-cell interference cancellation method.

This paper is organized as follows. Section 2 describes the system model and the useful notations. Section 3 presents the achievable rate with SSD when infinite-layer RS is applicable. In Section 4, we analyze the achievable sum capacity when finite-layer RS is assumed. The numerical results are presented in Section 5. Section 6 concludes this paper.

2. System Model

2.1. Multi-Layer Rate-Splitting for Interference Channel. We consider an IC model with two transmitters, i.e., Tx-i, $i = 1, 2$, and two receivers, i.e., Rx-j and $j = 1, 2$, where Rx-j is the target receiver of Tx-i when $j = i$. We assume the additive white Gaussian noise (AWGN) channel, as shown in Figure 2, where the channel gain between Tx-i and Rx-j is $h_{j,i}$, and the noise variance is N_0. The transmission power at Tx-i is P_i.

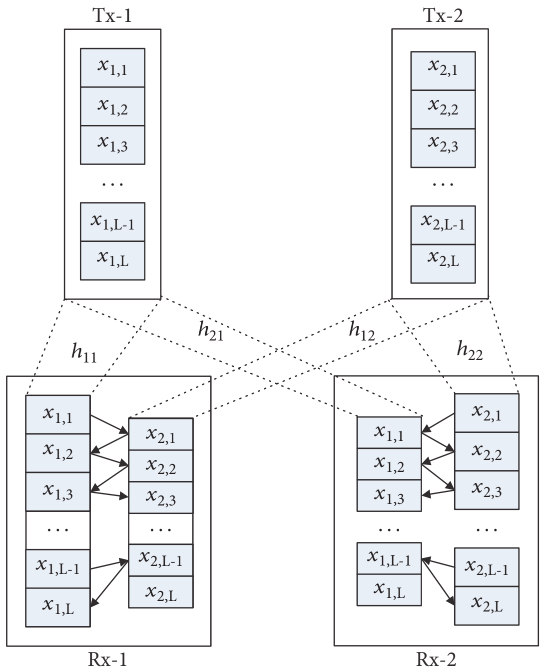


FIGURE 2: An illustration of multi-layer RS and SSD in IC, with the decoding order Π_{12}.

Without loss of generality, we assume $P_1 = P_2 = P$. Then, the received signals are given by

$$y_1 = \sqrt{h_{1,1}}x_1 + \sqrt{h_{1,2}}x_2 + n_1,$$
$$y_2 = \sqrt{h_{2,1}}x_1 + \sqrt{h_{2,2}}x_2 + n_2, \quad (1)$$

where x_i is the transmitting signal of Tx-i, with $\|x_i\|_2^2 = P_i$.

To exploit the potential of IC, we employ multi-layer RS at the transmitters. In the proposed scheme, Tx-i's total transmission data rate R_i is split into L_i *splits* by splitting the total transmission power into $[p_{i,1}, \cdots, p_{i,k} \cdots, p_{i,L_i}]$, where $p_{i,k}$ is the allocated power to k-th split of Tx-i and $\sum_{k=1}^{L_i} p_{i,k} = P$. We assume equal-power RS throughout this paper, i.e., $p_{i,k} = P/L_i = p_i$, unless otherwise stated, since equal-power allocation is usually applied as a baseline in analyzing the achievable capacities in different systems with RS [25, 26]. Correspondingly, the transmission signal of Tx-i can be represented as

$$x_i = \sum_{k=1}^{L_i} x_{i,k}, \quad (2)$$

where $\|x_{i,k}\|_2^2 = p_{i,k}$. We note that the elaborately designed power allocation among the message splits may further improve the system performance [27], which is also a promising future direction.

Without loss of generality, we assume $L_1 = L_2 = L$. It should be noted that IC can be regarded as two MACs from the point of view of the receivers and that the SD bound is derived by taking the minimum of the sum capacity of the two MACs, i.e.,

$$r_{\mathrm{SD}} = \min\left\{\log\left(1 + \frac{h_{1,1}P_1 + h_{1,2}P_2}{N_0}\right),\right.$$
$$\left.\log\left(1 + \frac{h_{2,1}P_1 + h_{2,2}P_2}{N_0}\right)\right\}. \quad (3)$$

To maintain low computational complexity, we apply SSD as well as interference cancellation at each receiver to sequentially decode the signal splits, where the interference splits are regarded as additive Gaussian white noise. Each receiver may first detect a signal split and then an interference split one after another. The successfully decoded splits are then cancelled from the received signal. In our system, we consider a fixed decoding order for a certain $\Pi_{m,l}$. The optimal decoding order and power allocation will be investigated in the future work. We define the notation $\Pi_{l,m}$, $l, m = 1, 2$, to represent the decoding order where at Rx-1, the successive decoding starts from $x_{l,1}$ and, at Rx-2, the successive decoding starts from $x_{m,1}$. Afterwards, the splits of both transmitters are decoded one after another. As an example, when $\Pi_{1,2}$ is assumed, Tx-1 successively decodes and cancels $x_{1,1}$, $x_{2,1}$, $x_{1,2}$, $x_{2,2}$..., and Tx-2 successively decodes and cancels $x_{2,1}$, $x_{1,1}$, $x_{2,2}$, $x_{1,2}$... Therefore, there are a total number of four decoding orders, i.e., $\Pi_{1,1}$, $\Pi_{1,2}$, $\Pi_{2,1}$, and $\Pi_{2,2}$. We visually illustrate this example in Figure 2, where $\Pi_{1,2}$ is assumed and the arrows indicate the decoding order.

The received signal to interference and noise ratio (SINR) of the kth split transmitted from Tx-i to Rx-j with decoding order $\Pi_{l,m}$ is denoted as $s_{j,i}^{\Pi_{l,m},(k)}$. Accordingly, we define the achievable rate of the kth split transmitted from Tx-i at Rx-j with decoding order $\Pi_{l,m}$ as $r_{j,i}^{\Pi_{l,m},(k)}$, which is given by

$$r_{j,i}^{\Pi_{l,m},(k)} = \log\left(1 + s_{j,i}^{\Pi_{l,m},(k)}\right). \quad (4)$$

The coding rate of the message in each split should be equal to the corresponding achievable channel capacity derived in (4) to ensure successful decoding. Assuming perfect interference cancellation, the SINR of each power split is calculated by dividing the received power of this split by noise plus all residual interference. As an example, when decoding order $\Pi_{1,2}$ is assumed, the SINR of each power split is formulated as follows:

$$s_{1,1}^{\Pi_{1,2},(k)} = \frac{h_{1,1}(P_1/L)}{h_{1,1}(P_1/L)(L-k) + h_{1,2}(P_2/L)(L-k+1) + N_0}, \quad (5)$$

$$s_{1,2}^{\Pi_{1,2},(k)} = \frac{h_{1,2}(P_2/L)}{h_{1,1}(P_1/L)(L-k) + h_{1,2}(P_2/L)(L-k) + N_0}, \quad (6)$$

$$s_{2,1}^{\Pi_{1,2},(k)} = \frac{h_{2,1}(P_1/L)}{h_{2,1}(P_1/L)(L-k) + h_{2,2}(P_2/L)(L-k) + N_0}, \quad (7)$$

$$s_{2,2}^{\Pi_{1,2},(k)} = \frac{h_{2,2}(P_2/L)}{h_{2,2}(P_2/L)(L-k) + h_{2,1}(P_1/L)(L-k+1) + N_0}. \quad (8)$$

Furthermore, we define the sum achievable rate of Tx-i's signal at Rx-j with decoding order $\Pi_{l,m}$ as

$$r_{j,i}^{\Pi_{l,m},[L]} = \sum_{k=1}^{L} r_{j,i}^{\Pi_{l,m},(k)}. \quad (9)$$

The decoding order may affect the receiving SINR of each split as well as the achievable rate. Therefore, at transmitter, it is necessary to consider the effect of the decoding order when assigning the rate to each split. Besides, due to the fact that some splits will be decoded by both receivers, the rates of these splits should be carefully assigned such that successful interference cancellations at two receivers can be carried out. With a fixed decoding order $\Pi_{l,m}$, we define a rate matching (RM) operation in this paper, which ensures that the maximum affordable transmission rate is assigned to each split such that the split can be successfully recovered by both Rx-1 and Rx-2. For example, when RM is employed, the transmission rate of kth split of Tx-i, with the decoding order $\Pi_{l,m}$, is defined as $\hat{r}_i^{\Pi_{l,m},(k)}$, which is given by

$$\hat{r}_i^{\Pi_{l,m},(k)} = \min\left\{r_{1,i}^{\Pi_{l,m},(k)}, r_{2,i}^{\Pi_{l,m},(k)}\right\}. \quad (10)$$

2.2. Notations. Recall that, in this paper, we aim to study whether RS and SSD can achieve the SD bound, when large even infinite number of layers is available. However, it is nontrivial to directly compare their performances. Hence, the analysis in this paper is organized in incremental steps, as illustrated in the following.

We start with the case where RM is not conducted; i.e., the data rate of each split is only decided by the received SINR of the target receiver with a given SSD order. We denote this scheme where infinite-layer RS and SSD are applied without RM as **EPRSO** (as a short notation of *Equal-Power Rate Splitting without RM*). We note that this scheme is not realistic, since RM is not employed to ensure the success of SSD. To analyze EPRSO, we propose a genie-aided model, where the interference splits are decoded with the help of genie transmitters. Then we study the realistic settings by considering in RM operations. And we denote the scheme applying infinite-layer RS and SSD with RM as **EPRSW** (as a short notation of *Equal-Power Rate Splitting with RM*). Obviously, EPRSO achieves the upper bound capacity performance of EPRSW. Besides, we define the scheme named **f-EPRSW** (as a short notation of *finite-layer Equal-Power Rate Splitting with RM*) where finite-layer RS and SSD with RM are assumed. In the following sections, we first analyze the gap of the achievable sum capacity between SD and EPRSO and then analyze the gap between EPRSO and EPRSW by taking EPRSO as a bridge in comparing SD and EPRSW. Finally, we compare the performance between SD and f-EPRSW.

The performance metric used to compare SD, EPRSO, EPRSW, and f-EPRSW is the achievable sum capacity at the receivers [22, 24]. Note that when the sum rate of the proposed scheme, i.e., EPRSW, is exactly the same as SD, then through time sharing technique and regarding interference as noise, the proposed scheme can also reach other points in the capacity region. Therefore, it is sufficient to study the achievable sum capacity.

3. Performance Analysis of Infinite-Layer RS

In this section, we analyze whether EPRSO and EPRSW can approach SD bound when the splitting number approaches infinity. To begin with, we present some preliminary Lemmas and Theorems.

3.1. Preliminary. In this paragraph, we do not assume that $P_1 = P_2 = P$, since the results derived in the following Lemmas and Theorems still hold with arbitrary P_i. We first show the existence of the limit of $r_{j,i}^{\Pi_{l,m},[L]}$ when $L \longrightarrow +\infty$ according to Lemmas 1 and 2.

Lemma 1 (monotonicity). *Given the decoding order $\Pi_{l,m}$, $r_{j,i}^{\Pi_{l,m},[L]}$ increases with L if $j = 1$ and $l \neq i$, or if $j = 2$ and $m \neq i$, and $r_{j,i}^{\Pi_{l,m},[L]}$ decreases with L if $j = 1$ and $l = i$, or if $j = 2$ and $m = i$.*

Proof. Without loss of generality, we take $\Pi_{1,2}$ as an example, and aim to prove that $r_{1,1}^{\Pi_{1,2},[L]}$ increases with L by mathematical deduction method. The proof consists of two steps, i.e., the base step and the induction step, where $r_{1,1}^{\Pi_{1,2},[L]} < r_{1,1}^{\Pi_{1,2},[L+1]}, \forall L$.

In base step, we aim to prove that $r_{1,1}^{\Pi_{1,2},[1]} < r_{1,1}^{\Pi_{1,2},[2]}$. We first calculate $r_{1,1}^{\Pi_{1,2},[1]}$ and $r_{1,1}^{\Pi_{1,2},[2]}$ by assuming $L = 1$ and 2, respectively. $r_{1,1}^{\Pi_{1,2},[1]}$ and $r_{1,1}^{\Pi_{1,2},[2]}$ are given, respectively, by

$$\begin{aligned} &\log\left(\frac{h_{1,1}P_1}{h_{1,2}P_2 + N_0}\right), \\ &\log\left(\frac{h_{1,1}P_1/2}{h_{1,1}P_1/2 + h_{1,2}P_2/2 + N_0}\right) \\ &\quad + \log\left(\frac{h_{1,1}P_1/2}{h_{1,2}P_2/2 + N_0}\right). \end{aligned} \quad (11)$$

Hence, $r_{1,1}^{\Pi_{1,2},[2]} - r_{1,1}^{\Pi_{1,2},[1]}$ is given by

$$r_{1,1}^{\Pi_{1,2},[2]} - r_{1,1}^{\Pi_{1,2},[1]} = \log\left(\frac{(1/2)\,h_{1,2}h_{11}P_1P_2 + \mho}{(1/4)\,h_{1,2}h_{11}P_1P_2 + \mho}\right) > 0, \quad (12)$$

where $\mho$ is the same term appeared in both numerator and denominator. The proof of the induction step is similar, which is omitted for brevity. Therefore, by mathematical deduction, the statement in Lemma 1 holds. □

Lemma 2 (upper bound). *Given the decoding order $\Pi_{l,m}$, $\lim_{L\longrightarrow+\infty} r_{j,i}^{\Pi_{l,m},[L]}$ is upper bounded by $h_{j,i}P_i/N_0$, if $j = 1$ and $l \neq i$, or if $j = 2$ and $m \neq i$ (or lower bounded by $h_{j,i}P_i/N_0$, if $j = 1$ and $l = i$ or $j = 2$ and $m = i$).*

Proof. We take $r_{1,1}^{\Pi_{1,2},[L]}$ as an example. $r_{1,1}^{\Pi_{1,2},[L]}$ is upper bounded by $\hat{r}_{1,1}^{\Pi_{1,2},[L]}$, which is given by

$$\begin{aligned}\hat{r}_{1,1}^{\Pi_{1,2},[L]} &= \sum_{k=1}^{L} \log\left(1+\frac{h_{1,1}P_1/L}{N_0}\right) \\ &= \log\left(1+\frac{h_{1,1}P_1/L}{N_0}\right)^L.\end{aligned} \tag{13}$$

Let $L \longrightarrow +\infty$; then we have

$$r_{1,1}^{\Pi_{1,2},[L]} < \hat{r}_{1,1}^{\Pi_{1,2},[L]} = \frac{h_{11}P_1}{N_0}, \tag{14}$$

where $\lim_{x\longrightarrow+\infty}\log(1+1/x)^x = 1$. □

According to Lemmas 1 and 2 and the theorem of supremum, there exists a limit of $r_{j,i}^{\Pi_{l,m},[L]}$ when $L \longrightarrow +\infty$, with decoding order $\Pi_{l,m}$. Define

$$r_{j,i}^{\Pi_{l,m},[+\infty]} = \lim_{L\longrightarrow+\infty} r_{j,i}^{\Pi_{l,m},[L]}. \tag{15}$$

where $r_{j,i}^{\Pi_{l,m},[+\infty]}$ is given by the following Theorem 3.

Theorem 3 (limit). *When $L \longrightarrow +\infty$, $r_{j,i}^{\Pi_{l,m},[L]}$ converges to $r_{j,i}^{[+\infty]}$, $i, j = 1, 2$, with any choice of $\Pi_{l,m}$,*

$$r_{1,1}^{[+\infty]} = \frac{h_{1,1}P_1}{h_{1,1}P_1+h_{1,2}P_2}\log\left(1+\frac{h_{1,1}P_1+h_{1,2}P_2}{N_0}\right), \tag{16}$$

$$r_{1,2}^{[+\infty]} = \frac{h_{1,2}P_2}{h_{1,1}P_2+h_{1,2}P_2}\log\left(1+\frac{h_{1,1}P_1+h_{1,2}P_2}{N_0}\right), \tag{17}$$

$$r_{2,1}^{[+\infty]} = \frac{h_{2,1}P_1}{h_{2,1}P_1+h_{2,2}P_2}\log\left(1+\frac{h_{2,1}P_1+h_{2,2}P_2}{N_0}\right), \tag{18}$$

$$r_{2,2}^{[+\infty]} = \frac{h_{2,2}P_2}{h_{2,1}P_1+h_{2,2}P_2}\log\left(1+\frac{h_{2,1}P_1+h_{2,2}P_2}{N_0}\right). \tag{19}$$

Proof. We take $r_{1,1}^{\Pi_{1,m},[+\infty]}$ as an example. We note that $\log(1+x) \longrightarrow x$ when $x \longrightarrow 0$. Hence, we can remove the log terms in (4), and the limit of $r_{1,1}^{\Pi_{1,m},[L]}$ is given by (20). We define $Z_1 = (h_{1,1}P_1+h_{1,2}P_2)/(h_{1,1}P_1+h_{1,2}P_2+N)$, and $Z_2 = h_{1,1}P_1/(h_{1,1}P_1+h_{1,2}P_2+N)$,

$$\begin{aligned}\lim_{L\longrightarrow+\infty} r_{1,1}^{\Pi_{1,m},[L]} &= \lim_{L\longrightarrow+\infty}\sum_{l=0}^{L-1}\frac{h_{1,1}P_1}{L(h_{1,1}P_1+h_{1,2}P_2+N_0)-l(h_{1,1}P_1+N_0+h_{1,2}P_2)-h_{1,1}P_1} \\ &= \lim_{L\longrightarrow+\infty}\sum_{l=0}^{L-1}\frac{h_{1,1}P_1}{L(h_{1,1}P_1+h_{1,2}P_2+N_0)}\frac{1}{1-(l/L)((h_{1,1}P_1+h_{1,2}P_2)/(h_{1,1}P_1+h_{1,2}P_2+N_0))-(h_{1,1}P_1/L(h_{1,1}P_1+h_{1,2}P_2+N_0))}\end{aligned} \tag{20}$$

Equation (20) can be rewritten as

$$\begin{aligned}\lim_{L\longrightarrow+\infty} r_{1,1}^{\Pi_{1,m},[L]} &= \lim_{L\longrightarrow+\infty}\sum_{l=0}^{L-1}\frac{Z_2}{L}\frac{1}{1-((l+1)/L)Z_1} \\ &= \frac{Z_2}{Z_1}\lim_{L\longrightarrow+\infty}\sum_{l=0}^{L-1}\frac{Z_1}{L}\times\frac{1}{1-(Z_1/L)(1+l)} \\ &= \frac{Z_2}{Z_1}\int_0^{Z_1}\frac{1}{1-x-o(x)}dx \\ &= -\frac{Z_2}{Z_1}\log(1-x-o(x))\Big|_0^{Z_1} \\ &= -\frac{h_{1,1}P_1}{h_{1,1}P_1+h_{1,2}P_2}\log\left(1-\frac{h_{1,1}P_1+h_{1,2}P_2}{h_{1,1}P_1+h_{1,2}P_2+N_0}\right) \\ &= \frac{h_{1,1}P_1}{h_{1,1}P_1+h_{1,2}P_2}\log\left(1+\frac{h_{1,1}P_1+h_{1,2}P_2}{N_0}\right) \\ &= r_{1,1}^{[+\infty]}.\end{aligned} \tag{21}$$

Interestingly, we see that $\Pi_{1,m}$ does not affect the asymptotic behavior of $r_{1,1}^{\Pi_{1,m},[L]}$ when $L \longrightarrow +\infty$. Thus, we can omit the notation of $\Pi_{l,m}$. With the similar approach, we can derive $r_{1,2}^{[+\infty]}$, $r_{2,1}^{[+\infty]}$, and $r_{2,2}^{[+\infty]}$. □

3.2. Analysis of EPRSO. As aforementioned, it is nontrivial to directly find the relationship between the achievable sum capacity between EPRSW and SD, so we firstly study EPRSO as a bridge. As shown in Figure 3, the original IC is decomposed into two virtual MACs with the help of genie Tx-1 and genie Tx-2, and the two MACs do not interfere with each other. We note that the genie transmitters are introduced to convert the original IC to two virtual MACs, where the achievable rates are easier to be computed. With the given channel conditions and decoding order $\Pi_{l,m}$, the achievable rate of the two transmitters in virtual MAC-j, $j = 1, 2$, is given by $r_{j,1}^{\Pi_{l,m},[L]}$ and $r_{j,2}^{\Pi_{l,m},[L]}$, respectively. Meanwhile, Tx-i will have two capacities, i.e., $r_{1,i}^{\Pi_{l,m},[L]}$ and $r_{2,i}^{\Pi_{l,m},[L]}$, dedicated for virtual MAC-1 and 2, respectively. Therefore, the total achievable rate is derived as

$$\begin{aligned}r_{\text{EPRSO}}^{[L]} &= \min\left\{r_{1,1}^{\Pi_{l,m},[L]}, r_{2,1}^{\Pi_{l,m},[L]}\right\} \\ &\quad + \min\left\{r_{1,2}^{\Pi_{l,m},[L]}, r_{2,2}^{\Pi_{l,m},[L]}\right\},\end{aligned} \tag{22}$$

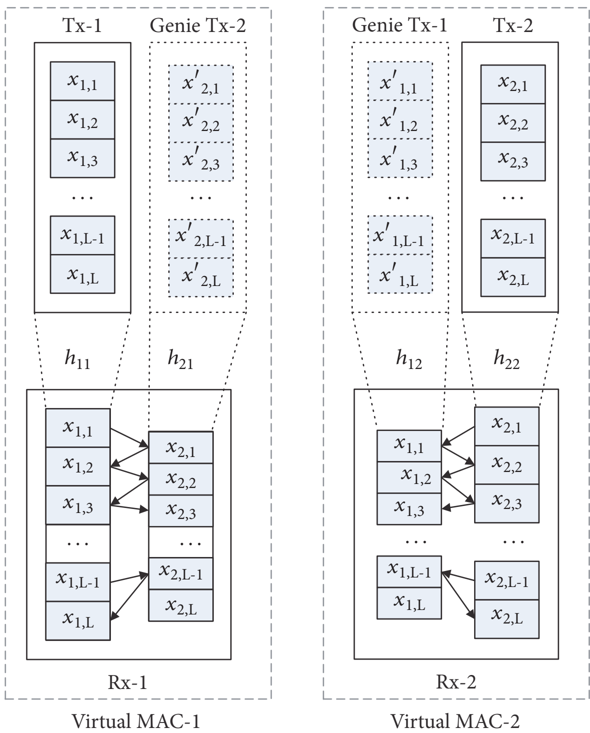


FIGURE 3: Decomposition of IC into two MACs, with the help of two genie transmitters.

when $L \longrightarrow +\infty$, $r_{\text{EPRSO}}^{[+\infty]}$ is derived as

$$r_{\text{EPRSO}}^{[+\infty]} = \min\left\{r_{1,1}^{[+\infty]}, r_{2,1}^{[+\infty]}\right\} + \min\left\{r_{1,2}^{[+\infty]}, r_{2,2}^{[+\infty]}\right\}. \quad (23)$$

The following theorem demonstrates the sufficient conditions of the channel coefficients, where EPRSO asymptotically approaches the performance of SD.

Theorem 4 (EPRSO). *EPRSO achieves the same performance as SD if both ($h_{1,1} \le h_{2,1}$) and ($h_{1,2} \le h_{2,2}$) hold, or if both ($h_{1,1} \ge h_{2,1}$) and ($h_{1,2} \ge h_{2,2}$) hold.*

Proof. We take the first condition as an example. When ($h_{1,1} \le h_{2,1}$) and ($h_{1,2} \le h_{2,2}$) hold, we have

$$\begin{aligned} r_{1,1}^{\Pi_{l,m},[+\infty]} &\le r_{2,1}^{\Pi_{l,m},[+\infty]}, \\ r_{1,2}^{\Pi_{l,m},[+\infty]} &\le r_{2,2}^{\Pi_{l,m},[+\infty]}, \end{aligned} \quad (24)$$

Therefore, the achievable rate is derived as

$$r_{\text{EPRSO}}^{[+\infty]} = r_{1,1}^{[+\infty]} + r_{1,2}^{[+\infty]} = \log\left(1 + \frac{h_{1,1}P + h_{1,2}P}{N_0}\right), \quad (25)$$

which exactly follows the expression of the SD bound in (3). □

The gap between EPRSO and SD is also calculated as

$$r_{\text{SD}} - r_{\text{EPRSO}}^{[+\infty]} = \min\left\{\left|r_{i,j}^{[+\infty]}, r_{j,j}^{[+\infty]}\right|, \left|r_{i,i}^{[+\infty]}, r_{j,i}^{[+\infty]}\right|\right\}, \quad (26)$$

where $i, j = 1, 2$.

Remark 5. When symmetric IC is assumed, i.e., $h_{1,1} = h_{2,2}$ and $h_{1,2} = h_{2,1}$, the gap between EPRSO and SD is derived as

$$r_{\text{SD}} - r_{\text{EPRSO}}^{[+\infty]} = \frac{h_{1,2} - h_{1,1}}{h_{1,2} + h_{1,1}} \log\left(1 + \frac{(h_{1,2} + h_{1,1})P}{N_0}\right). \quad (27)$$

As an example, when $h_{1,1} = 1$ and $h_{1,2} = 0.9$, the loss of EPRSO is about 5% compared to SD.

Remark 6. In symmetric IC, $r_{\text{EPRSO}}^{[+\infty]}$ equals r_{SD} if and only if $h_{1,1} = h_{1,2} = h_{2,1} = h_{2,2}$.

3.3. *Analysis of EPRSW.*

Compared with EPRSO, EPRSW satisfies the individual rate constraint for each power split by employing RM operation. Therefore, the sum rate constraint in (23) is not sufficient. The achievable sum rate of IC with infinite-layer RS, SSD, and RM; i.e., EPRSW, is denoted as $r_{\text{EPRSW}}^{[L]}$

$$\begin{aligned} r_{\text{EPRSW}}^{[L]} = \sum_{k=1}^{L} \Big(&\min\left\{r_{1,1}^{\Pi_{l,m},(k)}, r_{2,1}^{\Pi_{l,m},(k)}\right\} \\ &+ \min\left\{r_{1,2}^{\Pi_{l,m},(k)}, r_{2,2}^{\Pi_{l,m},(k)}\right\}\Big), \end{aligned} \quad (28)$$

where L is the splitting number. The following theorem demonstrates the sufficient conditions where EPRSW approaches EPRSO.

Theorem 7 (EPRSW). *EPRSW achieves the same performance of EPRSO if there exists $\Pi_{l,m}$, such that, for any k_1, k_2, the following two conditions are satisfied:*

(1)

$$\left(r_{1,1}^{\Pi_{l,m},(k_1)} - r_{2,1}^{\Pi_{l,m},(k_1)}\right)\left(r_{1,1}^{\Pi_{l,m},(k_2)} - r_{2,1}^{\Pi_{l,m},(k_2)}\right) \ge 0, \quad (29)$$

(2)

$$\left(r_{1,2}^{\Pi_{l,m},(k_1)} - r_{2,2}^{\Pi_{l,m},(k_1)}\right)\left(r_{1,2}^{\Pi_{l,m},(k_2)} - r_{2,2}^{\Pi_{l,m},(k_2)}\right) \ge 0, \quad (30)$$

Proof. When the above conditions are satisfied, the min operator in (28) can be taken out of $\sum$ and then (28) is exactly the same as (23). □

Remark 8. In symmetric IC where $h_{11} \ge h_{12}$, we readily see that $r_{1,1}^{\Pi_{l,m},(k_1)} \ge r_{2,1}^{\Pi_{l,m},(k_1)}$ and $r_{2,2}^{\Pi_{l,m},(k_1)} \ge r_{1,2}^{\Pi_{l,m},(k_1)}$, when $\Pi_{1,2}$ is applied. In this case, the sufficient conditions in Theorem 7 are satisfied, which means that no gap exists between EPRSW and EPRSO in symmetric IC and the gap between EPRSW and SD also follows (27). Otherwise, in symmetric IC where $h_{11} \le h_{12}$, the conditions in Theorem 7 also hold, if the decoding order $\Pi_{2,1}$ is assumed. According to the above analysis, we are ready to say that, infinite-layer RS and SSD can achieve the same capacity region as SD, if the sufficient conditions in both Theorems 4 and 7 are satisfied.

However, due to the implicit expressions of the conditions given in Theorem 7, it is not straightforward to conclude the channel gain settings where EPRSWs achieve the same performance as SD. In the following, we show that, in most channel gain settings, the conditions of Theorems 4 and 7 cannot be satisfied simultaneously.

When L-layer RS is employed, the gap between EPRSO and EPRSW is derived in (31),

$$\begin{aligned}\Delta_{\Pi_{l,m}}^{L} &= \sum_{k=1}^{L}\Bigg(\left|r_{1,1}^{\Pi_{l,m},(k)} - r_{2,1}^{\Pi_{l,m},(k)}\right| \\ &\quad \cdot \mathrm{I}\left(-\left(r_{1,1}^{\Pi_{l,m},(k)} - r_{2,1}^{\Pi_{l,m},(k)}\right)\left(r_{1,1}^{+\infty} - r_{2,1}^{+\infty}\right)\right) \\ &\quad + \left|r_{1,2}^{\Pi_{l,m},(k)} - r_{2,2}^{\Pi_{l,m},(k)}\right| \\ &\quad \cdot \mathrm{I}\left(-\left(r_{1,2}^{\Pi_{l,m},(k)} - r_{2,2}^{\Pi_{l,m},(k)}\right)\left(r_{1,2}^{+\infty} - r_{2,2}^{+\infty}\right)\right)\Bigg) \\ &= \sum_{k=1}^{L}\sum_{c=1}^{2}\left|r_{1,c}^{\Pi_{l,m},(k)} - r_{2,c}^{\Pi_{l,m},(k)}\right| \cdot \mathrm{I}\Big(-\Big(r_{1,c}^{\Pi_{l,m},(k)} \\ &\quad - r_{2,c}^{\Pi_{l,m},(k)}\Big)\left(r_{1,c}^{+\infty} - r_{2,c}^{+\infty}\right)\Big),\end{aligned} \tag{31}$$

where $\mathrm{I}(\cdot)$ is an indication function, i.e., $\mathrm{I}(+) = 1$ and $\mathrm{I}(-) = 0$. Without loss of generality, we assume $h_{1,1} > h_{2,1}$, $h_{1,2} > h_{2,2}$, which is the sufficient condition of EPRSO achieving the SD bound. Furthermore, we assume that the receivers follow the decoding order $\Pi_{1,2}$. Since $r_{1,1}^{[+\infty]} \geq r_{2,1}^{[+\infty]}$, to ensure that EPRSW and EPRSW have the same performance, $\Delta_{\Pi_{1,2}}^{L}$ must be equal to 0; i.e., the following condition must hold:

$$r_{1,1}^{\Pi_{1,2},(k)} \geq r_{2,1}^{\Pi_{1,2},(k)}, \quad 1 \leq k \leq K, \tag{32}$$

i.e.,

$$\frac{h_{1,2}}{h_{1,1}}(L-l+1) + \frac{L}{h_{1,1}} \geq \frac{h_{2,2}}{h_{2,1}}(L-l) + \frac{L}{h_{2,1}}, \quad 1 \leq k \leq K. \tag{33}$$

When $k = L - 1$, (33) is simplified to

$$2\frac{h_{1,2}}{h_{1,1}} + \frac{L}{h_{1,1}} \geq \frac{h_{2,2}}{h_{2,1}} + \frac{L}{h_{2,1}}. \tag{34}$$

For $L \longrightarrow +\infty$, the terms $h_{1,2}/h_{1,1}$ and $h_{2,2}/h_{2,1}$ can be ignored, and a necessary condition of (34) is $h_{2,1} \geq h_{1,1}$, which contradicts the assumption.

According to the above analysis, with equal-power infinite-layer RS, the sufficient conditions in Theorems 4 and 7 are usually contradictory; i.e., when EPRSO achieves the SD bound, the gap between EPRSO and EPRSW, i.e., $\Delta_{\Pi_{l,m}}^{L}$ is always non-zero. Hence, EPRSW performs slightly worse than SD in general, even if the split number approaches infinite. Besides, in symmetric IC, the achievable sum capacities of EPRSO/EPRSW are no larger than that of SD. Nevertheless, we find that the performance gap between EPRSO/EPRSW and SD is usually pretty small, as further illustrated in Section 5, which makes its finite-layer variant a good tradeoff between complexity and performance.

4. Performance Analysis of Finite-Layer RS

In the previous section, we have analyzed the asymptotic achievable rate of multi-layer RS and SSD in IC, by making an unrealistic assumption where each transmitter employs infinite-layer RS. In this section, we consider the achievable rate where only finite-layer RS is allowed.

4.1. Achievable Sum Capacity with Finite-Layer RS. To analyze the achievable sum capacity with finite-layer RS, we first define the discrete sequence

$$\boldsymbol{r}_i^{\Pi_{l,m}} = \left[r_i^{\Pi_{l,m}}[1], \cdots, r_i^{\Pi_{l,m}}[L], \cdots\right], \quad L \in \mathbb{Z}^+, \tag{35}$$

where its Lth element is given by

$$r_i^{\Pi_{l,m}}[L] = \min\left\{r_{1,i}^{\Pi_{l,m},[L]}, r_{2,i}^{\Pi_{l,m},[L]}\right\}. \tag{36}$$

Then we define the discrete sequence

$$\boldsymbol{r}_{\mathrm{sum}}^{\Pi_{l,m}} = \left[r_{\mathrm{sum}}^{\Pi_{l,m}}[1], \cdots, r_{\mathrm{sum}}^{\Pi_{l,m}}[L], \cdots\right], \tag{37}$$

where $r_{\mathrm{sum}}^{\Pi_{l,m}}[L]$ represents the achievable sum capacity with L-layer equal-power RS and is given by

$$r_{\mathrm{sum}}^{\Pi_{l,m}}[L] = r_1^{\Pi_{l,m}}[L] + r_2^{\Pi_{l,m}}[L]. \tag{38}$$

We note that $r_{\mathrm{sum}}^{\Pi_{l,m}}[L]$ describe the relationship between the achievable sum capacity and the layer number of RS. For illustration purpose, we, respectively, interpolate $r_1^{\Pi_{l,m}}[L]$ and $r_2^{\Pi_{l,m}}[L]$ into two continuous functions, namely, $r_1^{\Pi_{l,m}}(\widehat{L})$ and $r_2^{\Pi_{l,m}}(\widehat{L})$, $\widehat{L} \in \mathbb{R}^+$. And we define $r_{\mathrm{sum}}^{\Pi_{l,m}}(\widehat{L}) = r_1^{\Pi_{l,m}}(\widehat{L}) + r_2^{\Pi_{l,m}}(\widehat{L})$. $r_{\mathrm{sum}}^{\Pi_{l,m}}(\widehat{L})$ can be either monotonically increasing, monotonically decreasing, or convex with an extreme point.

According to Lemma 1, $r_{1,i}^{\Pi_{l,m},[L]}$ increases/decreases with L if $i = l/i \neq l$. Thus, by varying decoding order (note that we have four decoding orders), a total number of four cases exist with respect to the monotonicity of $r_{1,i}^{\Pi_{l,m},[L]}$ and $r_{2,i}^{\Pi_{l,m},[L]}$; i.e., the two variables both increase and both decrease, the former increases and the latter decreases, or the former decreases and the latter increases, with respect to L. We illustrate this in Figure 4. The first two cases in Figure 4 may have two subvariants, according to whether $r_{1,i}^{\Pi_{l,m},[L]}$ and $r_{2,i}^{\Pi_{l,m},[L]}$ intersect. For cases 1-3, $r_i^{\Pi_{l,m}}(\widehat{L})$ is monotone. However, for case 4, there exists an extreme point, denoted as L^*.

As an example, we assume $\Pi_{1,2}$ is applied and the possible shapes of $r_{\mathrm{sum}}^{\Pi_{1,2}}(\widehat{L})$ are illustrated in Figure 5. From Figure 5, we observe that $r_{\mathrm{sum}}^{\Pi_{l,m}}(\widehat{L})$ increases with L when $L \leq L_1^*$. However, when $L > L_1^*$, increasing the number of splitting layers does not always lead to capacity gain (as shown in the right part of Figure 5). According to the above qualitative analysis, we conclude that infinite-layer RS is not always better than finite-layer RS.

4.2. Analysis of EPRSW with Finite-Layer RS. Recall that, in last section, we concluded that the achievable sum capacity of

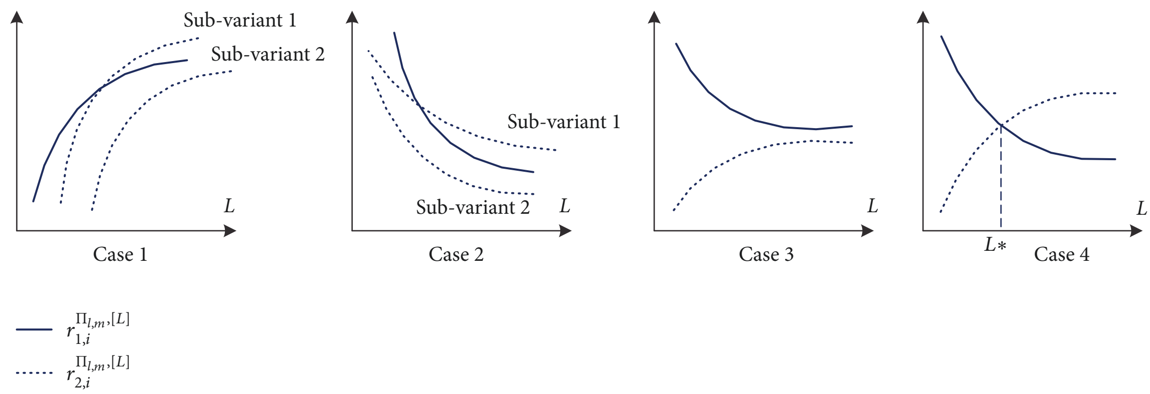


FIGURE 4: Illustrations of the increasing/decreasing property of $r_{1,i}^{\Pi_{l,m},[L]}$ and $r_{2,i}^{\Pi_{l,m},[L]}$ with various decoding orders. For example, if $i = 1$, the decoding orders of the four cases are $\Pi_{1,1}$, $\Pi_{2,2}$, $\Pi_{1,2}$, and $\Pi_{2,1}$, respectively.

EPRSW is usually smaller than that of SD. However, we show in the following that EPRSW with finite-layer RS can actually outperform SD in certain channel conditions.

Consider EPRSW with L-layer RS, as shown in Figure 2. Since the decoding order is set as $\Pi_{1,2}$, the last split of Tx-2, i.e., $x_{2,L}$, does not need to be decoded by Rx-1. Similarly, the last split of Tx-1, i.e., $x_{1,L}$, does not need to be decoded by Rx-2. Therefore, there is no need to conduct RM on the transmission rates of these two splits as defined in (10); i.e., the transmission rates are directly given by $\tilde{r}_i^{\Pi_{l,m},(k)} = r_{i,i}^{\Pi_{l,m},(k)}$, $i = 1, 2$, which is strictly larger than $\tilde{r}_i^{\Pi_{l,m},(k)}$. We note that when $L \longrightarrow +\infty$, the gap between $\tilde{r}_i^{\Pi_{l,m},(k)}$ and $\tilde{r}_i^{\Pi_{l,m},(k)}$ approaches zero, due to the infinitely-small SNR. However, this gap cannot be ignored with finite value of L.

Observing this fact, the achievable sum capacity of EPRSW with finite-layer (in short f-EPRSW), i.e., $r_{\text{f-EPRSW}}^{\Pi_{l,m},[L]}$, is given i

$$r_{\text{f-EPRSW}}^{\Pi_{l,m},[L]} = \begin{cases} \sum_{k=1}^{L-1} \left(\min\left\{ r_{1,2}^{\Pi_{l,m},(l)}, r_{2,2}^{\Pi_{l,m},(l)} \right\} + \min\left\{ r_{1,1}^{\Pi_{l,m},(l)}, r_{2,1}^{\Pi_{l,m},(l)} \right\} \right) + r_{2,2}^{\Pi_{l,m},(L)} + \min\left\{ r_{1,1}^{\Pi_{l,m},(L)}, r_{2,1}^{\Pi_{l,m},(L)} \right\}, & l = 1,\ m = 1 \\ \sum_{k=1}^{L-1} \left(\min\left\{ r_{1,2}^{\Pi_{l,m},(l)}, r_{2,2}^{\Pi_{l,m},(l)} \right\} + \min\left\{ r_{1,1}^{\Pi_{l,m},(l)}, r_{2,1}^{\Pi_{l,m},(l)} \right\} \right) + r_{2,2}^{\Pi_{l,m},(L)} + r_{1,1}^{\Pi_{l,m},(L)}, & l = 1,\ m = 2 \\ \sum_{k=1}^{L} \left(\min\left\{ r_{1,2}^{\Pi_{l,m},(l)}, r_{2,2}^{\Pi_{l,m},(l)} \right\} + \min\left\{ r_{1,1}^{\Pi_{l,m},(l)}, r_{2,1}^{\Pi_{l,m},(l)} \right\} \right), & l = 2,\ m = 1 \\ \sum_{k=1}^{L-1} \left(\min\left\{ r_{1,2}^{\Pi_{l,m},(l)}, r_{2,2}^{\Pi_{l,m},(l)} \right\} + \min\left\{ r_{1,1}^{\Pi_{l,m},(l)}, r_{2,1}^{\Pi_{l,m},(l)} \right\} \right) + r_{1,1}^{\Pi_{l,m},(L)} + \min\left\{ r_{1,2}^{\Pi_{l,m},(L)}, r_{2,2}^{\Pi_{l,m},(L)} \right\}, & l = 2,\ m = 2. \end{cases} \tag{39}$$

The expression of $r_{\text{f-EPRSW}}^{\Pi_{l,m},[L]}$, for arbitrary $l, m \in \{1, 2\}$, can be further synthesized as

$$\begin{aligned} r_{\text{f-EPRSW}}^{\Pi_{l,m},[L]} = \sum_{k=1}^{L-1} \Bigg(& \min\left\{ r_{1,2}^{\Pi_{l,m},(l)}, r_{2,2}^{\Pi_{l,m},(l)} \right\} \\ & + \min\left\{ r_{1,1}^{\Pi_{l,m},(l)}, r_{2,1}^{\Pi_{l,m},(l)} \right\} \Bigg) + \left(r_{2,2}^{\Pi_{l,m},(l)} \right)^{2-l} \\ & \cdot \left(\min\left\{ r_{1,2}^{\Pi_{l,m},(l)}, r_{2,2}^{\Pi_{l,m},(l)} \right\} \right)^{l-1} + \left(r_{1,1}^{\Pi_{l,m},(l)} \right)^{m-1} \\ & \cdot \left(\min\left\{ r_{1,1}^{\Pi_{l,m},(l)}, r_{2,1}^{\Pi_{l,m},(l)} \right\} \right)^{2-m}. \end{aligned} \tag{40}$$

In the following, the sufficient channel conditions where f-EPRSW outperforms SD are given in Theorem 9.

Theorem 9 (f-EPRSW). *The proposed f-EPRSW outperforms EPRSO when the conditions (1)-(3) and one of the conditions (4)-(5) are satisfied:*

(1) $\Pi_{l,m} \neq \Pi_{2,1}$;

(2) *The channel coefficients can satisfy the conditions in Theorem 4;*

(3) *The channel coefficients can satisfy the conditions in Theorem 7;*

(4)

$$\frac{1}{\left(P + L/h_{1,2}\right)\left(P + L/h_{1,1}\right)} > \frac{h_{1,2}}{h_{1,1} + h_{1,2}} \log\left(1 + \left(h_{1,1} + h_{1,2}\right) P\right)$$

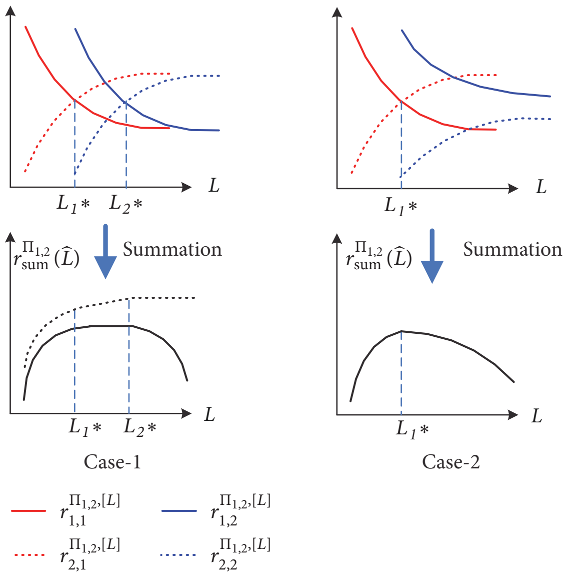


FIGURE 5: Illustrations of $r_{\text{sum}}^{\Pi_{l,m}}(\hat{L})$. Case-1 and Case-2 show two examples where increasing L does not increase the achievable sum capacity.

$$-\sum_{k=1}^{L}\log\left(1+\frac{h_{1,1}P}{(h_{1,1}+h_{1,2})(L-k)P+L}\right) \tag{41}$$

(5)

$$\begin{aligned}&\frac{1}{(P+L/h_{2,2})(P+L/h_{2,1})}\\ &\quad>\frac{h_{2,2}}{h_{2,2}+h_{2,1}}\log\left(1+(h_{2,2}+h_{2,1})P\right)\\ &\quad-\sum_{k=1}^{L}\log\left(1+\frac{h_{2,2}P}{(h_{2,2}+h_{2,1})(L-k)P+L}\right)\end{aligned} \tag{42}$$

Proof. Without loss of generality, we assume conditions (1)-(4) are satisfied. According to (1) and (2), $\forall\epsilon$, $\exists L^*$, when $L > L^*$, $|r_{\text{EPRSW}}^{[L]} - r_{\text{SD}}| < \epsilon$. Furthermore, according to (4), we readily see that

$$\begin{aligned}&r_{\text{EPRSW}}^{[L^*]}+\left(r_{2,2}^{\Pi_{l,m},(l)}\right)^{2-l}\left(\min\left\{r_{1,2}^{\Pi_{l,m},(l)},r_{2,2}^{\Pi_{l,m},(l)}\right\}\right)^{l-1}\\ &\quad-\min\left\{r_{1,2}^{\Pi_{l,m},(k)},r_{2,2}^{\Pi_{l,m},(k)}\right\}>r_{\text{SD}},\end{aligned} \tag{43}$$

which indicates that f-EPRSW achieves better capacities than SD. □

The reason why f-EPRSW outperforms SD is because RS and SSD can *transform* the underlying physical channel conditions by recovering and cancelling some signal splits. Furthermore, some splits of the transmitters may not be decoded by the unexpected receivers f-EPRSW, which relaxes the RM requirements and thus enables higher data rates on these splits. As an instance, assume that multi-layer RS is applied, with several iterations of SSD, and the achievable sum capacity is already quite close to the SD bound. Then if we remove the RM requirement on the last split, as is done in f-EPRSW, the achievable rate can surpass the SD bound. Nevertheless, it is still pretty hard to provide the exact expression of the capacity gain, which may be an interesting future research aspect.

5. Numerical Results

In this section, we present some numerical results to verify the above analysis. We assume a two-transmitter two-receiver IC model, where the transmission power of each transmitter is $P = 10$ and the noise variance is $N_0 = 1$. First of all, we show the relationship between the achievable rate of EPRSW and the number of layers in RS, with some typical channel gain settings. Then we compare the achievable sum capacities of EPRSW and SD with general channel gain settings. Finally, we show some special cases where f-EPRSW outperforms SD.

Figure 6 shows the relationship between the achievable rate and the number of layers in RS in different decoding orders and compares the achievable sum capacities of EPRSO, EPRSW, and SD. The channel coefficients are set as $h_{1,1} = h_{2,2} = 1, h_{1,2} = h_{2,1} = 0.9$, which constitute a typical symmetric IC with strong but not very strong interference. Taking Figure 6(a) as an example, we can observe that $r_{2,1}^{\Pi_{1,2},[L]}$ and $r_{1,2}^{\Pi_{1,2},[L]}$ increase with L and are upper bounded by $h_{2,1}(P/L)/N_0$ and $h_{1,2}(P/L)/N_0$, respectively. Meanwhile, $r_{1,1}^{\Pi_{1,2},[L]}$ and $r_{2,2}^{\Pi_{1,2},[L]}$ decrease with L and are lower bounded by $h_{1,1}(P/L)/N_0$ and $h_{2,2}(P/L)/N_0$, respectively. From Figure 6, we also see that $r_{j,i}^{\Pi_{l,m},[L]}$ is independent of $\Pi_{l,m}$ when $L \longrightarrow +\infty$. These results exactly follow the analysis in Section 3.1. Besides, with appropriately decided decoding orders, i.e., $\Pi_{1,2}$ and $\Pi_{2,1}$, the achievable sum capacities increase with L in both EPRSO and EPRSW schemes. However, with $\Pi_{1,1}$ or $\Pi_{2,2}$, RS does not increase the achievable sum capacity. At some L, EPRSO can outperform EPRSW due to the absence of RM. However, there is always a gap between EPRSW and SD, which coincides the conclusion derived in Section 3.3.

In Figure 7, we present the achievable sum capacities of EPRSW versus different L (as shown by the red cycles) as well as the SD capacity region (as shown by the black dotted area). The black arrow indicates the direction of the growth of L. Figure 7(a) shows that larger L leads to better capacity. Meanwhile, Figure 7(b) also shows that better fairness can be achieved with larger L, if $\Pi_{1,1}$ or $\Pi_{2,2}$ is assumed as the decoding orders. The orthogonal multiple access (OMA) achievable rate region is highlighted in Figure 7, which shows that OMA is strictly suboptimal compared with SD and EPRSW.

Figure 8 shows the performance gap between EPRSW and SD with extensive channel gain settings, where $h_{1,1} = 1$ and $h_{1,2}, h_{2,1}$, and $h_{2,2}$ take values within [0.5, 1.5]. Each black cycle represents a set of channel coefficients where EPRSW can achieve the SD bound with a maximum of X% loss, where X=0.5, 1, 5, and 10, respectively. We can observe that when 5% loss is allowed, EPRSW performs almost as good as SD in

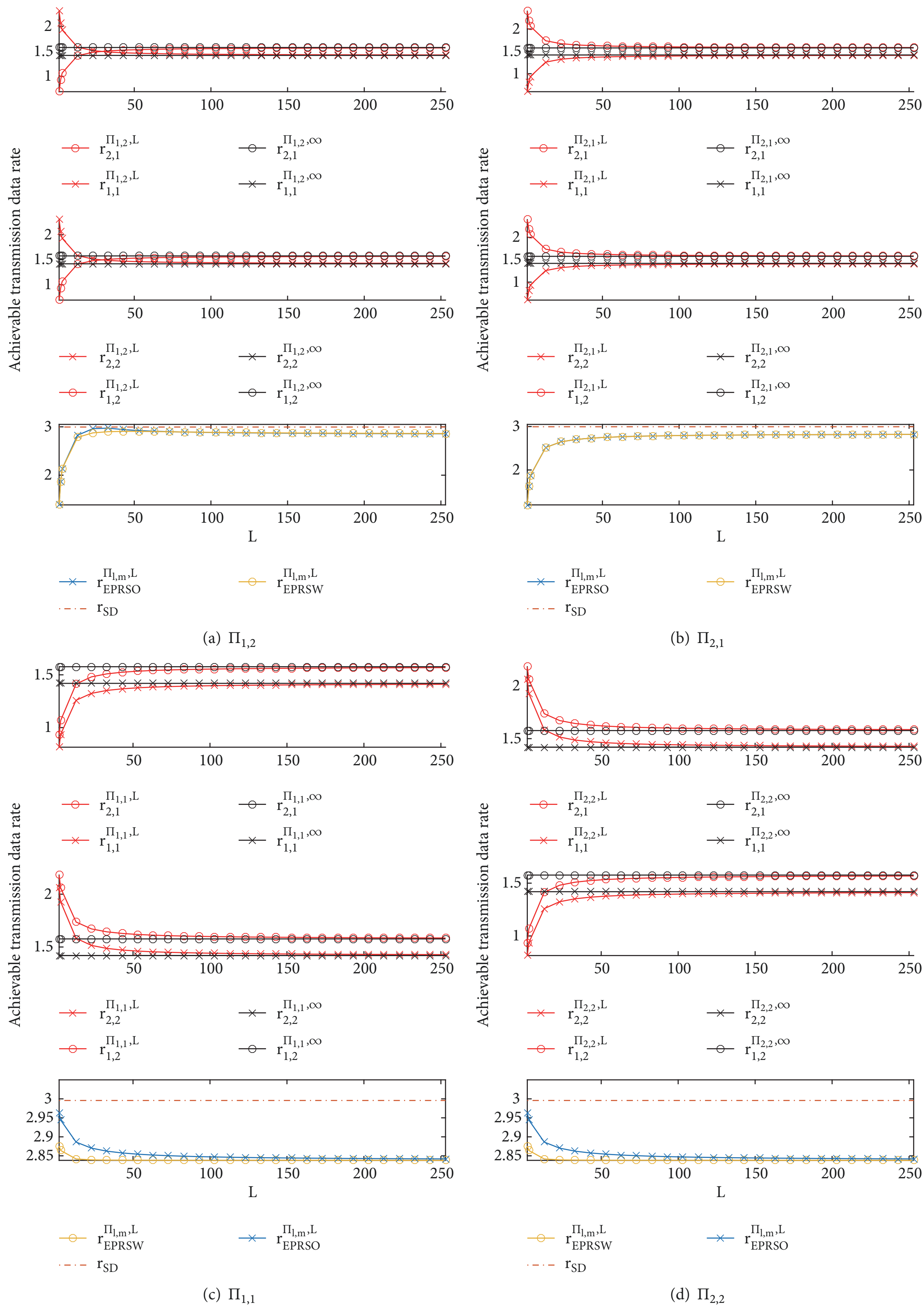
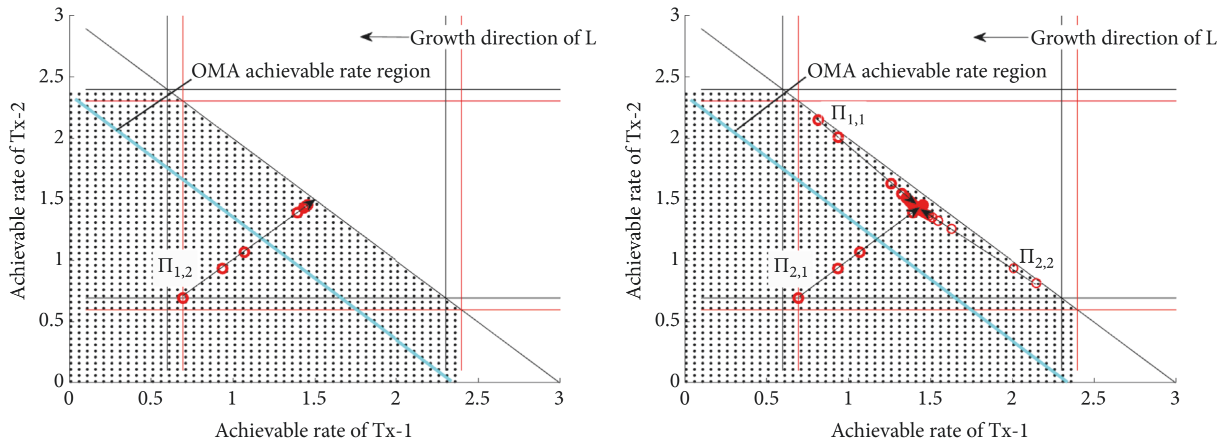


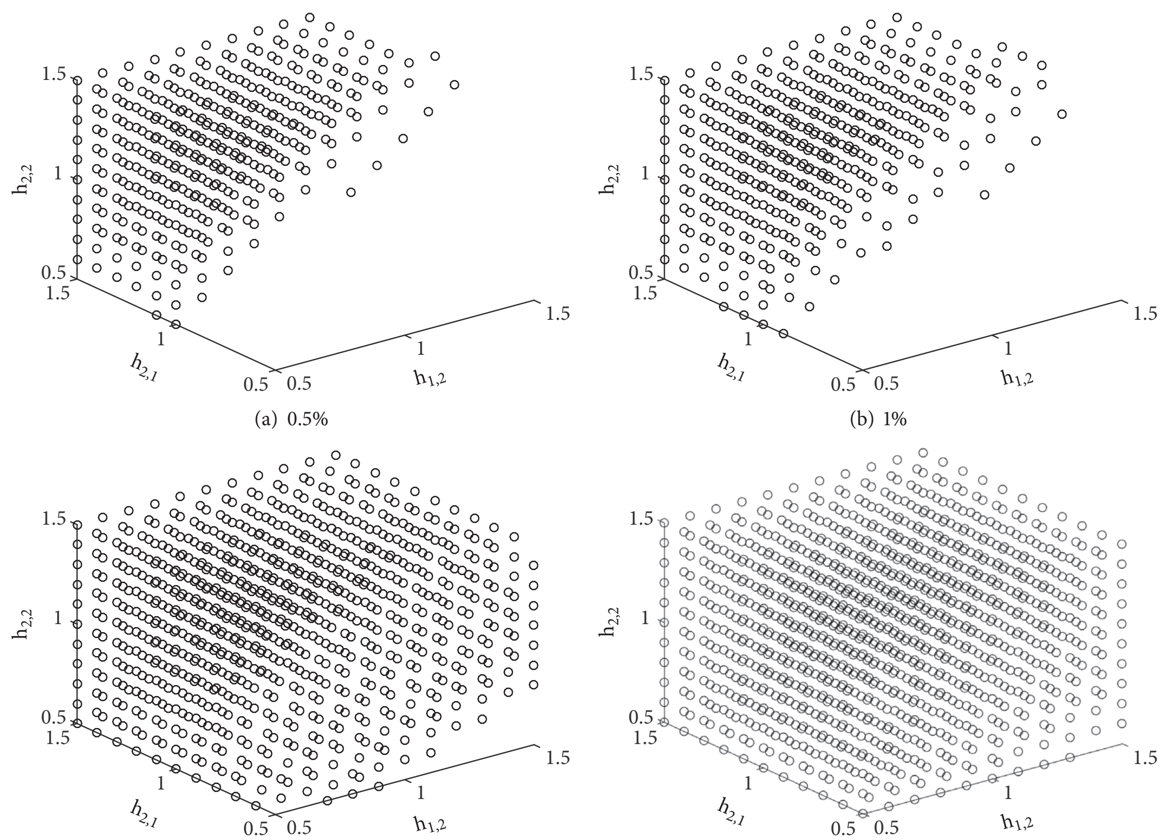
Figure 6: Achievable rate vs. the number of layers in RS, with varying decoding order. The channel coefficients are set with $h_{1,1} = h_{2,2} = 1, h_{1,2} = h_{2,1} = 0.9$.



(a) Achievable rate vs. the number of layers in RS with $\Pi_{1,2}$

(b) Achievable rate vs. the number of layers in RS with $\Pi_{1,1}$, $\Pi_{2,1}$, and $\Pi_{2,2}$

FIGURE 7: Achievable rate regions of SD and EPRSW, with different decoding orders. The black dotted area indicates the SD bound, and the black arrows indicates the growth direction of L.



(a) 0.5%

(b) 1%

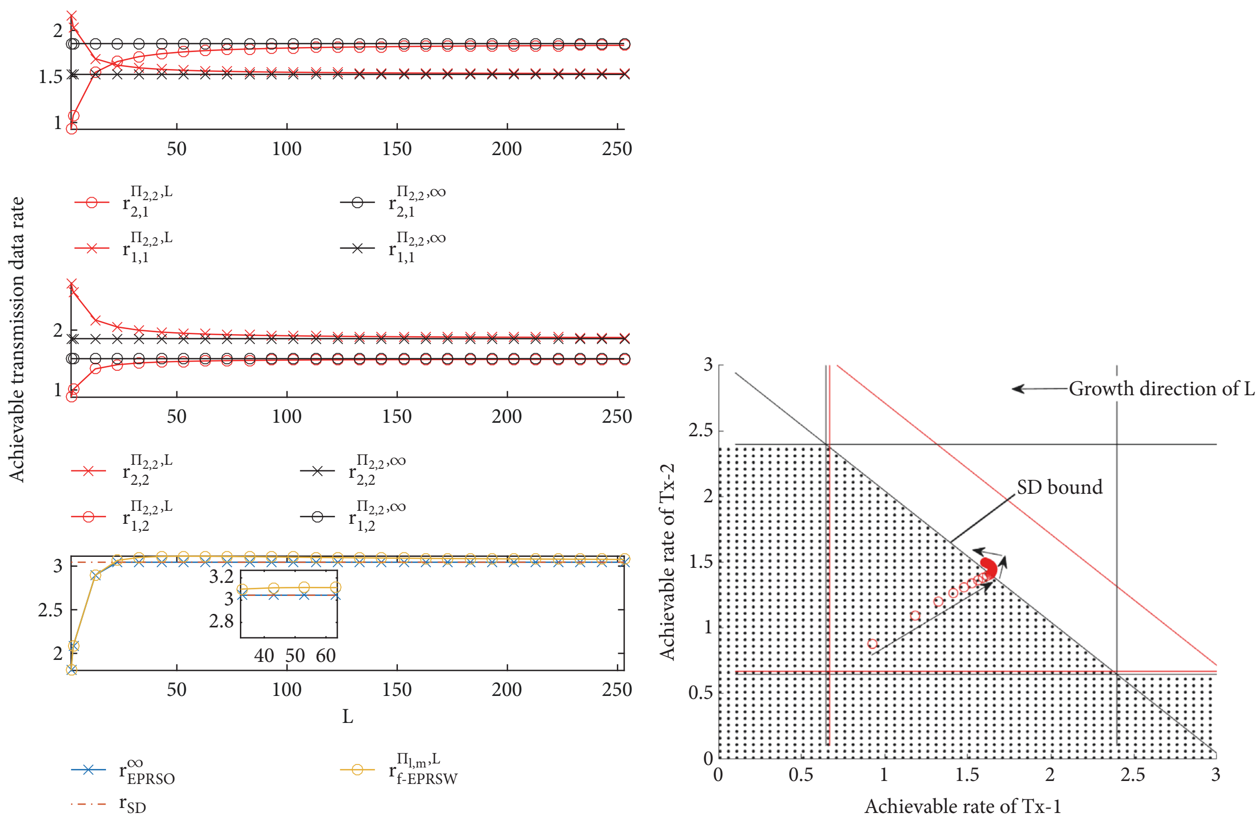
(c) 5%

(d) 10%

FIGURE 8: A plot of the channel gain settings where EPRSW can achieve the SD bound with a maximum of X% loss (X=0.5, 1, 5, and 10). The channel coefficients are set as follows: $h_{1,1} = 1$ and $h_{1,2}, h_{2,1}, h_{2,2}$ take values within [0.5, 1.5].

most channel gain settings. This reflects that the performance gap between EPRSW and SD is rather small.

In Figure 9, we present a case where f-EPRSW can outperform SD. The channel coefficients are set as $h_{1,1} = h_{1,2} = 2$ and $h_{2,1} = h_{2,2} = 1$, which indicates the strong interference situation. The achieved capacity of f-EPRSW with SSD surpasses the boundary of SD region, with the increase of L. When channel coefficients vary slightly from



(a) Achievable rate vs. the number of layers in RS with $\Pi_{1,2}$

(b) Comparison between the achievable rate region of SD and that of f-EPRSW, varying L

Figure 9: A case where f-EPRSW outperforms SD, with $h_{1,1} = 2, h_{1,2} = 2, h_{2,1} = 1$, and $h_{2,2} = 1$.

this setting, it is also observed that f-EPRSW can outperform SD. Hence, the case in Figure 9 is not an isolated evidence.

6. Conclusion and Future Work

In this paper, we have studied a fundamental problem in the Gaussian IC: whether multi-layer RS and SSD can achieve the SD capacity bound. The analysis in this paper shows that the achievable sum capacity of the EPRSW scheme with equal-power infinite-layer RS and SSD cannot reach, but can be pretty close to the SD achievable bound in IC. The exact capacity loss of EPRSW compared with SD was derived in symmetric IC. Nevertheless, the proposed f-EPRSW scheme, which employs equal-power finite-layer RS, SSD, and suitable transmission rate assignment, can even outperforms SD in certain channel gain settings. Therefore, we can conclude that applying RS and SSD is not always weaker than SD, at least when multiple layers and suitable assignment method are employed. At last, we note that extending the proposed scheme and the analysis into the multiuser case would be an interesting future research direction.

Data Availability

No data were used to support this study.

Conflicts of Interest

The authors declare that there are no conflicts of interest regarding the publication of this paper.

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