Research Article

Generalized Complex Quadrature Spatial Modulation

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Spatial modulation (SM) is a multiple-input multiple-output (MIMO) system that achieves a MIMO high spectral efficiency while maintaining the transmitter computational complexity and requirements as low as those of the single-input systems. The complex quadrature spatial modulation (CQSM) builds on the QSM scheme and improves the spectral efficiency by transmitting two signal symbols at each channel use. In this paper, we propose two generalizations of CQSM, namely, generalized CQSM with unique combinations (GCQSM-UC) and with permuted combinations (GCQSM-PC). These two generalizations perform close to CQSM or outperform it, depending on the system parameters. Also, the proposed schemes require much less transmit antennas to achieve the same spectral efficiency of CQSM, for instance, assuming 16-QAM, GCQSM-PC, and GCQSM-UC require 10 and 15 transmit antennas, respectively, to achieve the same spectral of CQSM which is equipped with 32 antennas.

1. Introduction

Multiple-input multiple-output (MIMO) techniques are capable of satisfying the continuous demand in high data rates in communication systems. Index modulation (IM) is a group of MIMO techniques that use the indices of a given resource(s) of the communication system to convey additional information, besides that carried by the signal symbols [1, 2]. These indices represent distinct antennas [3], spreading codes [4], polarities [5], subcarriers [6], and combinations of angles [7], among others. A virtual spatial modulation (VSM) was also proposed in [8], where index modulation is performed on the parallel channels resulting from the singular value decomposition of the MIMO channel matrix.

Spatial modulation (SM) has attracted an increasing interest from researchers due to its low-complexity and high data rates. In SM, information is transmitted through the signal symbols, drawn from any conventional constellation sets, and the index of the antenna from which the signal symbol is transmitted [3]. To achieve a high spectral efficiency, SM requires relatively high number of transmit antennas. A parallel SM, denoted by half-full transmit-diversity SM (HFTD-SM), is proposed recently, where antennas are grouped into two-antenna groups. The conventional SM is then performed using the same signal symbol in each group [9, 10].

A conventional generalized SM (GSM) transmits the same signal symbol from a combination of antennas, rather than one as in SM, leading to a reduction in the number of transmit antennas required to achieve a given spectral efficiency [11]. A different implementation of the GSM was proposed in [12], referred to as MA-SM, in which each activated antenna in a given combination transmits a different signal symbol. This improves the spectral efficiency at the cost of additional hardware requirements and computational complexity. GSM was extended to the multiuser scenario in [13].

Quadrature spatial modulation (QSM) is another technique that extends the spatial constellation into in-phase and quadrature dimensions that are used to transmit the real and imaginary part, respectively, of a single signal symbol [14]. In [15], a generalized QSM (GQSM) that combines the benefits of spatial multiplexing and QSM was proposed. Antennas are grouped and independent QSM modulation is performed in each group. The hardware design of the generalized quadrature space shift keying modulation (GQSSK) and generalized QSM (GQSM) using a single RF chain was investigated in [16].
Building on the QSM, a complex QSM (CQSM) improves the spectral efficiency by transmitting two signal symbols at each channel use from antennas whose indices also carry information [17]. When both signal symbols are transmitted from the same antenna, the resulting combination is drawn from the Minkowski sum of the two sets from which the symbols are drawn. The second modulation set is designed as a rotated version of the first, where the rotation angle is optimized to minimize the average bit-error rate. To avoid transmitting the two signal symbols from the same antenna, the transmitter is equipped with an additional antenna that is used to transmit the second symbol only when both symbols are supposed to be transmitted from the same antenna in CQSM. The proposed modification is named improved CQSM (ICQSM) in [18]. The modulation set design of the ICQSM was addressed in [19]. Notice that the CQSM can be conceptualized based on the conventional SM or the GSM, and not as an extension of the QSM. While this approach is correct, we built CQSM as an extension of the QSM where instead of transmitting the real and imaginary parts of a single signal symbol from the in-phase and quadrature spatial dimensions, two complex-valued signal symbols are transmitted through the two spatial dimensions. To be consistent with our previous work, we keep the same name that explicitly includes QSM.

**Contributions.** The contributions of this paper are summarized as follows:

1. We propose two generalized CQSM schemes. The first is GCQSM with unique combinations (GCQSM-UC), where the spatial symbols are generated as in generalized GSM. Each resulting combination is then split into two subsets. The first subset of antennas is used to transmit the first symbol; the second subset is used to transmit the second signal symbol. The second scheme is named GCQSM with permuted combinations (GCQSM-PC). In GCQSM-PC the antenna combinations from which the first symbol is transmitted are generated as the full set of combinations of a given length. For each combination associated with the first symbol, the corresponding list of combinations for the second symbol is generated such that no overlap occurs between antennas used for transmitting the two symbols. This algorithm reduces the number of transmit antennas required to achieve a given spectral efficiency.

2. We also propose an analytical method to optimize the rotation angle for CQSM and ICQSM systems. The obtained rotation angle minimizes the upper-bound of pairwise error probability.

Based on the simulation results, the proposed schemes perform very close to CQSM, while requiring much smaller number of antennas to achieve the same spectral efficiency. For instance, CQSM, GCQSM-UC, and GCQSM-PC require 32, 15, and 10 transmit antennas to achieve a spectral efficiency of 14 bits per channel use (bpcu), assuming 2 bits per signal symbol.

**Notations.** We denote the spectral efficiency achieved by spatial and signal symbols ES\(_{\text{spa}}\) and ES\(_{\text{sig}}\), respectively, and the total spectral efficiency ES = ES\(_{\text{sig}}\) + ES\(_{\text{spa}}\). The vector \(e^a\) is the \(a\)th column of the \(b \times b\) identity matrix. \(A = \{a_1, \ldots, a_n\}\) is a set, where the order of elements does not matter, and \(B = (b_1, \ldots, b_K)\) is a tuple where order matters. In the following, \((\cdot)^T\) and \((\cdot)^H\) denote the matrix/vector transpose and Hermitian transpose, respectively. \(Q(\cdot)\) is the Gaussian tail function, or simply the \(Q\)-function. A signal symbol is a complex element drawn from a quadrature amplitude modulation (QAM) or phase shift keying (PSK) set. The number of bits carried by each signal symbol is equal to \(q\), where \(2^q\) is the cardinality of the modulation set. A spatial symbol is the index(es) of the antenna(s) from which a single or several signal symbols are transmitted.

The rest of this paper is organized as follows. In Section 2, we describe the system model and review related works. In Section 3, we introduced the proposed generalized CQSM techniques and analyze their error performance in Section 4. In Section 5, we formulate the search of the optimal rotation angle for CQSM and ICQSM as an optimization problem that reduced the asymptotic upper-bound on the error probability. The optimization of the rotation angle of the proposed generalized schemes is addressed in Section 6. The simulation results are given in Section 7 and conclusions are drawn in Section 8.

### 2. System Model and Related Works

#### 2.1. System Model

Consider a MIMO system with \(n_T\) transmit and \(n_R\) receive antennas. The system equation is given as follows:

\[
y = Hs + n,
\]

where \(H\) and \(n\) are the channel matrix and the noise vector whose elements are i.i.d. centered circularly symmetric complex Gaussian and have a variance of one and \(\sigma^2\), respectively, and \(s\) is the transmitted vector. In SM techniques, the vector \(s\) contains a few nonzero elements.

#### 2.2. Spatial Modulation

In SM, both a signal symbol and the index of the antenna from which it is transmitted carry information. As such, SM achieves an improved spectral efficiency while keeping the transmitter as simple as that of the single-input communication systems. Accordingly, the SM system is modeled as follows:

\[
y = He^a_k s_k + n = h_k s_k + n.
\]

The spectral efficiency, which is equivalent to the capacity at high signal-to-noise ratio (SNR), is equal to \(q + N\) bits per channel use (bpcu), with \(N = \log_2(n_T)\).

#### 2.3. Generalized Spatial Modulation

In SM, the number of transmit antennas increases exponentially in terms of the number of bits carried by each spatial symbol. That is, \(n_T = 2^N\), where \(N\) is the number of bits per spatial symbol. GSM
reduces the number of transmit antennas by sending the same signal symbol from a combination of two or more antennas. Let \( n_T \) denote the number of active antennas at each channel use; then the number of spatial symbols, also referred to as combinations, for a given \( n_T \) is \( \binom{n_T}{m} \). Since the number of spatial symbols should be a power of two, only \( 2^{\text{ES}_{pa}} \) combinations can be used, where \( \text{ES}_{pa} = \lfloor \log_2(\binom{n_T}{m}) \rfloor \).

Assuming \( n_U = 2 \), GSM requires 7 transmit antennas to achieve \( \text{ES}_{pa} = 4 \) bits per spatial symbol, whereas SM requires 16 antennas to achieve the same spectral efficiency.

Let \( I = \{l_1, \ldots, l_{n_T}\} \in \mathbf{L} \) be a spatial symbol, where \( \mathbf{L} \) is the set of spatial symbols that can be used for transmission. Then the received vector in the case of GSM is given by

\[
y = \frac{1}{\sqrt{n_T}} \sum_{i \in I} \mathbf{h}_i + \mathbf{n}.
\]  

(3)

2.4. Multi-Active Spatial Modulation. In contrast to GSM, in which a single signal symbol is transmitted from a combination of antennas, MA-SM transmits a different signal symbol from each activated antenna. Let \( n_T \) be the number of activated antennas; then the received vector is given by

\[
y = \sum_{i \in \Omega} \mathbf{h}_i \mathbf{s}_i + \mathbf{n},
\]  

(4)

where \( I = \{l_1, \ldots, l_{n_T}\} \in \mathbf{L} \), and \( \mathbf{L} \) is the list of spatial symbols of MA-SM, which is equivalent to that of GSM. Additionally, a 3-dimensional constellation set is proposed in which each spatial symbol is associated with a unique rotation angle. Signal symbols transmitted from a given spatial symbol are rotated before transmission. According to this description, MA-SM benefits from the moderate computational complexity and low hardware requirements of SM and the high multiplexing gain of the vertical-Bell Labs layered space-time (V-BLAST) system. The spectral efficiency of MA-SM is equal to \( \text{ES}_{spa} = q \times n_T \), where \( \text{ES}_{spa} \) is defined in Section 2.3.

2.5. Complex Quadrature Spatial Modulation. QSM expands the spatial constellation into in-phase and quadrature dimensions. At each channel use, a single signal symbol is transmitted: the real part of the signal symbol is transmitted through the in-phase spatial dimension; the imaginary through the quadrature dimension. As such, the spectral efficiency of QSM is \( q + 2 \log_2(n_T) \), where \( \text{ES}_{spa} = 2 \log_2(n_T) \) and \( \text{ES}_{spa} = q \).

QCSM transmits two signal symbols at each channel use, leading to a spectral efficiency of \( 2q + 2 \log_2(n_T) \), where \( \text{ES}_{spa} = 2 \log_2(n_T) \) and \( \text{ES}_{spa} = 2q \). The first signal symbol \( \mathbf{s}_{k_1} \in \Omega_{k_1} \) is transmitted from the \( i_1 \)th transmit antenna, and the second signal symbol \( \mathbf{s}_{k_2} \in \Omega_{k_2} \) is transmitted from the \( i_2 \)th antenna. To make the distinction between \( \mathbf{s}_{k_1} \) and \( \mathbf{s}_{k_2} \) at the receiver side, \( \Omega_{k_1} \) is obtained as follows:

\[
\Omega_{k_1} = \{ s_{k_1} | s_{k_1} = s_{k_1} e^{j\theta}, s_{k_1} \in \Omega_{k_1} \}. \quad (5)
\]

The rotation angle \( \theta \) is optimized such that the probability of error is reduced. The received vector of the CQSM is given by

\[
y = \mathbf{Hs} + \mathbf{n} = \mathbf{h}_{i_1} s_{k_1} + \mathbf{h}_{i_2} s_{k_2} + \mathbf{n},
\]  

(6)

which is expanded to

\[
y = \begin{cases} 
\mathbf{h}_i s_{k_i} + \mathbf{h}_{i_2} s_{k_2} + \mathbf{n} & \text{if } i_1 \neq i_2 \\
\mathbf{h}_i (s_{k_1} + s_{k_2}) = \mathbf{h}_i s_{k_i} & \text{if } i_1 = i_2 = i
\end{cases}
\]  

(7)

where \( s_{k_i} \in \Omega_{k_i} \) and \( \Omega_{k_i} = \Omega_u \oplus \Omega_b \), with \( \oplus \) denoting the Minkowski sum [20].

The full modulation set \( \Omega = \Omega_k \cup \Omega_b \cup \Omega_u \) is of size \(|\Omega_k| + |\Omega_b| + |\Omega_u|\). For instance, assuming \(|\Omega_k| = |\Omega_b| = 16\), the number of symbols in \( \Omega_{k} \) becomes 288. This high density of symbols reduces the Euclidean distance among signal symbols, leading to degradation in the error performance.

2.6. Improved Complex Quadrature Spatial Modulation. In ICQSM, the transmitter is equipped with an additional antenna, indexed \( n_K \) with \( n_K = n_T + 1 \), which is used only when \( i_1 = i_2 \). In this case, the first signal symbol is transmitted from its designated antenna and the second symbol is transmitted from the additional antenna. Accordingly, the ICQSM system is modeled as follows:

\[
y = \begin{cases} 
\mathbf{s}_{k_1} \mathbf{h}_{i_1} + \mathbf{s}_{k_2} \mathbf{h}_{i_2} + \mathbf{n} & \text{if } i_1 \neq i_2 \\
\mathbf{s}_{k_1} \mathbf{h}_{i_1} + \mathbf{s}_{k_2} \mathbf{h}_{i_2} + \mathbf{n} & \text{if } i_1 = i_2
\end{cases}
\]  

(8)

This strategy reduces the size of the total modulation set \( \Omega_{k} \) from \(|\Omega_k| + |\Omega_u| + |\Omega_b| \times |\Omega_u| \) in the case of CQSM to only \(|\Omega_k| + |\Omega_b| \) in ICQSM. This increases the Euclidean distance among the signal symbols and hence improves the error performance.

Based on the above descriptions, QCSM activates either one or two antennas, whereas ICQSM always has two active antennas. To perform a fair comparison between these two systems and MA-SM, we assume that \( n_T = 2 \). That is, the hardware requirements of the transmitter and the computational complexity of the detection algorithm are equivalent. Since both systems transmit two signal symbols at each channel use, the comparison is conducted in terms of the spectral efficiency achieved by spatial symbols. SM, on the other hand, transmits a single signal symbol at each channel use. Table 1 lists the number of transmit antennas required to achieve a given number of bits per spatial symbol (SE\(_{spa}\)) by several systems. For instance, CQSM and ICQSM require 3 and 2 antennas less than MA-SM to achieve the same SE\(_{spa}\) of 4 bpcu. This gap increases for higher SE\(_{spa}\).

3. Generalized Complex Quadrature Spatial Modulation

Both CQSM and ICQSM require the same number of RF chains, and ICQSM requires one more physical transmit antenna compared to CQSM. Transmitting each signal symbol from a combination of antennas, instead of a single antenna, reduces the number of transmit antennas required to achieve a given SE\(_{spa}\). In the following subsections, we introduce two generalizations of CQSM, namely, generalized CQSM with unique combinations (GCGSM-UC) and generalized CQSM with permuted combinations (GCQSM-PC).
Let $m$ matters but the order of the elements in each set does not.

In the following, we denote by $i_1$ and $i_2$ the set of antennas used for transmitting the first and second signal symbols, respectively. In the sequel, the tuple $A = (B, C)$ is composed of the two sets $B$ and $C$, where the order of the sets in the tuple matters but the order of the elements in each set does not.

### 3.1. Generalized CQSM with Unique Combinations

Let $n_{sp}$ be the number of antennas from which each of the two signal symbols is transmitted; then for a given $n_T$, the number of combinations that can be used for transmission is $\binom{n_T}{n_{sp}}$. Assuming that $i = \{i_1, i_2\}$ is a spatial symbol, i.e., a combination of antennas of length $2n_{sp}$, then the first and second $n_T$ antennas are used to transmit the first and second signal symbols, respectively. Table 2 depicts an example of the spatial symbols for $n_T = 6$ and $n_{sp} = 2$. According to this description, there will be no overlap between the sets of antennas used to transmit the first and second signal symbols. Since the number of spatial symbols should be a power of two, then only $2^{SE_{sp}}$ signal symbols can be used for transmission, where $SE_{sp} = \left\lfloor \log_2 \left( \binom{n_T}{n_{sp}} \right) \right\rfloor$.

<table>
<thead>
<tr>
<th>$SE_{sp}$</th>
<th>SM</th>
<th>MA-SM</th>
<th>ICQSM</th>
<th>CQSM</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>4</td>
<td>4</td>
<td>3</td>
<td>2</td>
</tr>
<tr>
<td>4</td>
<td>16</td>
<td>7</td>
<td>5</td>
<td>4</td>
</tr>
<tr>
<td>6</td>
<td>64</td>
<td>12</td>
<td>9</td>
<td>8</td>
</tr>
</tbody>
</table>

Table 2: An example of the spatial symbols $(i_1, i_2)$ that can be used for transmission by GCQSM-UC, assuming $n_T = 6$ and $n_{sp} = 2$.

Let $s_{k_1} \in \Omega^g_a$ and $s_{k_2} \in \Omega^g_b$ be the two signal symbols to be transmitted at a given channel use; then the received vector is given by

$$ y = \frac{s_{k_1}}{\sqrt{n_U}} \sum_{i \in i_1} h_i + \frac{s_{k_2}}{\sqrt{n_U}} \sum_{i \in i_2} h_i + n. $$

The receiver employs the maximum-likelihood principle to recover the two signal symbols and the spatial symbol as follows:

$$ (i^*, s_{k_1}^*, s_{k_2}^*) = \arg\min_{s_{k_1} \in \Omega^g_a, s_{k_2} \in \Omega^g_b} \| y - g \|^2 $$

$$ = \arg\min_{s_{k_1} \in \Omega^g_a, s_{k_2} \in \Omega^g_b} \| g \|^2 - 2 \Re (y^H g), $$

where

$$ g = \frac{s_{k_1}}{\sqrt{n_U}} \sum_{i \in i_1} h_i + \frac{s_{k_2}}{\sqrt{n_U}} \sum_{i \in i_2} h_i. $$

GCQSM-UC can be easily extended to transmitting more than two signal symbols per channel use at the cost of requiring more RF chains at the transmitter side. This is possible because the error performance of the system does not depend on the rotation angle $\theta$. This will be addressed in more detail in subsequent sections.

### 3.2. Generalized CQSM with Permutated Combinations

To reduce the number of transmit antennas required to achieve a given spectral efficiency, we use the structure of the transmitted vector to increase the number of spatial symbols. Let $i = (i_1, i_2)$ be a spatial symbol, where the first signal symbol $s_{k_1}$ is transmitted from the $i_1$ combination of antennas and the second signal symbol $s_{k_2}$ from the combination $i_2$. For a given channel realization, the received noiseless vector $g$ is given by

$$ g = \frac{s_{k_1}}{\sqrt{n_U}} \sum_{i \in i_1} h_i + \frac{s_{k_2}}{\sqrt{n_U}} \sum_{i \in i_2} h_i $$

$$ = \frac{s_{k_1}}{\sqrt{n_U}} \sum_{i \in i_1} h_i + \frac{s_{k_2}}{\sqrt{n_U}} \sum_{i \in i_2} e^{j\theta} h_i, $$

where $s_{k_1} = s_{k_1} e^{j\theta}$ and $s_{k_1}, s_{k_2} \in \Omega^{g}_a$. To obtain $i_1$ and $i_2$, we impose the following conditions:

(i) $i_1 \cup i_2 = \emptyset$: this implies that the set of antennas from which the first signal symbol and that from which the second symbol are transmitted do not overlap. This condition reduces the detection ambiguity at the receiver.

(ii) $(i_1, i_2) \neq (i_2, i_1)$: as described earlier, the first and second symbols are drawn from different constellation sets. Therefore, for given signal symbols’ indices $k_1$ and $k_2$, the following two received lattice points—noiseless received vectors—are distinguishable:

$$ g_1 = \frac{s_{k_1}}{\sqrt{n_U}} \sum_{i \in i_1} h_i + \frac{s_{k_2}}{\sqrt{n_U}} \sum_{i \in i_2} e^{j\theta} h_i, $$

$$ g_2 = \frac{s_{k_1}}{\sqrt{n_U}} \sum_{i \in i_1} h_i + \frac{s_{k_2}}{\sqrt{n_U}} \sum_{i \in i_2} e^{j\theta} h_i. $$

Input: \( A = \{1, \ldots, n_T\}, n_U, n_r \) \\
Output: The set of spatial symbols \( \mathcal{F} \) \\
\( \mathcal{G} = \text{COMB}(A, n_U) \) \\
for \( i_1 \in \mathcal{G} \) do \\
\( B = A_{i_1} \) \\
\( \mathcal{D} = \text{COMB}(B, n_r) \) \\
for \( i_2 \in \mathcal{D} \) do \\
APPND(\( (i_1, i_2) \)) \\
end \\
end

Algorithm 1: Generation of the spatial symbols, assuming \( n_r \) transmit antennas and that each signal symbol is transmitted from a combination of \( n_U \) antennas.

Table 3: An example of the spatial symbols \((i_1, i_2)\) that can be used for transmission by GCQSM-PC, assuming \( n_r = 5 \) and \( n_U = 2 \).

<table>
<thead>
<tr>
<th>( i_1 )</th>
<th>( i_2 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>([1, 2])</td>
<td>([3, 4], [3, 5], [4, 5])</td>
</tr>
<tr>
<td>([1, 3])</td>
<td>([2, 4], [2, 5], [4, 5])</td>
</tr>
<tr>
<td>([1, 4])</td>
<td>([2, 3], [2, 5], [3, 5])</td>
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<tr>
<td>([1, 5])</td>
<td>([2, 3], [2, 4], [3, 4])</td>
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<tr>
<td>([2, 3])</td>
<td>([1, 4], [1, 5], [4, 5])</td>
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<td>([2, 4])</td>
<td>([1, 3], [1, 5], [3, 5])</td>
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<td>([2, 5])</td>
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<td>([3, 4])</td>
<td>([1, 2], [1, 5], [2, 5])</td>
</tr>
<tr>
<td>([3, 5])</td>
<td>([1, 2], [1, 4], [2, 4])</td>
</tr>
<tr>
<td>([4, 5])</td>
<td>([1, 2], [1, 3], [2, 3])</td>
</tr>
</tbody>
</table>

This is rendered possible using the rotation angle \( \theta \).

The optimization of the rotation angle is addressed in a following section.

Algorithm 1 depicts a pseudocode of generating the set of spatial symbols \( \mathcal{F} \) that can be used for transmission. Therein \( \text{COMB}(A, n_U) \) generates all combinations of length \( n_U \) of the elements of the set \( A \). Also, \( \text{APPND}(\mathcal{F}, (i_1, i_2)) \) appends the tuple \((i_1, i_2)\) to the set \( \mathcal{F} \). \( B = A_{i_1} \) is the difference of the two sets \( A \) and \( i_1 \). Accordingly, the number of spatial symbols that can be used for transmission is given by

\[
C_{\text{spa}} = \binom{n_T}{n_U} \times \binom{n_T - n_U}{n_U}.
\] (15)

Table 3 depicts an example of the obtained spatial symbols, assuming \( n_r = 5 \) and \( n_U = 2 \). In this case, GCQSM-PC obtains 30 spatial symbols, whereas GCQSM-UC obtains only five.

Table 4 lists the number of transmit antennas required by each system to achieve a given spectral efficiency per spatial symbol. For instance, SM, GSM (and MA-SM), CQSM, GCQSM-UC, and GCQSM-PC require 1024, 46, 32, 15, and 10 transmit antennas, respectively, to achieve 10 bits per spatial symbol.

4. Performance Analysis of the Generalized CQSM

Let

\[
g_i = \frac{s_{k_1}}{\sqrt{n_U}} \sum_{i \in \tilde{i}_1} h_i + \frac{s_{k_2}}{\sqrt{n_U}} \sum_{i \in \tilde{i}_2} e^{j \theta_i} h_i
\] (16)

and

\[
g_k = \frac{s_{k_1}}{\sqrt{n_U}} \sum_{i \in \tilde{i}_1} h_i + \frac{s_{k_2}}{\sqrt{n_U}} \sum_{i \in \tilde{i}_2} e^{j \theta_i} h_i
\] (17)

be two noiseless received codewords corresponding to the transmitted symbol \((s_{k_1}, s_{k_2}, t_1, t_2)\) and \((s_{k_1}, s_{k_2}, \tilde{t}_1, \tilde{t}_2)\), where \( s_k \) is a signal symbol and \( t = \{t_1, \ldots, t_m\} \) is a spatial symbol. The corresponding received vectors are denoted by \( y_i \) and \( y_k \), respectively.

The conditional pairwise error probability (PEP) of the maximum-likelihood (ML) receiver is given by [21, 22]

\[
\Pr [g_i \rightarrow g_k \mid H] = Q \left( \frac{\|y_i - y_k\|^2}{2\sigma_n^2} \right),
\] (18)

where \( Q(\cdot) \) is the Gaussian tail probability function, or simply the Q-function, and \( d_{ik}(\cdot) = \|y_i - y_k\|^2 \) is the squared Euclidean distance between \( s_i \) and \( s_k \) at the receiver. Similarly, \( d_{ik}(tx) = \|s_i - s_k\|^2 \) is the squared Euclidean distance between \( s_i \) and \( s_k \) at the transmitter. Note that,

\[
\E_h \{d_{jj}^2\} = \E \left\{ (s_i - s_k)^H H^H H (s_i - s_k) \right\}
\] (19)

The unconditional PEP (UPEP), assuming \( n_r \) receives antennas, which is obtained by taking the expectation of (18) over the channel \( H \) is given by

\[
\Pr [g_i \rightarrow g_k] = \frac{n_r}{\mu_{i,j}} \sum_{l=0}^{n_r-1} (n_r - 1 + l) \left[ 1 - \mu_{i,j} \right]^l.
\] (20)

where

\[
\mu_{i,j} = \frac{1}{2} \left( 1 - \sqrt{\frac{\gamma \cdot d_{ik}^2(tx)}{4 + \gamma \cdot d_{ik}^2(tx)}} \right),
\] (21)

and \( \gamma = \sigma_s^2/\sigma_n^2 \) is the signal-to-noise ratio (SNR) with \( \sigma_s^2 = E(s^*s) \), the expected power of the signal symbol.

The union bound on the pairwise error probability is then given by averaging all the pairwise probabilities as follows;

\[
\Pr [e] \leq \frac{1}{2^M} \sum_{i=1}^{2^M} \sum_{k=1}^{2^M} \Pr [g_i \rightarrow g_k].
\] (22)
Accordingly, the average bit-error rate (BER) is upper-bounded by

\[
P_e \leq \frac{1}{M2^M} \sum_{i=1}^{2^M} \sum_{k=1}^{2^M} D_{s_i s_k} \Pr [\mathbf{g} \rightarrow \mathbf{g}_k] \tag{23}
\]

where \(D_{s_i s_k}\), the hamming distance between \(s_i\) and \(s_k\), is the number of errors associated with the event \([\mathbf{g} \rightarrow \mathbf{g}_k]\) and \(M\) is the spectral efficiency of the system. Note that in each transmitted vector symbol \(s\), there are exactly \((2 \times n_s)\) nonzero elements.

5. Optimization of the Rotation Angles of CQSM/ICQSM Revisited

In [17, 18], the optimal rotation angles applied to the second modulation set of the CQSM and ICQSM, respectively, are obtained through extensive Monte Carlo simulations. In this case, the obtained rotation angle corresponds to the minimum bit-error rate (BER) averaged over a large number of channel and noise realizations. Alternatively, the rotation angle can be obtained through minimizing the upper-bound of the unconditional pairwise error probability.

Let

\[
\mathbf{g} = \mathbf{H} s = s_1 \mathbf{h}_1 + s_2 \mathbf{h}_2
\]

\[
\mathbf{g}_k = \mathbf{H} s_k = s'_1 \mathbf{h}_1 + s'_2 \mathbf{h}_2,
\]

where \(s_y = \mathbf{g} + \mathbf{n}\) and \(s_k = \mathbf{g}_k + \mathbf{n}\) are the corresponding received vectors for a given noise vector \(\mathbf{n}\).

The PEP and UPEP of the CQSM and ICQSM can also be obtained using (18)–(21). The union bound on the pairwise error probability at high SNR values (a.k.a. asymptotic probability of error) is then given by

\[
\Pr [e] \leq \frac{1}{2^M} \sum_{i=1}^{2^M} \sum_{k=1}^{2^M} \Pr [\mathbf{g}_i \rightarrow \mathbf{g}_k] = \left( \frac{2n_R - 1}{n_R} \right)^{2^M} \sum_{i=1}^{2^M} \sum_{k=1}^{2^M} \left( d_{s_i s_k} \right)^{n_R} \tag{25}
\]

where the maximum value of \(B\) equals \(2^6\) as shown in [19]. The term \(\Omega_i\) is a function of the signal symbols \((s_k, s'_k, s_k', s'_k)\) and \(n_R\), and \(f_i\) is the frequency of \(\Omega_i\), where \(\sum_{i=1}^{B} f_i = n_R^2\). It is shown in [19] that \(B = 7\) in the case of ICQSM and it can be shown to be equal to 15 for CQSM. The rotation angle is then obtained by solving the following optimization problem:

\[
\text{argmin}_{0<\theta<\pi/2} \left( \sum_{i=1}^{B} f_i \Omega_i \right),
\]

where \(B = 15\) and 7 for CQSM and ICQSM, respectively.

Figure 1 depicts the cost function of (26) for CQSM and ICQSM and for several system configurations. The optimal rotation angle corresponds to the minimum value of the cost function. Assuming the case of CQSM and for the scenarios depicted in the figure, the obtained optimal rotation angles are very close to those obtained in [17]. Therefore, this small and tolerable degradation in the error performance comes at a high gain in the optimization time of the rotation angle. On the other hand, the obtained rotation angles for ICQSM are identical to those obtained in [18]. Assuming PSK modulation, the optimal rotation angle for the ICQSM is equal to \(\pi/L\), with \(L\) denoting the cardinality of the constellation set.

6. Optimization of the Rotation Angles for the Generalized CQSM

6.1. Rotation Angle of the GCQSM-UC. As explained earlier and based on the design of the spatial symbols of the GCQSM-UC, if \((i_1, i_2)\) is a spatial symbol that can be used for transmission then \((i_2, i_1)\) is not a valid spatial symbol. This implies that the first symbol \(s_{i_1}\) and \(s'_{i_2}\) are distinguishable using the indices of the antennas from which they are sent. As such, both symbols can be drawn from the same modulation set and hence the rotation angle will have no impact on the performance of the system. In the following \(\theta\) is set to zero for GCQSM-UC.

6.2. Rotation Angle of the GCQSM-PC. Opposed to the GCQSM-UC, both \((i_1, i_2)\) and \((i_2, i_1)\) are valid spatial symbols that can be used for transmission in GCQSM-PC. Therefore, the second symbol \(s'_{i_2}\) should be rotated so that it can be distinguished from the first signal symbol \(s_{i_1}\). The formulation of the upper-bound of the GCQSM-PC as in (22) is a hard problem. Alternatively, we obtained the rotation angle using simulations. Figure 2 depicts the BER of GCQSM-PC versus the rotation angle: the upper subfigure assumes QPSK modulation and the lower subfigure assumes 16-QAM. The optimal rotation angle is about \(\pi/4\) for all the simulated scenarios assuming QPSK modulation. Also, the optimal
rotation angle for the simulated scenarios using 16-QAM is approximately 23 degrees. These values of the optimal rotation angle are affected by the sets from which symbols are drawn, and the values of $n_T$ and $n_R$. As a future work, we would like to analytically derive a generic formula to obtain the optimal angle for arbitrary system configurations.

7. Simulation Results

The following results are obtained assuming that the channel state information (CSI) is known only at the receiver. To make a fair comparison, the rotation angles for the simulated scenarios are optimized using simulations for both CQSM and the generalized CQSM. The obtained rotation angles using simulations and the optimization method described in Section 5 for CQSM scheme have very similar values.

Figure 3 depicts comparisons between the MA-SM and the CQSM for several system configurations of $(n_T, q)$ and a fixed $n_R = 4$. We make the following comparisons.

(i) For the same $n_T$ and $q$ and different spectral efficiency (Figure 3(a)): in this case, MA-SM outperforms CQSM by about 2 dB for $n_T = 8$ and about 1.5 dB for $n_T = 16$. The gap is bridged between the two algorithms as $n_T$ becomes large. This decrease in the performance gap is due to the decrease in the probability of transmitting the two symbols from the same transmit antenna in the CQSM. This probability is equal to $1/n_T$. The comparison here is not fair because the CQSM achieves a spectral efficiency 2 bpcu higher than that of the MA-SM.

(ii) For the spectral efficiency and $q$ and different $n_T$ (Figure 3(b)): the performance of the two algorithms is depicted for an equal spectral efficiency assuming $n_T = 8, 16$ and $n_T = 12, 24$ for CQSM and MA-SM, respectively. For these two scenarios, CQSM requires 4 and 8 transmit antennas less than MA-SM. This huge reduction in the number of transmit antennas comes at a negligible degradation in the SNR for high $n_T$. This implies that the CQSM is more attractive for massive MIMO systems.

(iii) For the spectral efficiency and $n_T$ and different $q$ (Figure 3(c)): finally, the performance of the two algorithms is evaluated for the same spectral efficiency and number of transmit antennas. In this case, $q$ is set to 2 and 3 in the CQSM and MA-SM, respectively. At a target BER of $10^{-4}$, CQSM outperforms MA-SM by 2 and 2.4 dB, respectively, for the two depicted system configurations.

According to this analysis and results depicted in Figure 3, we conclude that CQSM can be used to reduce the number of transmit antennas at a marginal cost in the SNR for high $n_T$ or it can achieve an improvement in the SNR if the same number of antennas is deployed by the two systems.
Since at high $n_T$, CQSM requires one antenna less than ICQSM to achieve the same spectral efficiency at the cost of a moderate degradation in the BER performance, CQSM is used in the following comparisons.

Figure 4 depicts the BER performance of CQSM, GCQSM-UC, and GCQSM-PC for several system scenarios and $n_U = 2$. The title of each subfigure is a tuple that includes the number of transmit antennas required by CQSM, GCQSM-UC, and GCQSM-PC, respectively, to achieve the same spectral efficiency. The spectral efficiency $SE_{spa}$ achieved assuming the system’s parameters in Figure 4(a) is 6, in 4(b) is 8, in 4(c) is 10, and in 4(d) is 12 bpcu. For each of the system configurations $SE_{spa}$ is equal to 4 bpcu, leading to a total spectral efficiency $SE = SE_{spa} + SE_{sig}$ of 10, 12, 14, and 16 bpcu, respectively. The range over which the rotation angle is optimized is $[0, \pi/4]$. The rotation angles applied to the second signal symbol for the scenarios depicted in Figure 4 are listed in Table 5. The rotation angles are optimized through Monte Carlo simulations. Based on the results depicted in Figure 4, we make the following remarks in terms of the BER performance, the rotation angles, and the number of employed transmit antennas.

(i) For $n_R = 2$, the three techniques achieve the same BER performance regardless of the number of transmit antennas.

(ii) For a given value of $n_R$, the BER performance is degraded as the number of transmit antennas increases. The degradation is relatively small as $n_T$ increases. This is an expected behavior of SM techniques because the size of hypothesis set over which the ML detector performs the search increases as $n_T$ increases.

(iii) In Figure 4(a), GCQSM-PC slightly outperforms the other two techniques for $n_R = 4$ and 8. As $n_T$ gets larger, the probability that the two signal symbols are transmitted from the same antenna in CQSM reduces. Accordingly, the performance of CQSM and the two generalized algorithms almost coincide for $n_R = 4$ in Figures 4(b), 4(c), and 4(d). For $n_R = 8$, CQSM slightly outperforms GCQSM-UC and GCQSM-PU.

(iv) As shown in Figure 4(c), the generalized techniques perform close to the CQSM for $n_R = 4$. In this case, GCQSM-UC and GCQSM-PC require 17 and 22
transmit antennas less than CQSM while achieving the same spectral efficiency.

Finally, we compare the BER performance of SM, GSM, QSM, CQSM, and the proposed generalized CQSM techniques. We assume that all the schemes achieve the same $ES_{spa}$ and $ES_{sig}$, leading to the same total spectral efficiency of $ES = (ES_{spa} + ES_{sig})$. Also, the three generalized schemes use a combination of two antennas to transmit each signal symbol. Figures 5(a) and 5(b) depict the BER performance for a spectral efficiency of 16 and 18 bpcu, respectively. The number of transmit antennas and bits per signal symbol is represented as a tuple of the form $(n_T, q)$ for each of the systems. The rotation angles applied to CQSM and GCQSM-PC are 20 and 23 degrees, respectively, to obtain the results in the two subfigures. We make the following two remarks:

(i) CQSM, GCQSM-PC, and GCQSM-UC apply 16-QAM modulation, whereas SM, GSM, and QSM use 256-QAM to achieve an $ES_{sig}$ of 8 bpcu. The number of transmit antennas required to achieve the spectral efficiency of 16 and 18 bpcu is given in the second and third row, respectively, in Table 4. GCQSM-PC requires the least number of transmit antennas of 8 and 10 for the results in Figures 5(a) and 5(b), respectively. On the other hand, SM requires 256 and

Table 5: Range of optimal rotation angles used to obtain the BER results depicted in Figure 4, assuming QPSK modulation.

<table>
<thead>
<tr>
<th>$ES_{spa}$</th>
<th>$n_T$</th>
<th>$\theta_{CQSM}$</th>
<th>$\theta_{GCQSM-PC}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>6</td>
<td>2</td>
<td>$\geq 25$</td>
<td>$\geq 30$</td>
</tr>
<tr>
<td></td>
<td>4</td>
<td>$\geq 35$</td>
<td>$\geq 41$</td>
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<td>2</td>
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<td>$\geq 21$</td>
</tr>
<tr>
<td></td>
<td>4</td>
<td>$\geq 34$</td>
<td>$\geq 35$</td>
</tr>
<tr>
<td></td>
<td>8</td>
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<td>8</td>
<td>$\geq 38$</td>
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</table>
1024 antennas, respectively, to achieve the same SE. Compared to CQSM, GCQSM-PC requires 8 and 22 less antennas to achieve the same $SE_{\text{sp}}$ of 8 and 10 bpcu, respectively.

(ii) For both scenarios, GCQSM-UC achieves the best performance followed by GCQSM-PC and CQSM. In Figure 5(b), GCQSM-UC outperforms GCQSM-PC and CQSM by 1 and 1.6 dB, respectively. However, GCQSM-PC requires 5 less transmit antennas than GCQSM-UC. Therefore, this slight degradation in the SNR is tolerable in the case of GCQSM-PC given the high reduction in the number of transmit antennas used to achieve the same spectral efficiency.

Future Work. We would like to investigate the case of generalized CQSM where the number of transmit antennas used for transmitting each of the two signal symbols can be
variable. Accordingly, the total number of spatial symbols will be increased, leading to higher spectral efficiency.

8. Conclusions

In this paper, we introduced two generalizations of the complex quadrature spatial modulation, namely, GCQSM with unique combinations (GCQSM-UC) and with permuted combinations (GCQSM-PC). While the former generalization uses the conventional spatial symbol generation of the GSM scheme, the later scheme relies on the fact that the second signal symbol is distinguishable from the first through the angle rotation. This allows expanding the set of antenna combinations that can be used for transmission, leading to the reduction in the number of transmit antennas required to achieve a given spectral efficiency. The two proposed generalizations perform close to CQSM using QPSK and outperform it using higher order modulation schemes. Also, GCQSM-PC requires 10 antennas to achieve a spectral efficiency of 10 bcpu per spatial symbol. To achieve the same efficiency, GSM requires 46, and CQSM and QSM require 32 transmit antennas.

Data Availability

No data were used to support this study.

Conflicts of Interest

The author declares that there is no conflict of interest regarding the publication of this paper.

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