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Research Article

A Hybrid Predictive Strategy Carried through Simultaneously from Decision Space and Objective Space for Evolutionary Dynamic Multiobjective Optimization

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There are many issues to consider when integrating 5G networks and the Internet of things to build a future smart city, such as how to schedule resources and how to reduce costs. This has a lot to do with dynamic multiobjective optimization. In order to deal with this kind of problem, it is necessary to design a good processing strategy. Evolutionary algorithm can handle this problem well. The prediction in the dynamic environment has been the very challenging work. In the previous literature, the location and distribution of PF or PS are mostly predicted by the center point. The center point generally refers to the center point of the population in the decision space. However, the center point of the decision space cannot meet the needs of various problems. In fact, there are many points with special meanings in objective space, such as ideal point and CTI. In this paper, a hybrid prediction strategy carried through from both decision space and objective space (DOPS) is proposed to handle all kinds of optimization problems. The prediction in decision space is based on the center point. And the prediction in objective space is based on CTI. In addition, for handling the problems with periodic changes, a kind of memory method is added. Finally, to compensate for the inaccuracy of the prediction in particularly complex problems, a self-adaptive diversity maintenance method is adopted. The proposed strategy was compared with other four state-of-the-art strategies on 13 classic dynamic multiobjective optimization problems (DMOPs). The experimental results show that DOPS is effective in dynamic multiobjective optimization.

1. Introduction

In real life, there are many optimization problems which have multiple objectives but these objectives conflict with each other. These optimization problems are called multiobjective optimization problems (MOPs). However, many MOPs in the real world always contain uncertain and dynamic factors. For example, air traffic scheduling is easily affected by weather and some emergencies. So in that case the best solutions are hard to keep valid for a long time. If the objectives, constraints, or parameters of MOPs change with time, this kind of MOPs is called dynamic MOPs (DMOPs). Due to the dynamic character, DMOPs are harder to converge than MOPs. The mathematical formula is as follows:

$$\begin{aligned} \min_{x\in\Omega} \quad & F(x,t) = \left(f_1(x,t), f_2(x,t), \dots, f_m(x,t)\right)^T \\ s.t. \quad & g_i(x,t) \le 0, \quad i = 1,2,\dots,p; \\ & h_j(x,t) = 0, \quad j = 1,2,\dots,q \end{aligned} \tag{1}$$

where $x = (x_1, x_2, \ldots, x_n)$ is the n-dimensional decision vector whose domain of definition is Ω. t represents time variable. $F = (f_1, f_2, \ldots, f_m)$ is the m-dimensional objective vector. g represents p-dimensional inequality constraints, and h is q-dimensional equality constraints. The optimal tradeoff solution set is called Pareto set (PS) in decision space and Pareto front (PF) in the objective space.

In the recent literature, whether in industrial applications or scientific research, there is a lot of contents related to DMOPs [1–3]. Industrial applications involve design [4, 5], management [6, 7], scheduling [8–10], planning [11–14], and control [15, 16]. Scientific research includes constrained optimization [17, 18], machine learning [19, 20], and bilevel optimization [21]. In some fields [22–24], we will also try to use evolutionary algorithm to solve.

The integration of 5G network and Internet of things to build a future smart city will involve a lot of sensor installations and configurations as well as the priority processing of different tasks. In addition, a good resource allocation plan will greatly promote the construction of smart cities [25, 26]. These things have a lot to do with DOMPs. In the literature [27], a strategy based on centroid-based adaptation (CBA) is put forward to solve the problem of mission plan. In this case, two objectives need to be optimized: operation execution time and operation cost. In addition, there are time-varying constraints, including changes in execution times and task-related networks. The results show that CBA is effective in dealing with mission plan.

While handling DMOPs, the traditional static algorithms [28, 29] have not been suited. Because when using these methods, the population often converges to the optimum solutions in the current environment. And then the genes of the population will become single and lose diversity. When the environment varies, it is very difficult to find the optimal solutions. So, some researchers adjusted these static algorithms to solve DMOPs [8, 30].

Afterwards, other strategies such as the random initialization methods [24, 31–33], the memory strategies [34, 35], and prediction strategies [36–40] are introduced. The memory strategy can respond to the environmental changes by recording historical optimal solutions to converge fast. However, the memory strategies have the blind character dealing with DMOPs. When dealing with periodic problems, they can achieve good results. But the effect is bad for nonperiodic problems. The prediction strategy can take advantage of historical information to predict the optimal solutions of the new environment. The accuracy of prediction has been an important aspect on research. Hatzakis and Wallace proposed the feed-forward prediction strategy (FPS) [36] in 2009. In this strategy, the historical boundary points are used to predict the new boundary points in the new environment. By the new boundary points, the PF of the new environment is located. FPS has certain effect dealing with DMOPs, but only boundary points used are hard to trace PF or PS accurately. In 2013, Wu et al. proposed a predictive multiobjective genetic algorithm (PMGA) [39]. In this method, PF is clustered into multiple center centers to guide the rapid evolution of the population toward the optimal solutions. It is good for those problems whose decision variables have linear relationship between each other. However, for those problems whose decision variables have nonlinear relationship between each other, the effect is bad. And then, Zhou and Jin et al. proposed a population prediction strategy (PPS) [37]. PPS can not only predict the center point of the population of the next environment in the decision space, but also predict the shape distribution of the nondominated set. It has good effect for DMOPs, but it also has a drawback. In the front learning stage, the effect is bad due to lack of experience. In 2015, Wu and Jin et al. proposed a direct prediction strategy [38], which directly uses feed-forward center point to predict population optimal solutions of the next environment. It can achieve good results for DMOPs. However, it can introduce some useless individuals except nondominated individuals. And, in 2017, Jiang and Yang et al. proposed an algorithm based on steady state and generation [41], which can handle DMOPs well.

Many researchers try to find points or regions of special significance [42–44]. There are many special points that have been found, not only center points, such as boundary points and knee points [45–50]. In this paper, a hybrid prediction strategy based on center point and CTI, which can handle DMOPs from both decision space and objective space, is proposed. In the previous literature, the evolution direction of the population and the position and shape distribution of PF are judged according to the decision space in most cases. However, in this paper, we judge the evolution direction of population and distribution of optimal solutions by CTI in objective space. And then we combine it with the method based on center point to deal with DMOPs together. Then, memory strategy and adaptive diversity maintenance strategy are introduced to make up for the bad effect caused by inaccurate prediction in particularly complex problems.

The rest of this article is structured as follows. Section 2 introduces some related work. Section 3 describes DOPS in detail. Section 4 gives test problems and performance indicator. Section 5 gives experimental results and analysis. Section 6 does more discussion to further analyze the advantages and disadvantages of the prediction method based on the center point and that based on CTI. And, in the end, Section 7 gives conclusion and future work.

2. Related Work

2.1. A General Framework of Dynamic Multiobjective Genetic Algorithm. The general framework of dynamic multiobjective evolutionary algorithm is as follows.

(1) Initialize the population and set the relevant parameters.

(2) Detect the environmental change. When detecting the change, go to step (4); else, go to step (3).

(3) Optimization algorithm is used to handle the problem.

(4) Some response strategies are used, such as reinitialization, memory strategy, and prediction strategy, to respond to the environmental change.

(5) Judge the termination condition. If not terminated, go to step (2); if terminated, then exit.

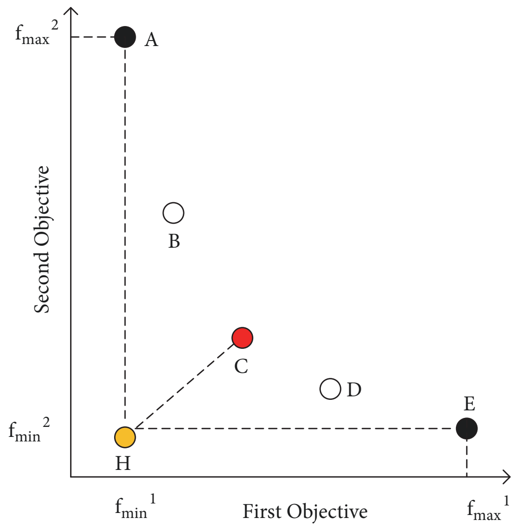


Figure 1: CTI schematic.

2.2. CTI. Many researchers have tried to find some points or regions with special meanings [36, 42–44]. Among them, ideal point and a particular point CTI [36, 45] extending from an ideal point have aroused great interest among the researchers.

The ideal point is a point composed of the smallest values of each dimension in the nondominated set. The mathematical definition is as follows:

$$H_i^* = \min\left\{f\left(NonP_i^1\right), f\left(NonP_i^2\right), \ldots, f\left(NonP_i^M\right)\right\}. \quad (2)$$

where $i = 1, 2, \ldots, m$. m is the dimension of the objective space. M is the size of the nondominated set. $f(NonP_i^M)$ denotes the value of the i-th dimension of the M-th individual in the nondominated set. As shown in Figure 1, A, B, C, D, and E are five individuals in nondominated set. Among them, A and E are boundary points. Thus, H is the ideal point.

In [36, 45], CTI is as the representative point to predict the location of PF. CTI is close-to-ideal point, which means that it is the closest individual to ideal point in nondominated set. As shown in Figure 1, C is CTI. C is the point whose Euclidean distance is shortest to ideal point H.

2.3. The Principle of the Prediction Model. In [34], the principle of the prediction model is described in detail. In the section, the principle is described in short. Supposing that $x_1, x_2, \ldots, x_t \in Q_i$, $i = 1, \ldots, t$, are a series of points, these points are n-dimensional decision vectors in the decision space which are used to describe the movement of PS. The general prediction model is defined as follows:

$$x_{t+1} = F\left(x_t, x_{t-1}, \ldots, x_{t-k+1}, t\right). \quad (3)$$

where K is the number of previous time steps. X_{t+1} is the individual in t+1 time step, which is also the individual to predict. Figure 2 is what happens when k is equal to 2. In this paper, a simple and widely used prediction model is adopted.

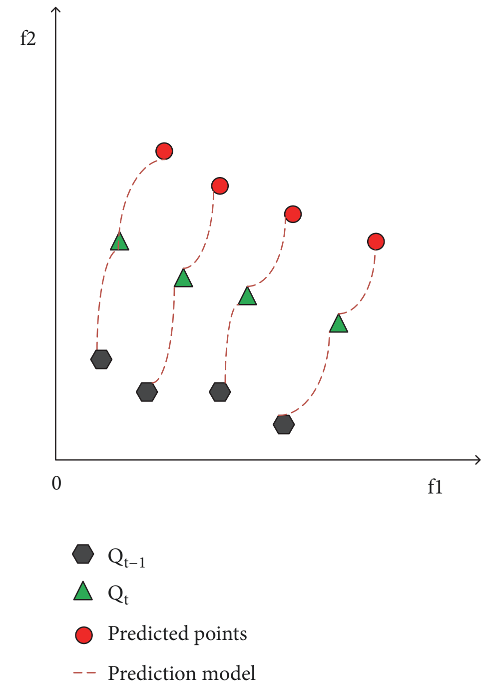


Figure 2: Prediction model.

$$x_{t+1} = x_t + \left(x_t - x_{t-1}\right) + Gauss\,(0, d). \quad (4)$$

where the direction and distance of prediction are got by $x_t - x_{t-1}$. Gauss(0,d) is a Gaussian perturbation added in order to avoid falling into a local optimum. d is the variance of disturbance.

3. A Hybrid Predictive Strategy Carried through Simultaneously from Decision Space and Objective Space

Prediction strategy has been used to predict moving PS or PF. In real world, there are all kinds of DMOPs. The change locations of DMOPs may be in PS or PF or both PS and PF [37, 52, 53]. Thus, to handle different kinds of DMOPs, a hybrid predictive strategy carried through simultaneously from decision space and objective space (DOPS) is proposed. The predictive strategy in the decision space is based on center point. And the predictive strategy in the objective space is based on CTI. Two kinds of predictive strategies can both predict the nondominated set. In the paper, the two predicted nondominated sets were halved and then merged into a new nondominated set. Then, several nondominated individuals in the last environment were introduced to the predicted population as the memory set. In the end, we adopted a kind of adaptive diversity maintenance strategy, which is similar to that in [46], to keep diversity. The diversity individuals produced by this diversity strategy are called random set. As shown in Figure 3, all predicted population consists of three parts.

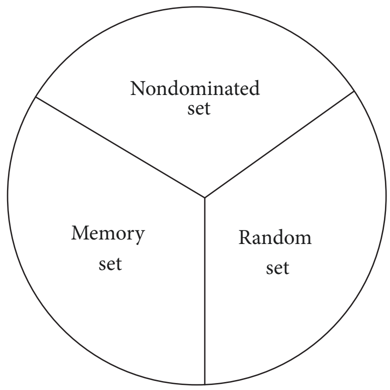


Figure 3: Population structure schematic.

3.1. Memory Set. In real world, there are all kinds of dynamic real problems. For some dynamic problems, the optimal solutions may appear periodically. The part of optimal solutions of the population can be recorded. If the same environment occurs in the future, then the optimal solutions recorded are also valid. In this section, a kind of simple memory strategy is selected. If the number of the optimal solutions to be memorized is $Nmem$, $Nmem$ individuals are randomly selected from the optimal individuals in the current population to be memorized, and the $Nmem$ individuals make up the memory set.

3.2. Nondominated Set. The predicted nondominated set is got by hybrid prediction strategy. It contains two prediction substrategies: one is the strategy based on the center point of decision space, and the other one is based on the CTI in objective space.

3.2.1. The Prediction Strategy Based on Center Point. The prediction strategy based on center point is a widely used strategy [37, 38, 40, 41, 45, 46, 54, 55]. The center point is defined as follows:

$$C_k^t = \frac{1}{\left|P_{Non-dom}^t\right|} \sum_{NonInd_k^t \in P_{Non-dom}^t} NonInd_k^t. \quad (5)$$

where $|P_{Non-dom}^t|$ represents the cardinality of the nondominated set. $P_{Non-dom}^t$ represents the nondominated set in t time step. $NonInd_k^t$ represents a nondominated individual in t step and in the k-th dimension. Based on the prediction model in Section 2.3, the prediction model based on center point can be expressed as

$$pop_k^{t+1} = pop_k^t + \left(C_k^t - C_k^{t-1}\right) + Gauss\,(0, d)\,. \quad (6)$$

where C_k^t and C_k^{t-1} show the center points in k dimension in t and t-1 time step, respectively. pop_k^t denotes the individuals of the nondominated set in k dimension and t time step. And pop_k^{t+1} denotes the predicted individuals in k dimension and in t+1 time step. As shown in Figure 4, the nondominated set of t+1 step can be predicted by using the nondominated set of t plus evolution direction got by center points of t and t-1 time step. The detailed description is in Algorithm 1.

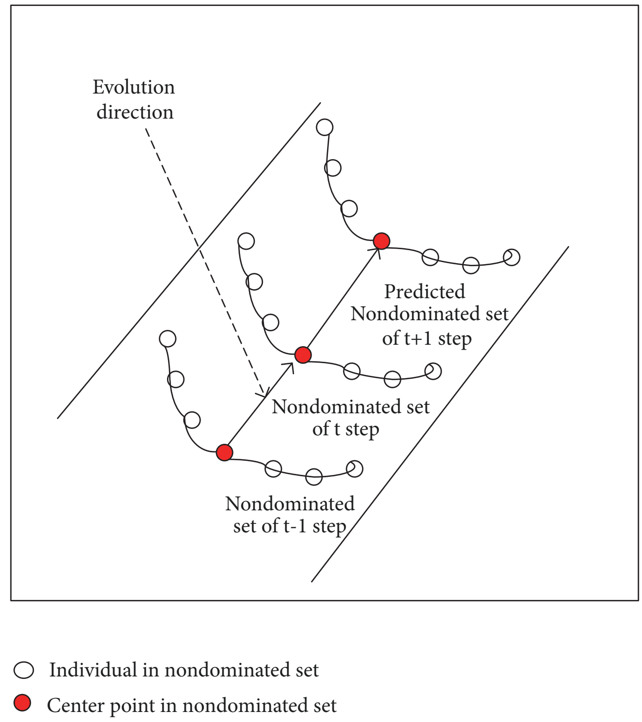


Figure 4: Center point prediction schematic.

3.2.2. The Prediction Strategy Based on CTI. CTI is a special point of nondominated sets in the objective space and has been widely used. However, in most cases, CTI is used to be a representative point to predict PF and to judge the position of PF [36, 45]. In this paper, we use CTI not only to judge the evolution direction of the population, but also to judge the position and shape of PF. This strategy is based on the prediction model in Section 2.3. The mathematical form is similar to formula (6), as follows:

$$pop_k^{t+1} = pop_k^t + \left(CTI_k^t - CTI_k^{t-1}\right) + Gauss\,(0, d) \quad (7)$$

where pop_k^t and Gauss(0,d) are the same as formula (6). CTI_k^t and CTI_k^{t-1} denote CTI in k dimension in t and t-1 time step, respectively. As shown in Figure 5, the nondominated set of t+1 step can be predicted by using the nondominated set of t plus evolution direction got by CTIs of t and t-1 time step. The detailed description is in Algorithm 2.

3.2.3. The Hybrid Prediction Strategy. The hybrid prediction strategy is a mixture of the prediction strategy based on the center point and the prediction strategy based on CTI. In theory, the final nondominated set, which consists of two nondominated sets got by two prediction strategies, respectively, is formed by cutting them in half and then merging them. But, in fact, it is related to whether the capacity of the nondominated set of the current population is odd or even, and whether $Nmem + Nnondom_t$ is greater than the population capacity $psize$. In this case, it is a little more complicated. The detailed process is shown in Algorithms 3 and 4.

Input: Pop_{t-1} and Pop_t, the population in t-1 and t time step.
Output: *Non-dom*1, the predicted non-dominated set in t+1 time step.
Step 1: Layer the populations, Pop_{t-1} and Pop_t, in t-1 and t time step, and then select the individuals in the first layer as non-dominated set, $Non\text{-}dom_{t-1}$ and $Non\text{-}dom_t$;
Step 2: According to formula (5), calculate the center points of $Non\text{-}dom_{t-1}$ and $Non\text{-}dom_t$, C_k^{t-1} and C_k^t, respectively.
Step 3: According to formula (6), predict the non-dominated set in the t+1 time step, *Non-dom*1.

Algorithm 1: Calculating the nondominated set by the prediction strategy based on center point.

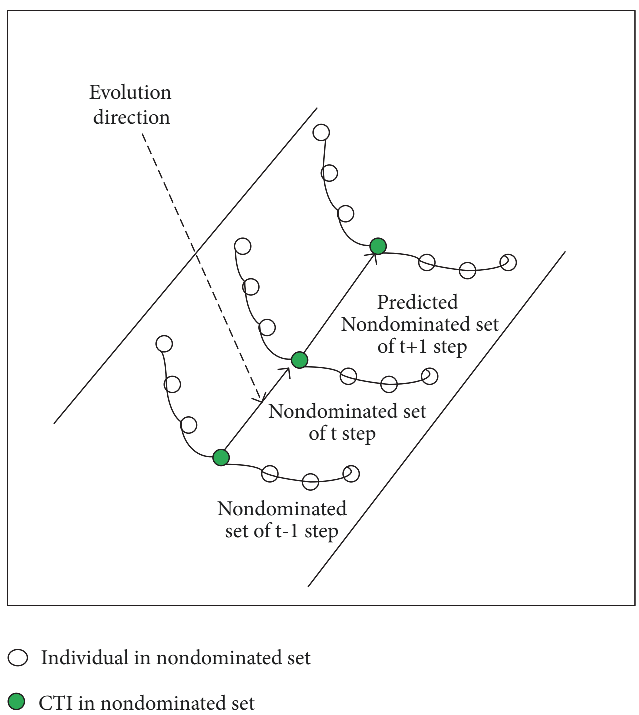


Figure 5: CTI prediction schematic.

3.3. Random Set. Random set is derived from the adaptive population diversity maintenance strategy. In dynamic optimization, random set plays a very important role. It is based on two reasons.

(1) In the environment, only the fittest individuals can survive. The genes of these individuals are suitable for the environment. Thus, in the environment, the genes of the individuals that survive will gradually become similar, which means that the genes of the population will become single. When the environment changes, these genetically unitary individuals who are used to the previous environment will be easily eliminated. Therefore, we need to increase the diversity in the population. When the environment changes, those individuals whose genes are suitable for the new environment will easily survive and reproduce.

(2) When dealing with dynamic problems with prediction strategies, especially for particularly complex problems, it is easy for prediction strategies to cause inaccurate prediction. Increasing the diversity of the population at this time will make up for the bad effect caused by inaccurate prediction.

In this paper, an adaptive population diversity maintenance strategy similar to that in [46] was adopted. The strategy adaptively introduced a corresponding number of random individuals according to the difficulty of the problem to be solved. Generally, the more complex the problem is, the greater the possibility of inaccurate prediction will be. Therefore, more random individuals need be introduced to make up for the bad influence brought by inaccurate prediction to a greater extent. And, at this time in DOPS, there were a big number of random individuals produced. On the contrary, the simpler the problem is, the more accurate the prediction is. At this time, the introduction of more random individuals will bring bad results. At that time in DOPS, the number of random individuals introduced happened to be small, even 0. Algorithm 5 describes the process in detail.

3.4. The Detailed Description of DOPS. DOPS is under the dynamic framework in Section 2.1. When the environment changes, DOPS reinitializes the population to respond quickly to the environmental change. The detailed description of DOPS is shown in Algorithm 6.

4. Test Problems and Performance Indicator

4.1. Test Problems. In [52], DMOPs are generally divided into 4 categories according to the location of changes in the test problems as follows:

(1) PS changes, but PF does not change.
(2) PS does not change, but PF changes.
(3) PS and PF change together.
(4) PS and PF do not change.

However, the first three are often encountered. These test problems come from three test suites: FDA test suite [52], dMOP test suite [53], and F5-F10 [37]. Among them, FDA4 and F8 have three objectives, but the other problems have two objectives. In addition, the decision variables of FDA and dMOP suites are linearly correlated between each other, while the decision variables of F series are nonlinearly correlated between each other. Therefore, the test problems in F suite are more difficult to optimize than those in FDA and dMOP series. In the F suite, F9 and

Input: Pop_{t-1} and Pop_t, the population in t and t-1 time step.
Output: $Non\text{-}dom2$, the predicted t+1 time step non-dominated set.
Step 1: Layer the population in t-1 and t time step, Pop_{t-1} and Pop_t and then select the individuals in the first layer as non-dominated set, $Non\text{-}dom_{t-1}$ and $Non\text{-}dom_t$;
Step 2: Calculate the boundary points of the non-dominated set in t-1 and t time step.
Step 3: Calculate the ideal point H_{t-1} and H_t, according to formula (2).
Step 4: The nearest individuals to H_{t-1} and H_t are identified as CTI_{t-1} and CTI_t in $Non\text{-}dom_{t-1}$ and $Non\text{-}dom_t$, respectively.
Step 5: According to formula (7), predict the non-dominated set in the t+1 time step, $Non\text{-}dom2$.

ALGORITHM 2: Calculating the nondominated set by the prediction strategy based on CTI.

Input: $Nmem$, the capacity of memory set; $psize$, the size of the population; $Non\text{-}dom1$ and $Non\text{-}dom2$, the non-dominated sets obtained by the prediction methods based on the center point and CTI, respectively; $Nnondom_t$, the capacity of the non-dominated set in t time step.
Output: The final non-dominated set in t+1 time step, $Non\text{-}dom$ and its size, $Nnondom$.
if $Nnondom_t + Nmem <= psize$ **then**
  if $Nnondom_t\%2 = 0$ **then**
    Select ($Nnondom_t/2$) individuals from $Non\text{-}dom1$, as $Non\text{-}dom1'$;
    Select ($Nnondom_t/2$) individuals from $Non\text{-}dom2$, as $Non\text{-}dom2'$;

$$Non\text{-}dom := Non\text{-}dom1' + Non\text{-}dom2'.$$

  end if
  if $Nondom_t\%2 = 1$ **then**
    Select $((Nnondom_t + 1)/2)$ individuals from $Non\text{-}dom1$, as $Non\text{-}dom1^*$;
    Select $((Nnondom_t - 1)/2)$ individuals from $Non\text{-}dom2$, as $Non\text{-}dom2^*$;

$$Non\text{-}dom := Non\text{-}dom1^* + Non\text{-}dom2^*.$$

  end if
  $Nnondom := Nnondom_t$.
end if
if $Nnondom_t + Nmem > psize$ **then**
  if $(psize - Nmem)\%2 = 0$ **then**
    Select $((psize - Nmem)/2)$ individuals from $Non\text{-}dom1$, as $Non\text{-}dom1'$;
    Select $((psize - Nmem)/2)$ individuals from $Non\text{-}dom2$, as $Non\text{-}dom2'$;

$$Non\text{-}dom := Non\text{-}dom1' + Non\text{-}dom2'.$$

  end if
  if $(psize - Nmem)\%2 = 1$ **then**
    Select $(psize - Nmem + 1)/2$ individuals from $Non\text{-}dom1$, as $Non\text{-}dom1^*$;
    Select $(psize - Nmem - 1)/2$ individuals from $Non\text{-}dom2$, as $Non\text{-}dom2^*$;

$$Non\text{-}dom := Non\text{-}dom1^* + Non\text{-}dom2^*.$$

  end if
  $Nnondom := psize - Nmem$.
end if

ALGORITHM 3: Calculating the final nondominated set in t+1 time step and its size.

F10 are two of the most difficult problems to optimize. In F9, for the most part, the environment changes slightly, but sometimes PS can occasionally jump from one area to another. In F10, the geometry of two consecutive PFs is different.

4.2. Performance Indicator. Two performance metrics, MIGD and MHVD, were used to evaluate the performance of these strategies regarding convergence and distribution.

4.2.1. Modified Inverted Generational Distance (MIGD). MIGD [37, 41, 45, 46] is a widely used indicator. Before MIGD, IGD metric is described firstly [37, 51, 56]. Here, let PF_t be a set of uniformly distributed Pareto optimal points of PF in t time and let P_t be an approximation set of PF in t time. The formula is as follows:

$$IGD(PF_t, P_t) = \frac{\sum_{v \in PF_t} d(v, P_t)}{|PF_t|} \quad (8)$$

Input: *Nmem*, the capacity of memory set; *psize*, the size of the population; Pop_{t-1} and Pop_t, the population in t-1 and t time step.
Output: The final non-dominated set, *Non-dom* and its size, *Nnondom*.
Step 1: Layer the Pop_t, and select the individuals in the first layer as the non-dominated set, $Non\text{-}dom_t$, and get its capacity, $Nnondom_t$;
Step 2: According to Algorithm 1, calculate the non-dominated set, *Non-dom*1;
Step 3: According to Algorithm 2, calculate the non-dominated set, *Non-dom*2;
Step 4: According to Algorithm 3, calculate the final non-dominated set, *Non-dom*, and its size, *Nnondom*;

Algorithm 4: Hybrid predictive strategy.

Input: *Nmem*, the capacity of memory set; *psize*, the size of the population; *Nnondom*, the capacity of the non-dominated set; *Low* and *Up*, the upper and lower bounds of the decision vector; *m*, the dimensions of the decision space.
Output: Pop_{rand}, the random set and its size, *Nrand*.

```
if Nnondom + Nmem <= psize then
  Nrand := psize − Nnondom − Nmem
  for all Ind ∈ Pop_rand do
    i:=0;
    while i < m do
      Ind_i := rand(Low_i, Up_i)
    end while
  end for
else
  Nrand := 0
  Pop_rand := NULL
end if
```

Algorithm 5: An adaptive diversity maintenance strategy.

where $d(v, P_t) = \min_{u \in P_t} \|F(v) - F(u)\|$ is the distance between v and P_t. $|PF_t|$ is the cardinality of PF_t.

Because the environment is dynamic, if only using simply IGD, it is hard to judge which algorithm is better between two algorithms in some situations [34]. MIGD is a modified version of IGD, which is defined as the average value of the IGD values in some time steps over a run.

$$MIGD = \frac{1}{|T|} \sum_{t \in T} IGD\left(PF_t, P_t\right) \tag{9}$$

where T is a set of discrete time points in a run and $|T|$ is the cardinality of T. MIGD is also a comprehensive performance metric to evaluate the performance of algorithms, such as convergence and distribution.

The smaller the value of MIGD is, the better the performance of the algorithm is.

4.2.2. Modified Hypervolume Difference (MHVD). The Hypervolume Difference (HVD) [34, 41] measures the gap between the hypervolumes of the got PF and the true PF. PF_t denotes a set of uniformly distributed Pareto optimal points of the PF in t time step and P_t is an approximation set of PF in t time step.

$$HVD\left(PF_t, P_t\right) = HV\left(PF_t\right) - HV\left(P_t\right) \tag{10}$$

where $HV(S)$ denotes the hypervolume of a set S.

MHVD is got by modifying HVD like MIGD to IGD. MHVD shows the average of the HVD values in some time steps over a run.

$$MHVD = \frac{1}{|T|} \sum_{t \in T} HVD\left(PF_t, P_t\right) \tag{11}$$

where T is a set of discrete time points over a run and $|T|$ is the cardinality of T. The reference point for the computation of hypervolume is $(Z_1^t + 0.5, Z_2^t + 0.5, \ldots, Z_M^t + 0.5)$, where Z_j^t is the maximum value of the j-th objective of the true PF in t time; M is the number of objectives. MHVD is also a comprehensive indicator. The smaller the value is, the better the performance of the algorithm is.

5. Experimental Results and Analysis

In this paper, we chose four commonly used strategies for comparison: (1) random initialization strategy (RIS), which will randomly initialize the population when the environment changes, (2) feed-forward prediction strategy (FPS) [36], (3) predictive multiobjective genetic algorithm (PMGA) [39], and (4) population prediction strategy (PPS) [37]. RM-MEDA [51] was used as the optimization algorithm.

5.1. Parameter Setting. In this article, set the number of individuals in the population to 100. The running generation

Initialization: number of time change, $t \coloneqq 0$; generation counter, $gt \coloneqq 0$; total generation number, $gmax$.
Step 1: Initialize the population, Pop_t.
Step 2: Detect the environmental change. If no change, go to step 8;
else, calculate the non-dominated set in the current population.
Step 3: If t=1, go to step 1.
Step 4: Select randomly $Nmem$ individuals from non-dominated set, as memory set, Pop_{mem};
Step 5: According to Algorithm 4, get the non-dominated set in t+1 time step,
$Non\text{-}dom$ and its size, N_{nondom}.
Step 6: According to Algorithm 5, get the random set, Pop_{rand}.
Step 7: Get the predicted population in the new environment by equation (∗).

$$Pop_{t+1} = Pop_{mem} + Non\text{-}dom + Pop_{rand}. \qquad (*)$$

Step 8: The optimization algorithm $RM - MEDA$ [51] is used to optimize the problem.
Step 9: If $gt > gmax$, output Pop_t, and then end; else, gt:=gt+1; go to step 2.

Algorithm 6: A hybrid predictive strategy carried through simultaneously from decision space and objective space.

number of the algorithm is set as 2500. And the environment changes once in 25 generations, so the number of environmental changes is 100. The dimension of the decision space is set as 20.

The Parameters in FPS. The number of the predicted individuals is 3(m+1) and m is the dimensions of the objective space. Seventy percent of the remaining individuals are inherited from the previous environment. The remaining 30% are randomly generated within the scope of the decision space.

The Parameters in PPS. The length of the historical sequence M is 23; the parameter p is 3.

The Parameters in DOPS. The size of memory set, $Nmem$, is set to 10.

Change Detection. Five percent of the individuals were randomly selected to detect environmental changes. If the objective value of an individual is detected to be inconsistent with the original objective value, the environment is considered to have changed.

In the experiment, we ran the algorithms 20 times on average and then took the average values to reduce the experimental errors.

5.2. Comparison on Performance Evaluation Results. In a dynamic environment, in some cases, when we compare the performance of different strategies, we need to analyze them at different time periods [34, 37, 54, 57]. In this paper, 100 environmental changes were divided into three stages. The first 20 environmental changes were the first stage; the middle 40 were the second stage, and the last 20 were the third stage. The mean and standard deviation of the MIGD values were calculated to compare the performance of the algorithms where the data marked black indicates that the MIGD value of the strategy is the minimum, which means that the strategy has the best performance. Wilcoxon rank sum test [58] was used to analyze the significance of different results at the significance level of 0.05.

5.2.1. Performance Comparison on FDA and dMOP Test Suites. As shown in Table 1, the five strategies were compared for the MIGD values in the FDA and dMOP test suites. In each small cell, this value represents the mean and standard deviation of the MIGD value. Total represents all environmental changes, which means 100 environmental changes.

(1) For the total stage, DOPS is slightly worse than with FPS on FDA2 and dMOP1. But, on other problems, DOPS is better than all other strategies. This shows that DOPS has good overall performance. Among them, RIS has the worst effect; that is, these prediction strategies are better than the random algorithm. This shows that the prediction strategy is not blind and effective in dealing with DMOPs.

(2) For the 1st stage, DOPS was slightly worse than FPS for FDA2 and dMOP1, but better than the other strategies for FDA3 and dMOP2. This shows that DOPS has the ability to respond quickly to environmental changes.

(3) For 2nd and 3rd stages, DOPS was slightly worse than PPS on test problems FDA1, dMOP2, and dMOP3.

(4) For 3-dimensional problem, FDA4, DOPS is better than other prediction strategies at total and 1st stages. This shows that DOPS is effective for multiobjective optimization problems whose objective number is bigger than 2.

MHVD values are shown in Table 2. The results in Table 2 are almost the same with Table 1.

From these experimental results, the reasons can be guessed. PPS needs to learn for a long time to accumulate experience, resulting in the fact that the effect is not good in the early stage. DOPS, on the other hand, does not require too long time to learn but processes simultaneously from decision space and objective space, so that it has a better prediction effect. However, when PPS accumulates enough experience, the prediction effect will be better. The specific performance in 2nd and 3rd stages was better than DOPS on some test problems. FPS showed good effect on FDA1 and dMOP1, because both FDA1 and dMOP1 were invariant PS problems. In FPS, 70% of the newly generated individuals were inherited from the optimal solutions in the previous environment, so FPS showed better results on the two test problems than the other four strategies.

Table 1: Mean and Standard Deviation of MIGD values of five strategies on FDA and dMOP test suites. The values in bold face denote having the best effect in these five strategies. ‡ and † indicate that DOPS is significantly better than and is equivalent to the corresponding strategy, respectively.

Problems	Statistic	RIS	FPS	PMGA	PPS	DOPS
FDA1	Total	1.3155(0.03030)‡	0.0516(0.00864)‡	0.0581(0.00316)‡	0.0528(0.00913)‡	**0.0303(0.00623)**
	1st stage	1.2215(0.07518)‡	0.2090(0.04391)‡	**0.0634(0.00658)**	0.2406(0.04615)‡	0.1102(0.03141)
	2nd stage	1.3310(0.04145)‡	0.0151(0.00117)‡	0.0567(0.00606)‡	**0.0102(0.00103)**	0.0113(0.00021)
	3rd stage	1.3447(0.04696)‡	0.0134(0.00084)‡	0.0581(0.00316)‡	**0.0062(0.00008)**	0.0113(0.00028)
FDA2	Total	0.0500(0.00078)‡	**0.0085(0.00068)**	0.4435(0.01275)‡	0.0097(0.00075)	0.0100(0.00042)
	1st stage	0.0491(0.00115)‡	**0.0198(0.00332)**	0.5137(0.04302)†	0.0232(0.00317)†	0.0213(0.00237)
	2nd stage	0.0503(0.00123)‡	**0.0060(0.00034)**	0.4269(0.01171)‡	0.0070(0.00068)	0.0073(0.00015)
	3rd stage	0.0501(0.00130)‡	**0.0056(0.00003)**	0.4435(0.01275)‡	0.0060(0.00004)	0.0074(0.00009)
FDA3	Total	1.7564(0.06554)‡	0.0645(0.00927)‡	0.7617(0.00886)‡	0.0941(0.01582)‡	**0.0388(0.00442)**
	1st stage	1.5737(0.12477)‡	0.2084(0.04439)‡	0.7552(0.01274)‡	0.3209(0.07801)‡	**0.1302(0.02582)**
	2nd stage	1.7709(0.11832)‡	0.0305(0.00339)‡	0.7610(0.01097)‡	0.0420(0.00821)‡	**0.0170(0.00160)**
	3rd stage	1.8287(0.11044)‡	0.0303(0.00421)‡	0.7617(0.00886)‡	0.0384(0.00675)‡	**0.0171(0.00207)**
FDA4	Total	0.4566(0.00922)‡	0.1414(0.00337)‡	3.7558(0.00341)‡	0.1307(0.00205)‡	**0.0171(0.00207)**
	1st stage	0.4390(0.02098)‡	0.1629(0.00852)‡	0.1583(0.00239)‡	0.1660(0.00816)‡	**0.1368(0.00365)**
	2nd stage	0.4594(0.01512)‡	0.1376(0.00427)‡	4.6121(0.01405)‡	**0.1247(0.00278)**‡	0.1368(0.00365)
	3rd stage	0.4622(0.01131)‡	0.1386(0.00340)‡	3.7558(0.00341)‡	**0.1231(0.00283)**‡	0.1368(0.00365)
dMOP1	Total	0.6386(0.01427)‡	**0.0072(0.00115)**	0.2437(0.01891)‡	0.0379(0.05152)†	0.0105(0.00102)
	1st stage	0.6486(0.03460)‡	**0.0195(0.00610)**	0.2037(0.02545)‡	0.1413(0.21119)†	0.0234(0.00512)
	2nd stage	0.6383(0.02790)‡	**0.0043(0.00008)**	0.2724(0.02718)‡	0.0209(0.02865)‡	0.0075(0.00012)
	3rd stage	0.6341(0.01865)‡	**0.0043(0.00007)**	0.2437(0.01891)‡	0.0057(0.00002)	0.0074(0.00018)
dMOP2	Total	1.6968(0.05407)‡	0.0622(0.00788)‡	0.1451(0.00650)‡	0.0607(0.01023)‡	**0.0379(0.00504)**
	1st stage	1.6332(0.08749)‡	0.2552(0.03931)‡	0.1774(0.01274)‡	0.2799(0.05178)‡	**0.1416(0.02592)**
	2nd stage	1.7087(0.09220)‡	0.0170(0.00113)‡	0.1384(0.00727)‡	**0.0111(0.00103)**	0.0131(0.00029)
	3rd stage	1.7150(0.06717)‡	0.0159(0.00080)‡	0.1451(0.00650)‡	**0.0061(0.00006)**	0.0134(0.00020)
dMOP3	Total	1.3215(0.03752)‡	0.0523(0.00654)‡	0.0581(0.00316)‡	0.0527(0.01084)‡	**0.0312(0.00889)**
	1st stage	1.2558(0.08243)‡	0.2124(0.03335)‡	**0.0634(0.00658)**	0.2403(0.05530)‡	0.1158(0.04631)
	2nd stage	1.3319(0.05383)‡	0.0149(0.00077)‡	0.0567(0.00606)‡	**0.0101(0.00093)**	0.0112(0.00022)
	3rd stage	1.3423(0.06014)‡	0.0136(0.00076)‡	0.0581(0.00316)‡	**0.0062(0.00010)**	0.0111(0.00035)

5.2.2. Performance Comparison on F5-F10. As shown in Table 3, mean values and standard deviation of MIGD values are put into it. From Table 3, some things can be seen.

For two-dimensional problems, F5, F6, and F7, DOPS is better than the other four strategies in total, 1st,and 2nd stages. This shows that DOPS still has better performance on problems where there are the nonlinear relationships between decision variables. But, in the third stage, the effect is slightly worse than PPS. The explanation in Section 5.2.1 can be used here. PPS needs to accumulate experience; when accumulating enough experience, it will show better results. For particularly complex problems F9 and F10, DOPS performed better at all stages than the other four strategies, possibly because the problems were so complex that the predictions of the FPS and PPS were inaccurate. The prediction in DOPS carried through simultaneously from the decision space and objective space can alleviate this situation. Coupled with the adaptive diversity maintenance strategy in DOPS, the effect of DOPS is better. In addition, in Tables 1 and 3, the standard deviation of DOPS is relatively small, which means that DOPS is robust. This is because DOPS simultaneously predicts from decision space and objective space, so that the two predictions are complementary to each other, making DOPS more robust to deal with various problems.

Table 4 shows the MHVD values on F5-F10. The results are almost identical with Table 3, but it seems better on F5. For F5, DOPS is better than the other strategies on all stages.

5.3. Final Population Diagram. In order to compare the performance of various strategies more intuitively, we selected the final population distribution diagram of five strategies on four test problems with different characteristics. As shown in Figures 6–9, the blue line represents the true PF of the population at time t, and every red dot represents an individual in the population. As the test problem is FDA1 in Figure 6, the PF of the test problem is invariant, so 6 moments were selected to observe the effect, while in other problems 8 moments were selected.

At certain moment, the more individuals are closer to the PF at that moment, indicating that the effect is better. As shown in Figures 6 and 7, the effect is almost identical to that shown in Tables 1 and 2. In the early stage, many individuals in DOPS converge to the PF surface. While in other strategies fewer individuals converge to the PF. In the

Table 2: Mean and Standard Deviation of MHVD values of five strategies on FDA and dMOP test suites. The values in bold face denote having the best effect in these five strategies. ‡ and † indicate that DOPS is significantly better than and is equivalent to the corresponding strategy, respectively.

Problems	Statistic	RIS	FPS	PMGA	PPS	DOPS
FDA1	Total	1.2328(0.01073)‡	0.0968(0.01053)‡	0.1326(0.00669)‡	0.0948(0.01448)‡	**0.0507(0.00401)**
	1st stage	1.2145(0.01900)‡	0.3680(0.05223)‡	**0.1424(0.01292)**	0.4192(0.07139)‡	0.1566(0.01827)
	2nd stage	1.2353(0.01564)‡	0.0342(0.00249)‡	0.1302(0.00557)‡	**0.0222(0.00227)**	0.0255(0.00062)
	3rd stage	1.2390(0.01275)‡	0.0305(0.00222)‡	0.1301(0.01206)‡	**0.0133(0.00022)**	0.0257(0.00062)
FDA2	Total	0.0714(0.00125)‡	**0.0320(0.00079)**	0.7283(0.01669)‡	0.0325(0.00072)	0.0330(0.00067)
	1st stage	0.0719(0.00221)‡	**0.0372(0.00413)**	0.9557(0.05030)‡	0.0405(0.00408)†	0.0396(0.00322)
	2nd stage	0.0714(0.00209)‡	0.0306(0.00026)	0.6743(0.00871)‡	**0.0301(0.00029)**	0.0314(0.00014)
	3rd stage	0.0711(0.00188)‡	**0.0310(0.00004)**	0.6742(0.00871)‡	**0.0310(0.00005)**	0.0315(0.00010)
FDA3	Total	1.9361(0.01767)‡	0.7761(0.01825)‡	0.8604(0.00959)‡	0.8420(0.02751)†	**0.6712(0.00683)**
	1st stage	1.9195(0.04820)‡	1.0479(0.05214)†	0.9852(0.01236)‡	1.2829(0.14896)†	**0.7806(0.00898)**
	2nd stage	1.9387(0.02533)‡	0.7167(0.01667)‡	0.8310(0.01652)‡	0.7436(0.01854)‡	**0.6377(0.01288)**
	3rd stage	1.9414(0.02960)‡	0.7064(0.01855)‡	0.8310(0.02220)‡	0.7310(0.02692)‡	**0.6528(0.00784)**
FDA4	Total	1.3838(0.01837)‡	0.4249(0.01308)‡	1.3108(0.01204)‡	0.3818(0.00756)‡	**0.3410(0.00648)**
	1st stage	1.3392(0.03975)‡	0.4993(0.03182)‡	0.4503(0.00852)‡	0.5100(0.02893)‡	**0.4030(0.01961)**
	2nd stage	1.3942(0.02764)‡	0.4051(0.01404)‡	1.5138(0.01953)‡	0.3541(0.01032)‡	**0.3239(0.00633)**
	3rd stage	1.3945(0.02189)‡	0.4093(0.01419)‡	1.5165(0.01596)‡	0.3487(0.01104)‡	**0.3287(0.00605)**
dMOP1	Total	1.1531(0.01823)‡	**0.1501(0.00097)**	0.4267(0.03460)‡	0.1688(0.03108)†	0.1505(0.00098)
	1st stage	1.0885(0.04142)‡	0.1180(0.00484)†	0.3385(0.05409)‡	0.1715(0.09166)†	**0.1178(0.00341)**
	2nd stage	1.1718(0.03312)‡	**0.1577(0.00018)**	0.3985(0.02189)‡	0.1780(0.03702)†	0.1586(0.00044)
	3rd stage	1.1651(0.02816)‡	**0.1576(0.00020)**	0.4968(0.04069)‡	0.1583(0.00015)†	0.1580(0.00086)
dMOP2	Total	1.2672(0.01126)‡	0.2156(0.01314)‡	0.2138(0.01806)‡	0.2190(0.01537)‡	**0.1718(0.00203)**
	1st stage	1.1689(0.02085)‡	0.4303(0.06200)‡	0.2783(0.02538)‡	0.4484(0.07664)‡	**0.2107(0.01138)**
	2nd stage	1.2967(0.02139)‡	0.1656(0.00309)‡	0.1933(0.01614)‡	0.1709(0.00237)‡	**0.1619(0.00049)**
	3rd stage	1.2844(0.02055)‡	0.1636(0.00140)†	0.2036(0.02210)‡	**0.1581(0.00015)**	0.1632(0.00062)
dMOP3	Total	1.2282(0.00948)‡	0.0972(0.00848)‡	0.1326(0.00669)‡	0.0947(0.01470)‡	**0.0521(0.00452)**
	1st stage	1.2046(0.02570)‡	0.3695(0.04400)‡	**0.1424(0.01292)**	0.4193(0.07355)‡	0.1654(0.02333)
	2nd stage	1.2317(0.01575)‡	0.0340(0.00172)‡	0.1302(0.00557)‡	**0.0219(0.00206)**	0.0254(0.00040)
	3rd stage	1.2359(0.02242)‡	0.0312(0.00184)‡	0.1302(0.01206)‡	**0.0133(0.00024)**	0.0250(0.00069)

latter stages, DOPS and PPS work best. As can be seen from Figures 8 and 9, the effect of the final population distribution diagram is almost consistent with that in Tables 3 and 4. Both F6 and F10 are complex problems, especially F10. As can be seen from Figure 9, on RIS, FPS, PMGA, and PPS, few individuals converge on PF, while there are more in DOPS, indicating that DOPS has better effect on complex problems.

6. More Discussion

From the previous sections to see, DOPS is effective for DMOPs. However, DOPS is a hybrid prediction strategy: it includes the prediction strategy based on the center point and the prediction strategy based on CTI. In order to more accurately judge the role of the two strategies playing in dealing with DMOPs, the two strategies were compared experimentally. Here, only two strategies are compared and the memory set and the random set are no longer needed. The forms of the two strategies here are similar to formulas (6) and (7). But in formulas (6) and (7) the meanings of Pop_k^t and Pop_k^{t+1} have changed. Instead of representing individuals in a nondominated set, they represent individuals in the whole population. The prediction strategy based on the center point is called CPS, while the prediction strategy based on CTI is called IPS. CPS and IPS were compared on five test problems with different characteristics. The characteristics of the five test questions are shown in Table 5. As shown in Table 5, the number of objectives, change types, and relationship between decision variables are listed.

The MIGD comparison results of CPS and IPS on the five problems are shown in Table 6. From Table 6, we can observe the following.

(1) On dMOP3, the MIGD values of IPS are lower than CPS at all stages. This means that IPS is better than CPS in the problem of fixed PF. This may be because CTI is a special point on PF. When PF is constant, CTI can be used to judge the evolution direction of the population and the location and distribution of PF in the new environment more accurately.

(2) On F5, the MIGD values of CPS are smaller than IPS at all stages. This shows that the effect of CPS is better than effect of IPS on the problem with nonlinear relationship between decision variables; that is, on relatively complex problems, CPS is better than IPS. This may be because when dealing with nonlinear problems, only CTI is not enough to

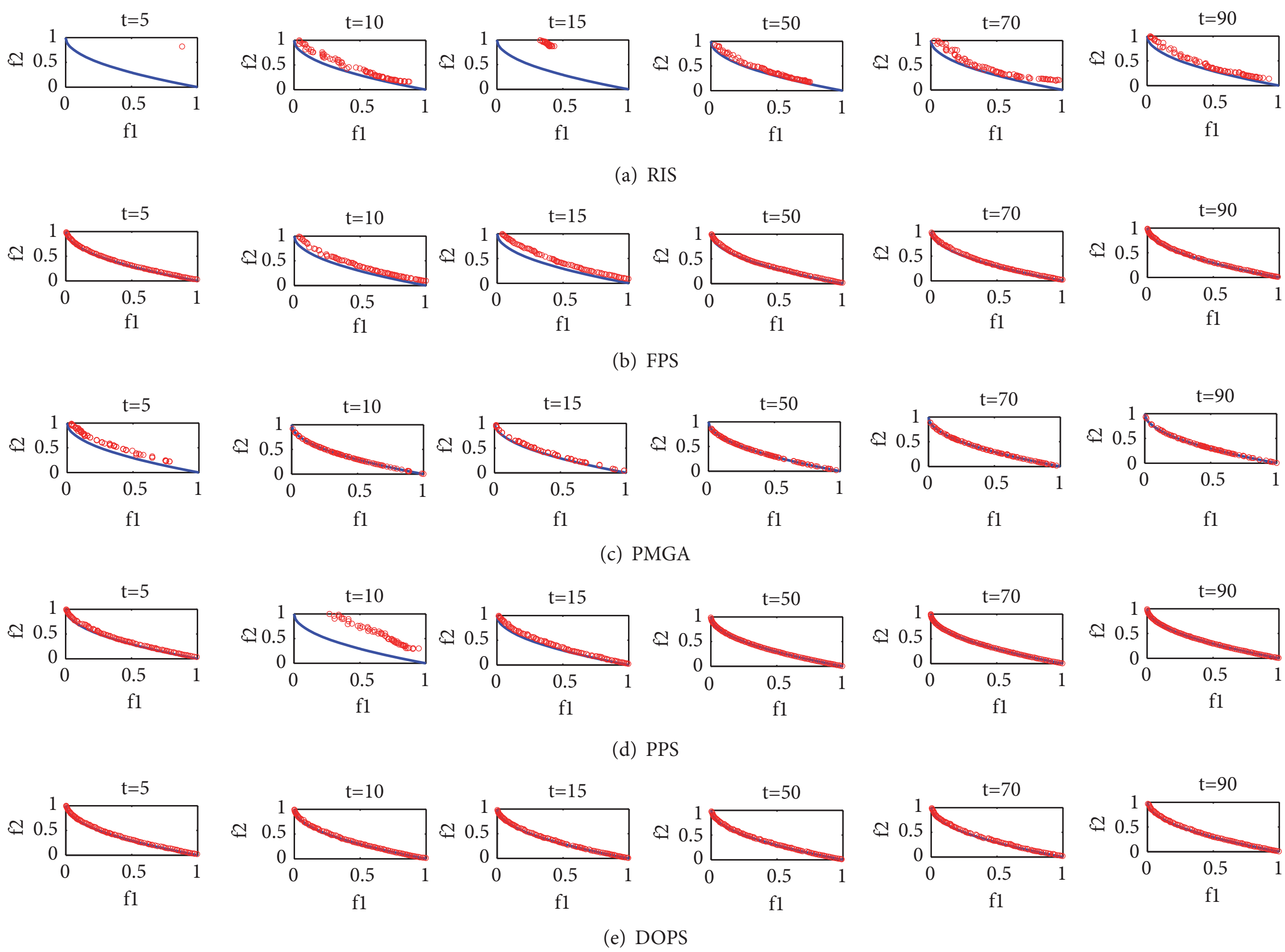


Figure 6: Final population distribution of the five strategies on FDA1.

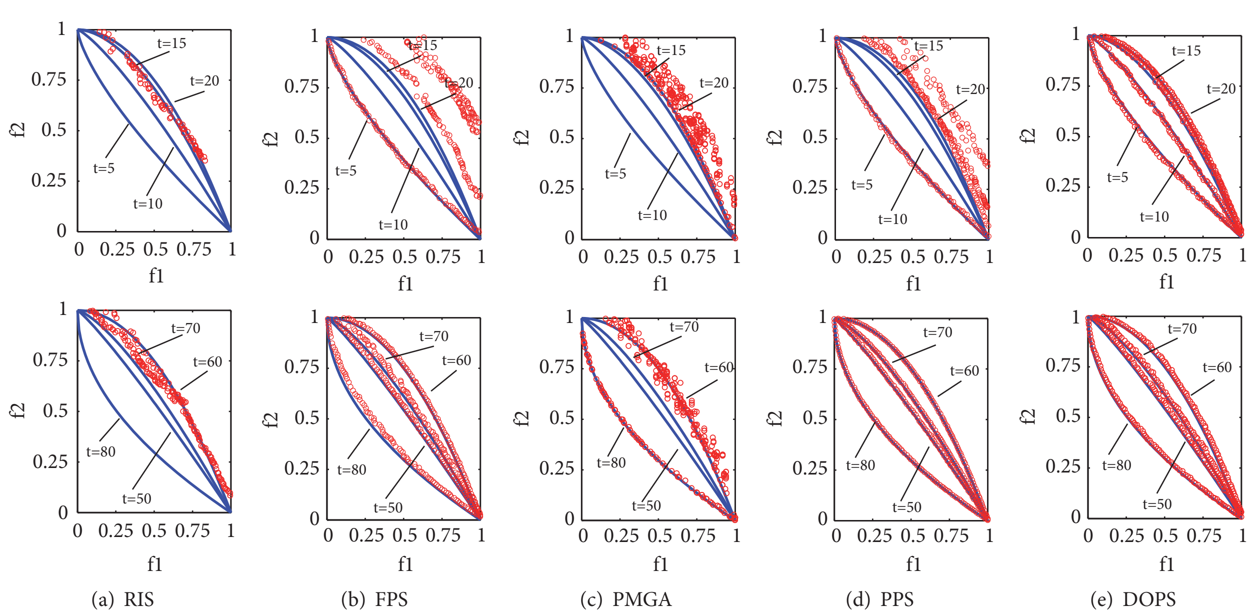
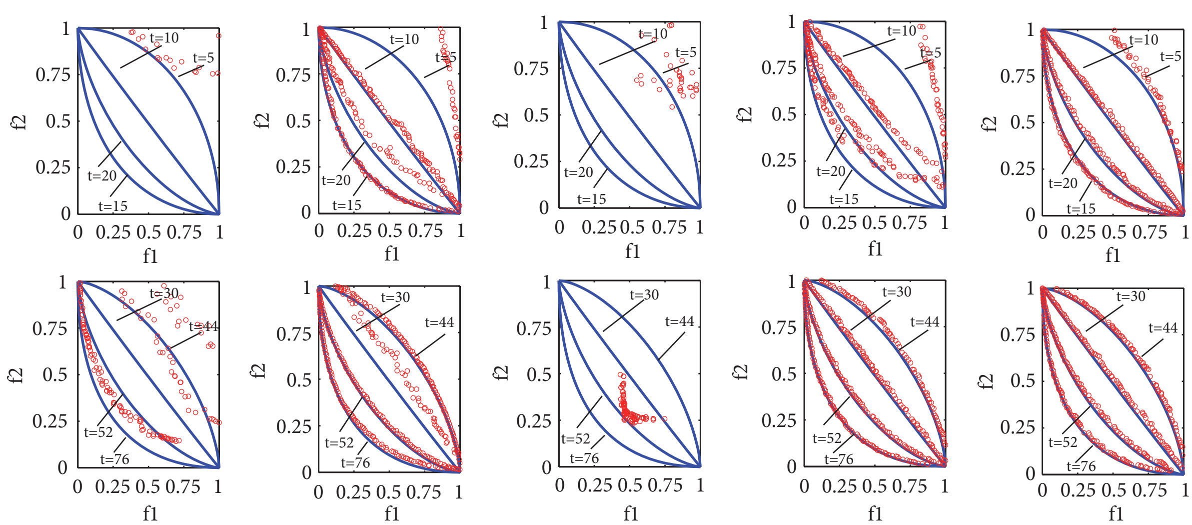


Figure 7: Final population distribution of the five strategies on dMOP2.

Table 3: Mean and Standard Deviation of MIGD values of five strategies on F5-F10. The values in bold face denote having the best effect in these five strategies.‡ and † indicate that DOPS is significantly better than and is equivalent to the corresponding strategy, respectively.

Problems	Statistic	RIS	FPS	PMGA	PPS	DOPS
F5	Total	1.1439(0.04184)‡	0.1852(0.08194)‡	3.1110(0.07348)‡	0.2323(0.07728)‡	**0.0442(0.01145)**
	1st stage	1.1844(0.06775)‡	0.5886(0.41127)‡	2.4771(0.36662)‡	1.0473(0.36248)‡	**0.1467(0.05625)**
	2nd stage	1.1344(0.04050)‡	0.1088(0.02892)‡	3.2192(0.24029)‡	0.0664(0.02698)‡	**0.0201(0.00147)**
	3rd stage	1.1341(0.06016)‡	0.0746(0.02526)‡	3.1110(0.07348)‡	**0.0169(0.00131)**	0.0197(0.00085)
F6	Total	0.5399(0.01228)‡	0.0548(0.01680)‡	1.2521(0.01678)‡	0.0751(0.04238)‡	**0.0241(0.00281)**
	1st stage	0.6958(0.03924)‡	0.1291(0.07744)‡	1.2312(0.06075)‡	0.3084(0.20490)‡	**0.0515(0.01462)**
	2nd stage	0.5103(0.01898)‡	0.0404(0.00820)‡	1.2799(0.03064)‡	0.0269(0.00916)‡	**0.0171(0.00035)**
	3rd stage	0.4956(0.01674)‡	0.0352(0.00504)‡	1.2521(0.01678)‡	**0.0143(0.00050)**	0.0181(0.00060)
F7	Total	0.6165(0.01535)‡	0.1273(0.02343)‡	1.2892(0.08301)‡	0.1006(0.04024)‡	**0.0451(0.01848)**
	1st stage	0.6764(0.03353)‡	0.3499(0.10311)‡	1.3816(0.11334)‡	0.4575(0.20361)‡	**0.0451(0.01848)**
	2nd stage	0.6009(0.01925)‡	0.0879(0.02554)‡	1.2468(0.09617)‡	0.0208(0.00415)‡	**0.0160(0.00054)**
	3rd stage	0.6037(0.02552)‡	0.0642(0.02467)‡	1.2892(0.08301)‡	**0.0133(0.00055)**	0.0165(0.00037)
F8	Total	0.9083(0.02482)‡	0.1418(0.00363)†	0.3795(0.00647)‡	0.1455(0.00463)‡	**0.1408(0.00249)**
	1st stage	0.7666(0.04113)‡	0.1944(0.01645)†	0.3978(0.02375)‡	0.2106(0.02369)‡	**0.1760(0.01330)**
	2nd stage	0.9473(0.04443)‡	**0.1313(0.00293)**	0.3708(0.00756)†	0.1335(0.00198)†	0.1316(0.00167)
	3rd stage	0.9366(0.02643)‡	0.1309(0.00169)	0.3795(0.00647)‡	**0.1301(0.00250)**	0.1333(0.00238)
F9	Total	1.1923(0.03253)‡	0.3542(0.06750)‡	2.5320(0.11486)‡	0.6186(0.19477)‡	**0.1305(0.02763)**
	1st stage	1.2308(0.10033)‡	0.9770(0.19262)‡	4.2814(0.09784)‡	2.5294(0.95190)‡	**0.4243(0.20966)**
	2nd stage	1.1787(0.06723)‡	0.2203(0.07491)‡	1.2583(0.00682)‡	0.2632(0.09615)‡	**0.0613(0.02554)**
	3rd stage	1.1875(0.04852)‡	0.2012(0.08490)‡	2.5320(0.11486)‡	0.0819(0.01601)‡	**0.0602(0.02883)**
F10	Total	1.0691(0.04720)‡	0.4280(0.05312)‡	0.7665(0.05569)‡	0.5097(0.09978)‡	**0.1388(0.03141)**
	1st stage	1.1510(0.07724)‡	0.6341(0.12031)‡	1.1785(0.25386)‡	1.5974(0.44011)‡	**0.2993(0.09248)**
	2nd stage	1.0608(0.04913)‡	0.4188(0.05751)‡	0.6958(0.01573)‡	0.3004(0.08774)‡	**0.1162(0.03636)**
	3rd stage	1.0385(0.07501)‡	0.3499(0.06662)‡	0.7665(0.05569)‡	0.2150(0.04395)‡	**0.0852(0.01222)**



(a) RIS (b) FPS (c) PMGA (d) PPS (e) DOPS

Figure 8: Final population distribution of the five strategies on F6.

accurately judge the PF or PS in the new environment, and the error will be smaller if the center point is used.

(3) For FDA2, CPS performed better in the total and 1st stages than IPS. This indicates that CPS is faster and more accurate than IPS in responding to environmental changes with invariant PS problems. But in the 2nd and 3rd stages, CPS is slightly less effective than IPS. For 3-dimensional problem, FDA4, the effect is similar to that of FDA2. This may be because, with the progress of the prediction process, the judgment of CTI will become more and more accurate,

Table 4: Mean and Standard Deviation of MHVD values of five strategies on F5-F10. The values in bold face denote having the best effect in these five strategies. ‡ and † indicate that DOPS is significantly better than and is equivalent to the corresponding strategy, respectively.

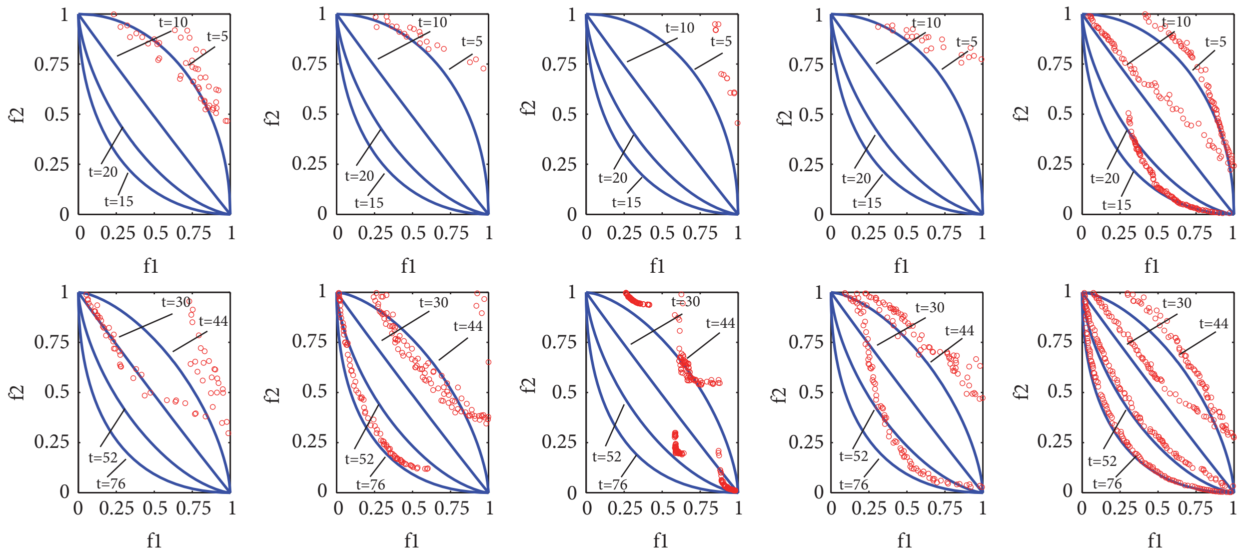
Problems	Statistic	RIS	FPS	PMGA	PPS	DOPS
F5	Total	1.3849(0.01419)‡	0.4745(0.07067)‡	1.7364(0.01230)‡	0.4845(0.05354)‡	**0.2923(0.01296)**
	1st stage	1.5641(0.03251)‡	0.9525(0.29421)‡	1.8196(0.04988)‡	1.3391(0.21636)‡	**0.4868(0.06892)**
	2nd stage	1.3441(0.01687)‡	0.3951(0.05421)‡	1.7344(0.03510)‡	0.3148(0.04150)‡	**0.2463(0.00293)**
	3rd stage	1.3406(0.02808)‡	0.3269(0.04382)‡	1.6988(0.01811)‡	0.2483(0.00111)‡	**0.2459(0.00219)**
F6	Total	0.9995(0.01531)‡	0.3050(0.03246)‡	1.5555(0.00974)‡	0.3436(0.04867)‡	**0.2483(0.00122)**
	1st stage	1.3182(0.04079)‡	0.4923(0.14787)†	1.7031(0.03430)‡	0.7177(0.22655)‡	**0.3684(0.03515)**
	2nd stage	0.9273(0.02093)‡	0.2646(0.02006)‡	1.5148(0.01269)‡	0.2626(0.01394)‡	**0.2481(0.00091)**
	3rd stage	0.9204(0.02104)‡	0.2565(0.00858)†	1.5259(0.03027)‡	**0.2468(0.00105)**	0.2483(0.00122)
F7	Total	1.0988(0.01522)‡	0.4448(0.04393)‡	1.5220(0.02041)‡	0.3625(0.02877)‡	**0.2654(0.00732)**
	1st stage	1.3190(0.03876)‡	0.8952(0.17527)‡	1.7286(0.01664)‡	0.8413(0.14139)‡	**0.3448(0.03924)**
	2nd stage	1.0465(0.02740)‡	0.3599(0.05159)‡	1.4745(0.02450)‡	0.2509(0.00448)‡	**0.2469(0.00117)**
	3rd stage	1.0464(0.02178)‡	0.3157(0.04522)‡	1.4715(0.02634)‡	0.2467(0.00126)†	**0.2463(0.00204)**
F8	Total	2.2789(0.02309)‡	0.3797(0.00916)†	0.9777(0.02016)‡	0.3954(0.01577)‡	**0.3783(0.00758)**
	1st stage	2.1534(0.06297)‡	0.5431(0.04274)‡	1.0026(0.06811)‡	0.6076(0.07844)‡	**0.4932(0.03185)**
	2nd stage	2.3030(0.02905)‡	**0.3413(0.00900)**	0.9828(0.04316)‡	0.3504(0.00781)†	0.3474(0.00564)
	3rd stage	2.3144(0.03879)‡	0.3405(0.00750)	0.9609(0.01666)‡	**0.3397(0.00780)**	0.3547(0.00668)
F9	Total	1.3930(0.01395)‡	0.6393(0.08717)‡	1.7018(0.01392)‡	0.6481(0.07278)‡	**0.3643(0.00475)**
	1st stage	1.5966(0.03298)‡	1.2334(0.16517)‡	1.9478(0.00387)‡	1.6530(0.26199‡)	**0.7094(0.08916)**
	2nd stage	1.3371(0.01545)‡	0.5259(0.10356)‡	1.7470(0.02775)‡	0.5175(0.10465)‡	**0.2814(0.01612)**
	3rd stage	1.3523(0.03077)‡	0.4705(0.12764)‡	1.5398(0.00912)‡	0.3014(0.02611)†	**0.2834(0.02490)**
F10	Total	1.3534(0.01879)‡	0.8905(0.07059)‡	1.1394(0.02832)‡	0.7476(0.05592)‡	**0.4360(0.04524)**
	1st stage	1.5587(0.03333)‡	1.2692(0.11566)‡	1.7628(0.10585)‡	1.5369(0.09760)‡	**0.7386(0.11058)**
	2nd stage	1.3022(0.02225)‡	0.8541(0.09269)‡	1.0906(0.03339)‡	0.6028(0.09317)‡	**0.3880(0.06687)**
	3rd stage	1.3071(0.02902)‡	0.7471(0.10528)‡	0.8921(0.00826)‡	0.5175(0.07129)‡	**0.3403(0.01635)**

Table 5: Five test problems with different characteristics.

Problems	Number of objectives	Changes types	Relationship between decision variables
FDA2	2	PF changes; PS is fixed	Linear
FDA4	3	PF is fixed; PS changes	Linear
dMOP3	2	PF is fixed; PS changes	Linear
F5	2	PF changes; PS changes	Nonlinear
F10	2	PF changes; PS changes	Nonlinear

Table 6: Mean and Standard Deviation of MIGD values of CPS and IPS. The values in bold face denote having the better effect in these two strategies.

Problems	Strategy	Total	1st stage	2nd stage	3rd stage
FDA2	CPS	**0.0087(0.00138)**	**0.0165(0.00138)**	0.0073(0.00180)	0.0063(0.00120)
	IPS	**0.0087(0.00064)**	0.0187(0.00228)	**0.0068(0.00133)**	**0.0058(0.00048)**
FDA4	CPS	**0.1521(0.00348)**	**0.1679(0.00649)**	0.1489(0.00323)	0.1479(0.00513)
	IPS	0.1530(0.00408)	0.1797(0.00843)	**0.1468(0.00648)**	**0.1467(0.00477)**
dMOP3	CPS	0.1551(0.01011)	0.2850(0.03400)	0.1210(0.02425)	0.1275(0.01648)
	IPS	**0.1349(0.01312)**	**0.2160(0.04473)**	**0.1074(0.01234)**	**0.1239(0.02219)**
F5	CPS	**0.5321(0.07584)**	**0.6463(0.24566)**	**0.5106(0.11876)**	**0.4995(0.10182)**
	IPS	0.6699(0.21041)	0.7706(0.29603)	0.7780(0.41179)	0.5141(0.08521)
F10	CPS	2.1766(0.21428)	1.6762(0.30120)	2.4446(0.16529)	2.1462(0.31530)
	IPS	**1.9126(0.16360)**	**1.4834(0.31371)**	**2.0169(0.43282)**	**2.0120(0.45767)**



(a) RIS (b) FPS (c) PMGA (d) PPS (e) DOPS

Figure 9: Final population distribution of the five strategies on F10.

so the prediction with CTI will become more and more accurate.

(4) For F10, we can see that IPS is better than CPS at all stages. This is a very strange phenomenon because F10 is the most complex among these problems. Its two consecutive PFs have different shapes and CTI is a point on PF. IPS is better may be because of two reasons as follows.

(a) Due to the complexity of F10, the prediction method based on the center point is not effective.

(b) The PF shape of F10 is suitable for IPS to predict.

In order to more intuitively observe the effect of CPS and IPS, we drew the trend chart of IGD of two strategies on dMOP3 and F5 with the change of environment. The smaller the IGD value is, the better the algorithm performance is. As shown in Figure 10, the abscissa represents the time step, which means that in which change the environment lies, and the ordinate represents the IGD value at that time step. The effect shown in Figure 10 is almost identical to that shown in Table 6. For example, on dMOP3, in the early stage, IPS is much better than CPS; in the later stage, IPS is slightly better than CPS.

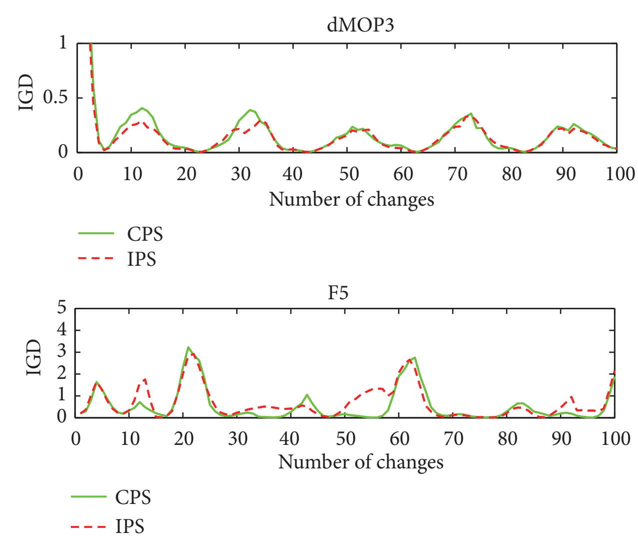


Figure 10: IGD trend comparison of CPS and IPS over number of changes for 20 runs on dMOP3 and F5.

7. Conclusion and Future Work

In this paper, a hybrid prediction strategy is proposed, which is carried through in decision space and objective space simultaneously. Mixed prediction strategy includes center point-based strategy and CTI-based strategy. The two strategies predict simultaneously from the decision space and objective space and obtain the predicted nondominant set of the next environment.

Nondominant set is the main body of the predicted population and plays an important role in guiding the evolution of population. In addition, in order to deal with the problem with periodic changes, some optimal individuals in the current population are introduced into the predicted population as memory set. Finally, a random set is introduced into the population. The number of the random individuals in the random set can vary adaptively according to the difficulty degree of the problem. The more difficult the problem is, the more random the individuals are in the random set and vice versa. Then, the two prediction strategies in the hybrid prediction strategy were compared to study the role of each strategy in solving the problem. Through experiments, it can be observed that both strategies have their own test problems that are suitable to solve. The two prediction strategies are just combined, so that they are complementary to each other to deal with DMOPs.

By comparing DOPS with four classical prediction strategies on 13 test problems with different characteristics, it can be seen that DOPS is effective in dealing with DMOPs. In addition, we will try to apply DOPS to practical problems, like scheduling resources in integrating 5G networks and the Internet of things, and make contributions to human development.

Data Availability

The data used to support the findings of this study are included within the article.

Ethical Approval

This article does not contain any studies with human participants or animals performed by any of authors.

Consent

Informed consent was obtained from all individual participants included in the study.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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