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Research Article

A User-Oriented Pricing Design for Demand Response in Smart Grid

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Demand response (DR) programs are designed to affect the energy consumption behavior of end-users in smart grid. However, most existing pricing designs for DR programs ignore the influence of end-users's diversity and personal preference. Thus, in this paper, we investigate an incentive pricing design based on the utility maximization rule with consideration of end-users' preference and appliances' operational patterns. In particular, the utility company determines the pricing policy by trading off the budget revenue and social obligation, while each end-user aims to maximize their own utility profits with high satisfaction level by scheduling multiclass appliances. We formulate the conflict and cooperative relationship between the utility company and end-users as a Stackelberg game, and the equilibrium points are obtained by the backward induction method, which exists and is unique. At the equilibrium, the utility company adopts real-time pricing (RTP) scheme to coordinate end-users to fulfill the benefit of themselves, i.e., under such price, end-users automatically maximize overall utility profits of the overall system. We propose a distributed algorithm and an adaptive pricing scheme for the utility company and end-users to jointly achieve the best performance of the entire system. Finally, extensive simulation results based on real operation data show the effectiveness of the proposed scheme.

1. Introduction

Demand response (DR) program plays an important role in peak shaving, demand leveling, and load consumption reduction in distribution network by affecting the energy consumption of end-users [1, 2]. As an essential element of a smart grid, user-oriented DR allows heterogeneous end-users participate in the electricity markets to earn customized utility by coordinating interaction between utility companies [3, 4]. The utility company offers DR service to improve customers' satisfaction, and in turn, the amount of power consumption requested by customers affects the DR policies. Furthermore, in a modernized distribution system, elastic appliances (e.g., energy storage system (ESS)) are deployed closer to end-users. The operational patterns of elastic appliances can be combined with DR services to increase utility benefits and reliability of the smart distribution grid [4–6]. To complement with customers' preference and appliances' operational patterns, user-oriented pricing design for DR programs has recently been regarded as a promising solution in smart grid.

Recently, in user-oriented DR programs, the utility company offers a variety of utility benefits, such as bill savings for participants, quality of satisfactions improvement, and better market performance. In practice, it is a challenging problem for a utility company to design appropriate DR programs. The utility company cares not only about what the customers pay but also about social obligation, e.g., social fairness that addressed customers' payment based on their income levels. There are several benchmark designs focusing on billing mechanism in which customers are rewarded by their contribution in achieving

system-wide welfare [7]. In general, the utility company adopts adequate incentive pricing schemes to achieve a trade-off between minimizing the electricity payment and maximizing the satisfaction levels of end-users as well as to achieve a minimum peak-to-average ratio (PAR) to help shift usage away from peak hours to less congested times [8, 9]. In recent years, user-oriented pricing design has attracted growing attention both in academia and in industry, especially with the emerging development of smart grid [10]. Mostly, flat pricing, time-of-use (TOU) pricing (e.g., peak time vs. off-peak time), and real-time pricing (RTP) are among the most popular pricing schemes. Flat pricing scheme refers to those methods where the utility company announces a fixed price for all time periods. In TOU pricing scheme, the intended time cycles are divided into several time periods and the distinct price value is set to encourage users to shift demand to off-peak hours and further reduce the demand deviation.

In smart grid environment, techniques such as smart metering and domestic energy management devices are available for end-users to collect energy consumption data in real-time fashion, which enables end-users pay much more attention to their energy consumption patterns. However, the different characteristic type of customers and appliances' operational patterns affect the responses of end-users. Some previous works have studied how to classify appliances into several categories based on their energy consumption patterns, but it is difficult to consider the proper appliance classes to develop a more efficient and reliable DR program, and there are great needs to combine appliances' power scheduling with end-users' preference jointly to design user-oriented pricing DR programs [11–13]. Consequently, balancing end-users' preference and appliances' operational scheduling is another unresolved challenge. In addition, the different responses or sensitivities to different incentive pricing scenarios are modeled as several levels of satisfaction by adopting the concept of utility function from microeconomics [8], but lack of knowledge about how to formulate end-users' preference is a barrier for user-oriented DR program design. Also, the adverse impact of pricing manipulation from the utility company is still challenging for user-oriented DR programs, which may diminish the economic saving effort of an individual [14].

In this paper, we study a user-oriented pricing for DR with consideration of end-users' preference and appliances' operational patterns. In the user-oriented DR mechanism, the utility company plays a key role, i.e., designing incentive pricing policies and balancing social revenue and power network stability, which are important adaptive processes to maintain the reliability and resilience of the power grid in face of unanticipated energy consumption demand. End-users respond to price policies and maximize their own utility profits with high satisfaction level by scheduling multiclass appliances. The complicated interactions and inherent hierarchical relationship among the utility company and end-users motivate us to use a game theoretical framework. Therefore, we propose a two-stage Stackelberg game model to capture the interactions, in which the utility company is the leader and end-users are followers. In order to eliminate the adverse effect of pricing manipulation from the utility company, we propose an adaptive balancing scheme for the utility company to explore the trade-off between the revenue and the stability of power grid over peak/off-peak power demand time.

Our contributions can be summarized as follows:

(i) We establish an interactive user-oriented pricing DR model with consideration of end-users' satisfaction and appliances' operational patterns by a Stackelberg game. At the Stackelberg equilibrium, the utility company and end-users achieve the optimal demand/response strategies with the best performance of the entire system.

(ii) We design a distributed algorithm for the utility company and end-users, where each entity can independently determine its best demand/response strategies with only local information sharing such that it protects end-users' privacy. We propose an adaptive balancing scheme for the utility company to explore the trade-off between budget revenue and the stability of power network over peak/off-peak power demand time.

(iii) The effectiveness of the proposed pricing scheme is verified under a special case of adaptive balancing scenario by extensive simulations in terms of total utility profits at the system level and the stability of power network measurement.

The paper is organized as follows. We describe the current state of the art in Section 2. The mathematical models for the system, multiclass appliances, end-users' preference, and the profits utility function model are presented in Section 3. Next, in Section 4, we formulated the incentive pricing DR mechanism as a two-stage Stackelberg model and proposed a distributed algorithm and an adaptive pricing scheme. In Section 5, the numerical examples are demonstrated and analyzed. Finally, Section 6 concludes the whole paper.

2. Related Works

In user-oriented DR programs, the interactive profits between the utility company and end-users, two rational entities with conflicting objectives, are characterized as a nonlinear function of power transmission scheduling, such as end-users' satisfaction level improvement, operation cost reduction, bill saving, and demand and supply balance. Usually, end-users are eager to earn minimal billing payment and maximal satisfaction level which are modeled as the *utility function* [8]. However, due to the large and distributed deployment of appliances with different energy consumption patterns, it is uneasy to improve the economy and reliability of energy supply [6]. In addition, appliances with different operational patterns may respond to the incentive prices differently for the operation cost reduction [15]. Besides, the profits of the utility company are formulated as economy and reliability metrics, which are indicated by billing revenue and squared Euclidean

distance (SED) between the instantaneous energy demand and average demand of power system [5, 7]. To achieve both optimality and social obligation for the utility company, the weighted summation billing revenue and network stability metrics are pretty suit for the utility company, and the weights or balance criteria can be adjusted to the trade-off between billing revenue and peak load shaving [16]. However, all those above studies are limited in the sense that there is either only single type of end-users or multiclass appliances treated as one entity or absence of the adjustable balance criteria in the user-oriented incentive price designing. Differently in our study, we conclude the operational patterns of appliances and design a multitype end-user satisfaction function whose goal is to selfishly maximize its own utility profits while adapting to the adjustable balancing scheme for the utility company so as to achieve the trade-off between budget revenues and the stability of power network over peak/off-peak load demand time at the system level.

In most cases, the optimization problem of pricing design in DR program is formulated as a nonlinear optimization problem which can be solved by convex optimization methods [8, 17] or game theory methods [6, 18]. Na et al. approximated the utility maximization problem as a convex problem and designed a distributed algorithm for the utility company and end-users to jointly compute optimal price and demand schedule based on a prime-dual decomposition and gradient algorithm [8, 17]. While in practice, for different objectives, the strong duality of the objective function cannot always hold and thus the obtained solutions are near-optimal solutions. Game theory has been seen as a useful mathematical tool to handle these intractable problems even when the convexity of the objective function cannot be guaranteed. Specially, due to the inherent hierarchical relationship framework of the Stackelberg game model, various works have modeled the DR program as Stackelberg game [19, 20]. The authors introduced a distributed algorithm that schedules power loads to meet an optimization objective using game theoretical approach and showed the advantage of the particular effectiveness in capturing the interaction among customers [6, 18]. Considering inherent vulnerabilities of privacy and security issues, Sabita et al. proposed a scheme to eliminate the threat of price manipulation from data communications on the reliability of power supply [21], even though much more reliable encryption and communication technologies are available nowadays. It is observed that end-users' preference diversities in the non-cooperative subgame of the Stackelberg game could cause reactive fluctuations in terms of price response for the utility company [16]. For the virtue of privacy protection in the distributed algorithm, there is great motivation for us to design distributed methods exploited in the Stackelberg game to protect privacy and preference information for optimization from attackers in the data communications network. However, limited work has emphasized on the circumstance where the Stackelberg game is designed to protect privacy parameters of heterogeneous end-users and alleviate the impact of pricing manipulation from the utility company.

3. System Model

We consider a distribution network with one utility company; set $\mathcal{N}$ of heterogeneous end-users and numerous appliances as shown in Figure 1. The utility company collects DR information from customers through the DR aggregator. For each end-user, we assume there is an *energy management controller* (EMC) unit which is embedded in end-user's smart meters. EMC units are connected with each other including the utility company through a communication channel using wireless technologies, e.g., WiFi, WiMAX, or LTE, which play the role of controlling and coordinating. The intended time cycle for operation is divided into T time slots, where $T \triangleq \|\mathcal{T}\|$ and $\mathcal{T}$ is the set of time slots. The utility company adopts different pricing schemes (e.g., flat-rate pricing, TOU, and RTP), and p_t (cent/kWh) denotes the incentive price for per unit power in time slot t. The aggregator and EMC units coordinate incentive price p_t in each time slot, and the final decision is broadcast by the utility company. Customers operate a set of $\{\mathcal{A}_1, \mathcal{A}_2, \ldots\}$ appliances, and each appliance is controlled by an EMC unit. $x_{i,a}^t$ (kWh) $(i \in \mathcal{N}, t \in \mathcal{T}, a \in \mathcal{A})$ denotes power consumption rate of appliances a for customer i in time slot t. $l_i^t = \sum_{a\in\mathcal{A}} x_{i,a}^t$ (kWh) $(i \in \mathcal{N}, t \in \mathcal{T}, a \in \mathcal{A})$ denotes total power consumption rate for all appliances in time slot t. l_i^t also represents the actual power transmission rate between the utility company and customer i, which is constraint by maximal and minimal capacity constraint, i.e., $l_{i,\min}^t \le l_i^t \le l_{i,\max}^t$.

3.1. Operational Pattern Model of Appliances. In a distribution network, the total set of appliances is classified into three categories $\{\mathcal{A}_1, \mathcal{A}_2, \mathcal{A}_3\}$ according to their energy consumption patterns and operational properties. Here, we briefly introduce each appliance class and its power consumption rate constraints [15].

3.1.1. Inelastic Appliances. The amount of power consumption rate of inelastic appliances $a \in \mathcal{A}_1$ is fixed in each time slot. However, they can be intermittently turned on/off without any performance degradation. For example, appliances such as battery chargers with fixed charging rates and vacuum cleaners can be classified into this set. The power consumption rate $x_{i,a}^t, a \in \mathcal{A}_1$ is constrainted by the following equation:

$$0 \le x_{i,a}^t \le x_{i,a}^{t,\max}, \quad \forall a \in \mathcal{A}_1, i \in \mathcal{N}, t \in \mathcal{T}, \tag{1}$$

where $x_{i,a}^{t,\max}$ means the appliances a are always running during time slot t.

3.1.2. Elastic Appliances with Memoryless Property. Different from inelastic appliances, the power consumption rate of elastic appliances with memoryless property can be flexibly adjusted within the subinterval time slot, whose performance (or satisfaction) depends only on the current power consumption level, for example, appliances such as light bulbs with controllable brightness and electric fans with

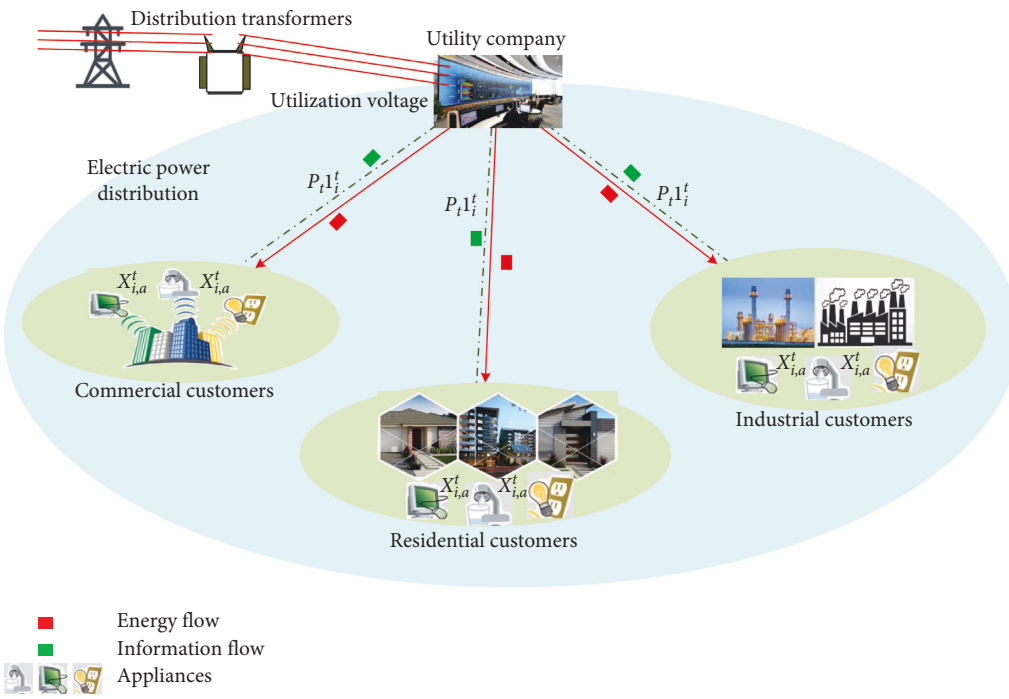

Figure 1: The illustration of system model.

controllable speeds. Appliances $a \in \mathscr{A}_2$ have the maximum and minimum values for power consumption rate in time slot t, that is,

$$x_{i,a}^{t,\min} \le x_{i,a}^t \le x_{i,a}^{t,\max}, \quad \forall a \in \mathscr{A}_2, i \in \mathscr{N}, t \in \mathscr{T}, \tag{2}$$

where the maximum power consumption rate $x_{i,a}^{t,\max}$ represents the power transmission level that $a \in \mathscr{A}_2$ are running with maximal power consumption rate. The minimum power consumption rate $x_{i,a}^{t,\min}$ represents the loads are always running during the day with lowest power rate requirements.

3.1.3. Elastic Appliances with Memory Property. The energy consumption rate of elastic appliances with memory property can also be flexibly adjusted within time slot $t \in \mathscr{T}$, whose performance depends not only on the total amount of power transmission but also on the previous memory level. For example, appliances such as ESS and EVs with controllable power charging/discharging rates usually have an upper bound on power charging rate (when $x_{i,a}^t > 0$), denoted by $x_{i,a}^{t,\max}$, and an upper bound on power discharging rate (when $x_{i,a}^t < 0$), denoted by $-x_{i,a}^{t,\min}$. Let $B_{i,a}^{\text{cap}}$ denote memory-based appliance capacity. Thus, the following constraints are present on power consumption rate of appliances $a \in \mathscr{A}_3$:

$$\begin{aligned} -x_{i,a}^{\min} &\le x_{i,a}^t \le x_{i,a}^{\max}, && \forall a \in \mathscr{A}_3, i \in \mathscr{N}, t \in \mathscr{T}, \\ 0 &\le B_{i,a}^t \le B_{i,a}^{\text{cap}}, && \forall a \in \mathscr{A}_3, i \in \mathscr{N}, t \in \mathscr{T}, \\ B_{i,a}^t &= \sum_{t=1}^{t} x_{i,a}^t + b_{i,a}^0, && \forall a \in \mathscr{A}_3, i \in \mathscr{N}, t \in \mathscr{T}, \end{aligned} \tag{3}$$

where $B_{i,a}^t$ denotes the dynamics of the memory-based energy storage level under the assumption that the power leakage is negligible and $b_{i,a}^0$ is the initial memory-based energy storage level.

Considering the high expense of memory-based elastic appliances, the extra economic damage is modeled by function $C_i(\mathbf{x}_{i,a})$ $(a \in \mathscr{A}_3)$, which captures the economic damage caused by charging/discharging operations, e.g., fast charging or deep discharging:

$$C_i(\mathbf{x}_{i,a}) = B_{\alpha_i} \sum_{t}^{T} x_{i,a}^{t^2} + B_{\gamma_i} \sum_{t=1}^{T} \left(\min\left(B_{i,a}^t - \delta_i B_{i,a}^{\text{cap}}, 0\right)\right)^2, \quad \forall a \in \mathscr{A}_3, i \in \mathscr{N}, t \in \mathscr{T}, \tag{4}$$

where $B_{\alpha_i}, B_{\gamma_i}, \delta_i > 0$ are weight factors that evaluate total extra cost of memory-based elastic appliances. By appropriately controlling the values of weight factors, customers can adjust the trade-off between its achieved utility and electric cost corresponding to fast charging and deep discharging. The cost function C_i is convex function of vector $\mathbf{x}_{i,a}$.

3.2. End-Users' Preference and Utility Function Model. The power transmission demands of end-users varied from different conditions, such as the type of end-users, the shape of supply-demand requirements, and sensitivity to incentive price. Meanwhile, end-users are independent entities in the local distribution network, e.g., household customers respond to the same incentive price differently from industrial customers. The reaction of end-users can be modeled as utility functions by adopting the concept of the *satisfaction*

function from microeconomics [22], and the corresponding satisfaction functions can be modeled as $s_i^t(l_i^t, \beta_i^t)$, where β_i^t is a preference parameter that highly depends on end-users' diversity and time. The satisfaction function $s_i^t(l_i^t, \beta_i^t)$ quantitatively models end-users' preference as a function of actual power transmission characterized by different features of various types of customers. Given the incentive price p_t, end-users are willing to maximize the summation of overall utility profits that is defined as utility function $U_i(l_i^t, x_{i,a}^t)$:

$$U_i^t\left(l_i^t, x_{i,a}^t\right) = -p_t l_i^t - C_i\left(x_{i,a}^t\right) + s_i^t\left(l_i^t, \omega_i^t\right), \quad \forall a \in \mathscr{A}, i \in \mathscr{N}, t \in \mathscr{T}. \tag{5}$$

For announced incentive price p_t, customer i tries to adjust power consumption rate $\left\{l_i^t, x_{i,a}^t, a \in \mathscr{A}\right\}$ to maximize their own utility, and the optimal power transmission scheduling is formulated by the following optimization problem:

$$\begin{aligned} \max_{\substack{\{l_i^t, x_{i,a}^t, p_t\} \\ l_{i,\min}^t \le l_i^t \le l_{i,\max}^t}} \quad & \sum_{i \in \mathscr{N}} U_i^t\left(l_i^t, x_{i,a}^t, p_t\right), \\ \text{subject to} \quad & (1) - (3). \end{aligned} \tag{6}$$

The objective function of (6) is concave, and the feasible set is convex. Therefore, an optimal point can be solved by *convex programming* techniques such as the interior point method (IPM) in a central fashion. However, the problem arising in (6) from a central manner is that the central processor needs to know the exact utility function parameters of customers, which violates customers' privacy. In Section 4, there is a distributed solution method that solves the optimization problem in local EMC, and the private parameters β_i^t are protected.

3.3. Utility Company's Utility Function Model. In a distribution network, the utility company serves as an intermediary that participates in wholesale markets, including day-ahead, real-time balancing, and ancillary services, to provide enough electricity supply from generators and then sell it to customers in a retail market. The utility company wishes to maximize the profit, as well as fulfill its obligation to serve the public and meet customers' satisfaction. Namely, the utility company is regulated so that not only do maximize its revenue through selling electricity, but induce customers to realize maximal social welfare. Hence, the utility company's utility function is defined as the weighted summation of economic revenue and SED [5]:

$$S_t\left(p_t, l_i^t\right) = \sum_{i \in \mathscr{N}} \left(w_u \sum_{t \in \mathscr{T}} (p_t - m_t) l_i^t - w_c\left(l_i^t - \bar{l}_i\right)^2 \right), \quad \forall i \in \mathscr{N}, t \in \mathscr{T}, \tag{7}$$

where m_t is the marginal price of electricity, $\bar{l}_i = (1/T)\sum_{t \in \mathscr{T}} l_i^t$ $(\forall i \in \mathscr{N})$ is the average power demand during the day, and $w_u \ge 0$ and $w_c \ge 0$ are weight factors measuring the total utility of budget and the deviation of customers' daily consumption, respectively. The optimization problem of utility company is formulated as

$$\begin{aligned} \max_{\substack{\{p_t, l_i^t\} \\ Q_t^{\min} \le \sum_{i \in \mathscr{N}} l_i^t \le Q_t^{\max}}} \quad & S_t\left(p_t, l_i^t\right), \\ \text{subject to} \quad & p_t^{\min} \le p_t \le p_t^{\max}, \forall t \in \mathscr{T}, \end{aligned} \tag{8}$$

where $Q_t^{\min}$ and $Q_t^{\max}$ are power transmission rate thresholds for all customers in local distribution network.

In this problem, if $w_u > 0$ and $w_c = 0$, the problem becomes a maximization problem of the total utility profit of revenue. On the other hand, if $w_u = 0$ and $w_c > 0$, the problem becomes a minimization problem of power supply deviation. By deciding weight factors appropriately, the utility company can adjust the trade-off between achievable budget revenue and the stability of power network.

4. Two-Stage Stackelberg DR Scheduling

In this section, we formulate a two-stage Stackelberg game model [23] to capture the interactions between the utility company and end-users. The first stage of this game is that the utility company determines the incentive pricing mechanism, and the second stage is that customers decide how much power to be consumed by appliances. It is natural to assume that the utility company is the first mover and customers are followers making their decision according to the price p_t. To obtain the Stackelberg equilibrium of the game, the backward induction method is used to derive the closed-form expressions of power transmission rate [23]. In particular, we first consider the power transmission rate $x_{i,a}^t$ determined by customers for given incentive price p_t.

4.1. Optimal Response of End-Users. Since not all customers respond to the incentive price in the same way, end-users' preference model in Section 2 can be extended to heterogeneous customer scenario. In this paper, we define customers' satisfaction function by different parameters as follows:

$$\begin{aligned} s_i^t\left(l_i^t, \beta_i^t\right) &= c_i l_{i,\max}^t f_i^t\left(l_i^t, \beta_i^t\right), \quad i \in \mathscr{N}, t \in \mathscr{T}, \\ f_i^t\left(l_i^t, \beta_i^t\right) &= \omega_i^{t^{\beta_i^t}}, 0 < \beta_i^t \le 1, \quad i \in \mathscr{N}, t \in \mathscr{T}, \end{aligned} \tag{9}$$

where $\omega_i^t = l_i^t / l_{i,\max}^t$ denotes the ratio between actual power transmission and maximal possible demand transmission with the utility company in normalized form. Let c_i denote valuation of unit power transmission for customer i and c_i be independent from time. c_i^t is varied from different customer types, e.g., valuation of industrial-type customers can be much greater than that of residential-type customers.

To analyze the Stackelberg equilibrium, we use backward induction and first consider the second stage of the game, i.e., given p_t, customers maximize their welfare function (6) by choosing the power transmission rate $l_i^t, i \in \mathscr{N}$. The optimal solution $l_i^{t*}(p_t)$ always exists and can be expressed as

$$l_i^{t*}(p_t) = \begin{cases} 0 \text{ or } l_{i,\max}^t, & \text{if } f_i^{t''}(\cdot) \geq 0, \\ \min\left\{l_{i,\max}^t, \omega_i^t f_i'^{-1}(p_t/c_i)\right\}, & \text{if } f_i^{t''}(\cdot) < 0, \end{cases} \tag{10}$$

where $f_i^{t_I^{-1}}(\cdot)$ is the inverse function of first-order derivative of the satisfaction function $f_i^t(.)$. When the satisfaction function is concave, the optimal solution is $\min\{l_{i,\max}^t, \omega_i^t f_i'^{-1}(p_t/c_i)\}$. On the other hand, the optimal total power transmission rate $L_t^*(p_t) = \sum_{i\in\mathcal{N}} l_i^{t*}$ of all customers in time slot t cannot exceed the power transmission rate capacity of utility company in a distribution network:

$$Q_t^{\min} \leq L_t^*(p_t) \leq Q_t^{\max}, \quad \forall t \in \mathcal{T}. \tag{11}$$

In addition, the second-order derivative of $\sum_{i\in\mathcal{N}} U_i^t$ is

$$\frac{\partial^2 U_i^t}{\partial l_i^t \partial l_k^t} = \begin{cases} \dfrac{c_i}{\omega_i^t} f_i^{t''}\left(\dfrac{l_i^t}{\omega_i^t}\right), & \text{when } k = i, \\ 0, & \text{when } k \neq i. \end{cases} \tag{12}$$

Since diagonal elements of the Hessian matrix are all negative, the off-diagonal elements are all zero. The Hessian matrix is positive definite, meaning that strong duality holds, and each customer and utility company can simply solve their local optimization problem determined by (10) and (11).

4.2. Optimal Response of Utility Company. In the above section, customers' optimal response $l_i^{t*}(p_t)$ and the total power transmission rate $L_t^*(p_t)$ are obtained. Next, the optimal pricing incentive strategy of the first stage will be analyzed based on the responses of customers. Knowing the optimal power transmission rate responses of customers, the utility company calculates the optimal incentive pricing by solving the optimization problem (8). Substituting (10) into (8), we obtain the utility function of the utility company corresponding to incentive price p_t as follows:

$$S_t(p_t) = \sum_{i\in\mathcal{N}} \left(w_u \sum_{t\in\mathcal{T}} (p_t - m_t) l_i^{t*}(p_t) - w_c \left(l_i^{t*}(p_t) - \bar{l}_i^*(p_t)\right)^2 \right). \tag{13}$$

Given the optimal response of customers (10), a function of incentive price p_t, the constraints on customers' power transmission rate can be rewritten as

$$p_t = l_i^{t*^{-1}}\left(l_i^{t*}\right) = c_i f_i'\left(\frac{l_i^{t*}}{\omega_i^t}\right), \quad \forall i \in \mathcal{N}, t \in \mathcal{T}. \tag{14}$$

Since (14) is a decreasing function with l_i^t, the constraints on incentive price p_t can be written as

$$p_t^{\min} \leq p_t \leq p_t^{\max}, \quad \forall t \in \mathcal{T}, \tag{15}$$

where $p_t^{\min} = \max\left\{m_t, c_i f_i'(l_{i,\max}^{t*}/\omega_i^t)\right\}$ and $p_t^{\max} = c_i f_i'(l_{i,\min}^{t*}/\omega_i^t)$, and the constraints are linear.

The optimization of $S(p_t)$ with respect to incentive price p_t now becomes

$$\begin{aligned} \max_{\{p_t\}} \quad & S_t(p_i^t), \\ \text{subject to} \quad & p_t^{\min} \leq p_t \leq p_t^{\max}. \end{aligned} \tag{16}$$

To ensure that the solution is the optimum, we need to check the negative-definiteness of the Hessian matrix. In this problem, the negative-definiteness of the Hessian matrix of p is parameter dependent and has been derived [18].

Theorem 1. *There exists a unique Nash equilibrium p_t^* and $\left\{l_i^{t*}, x_{i,a}^{t*}, \forall i, t\right\}$, and thereby a unique Stackelberg equilibrium.*

Proof. We write the summation of end-users and utility company's utility function from the overall system:

$$\begin{aligned} \max_{\substack{(l_i^t, x_{i,a}^t, p_t) \in \mathcal{X}, \\ Q_t^{\min} \leq \sum_{i\in\mathcal{N}} l_i^t \leq Q_t^{\max}}} \quad & \sum_{i\in\mathcal{N}} U_i^t\left(l_i^t, x_{i,a}^t, p_t\right) + S_t(p_t), \\ \text{subject to} \quad & (1) - (3), (15). \end{aligned} \tag{17}$$

The feasible set is convex set defined by constraints. Clearly, the optimal solution $(p_t^*, l_i^{t*}, \forall i \in \mathcal{N}, t \in \mathcal{T})$ exists. Moreover, there exist Lagrange multipliers $p_i^{t*}, \forall i, t$, such that (taking derivative with respect to l_i^t)

$$p_i^{t*} = U_i^{t'}\left(l_i^t, x_{i,a}^t, p_t\right) \geq 0. \tag{18}$$

Since the right-hand side is independent of customer i, the Lagrange multipliers $p_i^{t*} \geq 0$ for all customers. Check the KKT condition for system utility function, since both the utility problems of customers and utility company are concave, the KKT conditions are necessary and sufficient for the optimality. □

4.3. Distributed Pricing Incentive DR Algorithm. The above theorem derives the existence and uniqueness of an equilibrium in the pricing incentive DR problem and also motivates a distributed solution, that utility company coordinates customers to jointly compute an equilibrium based on utility functions. The operation of the proposed algorithm and the interaction between the utility company and end-users are depicted in Figure 2.

As described in Algorithm 1, within one loop, the utility company updates power transmission rate capacity $L_t^*(p_t)$ according to (11) and further calculates $p_t^{\min}$ and $p_t^{\max}$ at the beginning of time slot t and then constructs utility function optimization problem $S(p_t)$ according to (13). Eventually, the utility company figures out the optimal price by solving (16) and broadcasts new value of price p_t^* and keeps on receiving customers' response of l_i^{t*}. On the other hand, customers keep on receiving incentive price information from the utility company all the time. As described in Algorithm 2, at the beginning of time slot $t \in \mathcal{T}$, EMCs update the operational properties of appliances models. By solving the constructed utility function $U_i^t(l_i^t, x_{i,a}^t, p_t)$ locally, the EMC in customer i gets the optimal power rate $l_i^{t*}(p_t)$ and

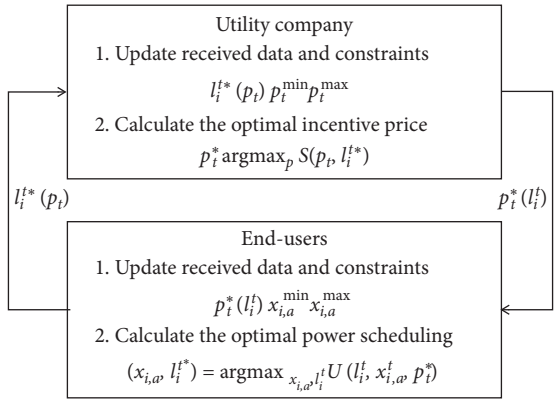

Figure 2: Illustration of operation of the proposed algorithm.

Input: w_u, w_c, m_t, $x_{i,a}^{t*}$, l_i^{t*}.
Output: p_t^*.
(1) Initialize: $L_t(p_t)$, p_t randomly.
(2) **while** receive information from customers, $t \in \mathcal{T}$ **do**
(3) Update power rate capacity $L_t(p_t)$ by (11).
(4) Update constraint boundary $p_t^{\min}$, $p_t^{\max}$ by (15).
(5) Construct utility function optimization problem (13).
(6) Calculate optimal value of price p_t^* by solving (16).
(7) Broadcast new value of price p_t^*.
(8) Receive customers' response of l_i^{t*}.
(9) **end while**
(10) **End**

Algorithm 1: Utility company algorithm.

Input: c_i^t, β_i^t, B_{α_i}, B_{γ_i}, δ_i, p_t^*.
Output: $x_{i,a}^{t*}$, l_i^{t*}.
(1) Initialize: $x_{i,a}^t$, l_i^t randomly.
(2) **while** receive information from utility company **do**
(3) Update optimal responsive price p_t^*.
(4) Update power rate constraints $x_{i,a}^t$ by (1)–(3).
(5) Construct customer i utility function $U_i^t(l_i^t, x_{i,a}^t, p_t)$.
(6) Calculate optimal response $l_i^{t*}(p_t)$ by (10).
(7) Send optimal response $l_i^{t*}(p_t)$ to utility company.
(8) Receive electricity price p_t^* from utility company.
(9) **end while**
(10) **End**

Algorithm 2: Customer $i \in \mathcal{N}$ algorithm.

$x_{i,a}^{t*}(p_t)$ corresponding to the given p_t^* and then sends the power rate information to the utility company. Until it converges, the EMC applies the power consumption rate scheduling to appliances.

The complexity of the algorithm is mainly driven by the number of iterations and the number of customers participating in the framework. Denote k as the number of iterations to satisfy the convergence with some precision; the complexity of the algorithm is $\mathcal{O}(kN)$. Indeed, the amount of k depends on the number of customers and the shape of supply and demand.

4.4. Proposed Adaptive Balancing Scheme. In the incentive price design process, the utility company has one important functionality to adjust the weights (w_c, w_u) of the summation in utility function (7) to make a balance of achievable revenue and stability of the power networks over peak/off-peak power demand time. While the bidirectional interaction between the utility company and end-users performs in a more timely and effective way, it is crucial for the utility company to carry out adaptive strategies for smoothing the fluctuation of energy demand. Next, we proceed to analyze the adaptive scheme with the goal of

achieving the trade-off between social revenue and stability of power supply. We define the balance criteria μ_t corresponding with time slot t as $\mu_t = w_c/w_u$, $t \in \mathcal{T}$. Using μ_t in (13), the Stackelberg equilibrium of Theorem 1 can be derived as the following proposition.

Proposition 1. *There exists a unique Nash equilibrium* $\{p_t^*, \mu_t^*, \forall t\}$ *and* $\{l_i^{t*}, x_{i,a}^{t*}, \forall i, t\}$, *and thereby a unique Stackelberg equilibrium.*

As proven in the proof of Theorem 1, there exists only one positive solution p_t^* for the incentive price design given by (17). Since $w_u \geq 0$ and $w_c \geq 0$, the utility company selects the balance criteria with the constraint of $\mu_t > 0$. Therefore, the Nash equilibrium in the adaptive balancing scheme scenario exists and is unique, and hence the Stackelberg game also admits a unique equilibrium $\{p_t^*, \mu_t^*, \forall t\}$ in Proposition 1.

Utility function (7) indicates that the deviation of energy supply will decrease (increase) if the balance criteria μ_t are increased (decrease) by the adaptive setting. In other words, when the utility company attempts to maintain the stability of the power supply, it might suffer economic revenue damage by updating the incentive price p_t^*. Theoretically, the utility company can acquire the best status of utility profits by balancing social revenue and power supply deviation at the equilibrium balance criteria point μ_t^*. In practice, it is impossible for most cases to adapt μ_t^* since the utility company has to fulfill the social obligation that the electricity price should be accepted by most end-users according to their income levels. Consequently, we propose interesting adaptive balancing schemes by setting μ_t:

(1) $\mu_t \gg \mu_t^*$: considering the worst case $\mu_t \propto \infty$ when $w_u = 0$, the optimal response of the utility company is merely to maintain the stability of the power network, while it may sacrifice the budget revenue from making energy transactions with end-users. In this scenario, it is appropriate for peak/off-peak power demand periods so as to minimize the deviation of power supply by setting relative larger balance criteria μ_t.

(2) $0 \leq \mu_t \ll \mu_t^*$: the worst impact that the adaptive scheme can cause is by setting μ_t so low that the stability measurement of power network occupies rarely any weight in the utility profit $S(p)$. In other words, out of peak/off-peak power demand time periods, to maximize budget revenue, the balance criteria should be set as smaller, as long as the deviation of power supply stays in a tolerable degree.

(3) $\mu_t \propto \mu_t^*$: when $\mu_t = \mu_t^*$, the adaptive balancing scheme reaches a possible best performance of the entire system, in which the utility company achieves a system-level balance of budget revenue and deviation of power supply.

5. Numerical Example

5.1. General Setup. We consider a distribution network with one utility company and set $\mathcal{N}$ of heterogeneous consumers and various appliances. The optimization period is set as 24 hours (i.e., $T = 24$), and the length of each time interval is 1 hour. We randomly generate the daily customers' demand data based on the daily report released by the U.S. Federal Energy Regulatory Commission (FERC) [24] for different types of end-users. Customers have a random number of appliances that follow the strict energy consumption scheduling constraints with power scheduling limits $x_{i,a}^{t,\max} = 20\,\text{kWh}$, $a \in \mathcal{A}$, and $B_{\alpha_i} = 5$, $B_{\gamma_i} = 5$, $\delta_i = 2$, $B_{i,a}^{\text{cap}} = 40\,\text{kWh}$, $\forall a \in \mathcal{A}_3$, unless mentioned otherwise. The valuation of unit power $c_i = 100$ is set equally for different types of end-users.

Due to the inherent diversity of customers, the customer preference model in Section 3 is extended to multiple-type features with preference parameter β_i^t. On the one hand, β_i^t indicates customer-type variation and illustrates the satisfaction preference for heterogeneous customers. Different preference parameter β_i^t reflects the different preferences of customers. For example, industrial customers will make more profits than commercial customers with the same power transmission rate. In other words, the satisfaction valuation can be ranked corresponding to β_i^t and smaller β_i^t represents customers earning high profits. On the other hand, ω_i^t indicates power transmission sensitivity. For instance, satisfaction valuation of industrial-type customers drops rapidly when ω_i^t decreases. This is because low power transmission rate will lead to low productive effect. In contrast, the satisfaction valuation of residential customers may still be high even if ω_i^t is low. This is because residential customers usually need a few amount of power to support daily life, and they can drain power from memory-based elastic appliances instead of transporting power from a utility company. Moreover, residential customers are much more sensitive to power transmission rate, and they are more flexible to coordinate power transmission rate to reduce daily bills, so the first-order derivative of satisfaction valuation to l_i^t is larger than other types of customers.

In this section, we consider three types of customers, i.e., residential-type customers, commercial-type customers, and small industrial-type customers. Different types of customers are characterized by satisfaction function with different parameters, i.e., customers are divided into three types evenly, $\beta_i^t \in (0.1, 0.3]$, $\beta_i^t \in (0.3, 0.6]$, and $\beta_i^t \in (0.6, 0.9]$, as shown in Figure 3. Preference parameter β_i^t of customer i is randomly chosen from the customer category range according to type-varying characteristics. Valuation of per power transmission rate c_i is randomly chosen from $[0, 1]$. Specially, the satisfaction function satisfies $f_i^t(0, \beta_i^t) = 0$ and $f_i^t(1, \beta_i^t) = 1$. Also, $f(\cdot)$ is a no-decreasing, twice-differentiable function, and it is concave in the interval $[0, 1]$. Thus, customers' valuation of power transmission rate during time slot t is fulfilling the assumed properties described in our previous work [16].

5.2. Discussion on Adaptive Balance Criteria. Since the major cost of utility company is on maintaining of stability of power grid, we ignore the cost on infrastructure constructions and stress on the trade-off between power transmission revenue and stability of distribution network. The balance criteria $\mu_t = w_c/w_u$ trade off between total budget revenue of

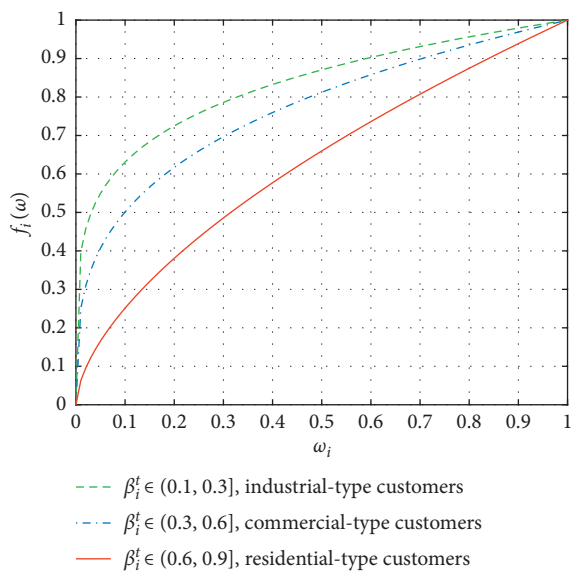

Figure 3: End-users' satisfaction level with different β_i.

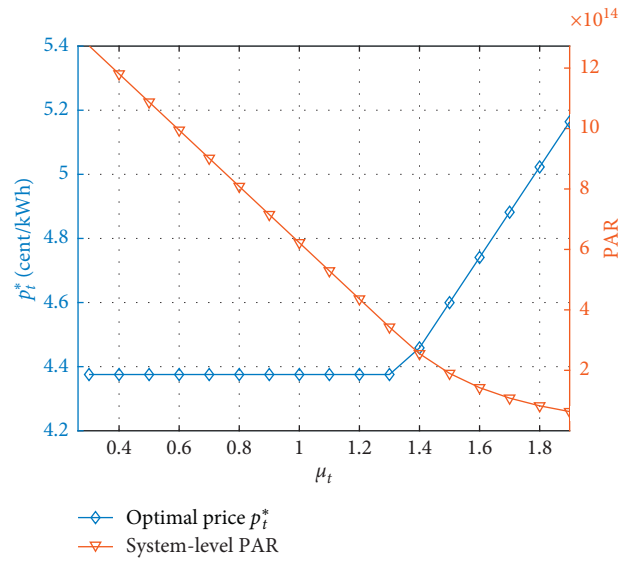

Figure 4: Optimal price and the stability measurement of power network with μ at equilibrium.

the utility company and the deviation of power supply. The utility company has the authority to decide the total revenue and coordinate provided power deviation by adopting different balance criteria μ_t. Based on this game framework, we ran the two-stage Stackelberg game algorithm and analyzed the Stackelberg equilibrium under different balance criteria μ_t via the utility profits at the system level.

We exploited how the change in incentive price p_t^* and the deviation of power network and, in turn, how the billing cost of end-users and utility profits of overall system are affordable and achievable at equilibrium, when the utility company set the balance criteria μ_t, which varied from 0.2 to 2. Figure 4 shows the optimal incentive price p_t^* and deviation of power network measured by PAR (i.e., $\max\{Q_t^*\}/(1/T)\sum_{t\in\mathcal{T}} Q_t^*$) at equilibrium corresponding to different balance criteria μ from the view of the utility company. It is important to note that the optimal incentive price increases and the deviation decreases corresponding to the increase of balance criteria μ. Therefore, larger balance criteria μ will incur a higher stability of the power network which verifies the effectiveness of the proposed adaptive balance criteria scheme in Section 4.4. From the view of customers, the billing cost increases linearly with the system scale when they purchase energy from the utility company, as depicted in Figure 5. However, it is not linear with the balance criteria μ_t for the different responses of energy usage amount Q_t^* from the end-users to the optimal incentive price p_t^* under balance criteria μ_t at equilibrium.

For the sake of illustration, we use $N = 10$ and $\mu \in [0, 2]$ with the granularity $\Delta\mu = 0.1$ for the plot. Figure 6 depicts the utility profits (i.e., $\sum_{i\in\mathcal{N}} U_i^* + S_t^*(p_t)$) of the overall system obtained by the two-stage Stackelberg game method at equilibrium versus μ_t compared with relevant utility maximization DR by distributed primal-dual decomposition methods [17]. It shows that whatever the pricing methods are studied, the proposed two-stage game theoretic approach outperforms the existing methods at equilibrium. And the

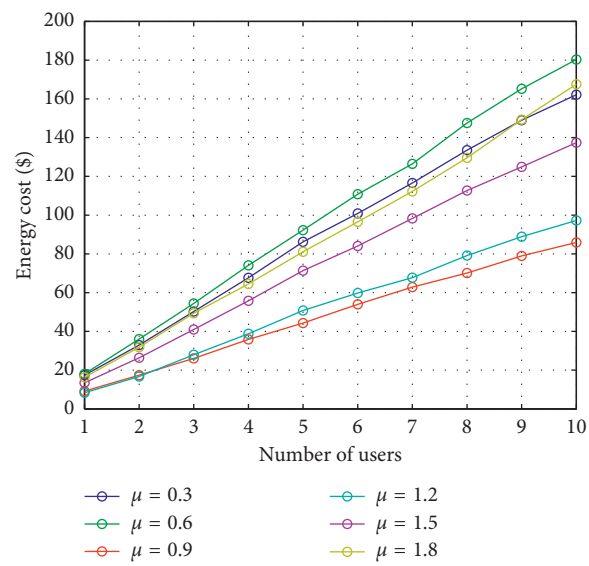

Figure 5: End-users' billing cost with μ at equilibrium.

utility profits reach the maximal value when the balance criteria are set $\mu_t = 0.9$. In other words, the optimal balance criteria $\mu_t^* = 0.9$ enable achieving the best possible performance trade-off between economic budgets cost and power supply deviation of the overall system.

5.3. Results with a Fixed Balancing Scheme. In order to analyze the impact of end-users' preference and power scheduling of appliances, we show the power response scheduling of three types of end-users (i.e., residential, commercial, and industrial customers) under a fixed balancing scheme with the optimal balance criteria $\mu_t = \mu_t^*$ and system scale $N = 200$. Figure 7 shows the load profile of the three regular end-users, which indicates that customers have shifted the peak load

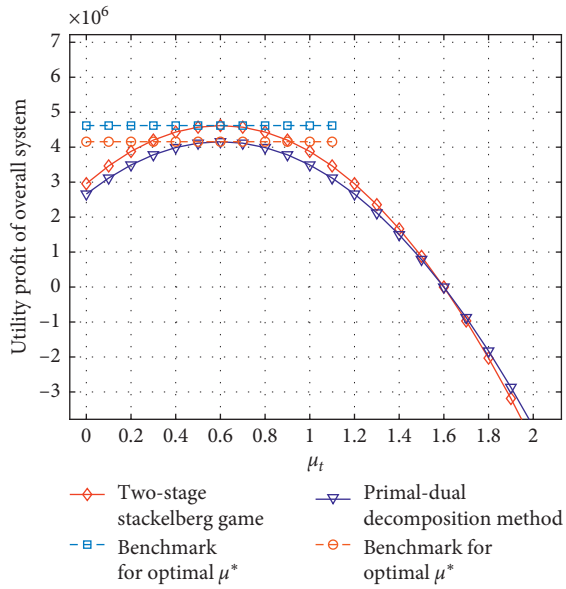

Figure 6: Utility profit of overall system with μ at equilibrium.

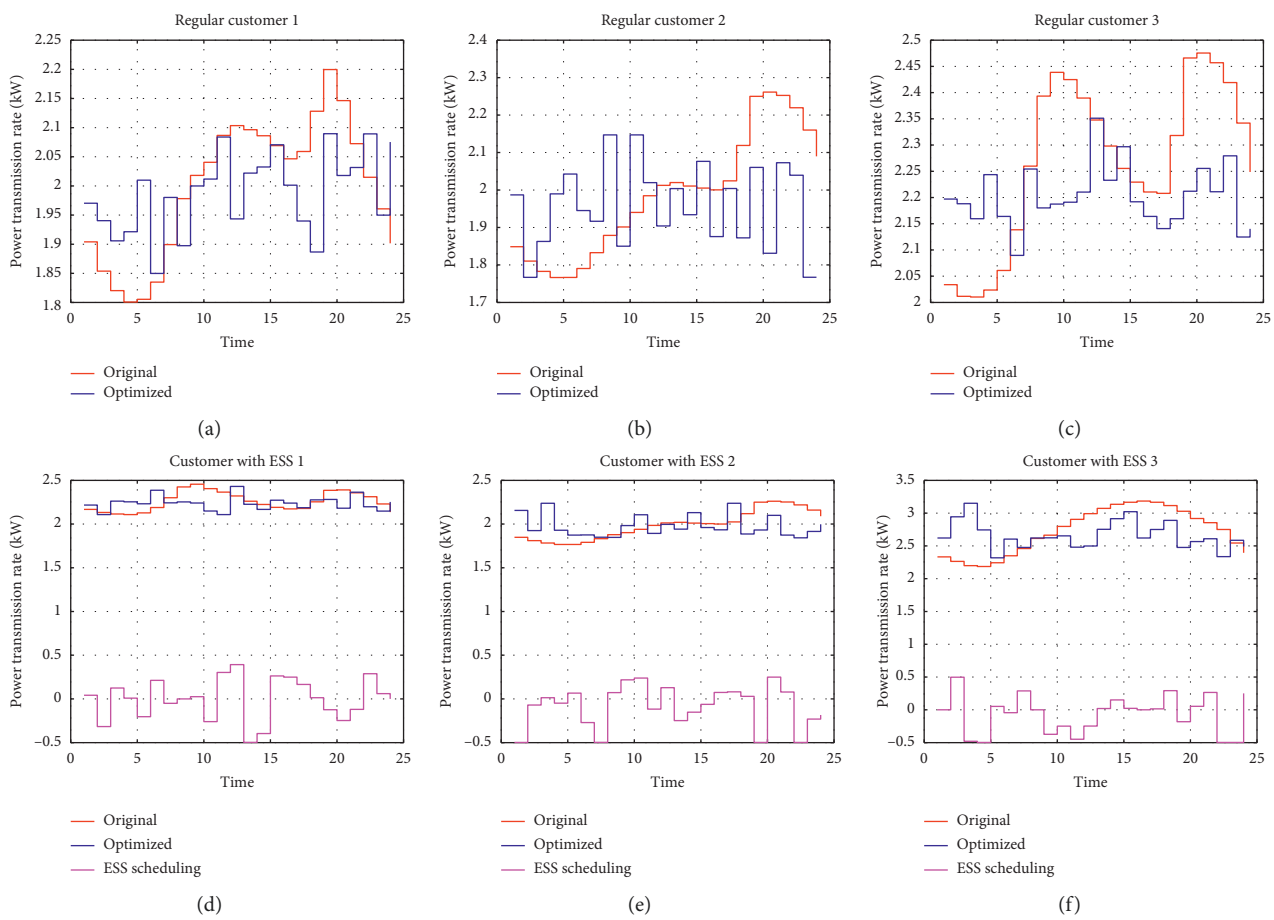

Figure 7: Power scheduling for single customer before and after optimization. (a) Residential customer. (b) Commercial customer. (c) Industrial customer. (d) Residential customer with ESS. (e) Commercial customer with ESS. (f) Industrial customer with ESS.

demand to off-peak time. However, the load curve of each customer is fluctuating over time. In practice, customers can decide their preference parameters and schedule the power consumed by appliances flexibly with more consideration of billing cost reduction or high-level satisfaction. Nevertheless, the aggregated power provision of the utility company is well

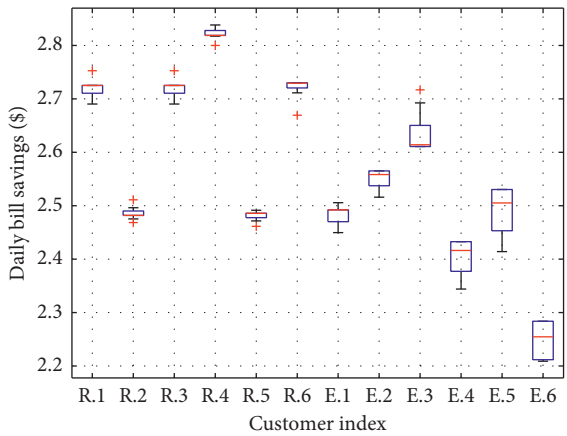

Figure 8: Billing cost reduction for 12 randomly selected end-users before and after optimization.

Table 1: Comparison of balancing mechanisms.

Schemes	Utility profit	Revenue (M·$)	PAR
No	9.236×10^6	5.382	1.275
Fixed	9.549×10^6	4.975	1.187
Adaptive	9.498×10^6	4.693	1.013

balanced, and the peak power demand has reduced from 447.04 kW to 433.99 kW and PAR reduced from 1.377 to 1.134. Considering the elasticity of energy storage facilities in achieving peak shaving and bill saving, Figure 8 using box plot depicts the statistics of daily bill savings for 12 end-users who are randomly selected and includes six regular customers and six customers with energy storage facilities. It shows that the absolute bill savings of customers with energy storage facilities are much higher than those of the regular customer without energy storage facilities as the energy storage facilities provide partial energy demand in peak/off-peak power demand times when the incentive price is much higher.

5.4. Results with Adaptive Balancing Scheme. In this section, we show numerical simulations by employing an adaptive balancing scheme. In the adaptive balancing scheme, the utility company set larger balance criteria μ_t (i.e., $\mu_t \gg \mu_t^*$) to maintain the stability of power network during peak/off-peak load demand times and much smaller $0 \le \mu_t \ll \mu_t^*$ out peak/off-peak load demand times. In this scheme, customers are induced by the utility company to adapt power consumption so as to make a contribution to flat power scheduling and, in turn, are rewarded with a reduction in billing cost. For comparison, the fixed balancing scheme is set with $\mu_t = \mu_t^*$ to indicate the best possible system-level performance status. Table 1 shows the comparative results between fixed and adaptive balancing mechanisms. It can be concluded from the table that even though the fixed balancing scheme achieves high utility profits of the overall system, the adaptive balancing scheme obtains high-level stability of power network with a relatively high utility profit.

6. Conclusion

In this paper, we propose a user-oriented pricing DR program based on the utility maximization rule with considerations of end-users' preference and appliances' operational patterns as well as the adaptive dynamic pricing scheme for the utility company. We formulate the conflict interaction between the utility company and end-users as a two-stage Stackelberg game, in which the utility company determines the pricing policy by trading off the budget revenue and the stability of the power network. End-users aim to minimize their own billing cost and operation cost of appliances with high satisfaction level by scheduling energy consumption of appliances. We design a distributed algorithm using the backward induction method to obtain the Stackelberg equilibrium for the utility company and end-users to jointly compute the best performance possible of the entire system. At the equilibrium, when end-users selfishly optimize their own utility profits, they automatically maximize the utility profit of the overall system. The effectiveness of our proposed pricing scheme is verified under a special case of adaptive balancing DR scenario in terms of utility profits at the system level. In the future, it will interesting to study whether this work can be improved by considering the competition among end-users with an oligopoly utility company market.

Data Availability

The data used to support the findings of this study are available from daily report released by the U.S. Federal Energy Regulatory Commission (FERC) [24].

Disclosure

A portion of this paper was presented at the Proceedings IEEE ICC, Kansas City, MO, USA, May 2018 [16].

Conflicts of Interest

The authors declare that they have no conflicts of interest.

Acknowledgments

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