Research Article
SIC-Coding Schemes for Underlay Two-Way Relaying Cognitive Networks

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In this paper, we propose an underlay two-way relaying scheme with the successive interference cancellation (SIC) solution in which two secondary sources transmit simultaneously their data to each other through secondary relays. The proposed scheme is operated in only two time slots and under an interference constraint of a primary receiver, denoted as the UTW-2TS scheme. In the UTW-2TS scheme, the secondary relays employ the SIC operation to decode successively the data from received broadcast signals and then encode these data by two techniques: digital network coding (DNC) enforced by XOR operations (denoted as the UTW-2TS-DNC protocol) and superposition coding (SC) enforced by power domain additions (denoted as the UTW-2TS-SC protocol). A selected secondary relay which subjects to maximize decoding capacities and to minimize collection time of channel state information in two protocols UTW-2TS-DNC and UTW-2TS-SC experiences residual interferences from imperfect SIC operations. Outage probabilities and throughputs are solved in terms of exact closed-form expressions to evaluate the system performance of the proposed protocols. Simulation and analysis results provide performance enhancement of the proposed protocols UTW-2TS-DNC and UTW-2TS-SC owing to increase the number of the cooperative secondary relays, the interference constraints, and the distances from the secondary network to the primary receiver. The best throughputs are pointed at optimal interference power allocation coefficients and optimal locations of the selected secondary relay. Considering the same power consumption, the UTW-2TS-DNC protocol outperforms the UTW-2TS-SC protocol. Finally, the simulation results are collected to confirm the exact analysis values of the outage probabilities and throughputs.

1. Introduction

In recent years, the radio spectrum has become scarce due to the increasing bandwidth demand for mobile multimedia services and the explosive development of next-generation wireless networks such as wireless sensor networks, Internet of Things (IoT), and fifth-generation (5G) networks. On the other hand, the utilization of the licensed frequency spectrum versus time and space is low [1]. In this context, cognitive radio was proposed as an effective spectrum sharing solution in which secondary users (SUs) coexist with primary users (PUs) [2]. The SUs can operate flexibly and intelligently in (interweave, overlay, and underlay) protocols to access the licensed frequency spectra of the PUs as long as quality of service (QoS) of the primary network is maintained [3, 4]. In the interweave cognitive radio [5], the SUs use spectrum sensing methods to detect the unoccupied spectra to avoid interference to the PUs. In an opposite way, the SUs in the overlay and underlay methods can access the licensed bands at the same time with the PUs [6–9]. The overlay approach requires the cooperation between the SUs and the PUs; i.e., the secondary transmitters have to combine the data of the SUs and the PUs and then send the combined data to the intended secondary and primary receivers [6]. The SUs in the underlay approach can access the spectra at any time provided that the interferences affected on the PUs must be below a tolerable interference level [7–9]. Based on the required QoS, the PUs can calculate the tolerable interference
level and send it to the secondary network so that the SUs can appropriately adjust their transmit powers.

Two-way communication networks have been much interested in switching data between interactive users in which many research studies have been launched to enhance performance [10]. Two-way cooperation solutions investigated in [10] have great attractions owing to increase significantly spectrum utilization efficiency. The operation of two-way relaying is to exchange data between users with the help of other intermediate users as amplify-and-forward (AF) and decode-and-forward (DF) devices, denoted as relays. In [11], an optimum relay is selected to decode received signals from the transmit users, mix these data by XOR operation, and then broadcast back to both users. The operation of the optimum relay in [11] is named as digital network coding (DNC).

1.1. Related Work and Motivation. Proposals for increasing performance in the underlay two-way relaying networks have been studied in [12–18]. The authors in [12] combined DNC and opportunistic relay selection (ORS) to decrease the outage performance of the secondary two-way relaying network under the interference constraint required by the primary receiver. Moreover, the best relay selection strategy in the third time slot, which follows a max-min criterion, was proposed in [12] to significantly increase the system outage probabilities, as compared with the traditional DNC approach [12]. By expanding the published work [12], Toan et al. in [13, 14] analyzed the performance of the underlay two-way relaying communication systems with the presence of multiple primary receivers. The authors in [15] evaluated the outage probabilities and symbol error probabilities of many primary one-way networks and a secondary two-way network operating on both the independent and identically distributed (i.i.d.) and independent but nonidentically distributed (i.i.d.) Nakagami-m fading channels. Imperfect channel state information (CSI) from the SUs to the PUs have been included in probability analyses [16]. The authors exploited both the direct and relaying links with appropriate diversity combinations in an underlay three-phase two-way scheme [17]. In [18], the authors considered an underlay wiretap cognitive two-way relaying network in which two SUs exchange their messages via multiple secondary DF relays in the presence of an eavesdropper. In [18], DNC, ORS, and artificial noises were used to mitigate eavesdropping attacks. However, above investigations perform on three orthogonal time slots, and hence, the bandwidth utilization efficiency is only 2/3 (two data per three time slots). This motivates us to propose new two-way relaying approaches for the underlay cognitive radio network, where only two time slots are used to obtain the data rate of 2/2.

In [19], the authors applied successive interference cancellation (SIC) technique to sequentially decode signals received from multiple sources. With SIC, the spectral efficiency of the two-way relaying systems is improved because the number of time slots decreases. Indeed, SIC is one of the core technologies in nonorthogonal multiple access (NOMA) systems which can assign nonorthogonal resources such as power, time, and code to different users [20]. For the NOMA systems operating in the power domain, multiple transmit signals that are allocated with different transmit powers are merged by the superposition coding (SC) before they are sent to intended receivers. At each receiver, the signal with higher transmit power is first decoded, and it is then removed from the received signal. This process, named SIC, is repeated until the receiver obtains the desired signal [21]. The authors in [19] pointed out some implementation problems of the SIC utilization such as complexity and residual interference after performing interference cancellations. These issues will lead to lower decoding capacities than expected ones. In [22], the authors evaluated two-way relaying NOMA systems fully in terms of the outage probability, outage floor, ergodic capacity, and power allocation. The system performance of the proposed protocol in [22] outperforms that of three-time slot two-way relaying systems with orthogonal multiple access. In addition, we have applied the SIC scheme with DNC and ORS at the last time slot (the second time slot) for traditional two-way relaying networks in [23]. The results obtained in [23] presented improvements of the system performances as well as bandwidth utilization efficiencies, as compared with the corresponding scenarios without using SIC. However, the previous published works [19, 22, 23] only considered the conventional wireless networks; i.e., the underlay cognitive network was not studied.

1.2. Contributions. Motivated by above issues, in this paper, we propose an underlay two-way relaying scheme with the SIC solution in which two secondary sources send concurrently their data to each other through multiple secondary relays under an interference constraint of a primary receiver. The proposed scheme is operated in two time slots, denoted as the UTW-2TS scheme. In the UTW-2TS scheme, the secondary relays employ the SIC operation to decode successively the data from received broadcast signals. The data carried by the stronger channel gains is decoded firstly and will be subtracted to detect the remaining data. These decoded data are encoded by two techniques: the DNC enforced by XOR operations (denoted as the UTW-2TS-DNC protocol) and the SC enforced by power domain additions (denoted as the UTW-2TS-SC protocol). In addition, because of imperfect operations of the SIC, the secondary relays experience residual interferences. A secondary relay is selected in two protocols UTW-2TS-DNC and UTW-2TS-SC which subjects to maximize decoding capacities and to minimize collection time of CSI. Outage probabilities and throughputs are solved in terms of exact closed-form expressions to evaluate the system performance of the proposed UTW-2TS-DNC and UTW-2TS-SC protocols. These outage probability and throughput expressions are proved by doing the Monte Carlo simulations. The contributions of this paper are cataloged as follows:

1. Proposing SIC-based underlay two-way relaying cognitive protocols which enhance the data rate, as compared with the corresponding ones without using the SIC.
(2) Exactly analyzing the outage probabilities and throughputs of two secondary sources in two operation protocols UTW-2TS-DNC and UTW-2TS-SC with the same two-way relaying model in which the SIC solution in the selected secondary relay is practically considered in the perfect and imperfect cases.

(3) The UTW-2TS-DNC protocol outperforms the UTW-2TS-SC protocol with the same power consumption.

(4) The proposed UTW-2TS-DNC and UTW-2TS-SC protocols can achieve the best throughputs at optimal interference power allocation coefficients and optimal locations of the selected secondary relay. In addition, we examine the throughputs of the UTW-2TS-SC protocol versus the changes of the power allocation coefficients.

(5) The system performances in terms of the outage probabilities and throughputs are improved when the number of the cooperative secondary relays, the interference constraints, and the distances from the secondary nodes to the primary receiver are increased as well as when the residual interference powers decrease.

1.3. Paper Organization and Notations. This paper is organized into sections as follows. Section 2 presents a system model of an underlay two-way relaying scheme with multiple secondary relays. Section 3 analyzes outage probabilities and throughputs of the proposed protocols UTW-2TS-DNC and UTW-2TS-SC. Results and discussions are shown in Section 4. Lastly, Section 5 summarizes contributions.

The notations used in this paper are denoted as follows: \( f_X(\cdot) \) and \( F_X(\cdot) \) denote, respectively, the probability density function (PDF) and the cumulative distribution function (CDF) of a random variable (RV) \( X \); \( \Pr \{ \Xi \} \) denotes the probability operation of an event \( \Xi \); \( E \{ \cdot \} \) denotes the expectation operator; \( \binom{p}{m} \) denotes the binomial coefficient \( \frac{M!}{p!(M - p)!} \); \( \oplus \) denotes XOR math operation; and \( \Gamma [u, v] \) is the upper incomplete Gamma function.

2. System Model

Figure 1 presents a system model of an underlay two-way relaying scheme. In this scheme, secondary sources \( S_1 \) and \( S_2 \) own data \( x_1 \) and \( x_2 \) (\( x_1 \in S_1 \) and \( x_2 \in S_2 \), respectively). The exchange of information is achieved between the \( S_1 \) and the \( S_2 \) through a cluster of \( M \) intermediate secondary relays \( R_i \), where \( i = \{1, 2, \ldots, M\} \). More specifically, the data \( x_1 \) of \( S_1 \) is exchanged with the data \( x_2 \) of \( S_2 \). The secondary network including \( S_1, S_2 \), and \( R_i \) suffers an interference constraint of a primary receiver (PR), denoted as \( I \). The secondary relays can perform the SIC technique to decode the received data-carried signals successively in an interval of a time slot.

We assume that a direct link between the secondary sources \( S_1 \) and \( S_2 \) does not exist due to the far distance or deep shadow fading \([18, 23, 24]\). It is also assumed that the secondary and primary nodes are installed with a single antenna, and additive noises at all the receivers are zero-mean Gaussian random variables (RVs) whose mean is zero, and variance is \( \sigma^2 \).

In Figure 1, \( h_{XY} \) and \( d_{XY} \) denote the flat-block Rayleigh fading channel coefficient and the normalized link distance of link \( X-Y \), respectively, where \( X, Y \in \{ S_1, S_2, R_i, PR \} \). Because the secondary relays \( R_i \) are located in the cluster, where the secondary relays \( R_i \) are close to each other, the normalized distances can be assumed to be identical, i.e., \( d_{S_i,R_i} = d_{R_i,S_i} = d_1 \), \( d_{S_i,R_1} = d_{R_1,S_i} = d_2 \), \( d_{R_i,PR} = d_3 \), \( d_{S_i,PR} = d_4 \), and \( d_{S_2,PR} = d_5 \). Because \( h_{XY} \) have Rayleigh distributions, the channel gains \( g_{XY} = |h_{XY}|^2 \) are exponential RVs with the PDFs \( f_{g_{XY}}(y) = \lambda_{XY} y e^{-\lambda_{XY} y} \) and the CDFs \( F_{g_{XY}}(y) = 1 - e^{-\lambda_{XY} y} \), where \( \lambda_{XY} = d_{XY} \), and \( \eta \) is a path loss exponent (see \([23, 27, 28]\)). Moreover, we have \( \lambda_{S_1,R_i} = \lambda_{R_i,S_i} = \lambda_1 \), \( \lambda_{S_2,R_i} = \lambda_2 \), \( \lambda_{R_i,PR} = \lambda_3 \), \( \lambda_{S_1,PR} = \lambda_4 \), and \( \lambda_{S_2,PR} = \lambda_5 \).

In the underlay cognitive radio network, the secondary network coexisted in the same frequency with the primary network and satisfies that the interference power at the primary receiver (PR) is less than or equal the constraint \( I \). [29, 30]. We have the inequality

\[
\begin{align*}
P_{S_1} g_{S_1,PR} + P_{S_2} g_{S_2,PR} & \leq I, \\
P_{R_i} g_{R_i,PR} & \leq I, \quad i = \{1, 2, \ldots, M\},
\end{align*}
\]

where \( P_{S_1}, P_{S_2}, \) and \( P_{R_i} \) are transmit powers of the secondary source \( S_1 \), the secondary source \( S_2 \), and the secondary relays \( R_i \), respectively.

From (1), the transmit powers of the secondary transmitters can be set as \( P_{S_1} = \alpha_1 I / g_{S_1,PR} \), \( P_{S_2} = \alpha_2 I / g_{S_2,PR} \), and \( P_{R_i} = I / g_{R_i,PR} \), where \( \alpha_1 \) and \( \alpha_2 \) are interference power allocation coefficients, \( 0 < \alpha_1, \alpha_2 < 1 \), and \( \alpha_1 + \alpha_2 = 1 \).
Comment 1. In practice, the transmit powers $P_X$ must be below a maximum power (denoted by $P_{X_{\text{max}}}$), where $X \in \{S_1, S_2, R_i\}$. Therefore, $P_X$ should be formulated as $P_X = \min\{P_{X_{\text{max}}}, u/g_{\text{PR}}\}$, where $u \in \{a_1, a_2, 1\}$. Suppose that $P_{X_{\text{max}}} \gg 1$ (e.g., the X nodes are near the PR node); hence, we can approximate $P_X$ by $P_X = u/g_{\text{PR}}$. It is worth noting that this assumption is used in many published literatures, i.e., [12, 26, 30–36].

The operation principle of the UTW-2TS schemes happens in two time slots as follows. In the first time slot, secondary sources $S_1$ and $S_2$ transmit simultaneously the data-carried signals $x_1$ and $x_2$, respectively, to all secondary relays under interference constraint of the primary receiver (PR). A selected secondary relay, denoted as $R_m$, where $m = \{1, 2, \cdots, M\}$, decodes the desired data by applying the SIC process. In the second time slot (last time slot), the $R_m$ creates the new data $x$ by the DNC technique, denoted as the UTW-2TS-DNC protocol, or by the SC technique, denoted as the UTW-2TS-SC protocol. The new data will be transmitted back to two secondary sources $S_1$ and $S_2$. Collection of CSI and system parameters for decoding data, cancelling interferences, subtracting self-interference components, and selecting the cooperative relay are performed by the medium access control (MAC) protocol as specified in [37].

The secondary sources $S_1$ and $S_2$ transmit simultaneously the data $x_1$ and $x_2$, respectively, to the secondary relays $R_i$ at the same time (the first time slot) and the same frequency as the uplink NOMA operation, where $i = \{1, 2, \cdots, M\}$. The received signal at the $R_i$ is expressed as

$$y_{R_i} = \sqrt{P_{S_1}} x_1 h_{S_1,R_i} + \sqrt{P_{S_2}} x_2 h_{S_2,R_i} + n_{R_i}, \quad (2)$$

where $E\{|x_1|^2\} = E\{|x_2|^2\} = 1$, and $n_{R_i}$ presents the Gaussian noises at the secondary relays $R_i$ with the same variance $\sigma^2$.

The secondary relays $R_i$ can compute the distances $d_{S,R_i}$ and $d_{S,R_i}$ by taking coordinate parameters in received request-to-send (RTS) messages of the secondary sources $S_1$ and $S_2$ in the setup phase [37]. Coordinates of the nodes can be received from the navigation systems. Hence, the secondary relays $R_i$ can make decoding decisions of the signals $x_1$ and $x_2$ in (2) by the SIC technique as follows.

Case 1. ($d_{S,R_i} \leq d_{S,R_2}$ or $E\{g_{S,R_i}\} \geq E\{g_{S,R_2}\}$ ($R_i$ is closer $S_1$ than $S_2$)).

In this case, the data $x_1$ in (2) is decoded by the secondary relay $R_i$ firstly whereas a signal $\sqrt{P_{S_2}} x_2 h_{S_2,R_i}$ containing $x_2$ is treated as interference. The received signal-to-interference-plus-noise ratios (SINRs) at the $R_i$ to decode $x_1$ are obtained from (2) as

$$\gamma_{R_i \rightarrow x_1}^{(1)} = \frac{P_{S_2} g_{S,R_i}}{P_{S_2} g_{S,R_i} + \sigma^2} = \frac{\alpha_2 Q g_{S,R_i} g_{S,PR}}{\alpha_2 Q g_{S,R_i} g_{S,PR} + g_{S,PR} g_{S,PR}} \left(1 - e^{-\lambda_1 \gamma_{\text{PR}}}\right), \quad (3)$$

where $Q = 1/\sigma^2$.

In (3), to increase the decoding capacity for the data $x_1$, decrease the collection of the CSI, and minimize help of the cooperative secondary relays, we only select one secondary relay among $M$ secondary relays. Indeed, the selected secondary relay $R_m$ is obtained as $R_m = \arg \max_{i=1,2,\cdots,M} g_{S,R_i}$. Then, the CDF and PDF of the RV $g_{S,R_m}$ are correspondingly expressed as ([28], eqs. 7–14)

$$F_{g_{S,R_m}}(y) = \left(1 - e^{-\lambda_1 \gamma_{\text{PR}}}\right) M = \sum_{p=0}^{M} \left(1 - p\right)^2 e^{-p \lambda_1 \gamma_{\text{PR}}}, \quad (4)$$

$$f_{g_{S,R_m}}(y) = M \lambda_1 e^{-\lambda_1 \gamma_{\text{PR}}} \left(1 - e^{-\lambda_1 \gamma_{\text{PR}}}\right)^{M-1}. \quad (5)$$

After decoding $x_1$ successfully, the inference component $\sqrt{P_{S_2}} x_2 h_{S_2,R_m}$ in (2) can be cancelled completely or partly by the SIC technique in which the transmit power $P_{S_2}$ and the channel coefficient $h_{S_2,R_m}$ were known in the setup phase [37]. The remaining received SINR at the selected secondary relay $R_m$ to decode $x_2$ is inferred as

$$\gamma_{R_m \rightarrow x_2}^{(1)} = \frac{P_{S_2} g_{S,R_m} g_{S,PR}}{\epsilon \lambda_1 g_{\text{PR}} + \sigma^2}. \quad (6)$$

where $\epsilon \lambda_1$ is a residual interference part at the secondary relay $R_m$ due to the imperfect SIC operations and can be modeled as an identical complex normal distribution $\epsilon \lambda_1 g_{\text{PR}} \sim \text{CN}(0, \Psi)$ [19] with zero mean and same variance $\Psi$ ($\Psi$ is also the power of the residual interference part), and hence, $g_{S,R_m} = |r_m|^2$ are also exponentially distributed RVs with the PDF as $f_{g_{S,R_m}}(y) = \Omega e^{-\Omega y}$ and the CDF as $F_{g_{S,R_m}}(y) = 1 - e^{-\Omega y}$ ([28], eqs. 6–68), where $\Omega = 1/\Psi$, $\epsilon = 0$ and $\epsilon = 1$ denote perfect and imperfect interference cancellation at the secondary relay $R_m$, respectively.

Substituting $P_{S_2} = \alpha_2 g_{S,R_m} g_{S,PR}$ into (6), $\gamma_{R_m \rightarrow x_2}^{(1)}$ is expressed as

$$\gamma_{R_m \rightarrow x_2}^{(1)} = \frac{\alpha_2 Q g_{S,R_m} g_{S,PR}}{g_{S,PR} (\epsilon Q g_{\text{PR}} + 1)}. \quad (7)$$

Case 2. ($d_{S,R_i} > d_{S,R_2}$ or $E\{g_{S,R_i}\} < E\{g_{S,R_2}\}$ ($R_i$ is closer $S_2$ than $S_1$)).

Similarly, the secondary relay $R_i$ decodes the data $x_2$ firstly with the interference signal $\sqrt{P_{S_2}} x_2 h_{S_2,R_i}$, where $i = \{1, 2, \cdots, M\}$. The SINRs at the $R_i$ for decoding $x_2$ and $x_1$ are expressed, respectively, as

$$\gamma_{R_i \rightarrow x_2}^{(2)} = \frac{P_{S_2} g_{S,R_i} g_{S,PR}}{P_{S_2} g_{S,R_i} g_{S,PR} + \sigma^2} = \frac{\alpha_2 Q g_{S,R_i} g_{S,PR}}{g_{S,PR} (\epsilon Q g_{\text{PR}} + 1)}. \quad (8)$$

$$\gamma_{R_i \rightarrow x_1}^{(2)} = \frac{P_{S_2} g_{S,R_i}}{\epsilon Q g_{\text{PR}} + \sigma^2} = \frac{\alpha_2 Q g_{S,R_i} g_{S,PR}}{g_{S,PR} (\epsilon Q g_{\text{PR}} + 1)}. \quad (9)$$
In the second time slot, if the selected secondary relay $R_m$ decodes successfully both data $x_1$ and $x_2$ in the first time slot, the $R_m$ creates a new data $x$ as

$$x = \begin{cases} \sqrt{\beta_1} x_1 + \sqrt{\beta_2} x_2, \quad \text{digital network coding (the UTW-2TS-DNC protocol)}, \\ \sqrt{\beta_1} x_1 + \sqrt{\beta_2} x_2, \quad \text{superposition coding (the UTW-2TS-SC protocol)}, \end{cases}$$

(10)

where $\beta_1$ and $\beta_2$ are power allocation coefficients to data $x_1$ and $x_2$, respectively, and satisfy conditions as [21]

$$\begin{align*}
\beta_1 + \beta_2 &= 1, \quad 0 \leq \beta_1 \leq 1, \quad 0 \leq \beta_2 \leq 1, \\
\beta_1 &\geq \beta_2, \\
\beta_1 < \beta_2, \\
\beta_1 &< \beta_2, \\
\beta_1 &> \beta_2.
\end{align*}$$

(11)

We remark that for the UTW-2TS-SC protocol as in (10) and (11), the selected secondary relay $R_m$ performs the downlink NOMA operation to allocate powers to data $x_1$ and $x_2$ as $x = \sqrt{\beta_1} x_1 + \sqrt{\beta_2} x_2$ in which $x_1$ and $x_2$ are the desired data of the secondary sources $S_k$ and $S_l$, respectively [21]. If the $S_k$ is closer to the $R_m$, the $R_m$ will allocate the lower power for the data $x_1$, where $k, l = \{1, 2\}$ and $l \neq k$ and vice versa. More specifically, in Case 1 ($d_{S_k, R_m} \leq d_{S_l, R_m}$), the $S_k$ is the nearby user and the $S_l$ is the distant user. Hence, the $R_m$ uses the lower power allocation coefficient $\beta_2$ ($\beta_2 \leq \beta_1$) to transmit the data $x_2$ to the $S_l$ and the higher power allocation coefficient $\beta_1$ to transmit the data $x_1$ to the $S_k$. In addition, we note that the secondary relay $R_m$ in Case 2 is selected by $R_m = \arg\max_{r \in \{1, 2, \ldots, M\}} g_{S_k, R_r}$.

The coded data $x$ in (10) will be broadcasted back to two secondary sources $S_1$ and $S_2$, and then the received signals at the secondary sources $S_k$ are expressed as

$$y_{S_k} = \sqrt{p_{R_m, k}} h_{R_m, S_k} x + n_{S_k},$$

(12)

where $n_{S_k}$ presents the Gaussian noises at the secondary sources $S_k$ with the same variance $\sigma^2$.

For the UTW-2TS-DNC protocol with $x = x_1 \oplus x_2$ in (12), the received SINRs at the secondary sources $S_k$ can be obtained from (12) to take $x$ as

$$\gamma_{S_k}^{\text{UTW-2TS-DNC}} = \frac{P_{R_m} |h_{R_m, S_k}|^2}{\sigma^2} = \frac{Qg_{R_m, S_k}}{g_{R_m, PR}}.$$  

(13)

The decoding method of the secondary sources $S_k$ to take the desired data is performed by decoding operations of the network coding; i.e., the $S_1$ takes the data $x_2$ by XOR operations of its data $x_1$ with the compressed data $x$ as $x_1 \oplus x = x_1 \oplus x_1 \oplus x_2 = x_2$.

For the UTW-2TS-SC protocol, the received signals at the secondary sources $S_k$ are shown more clearly as

$$y_{S_k} = \sqrt{\beta_1 p_{R_m, k}} h_{R_m, S_k} x_1 + \sqrt{\beta_2 p_{R_m, k}} h_{R_m, S_k} x_2 + n_{S_k},$$

(14)

In (14), the secondary source $S_k$ owns the data $x_k$ and does not decode this data. In addition, the $S_k$ can estimate the fading channel coefficient $h_{R_m, S_k}$ from receiving the setup messages of the selected secondary relay $R_m$ and takes the system parameters as the transmit power $p_{R_m}$ and the power allocation coefficient $\beta_k$ during the setup time, where $k = \{1, 2\}$, [37]. Hence, the $S_k$ knows the parameters $x_k$, $h_{R_m, S_k}$, $P_{R_m}$, and $\beta_k$, which then can cancel out the self-interference component $\sqrt{\beta_k p_{R_m, k}} h_{R_m, S_k} x_k$ in (14) [38, 39]. The received signals at the $S_k$ after subtracting the known self-interference components are obtained as

$$\tilde{y}_{S_k} = \sqrt{\beta_1 p_{R_m, k}} h_{R_m, S_k} x_1 + n_{S_k}.$$  

(15)

The received SINRs at the secondary sources $S_k$ can be computed to take the desired data $x_1$ as

$$\gamma_{S_k}^{\text{UTW-2TS-SC}} = \frac{P_{R_m, k} |h_{R_m, S_k}|^2}{\sigma^2} = \frac{Qg_{R_m, S_k}}{g_{R_m, PR}}.$$  

(16)

### 3. Outage Probability and Throughput Analyses

In this paper, it is assumed that an outage event occurs when the received SINR is less than a threshold SINR $\gamma_{th}$ [30, 40]. In addition, the data transmission in the secondary network is fixed to the delay-limited mode in which the secondary receivers such as the selected secondary relay $R_m$ and the secondary sources $S_1$ and $S_2$ decode the desired data block by block without using buffers [30, 40]. Throughput has been considered to characterize the spectrum utilization of the communication systems [30, 40, 41] (known as the mean spectral efficiency) and is related to outage analyses as

$$TP_{Z}^{(i)} = \frac{1}{2} (1 - \text{OP}_{Z-S_k}^{(i)}) R_{th} + \frac{1}{2} (1 - \text{OP}_{Z-S_l}^{(i)}) R_{th},$$

(17)

where 1/2 denotes that the $Z$ protocol operates in the two time slots, $Z \in \{\text{UTW-2TS-DNC, UTW-2TS-SC}\}$; $R_{th}$ is the threshold data rate and is given by $R_{th} = \log_2(1 + \gamma_{th})$ (bits/s/Hz) [30, 38, 40, 41]; $\text{OP}_{Z-S_k}^{(i)}$ is the outage probability at the secondary source $S_k$ in the case $I = \{l, k = \{1, 2\}\}$ and is defined as probability that the secondary source $S_k$ cannot decode successfully the desired data.
3.1. The UTW-2TS-DNC Protocol

3.1.1. Case 1: If, i = 1, 2, ..., M. The outage probability of the secondary source S_i occurs when the secondary source S_i cannot successfully decode the data x_i of the opposite secondary source S_j. Due to the operation of the SIC technique at the selected secondary relay R_m which was set up in the initial phase, the outage probability of the secondary source S_i happens when (1) the R_m fails to decode the data x_j; or (2) the R_m decodes the data x_j successfully but does not decode the data x_i; or (3) the R_m decodes both x_j and x_i successfully, but the S_i cannot decode the DNC-coded data x to get the desired data x_i in the second time slot. We express the outage probability of the secondary source S_i by the mathematical expression

\[ \text{OP}_{\text{UTW-2TS-DNC-S_i}}^{(1)} = \Pr \left\{ (y_{R_m}^{(1)} - y_{R_m}^{(2)}) \cap (y_{R_m}^{(1)} < y_{R_m}^{(2)}) \right\} \]

(18)

In (18), the probability \( \Phi_1 = \Pr \left\{ y_{R_m}^{(1)} < y_{R_m}^{(2)} \right\} \) is considered when the secondary relay R_m fails to decode the data x_j while applying the SIC technique. The probability \( \Phi_2 = \Pr \left\{ (y_{R_m}^{(1)} - y_{R_m}^{(2)}) \cap (y_{R_m}^{(1)} < y_{R_m}^{(2)}) \right\} \) is the decoding error of the data x_j at the secondary relay R_m. Finally, the probability \( \Pr \left\{ (y_{R_m}^{(1)} - y_{R_m}^{(2)}) \cap (y_{R_m}^{(1)} < y_{R_m}^{(2)}) \cap (y_{R_m}^{(1)} < y_{R_m}^{(2)}) \right\} \) measures the decoding error of the coded data x at the secondary source S_i. The event \( y_{R_m}^{(1)} < y_{R_m}^{(2)} \) occurs independently with the events \( y_{R_m}^{(1)} \geq y_{R_m}^{(2)} \) and \( y_{R_m}^{(1)} \geq y_{R_m}^{(2)} \); thus, we have an equivalent representation of (18) as

\[ \text{OP}_{\text{UTW-2TS-DNC-S_i}}^{(1)} = \Phi_1 + \Phi_2 + \Pr \left\{ (y_{R_m}^{(1)} - y_{R_m}^{(2)}) \cap (y_{R_m}^{(1)} \geq y_{R_m}^{(2)}) \right\} \]

(19)

To analyze the probabilities \( \Phi_1, \Phi_2, \) and \( \Pr \{ \gamma_{\text{UTW-2TS-DNC}} < \gamma_{\text{th}} \} \), we define \( \gamma_{\text{UTW-2TS-DNC}} = \gamma_{\text{th}} / \lambda_{\text{th}} \), where \( X_1, X_2, Y_1, Y_2 \in \{ S_1, S_2, R_1, \ldots, R_M \} \), \( \gamma_{\text{UTW-2TS-DNC}} \neq \gamma_{\text{th}} \), and \( i = \{ 1, 2, \ldots, M \} \); then, by referring from [30] (eqs. 24-25), the CDF and PDF of the RV \( \gamma_{\text{UTW-2TS-DNC}} \) are obtained as

\[ F_{\gamma_{\text{UTW-2TS-DNC}}} (z) = \frac{\lambda_{\text{UP}} z}{\lambda_{\text{UP}} + \lambda_{\text{S}} z}, \]

(20)

\[ f_{\gamma_{\text{UTW-2TS-DNC}}} (x) = \frac{\lambda_{\text{UP}} x \lambda_{\text{S}} z}{(\lambda_{\text{UP}} + \lambda_{\text{S}} z)^2}. \]

(21)

For the selected secondary relay R_m from the S_i-S_j links \( (m, i = \{ 1, 2, \ldots, M \}) \), the CDF of the RV \( G_{S_i R_m} / S_j \) is given as

\[ F_{G_{S_i R_m} / S_j} (z) = \Pr \left\{ \frac{G_{S_i R_m}}{G_{S_j}} < z \right\} = \Pr \left\{ \frac{g_{S_i R_m}}{g_{S_j}} < z \right\} \]

(22)

Substituting the CDF of the RV \( G_{S_i R_m} \) in (4) and the PDF of the RV \( g_{S_j} \) into (22), we have a final result as

\[ F_{G_{S_i R_m} / S_j} (z) = \int_0^\infty f_{g_{S_j}} (t) \times F_{g_{S_i R_m} / S_j} (zt) \, dt. \]

(23)

The probabilities \( \Phi_1, \Phi_2, \) and \( \Pr \{ \gamma_{\text{UTW-2TS-DNC}} < \gamma_{\text{th}} \} \) are solved by Lemmas 1 and 2 as follows.

Lemma 1. The probability \( \Phi_1 \) is solved as

\[ \Phi_1 = 1 + \lambda_2 \lambda_4 \lambda_5 \sum_{p=1}^{M} \left( \frac{p}{M} \right)^{-1} \exp \left\{ -\lambda_4 \lambda_5 \right\} \left( \frac{1}{\lambda_4} - \frac{u_1 (p)}{\lambda_2 \lambda_3 \lambda_4} \right) \]

(24)

where \( u_1 = \gamma_{\text{th}} \alpha_2 / \alpha_1, u_2 = \gamma_{\text{th}} \alpha_3 / \alpha_2, u_3 (p), \) and \( u_4 (p) \) are functions versus count variable \( p \) and are defined, respectively, as

\[ u_3 (p) = \lambda_1 + \rho \lambda_1 \lambda_2, \]

\[ u_4 (p) = \rho \lambda_1 u_3 (p). \]

Proof. (proven in Appendix A).
Lemma 2. The probability $\Phi_2$ is given in two cases of $\epsilon$ as follows:

For $\epsilon = 0$ (perfect SICs):

$$\Phi_2 = \frac{\lambda_3 \Phi_3}{\lambda_2 + \lambda_4 \Phi_3} - \lambda_2 \lambda_3 \sum_{p=0}^{M} \left( \frac{p}{M} \right) \Phi_3 \left\{ \frac{-1}{\lambda_2 \Phi_3(p) - \lambda_2 \Phi_3(p)} \right\} \times \left( \frac{\lambda_3 \Phi_3(p)}{\lambda_2 \Phi_3(p) - \lambda_2 \Phi_3(p)} \Phi_3 \left( \frac{(\lambda_3 + \lambda_4 \Phi_3) \times \Phi_3(p)}{\lambda_3 \Phi_3(p) + \Phi_3(p)} \right) \right) \right\}.$$  

(25)

For $\epsilon = 1$ (imperfect SICs):

$$\Phi_2 = 1 - \lambda_2 \epsilon^{(\lambda_3 + \lambda_2 \epsilon) \Omega} \sum_{p=0}^{M} \left( \frac{p}{M} \right) \Phi_3 \left\{ \frac{-1}{\lambda_2 \Phi_3(p) - \lambda_2 \Phi_3(p)} \right\} \times \left( \frac{\lambda_3 \Phi_3(p)}{\lambda_2 \Phi_3(p) - \lambda_2 \Phi_3(p)} \Phi_3 \left( \frac{(\lambda_3 + \lambda_2 \epsilon) \times \Phi_3(p)}{\lambda_3 \Phi_3(p) + \Phi_3(p)} \right) \right) \right\}.$$  

(26)

where $\Gamma(u, v)$ is the upper incomplete Gamma function ($\Gamma(u, v) = \int_{v}^{\infty} e^{-t} t^{u-1} dt$) ([42], eq. 8.350.2).

Proof. (see the proof and notations in Appendix B).

The last probability $\Pr \left\{ Y_{S_1, x}^{\text{UTW-2TS-DNC}} < Y_{th} \right\}$ in (19) is obtained by using formula (13) with $k = 1$ as

$$\Pr \left\{ Y_{S_1, x}^{\text{UTW-2TS-DNC}} < Y_{th} \right\} = \Pr \left\{ \frac{Qg_{S_1, S_2}}{g_{S_2, PR}} < Y_{th} \right\} = \Pr \left\{ \frac{G_{S_1, S_2}}{R_{S_2, PR}} < Y_{th} \right\} = \frac{F_{G_{S_1, S_2}/R_{S_2, PR}}(Y_{th})}{Q}.$$  

(27)

Substituting (27) into (19), the outage probability of the secondary source $S_1$ is obtained by the closed-form expression

$$\text{OP}^{(1)}_{\text{UTW-2TS-DNC-S}_1} = \frac{(\Phi_1 + \Phi_2) \lambda_3 Q + \lambda_1 Y_{th}}{\lambda_3 Q + \lambda_1 Y_{th}}.$$  

(28)

where $\Phi_1$ and $\Phi_2$ are given by closed-form expressions in Lemmas 1 and 2, respectively.

The secondary source $S_2$ expects to receive the desired data $x_2$ of the secondary source $S_1$. Hence, the outage probability of the secondary source $S_2$ occurs when (1) the selected secondary relay $R_m$ cannot decode the data $x_2$ or (2) the $S_2$ fails to decode the data transmitted by the $R_m$ after the data $x_1$ is decoded successfully. We formulate the outage probability of the secondary source $S_1$ as

$$\text{OP}^{(1)}_{\text{UTW-2TS-DNC-S}_2} = \Pr \left\{ \left( Y_{R_m, x_1}^{(1)} < Y_{th} \right) + \Pr \left\{ \left( Y_{R_m, x_1}^{(1)} > Y_{th} \right) \right\} \cap \left( Y_{S_2, x}^{\text{UTW-2TS-DNC}} < Y_{th} \right) \right\}.$$  

(29)

In (29), the probability part $\Pr \left\{ \left( Y_{R_m, x_1}^{(1)} > Y_{th} \right) \right\}$ shows transmission errors of the data $x_1$ from the selected secondary relay $R_m$ to the desired secondary source $S_2$. Because the event $Y_{S_2, x}^{\text{UTW-2TS-DNC}} < Y_{th}$ occurs independently, and with the help of (13) with $k = 1$, the outage probability of the secondary source $S_2$ is manipulated and solved as

$$\text{OP}^{(1)}_{\text{UTW-2TS-DNC-S}_2} = \Phi_1 + \Pr \left\{ \left( Y_{R_m, x_1}^{(1)} > Y_{th} \right) \right\} \times \Pr \left\{ Y_{S_2, x}^{\text{UTW-2TS-DNC}} < Y_{th} \right\} = \Phi_1 + \Pr \left\{ \frac{Qg_{R_m, S_2}}{g_{R_m, PR}} < Y_{th} \right\} = \Phi_1 + \frac{Qg_{R_m, S_2}}{g_{R_m, PR}}.$$  

(30)

where $\Phi_1$ is given exactly by Lemma 1.

Next, the throughput of the UTW-2TS-DNC protocol as in (17) is solved in Case 1 ($d_{S_2, R} \leq d_{S_1, R}$) by

$$\text{TP}^{(1)}_{\text{UTW-2TS-DNC-S}_1} = \frac{2}{\log_2(1 + Y_{th})} \left\{ 2 - \frac{\text{OP}^{(1)}_{\text{UTW-2TS-DNC-S}_1}}{\text{OP}^{(1)}_{\text{UTW-2TS-DNC-S}_2}} \right\} \times (2 - 2\Phi_1 - (1 - \Phi_1)) = \frac{\lambda_1 Y_{th}}{\lambda_3 Q + \lambda_1 Y_{th}}.$$  

(31)

where $\Phi_1$ and $\Phi_2$ are taken from Lemmas 1 and 2, respectively.

3.1.2. Case 2: $d_{S_2, R} > d_{S_1, R}$. In this case, a selected secondary relay $R_m$ must decode the data $x_2$ firstly. Because the system model of the UTW-2TS-DNC protocol in Figure 1 is
symmetric, we can model the outage probabilities in Case 1 as functions versus symmetric parameters as
\[ \text{OP}_{\text{UTW-2TS-DNC}}^{(1)}(S_1, S_2) = \text{OP}_{\text{UTW-2TS-DNC}}^{(1)}(\lambda_1, \alpha_1, \lambda_2, \alpha_2), \]
and
\[ \text{OP}_{\text{UTW-2TS-DNC}}^{(1)}(S_1, S_2) = \text{OP}_{\text{UTW-2TS-DNC}}^{(1)}(\lambda_1, \alpha_1, \lambda_2, \alpha_2). \]
Hence, in Case 2 \((d_{S,R} > d_{S,R})\), the outage probability of the secondary sources \(S_1\) and \(S_2\) can be inferred from Case 1 as follows:
\[ \text{OP}_{\text{UTW-2TS-DNC}}^{(2)}(S_1, S_2) = \text{OP}_{\text{UTW-2TS-DNC}}^{(1)}(\lambda_2, \alpha_2, \lambda_1, \alpha_1), \]
\[ \text{OP}_{\text{UTW-2TS-DNC}}^{(2)}(S_1, S_2) = \text{OP}_{\text{UTW-2TS-DNC}}^{(1)}(\lambda_2, \alpha_2, \lambda_1, \alpha_1), \]
where \(\text{OP}_{\text{UTW-2TS-DNC}}^{(1)}(\lambda_2, \alpha_2, \lambda_1, \alpha_1)\) and \(\text{OP}_{\text{UTW-2TS-DNC}}^{(1)}(\lambda_1, \alpha_1, \lambda_2, \alpha_2)\) are closed-form outage probability expressions of the secondary sources \(S_1\) and \(S_2\) in Case 1 with replacing \(\lambda_1 \leftrightarrow \lambda_2\) and \(\alpha_1 \leftrightarrow \alpha_2\). 

The throughput of the UTW-2TS-DNC protocol as in (17) is obtained in Case 2 by
\[ \text{TP}_{\text{UTW-2TS-DNC}}^{(2)} = \frac{\log_2(1 + \gamma_{th})}{2} - \left(2 - \text{OP}_{\text{UTW-2TS-DNC}}^{(2)} - \text{OP}_{\text{UTW-2TS-DNC}}^{(2)}\right), \]
where \(\text{OP}_{\text{UTW-2TS-DNC}}^{(2)}\) and \(\text{OP}_{\text{UTW-2TS-DNC}}^{(2)}\) are presented in (32) and (33), respectively.

3.2. The UTW-2TS-SC Protocol. Operation of the UTW-2TS-SC protocol is different to the UTW-2TS-DNC protocol in the time slot 2 (the last time slot) with power allocation coefficients \(\beta_1\) and \(\beta_2\) to decoded data \(x_1\) and \(x_2\). Similar to the UTW-2TS-DNC protocol for Case 1 \((d_{S,R} \leq d_{S,R})\) and Case 2 \((d_{S,R} > d_{S,R})\), the outage probability and corresponding throughput of the secondary sources \(S_1\) and \(S_2\) will be obtained briefly in the next subsections.

3.2.1. Case 1: \(d_{S,R} \leq d_{S,R}\). The decoding operation of the UTW-2TS-SC protocol is similar to that of the UTW-2TS-DNC protocol. One difference is that the secondary source \(S_1\) expects to decode the desired data \(x_2\) directly from the received signal \(y_{S_2}\) as in (14) where \(k = 1\) and \(l = 2\). Hence, the outage probability of the secondary source \(S_1\) (denoted as \(\text{OP}_{\text{UTW-2TS-SC-S1}}^{(1)}\)) is expressed in Case 1 \((d_{S,R} \leq d_{S,R})\) as
\[ \text{OP}_{\text{UTW-2TS-SC-S1}}^{(1)} = \Pr \left\{ y_{S_1, x_1}^{(1)} < y_{th}\right\} + \Pr \left\{ y_{S_1, x_1}^{(1)} < y_{th}\right\} \cap \left\{ y_{S_1, x_1}^{(1)} < y_{th}\right\} \]
\[ + \Pr \left\{ y_{S_1, x_1}^{(1)} < y_{th}\right\} \cap \left\{ y_{S_1, x_1}^{(1)} < y_{th}\right\} \]
\[ = \left(\Phi_1 + \Phi_2\right) \times \left(1 - \Pr \left\{ y_{S_1, x_1}^{(1)} < y_{th}\right\}\right) \]
where \(\Phi_1\) and \(\Phi_2\) are obtained from Lemmas 1 and 2.

In (35), the probability \(\Pr \left\{ y_{S_1, x_1}^{(1)} < y_{th}\right\} \cap \left\{ y_{S_1, x_1}^{(1)} < y_{th}\right\} \) directly calculates the decoding error of the data \(x_2\) at the secondary source \(S_1\).

Substituting (16) with \(k = 1\) and \(l = 2\) into the probability \(\Pr \left\{ y_{S_1, x_1}^{(1)} < y_{th}\right\} \) in (35) and performing similarly as (27), we have a result as
\[ \Pr \left\{ y_{S_1, x_1}^{(1)} < y_{th}\right\} = \Pr \left\{ \frac{Q_{R_1} g_{R_2}}{g_{R_2}} < y_{th}\right\} \]
\[ = \Pr \left\{ G_{R_1 R_2} < y_{th}, \beta \right\} \]
OP \text{UTW-2TS-SC}_1(\lambda_1, \alpha_1, \beta_1, \lambda_2, \alpha_2, \beta_2)$ and $\text{OP \text{UTW-2TS-SC}_2}(\lambda_1, \alpha_1, \beta_1, \lambda_2, \alpha_2, \beta_2)$. Because of the symmetric model of the UTW-2TS-SC protocol in Figure 1, the outage probability of the secondary sources $S_1$ and $S_2$ in Case 2 can be concluded from Case 1 as follows:

$$\text{OP \text{UTW-2TS-SC}_1}(\lambda_1, \alpha_1, \beta_1, \lambda_2, \alpha_2, \beta_2) = \text{OP \text{UTW-2TS-SC}_1}(\lambda_2, \alpha_2, \beta_2, \lambda_1, \alpha_1, \beta_1),$$

(39)

$$\text{OP \text{UTW-2TS-SC}_2}(\lambda_1, \alpha_1, \beta_1, \lambda_2, \alpha_2, \beta_2) = \text{OP \text{UTW-2TS-SC}_2}(\lambda_2, \alpha_2, \beta_2, \lambda_1, \alpha_1, \beta_1),$$

(40)

where $\text{OP \text{UTW-2TS-SC}_1}(\lambda_2, \alpha_2, \beta_2, \lambda_1, \alpha_1, \beta_1)$ and $\text{OP \text{UTW-2TS-SC}_2}(\lambda_2, \alpha_2, \beta_2, \lambda_1, \alpha_1, \beta_1)$ are closed-form outage probability expressions of the secondary sources $S_1$ and $S_2$ in Case 1 with replacing $\lambda_1 \mapsto \lambda_2$, $\alpha_1 \mapsto \alpha_2$, and $\beta_1 \mapsto \beta_2$.

The corresponding throughput of the UTW-2TS-SC protocol is obtained in Case 2 as

$$\text{TP \text{UTW-2TS-SC}} = \frac{\log_2(1 + y_{th})}{2} \cdot (2 - \text{OP \text{UTW-2TS-SC}_1} - \text{OP \text{UTW-2TS-SC}_2}),$$

(41)

where $\text{OP \text{UTW-2TS-SC}_1}$ and $\text{OP \text{UTW-2TS-SC}_2}$ are given in (39) and (40), respectively.

### 4. Results and Discussions

This section presents analysis and simulation results in terms of outage probabilities and throughputs of the protocols UTW-2TS-DNC and UTW-2TS-SC in the two-dimensional plane. These proposed protocols are considered in two cases of perfect SICs ($\epsilon = 0$, denoted by pSIC) and imperfect SICs ($\epsilon = 1$, denoted by ipSIC). The simulation results are performed by the Monte Carlo method to validate the analysis ones which are shown from exact closed-form expressions. Coordinates of the nodes $S_1$, $S_2$, and the cluster with $M$ secondary relays $R_i$ are set as $S_1(0, 0)$, $S_2(1, 0)$, $R_i(x_{PR}, y_{PR})$, and $R_i(x_{PR}, y_{PR})$, where $0 < x_{PR} < 1$ and $i = \{1, 2, \cdots, M\}$. The normalized distances are calculated from the coordinates as

$$d_1 = \sqrt{x_{PR}^2 + y_{PR}^2}, \quad d_2 = \sqrt{(1 - x_{PR})^2 + y_{PR}^2}, \quad d_3 = \sqrt{(x_{PR} - x_{PR})^2 + (y_{PR} - y_{PR})^2}, \quad d_4 = \sqrt{x_{PR}^2 + y_{PR}^2}, \quad d_5 = \sqrt{(1 - x_{PR})^2 + y_{PR}^2}.$$ 

It is assumed that the threshold SINR and the path loss exponent are fixed by $y_{th} = 3$ and $\eta = 3$, and $Q$ (dB) on the $x$-axis is defined as $Q = 10 \times \log_{10}(I/\sigma^2)$ (dB). Markers denote simulated results, and solid lines present analyzed ones.

Figure 2 shows the outage probabilities of the secondary sources $S_1$ and $S_2$ in the protocols UTW-2TS-DNC and UTW-2TS-SC versus $Q$ (dB) when $x_{PR} = 0.3$, $y_{PR} = 0$, $y_{PR} = 0.5$, $\Psi = 0$ (dB), $M = 3$, and interference allocation coefficients $\alpha_1 = \alpha_2 = 0.5$ [19, 30]. The normalized distances are calculated as $d_1 = 0.3$, $d_2 = 0.7$, $d_3 = 0.54$, $d_4 = 0.71$, and $d_5 = 0.71$; hence, the results are presented following Case 1.
Figure 3: Throughputs of the protocols UTW-2TS-DNC and UTW-2TS-SC versus $Q$ (dB) when $\alpha_1 = \alpha_2 = 0.5$, $\beta_1 = 0.7$, $\beta_2 = 1 - \beta_1 = 0.3$, $\Psi = 0$ (dB), $x_R = 0.3$, $y_R = 0$, $x_{PR} = y_{PR} = 0.5$, and $M = \{3, 12\}$.

Throughput (bits/s/Hz)

Figure 4 presents the throughputs of the protocols UTW-2TS-DNC and UTW-2TS-SC versus $Q$ (dB) when $x_R = 0.3$, $y_R = 0$, $x_{PR} = y_{PR} = 0.5$, $\Psi = 0$ (dB), $\alpha_1 = \alpha_2 = 0.5$, $\beta_1 = 0.7$, $\beta_2 = 1 - \beta_1 = 0.3$, and the number of intermediate secondary relays is set as $M \in \{3, 12\}$. Exact throughput curves of the UTW-2TS-DNC and UTW-2TS-SC protocols are obtained by theoretic analyses in (31) and (38). Simulation markers are carried out by (17) with separate outage probabilities for each protocol. From Figure 3, the throughputs of the UTW-2TS-DNC protocol are larger than those of the UTW-2TS-SC protocol with all the cases of SICs and $M$. In addition, the throughput of both protocols increase when the UTW-2TS systems use more secondary relays ($M$ is increased). These contributions are based on observations from SINRs (13) and (16). With $0 \leq \beta_1$ and $\beta_2 \leq 1$, the SINRs in (13) of the UTW-2TW-DNC protocol are always larger than those in (16) of the UTW-2TW-SC protocol. Furthermore, with more cooperative secondary relays, the diversity capacity of both protocols will be increased by the relay selection methods.

Figure 4 presents the throughputs of the protocols UTW-2TS-DNC and UTW-2TS-SC versus $Q$ when $Q = 10$ (dB), $\beta_1 = 0.7$, $\beta_2 = 1 - \beta_1 = 0.3$, $\Psi = -5$ (dB), $x_R = 0.3$, $y_R = 0$, $x_{PR} = y_{PR} = 0.5$, $M = 3$, and $\alpha_2$ can be set as $\alpha_2 = 1 - \alpha_1$. In Figure 4, the throughputs of the protocols UTW-2TS-DNC
and UTW-2TS-SC achieve the large values at an approximate interference power allocation coefficient $\alpha_1 \approx 0.6$ for both cases (perfect SICs and imperfect SICs). The value $\alpha_1 \approx 0.6$ is to balance constraints such as interferences from the secondary network to the primary network, locations of the secondary relays, and perfect/imperfect SIC operations.

Figure 5 shows the throughputs of the protocols UTW-2TS-DNC and UTW-2TS-SC versus $\beta_1$ when $Q = 10$ (dB), $\alpha_1 = 0.6$, $\alpha_2 = 1 - \alpha_1 = 0.4$, $\Psi = -5$ (dB), $x_R = 0.3$, $y_R = 0$, $x_{PR} = y_{PR} = 0.5$, $M = 3$, and $\beta_2 = 1 - \beta_1$. As observed from Figure 5, the UTW-2TS-DNC protocol is not affected by the power allocation coefficients $\beta_1$ and $\beta_2$, and the UTW-2TS-SC protocol reaches the largest throughputs at approximate values $\beta_1 = 0.7$ and $\beta_1 = 0.8$ corresponding to perfect SICs and imperfect SICs, respectively. These values $\beta_1 = 0.7$ and $\beta_1 = 0.8$ are to equalize the SINR qualities between two hops from the selected secondary relay to the secondary sources.

Figure 6 shows the throughputs of the protocols UTW-2TS-DNC and UTW-2TS-SC versus $\alpha_1$ and $\beta_1$ in the three-
Figure 6: Throughputs of the protocols UTW-2TS-DNC and UTW-2TS-SC versus $\alpha_1$ and $\beta_1$ when $Q = 10$ (dB), $\Psi = -5$ (dB), $x_R = 0.3$, $y_R = 0$, $x_{PR} = y_{PR} = 0.5$, $M = 3$, $\alpha_2 = 1 - \alpha_1$, and $\beta_2 = 1 - \beta_1$.

Figure 7: Throughputs of the protocols UTW-2TS-DNC and UTW-2TS-SC versus $x_R$ when $Q = 10$ (dB), $\alpha_1 = \alpha_2 = 0.5$, $\beta_1 = \beta_2 = 0.5$, $\Psi = -5$ (dB), $x_{PR} = y_{PR} = 0.5$, $M = 3$, and $y_R = 0$. 
dimensional plane when $Q = 10$ (dB), $\Psi = -5$ (dB), $x_R = 0.3$, $y_R = 0$, $x_{PR} = y_{PR} = 0.5$, $M = 3$, and $\alpha_1$ and $\beta_2$ are set as $\alpha_1 = 1 - \beta_1$ and $\beta_2 = 1 - \beta_1$. The value ranges of $\alpha_1$ and $\beta_1$ are established between 0.1 and 0.9. The results from Figure 6 confirm the contributions from Figures 4 and 5.

Figure 7 presents the throughputs of the protocols UTW-2TS-DNC and UTW-2TS-SC versus $x_R$ when $Q = 10$ (dB), $\alpha_1 = \alpha_2 = 0.5$, $\beta_1 = \beta_2 = 0.5$, $\Psi = -5$ (dB), $x_{PR} = y_{PR} = 0.5$, $M = 3$, and $y_{PR}$ is fixed as $y_R = 0$. In this simulation, the normalized distances $d_4$ and $d_5$ are fixed at the same value 0.7 and the remaining normalized distances $d_1$, $d_2$, and $d_3$ are changed from 0.1 to 0.9 for $d_1$, 0.9 to 0.1 for $d_2$, and 0.6 to 0.5 and back to 0.64 for $d_3$. From Figure 7, these proposed protocols UTW-2TS-DNC and UTW-2TS-SC achieve the largest throughputs at asymmetric locations of the selected secondary relay as $x_R \approx 0.2$ ($d_1 = 0.2$, $d_2 = 0.8$) and $x_R \approx 0.8$ ($d_1 = 0.8$, $d_2 = 0.2$) because of decoding the received data sequentially (SIC operations). In addition, at the symmetric location $x_R = 0.5$ ($d_1 = d_2 = 0.5$), the throughputs of both protocols again have the lowest values.

Figure 8 presents the throughputs of the protocols UTW-2TS-DNC and UTW-2TS-SC versus $y_{PR}$ when $Q = 10$ (dB), $\alpha_1 = \alpha_2 = 0.5$, $\beta_1 = 0.7$, $\beta_2 = 1 - \beta_1 = 0.3$, $\Psi = -5$ (dB), $x_R = 0.3$, $y_R = 0$, $M = 3$, and $x_{PR}$ is fixed at $x_{PR} = 0.5$. In this case, the normalized distances $d_1$ and $d_2$ are fixed to 0.3 and 0.7, respectively, and the primary receiver (PR) is moving farther the secondary network characterized by changing the normalized distances $d_3$, $d_4$, and $d_5$ from 0.22 to 0.92 for $d_4$ and from 0.51 to 1.03 for $d_3$ and $d_5$. As shown in Figure 8, the throughputs of the protocols UTW-2TS-DNC and UTW-2TS-SC are enhanced when the value of $y_{PR}$ increases because of low influence of the interference constraint to the sources and relays in the secondary network. Therefore, the secondary sources and relays can transmit with the maximum power, and interference cancelling and decoding capacity at the secondary relays and the secondary sources increase.

Figure 9 presents the throughputs of the protocols UTW-2TS-DNC and UTW-2TS-SC versus $\Psi$ when $Q = 10$ (dB), $\alpha_1 = \alpha_2 = 0.5$, $\beta_1 = 0.7$, $\beta_2 = 1 - \beta_1 = 0.3$, $x_R = 0.3$, $y_R = 0$, $x_{PR} = y_{PR} = 0.5$, and $M = 3$. Considering the case of the imperfect SICs for both protocols, the throughput performances decrease when the residual interference powers increase. Furthermore, the throughputs of the protocols UTW-2TS-DNC and UTW-2TS-SC in the
perfect SICs are upper limitations (expectations) of the ones in the imperfect SICs when $\Psi \to -\infty$ (dB).

5. Conclusions
In this paper, we proposed and analyzed the underlay two-way relaying scheme with two secondary sources and multiple secondary relays, known as the UTW-2TS scheme. The UTW-2TS scheme with the SIC solution operated in only two time slots and under an interference constraint of the primary receiver. The secondary relays decode successively the data transmitted by two secondary sources and then encode these data by two techniques: the DNC enforced by XOR operations (known as the UTW-2TS-DNC protocol) and the SC enforced by power domain additions (known as the UTW-2TS-SC protocol). A selected secondary relay which subjects to maximize the decoding capacities and to minimize the collection time of CSI in the proposed protocols UTW-2TS-DNC and UTW-2TS-SC suffered the residual interferences from the imperfect SIC operations. Exact outage probabilities and throughputs were derived to evaluate the system performance of the proposed UTW-2TS-DNC and UTW-2TS-SC protocols. Simulation and analysis results provided discoveries of the performance improvements by increasing of the number of the cooperative secondary relays, the interference constraints, and the distances from the secondary network to the primary receiver. In addition, the proposed UTW-2TS-DNC and UTW-2TS-SC protocols achieved the best throughputs at optimal interference power allocation coefficients and optimal locations of the selected secondary relay. Considering the same power consumption, the UTW-2TS-DNC protocol performed better than the UTW-2TS-SC protocol. Finally, the analysis results of the outage probabilities and throughputs were validated by the Monte Carlo simulations.

Appendix
A. Proof of Lemma 1
Substituting (3) into the formula of $\Phi_1$ as in (18), we have the expression

$$
\Phi_1 = \Pr \left\{ \frac{\alpha_1 Q g_{S,R_m} g_{S,PR}}{\alpha_2 Q g_{S,R_m} g_{S,PR} + g_{S,PR} g_{S,PR}} < \gamma_{th} \right\}
$$

$$
= \Pr \left\{ \frac{g_{S,R_m}}{g_{S,PR}} < \frac{\gamma_{th} \alpha_2 g_{S,R_m} + g_{S,PR}}{\alpha_1 Q g_{S,PR}} \right\}
$$

$$
= \Pr \left\{ \frac{G_{S,R_m}/S,PR}{G_{S,R_m}/S,PR} < \frac{\gamma_{th} \alpha_2}{\alpha_1 Q} \frac{g_{S,R_m} + g_{S,PR}}{g_{S,PR}} \right\}
$$

$$
= \int_0^{\gamma_{th}} f_{G_{S,R_m}/S,PR}(y) \times F_{G_{S,R_m}/S,PR}(y_1 + y_2)dy.
$$

(A.1)

From (23), we infer the CDF and the corresponding PDF of the RV $G_{S,R_m}/S,PR$ as

$$
F_{G_{S,R_m}/S,PR}(z) = \lambda_5 \sum_{p=0}^{\infty} \frac{(-1)^p}{\lambda_5 + p \lambda_2} = \frac{1}{\lambda_5 + \lambda_2 z}
$$

$$
f_{G_{S,R_m}/S,PR}(z) = \frac{\lambda_2}{\lambda_5 + \lambda_2 z}.
$$

(A.2)

$$
f_{G_{S,R_m}/S,PR}(z) = \frac{dF_{G_{S,R_m}/S,PR}(z)}{dz} = \frac{\lambda_2}{(\lambda_5 + \lambda_2 z)^2}.
$$

(A.3)

Substituting (23) and (A.3) into (A.1), we have the equivalent formula:

$$
\Phi_1 = \int_0^{\gamma_{th}} \frac{\lambda_2 \lambda_5}{(\lambda_5 + \lambda_2 z)^2} \sum_{p=0}^{\infty} \frac{p}{M} \left( \frac{(-1)^p}{\lambda_4 + p \lambda_1 \nu_1 + \nu_2} \right) dy
$$

$$
= \lambda_2 \lambda_5 \sum_{p=0}^{\infty} \left( \frac{p}{M} \right) \int_0^{\gamma_{th}} \frac{(-1)^p}{(\lambda_4 + p \lambda_1 \nu_1 + \lambda_2 \nu_2)} dy
$$

$$
= \lambda_2 \lambda_5 \sum_{p=0}^{\infty} \left( \frac{p}{M} \right) \int_0^{\gamma_{th}} \frac{(-1)^p}{(\lambda_4 + p \lambda_1 \nu_1 + \lambda_2 \nu_2)} dy.
$$

(A.4)

By performing variable transformations for the integral in (A.4) as $t = 1/(\lambda_2 + \lambda_4 \nu_2)$ and $y = \nu_1(t) + \lambda_2 \nu_1 \nu_2 - \lambda_5 \nu_2(t)$, where $\nu_1(t) = \lambda_4 + \lambda_1 \nu_1$ and $\nu_1(t) = \lambda_1 \nu_1$, Lemma 1 is proven completely.

B. Proof of Lemma 2
Substituting (3) and (6) into the formula of $\Phi_2$ as in (18), the probability $\Phi_2$ is expressed as

$$
\Phi_2 = \Pr \left\{ \left( \frac{G_{S,R_m}/S,PR}{G_{S,R_m}/S,PR} \gtrless \nu_1 \right) \cap \left( G_{S,R_m}/S,PR < \gamma_{th} \right) \right\}
$$

$$
= \Pr \left\{ \left( G_{S,R_m}/S,PR \gtrless \nu_1 \right) \cap \left( G_{S,R_m}/S,PR < \gamma_{th} \right) \right\}.
$$

(B.1)

In (B.1), we consider two cases of perfect SICs ($\varepsilon = 0$) and imperfect SICs ($\varepsilon = 1$) to solve the following:

Case $\varepsilon = 0$: formula (B.1) is expressed and manipulated as

$$
\Phi_2 = \Pr \left\{ \left( G_{S,R_m}/S,PR \gtrless \nu_1 \right) \cap \left( G_{S,R_m}/S,PR < \gamma_{th} \right) \right\}
$$

$$
= \int_0^{\gamma_{th}} f_{G_{S,R_m}/S,PR}(y) \times \left( 1 - F_{G_{S,R_m}/S,PR}(y_1 + y_2) \right) dy
$$

$$
= F_{G_{S,R_m}/S,PR}(y_2) - \int_0^{\gamma_{th}} f_{G_{S,R_m}/S,PR}(y) \times F_{G_{S,R_m}/S,PR}(y_1 + y_2) dy.
$$

(B.2)

where $F_{G_{S,R_m}/S,PR}(y)$ is the CDF of the RV $G_{S,R_m}/S,PR$ at a value $y_5 = \gamma_{th}/(\alpha_2 Q)$ (see (A.2)).
By using the PDF of the RV $G_{\xi_5,\nu_5,\nu_3,PR}$ as in (A.3) and the CDF of the RV $G_{\xi_5,\nu_5,\nu_3,PR}$ as in (23), formula (B.2) is rewritten as

$$\Phi_2 = F_{G_{\xi_5,\nu_5,\nu_3,PR}}(v_5) - \lambda_2 \lambda_4 \lambda_5 \sum_{p=0}^{M} \left( \frac{p}{M} \right) (-1)^p \int_0^{v_5} dy 
\left( \lambda_4 + p \lambda_4 v_2 + p \lambda_4 v_2 y \right) \left( \lambda_5 + \lambda_2 y \right)^2.$$  

(B.3)

Solving (B.3) also by variable transformations as in (A.4), $\Phi_2$ is presented as (25) in Lemma 2.

Case $\epsilon = 1$: formula (B.1) is performed as

$$\Phi_2 = \Pr \left\{ \left( G_{\xi_5,\nu_5,\nu_3,PR} \geq v_1 G_{\xi_5,\nu_5,\nu_3,PR} + v_2 \right) \cap \left( G_{\xi_5,\nu_5,\nu_3,PR} < v_5 + v_6 g_m \right) \right\}$$

$$= \int_0^{v_5 + v_6 g_m} \int_0^{v_1 + v_2} \left( 1 - F_{G_{\xi_5,\nu_5,\nu_3,PR}}(v_1 y + v_2) \right) \times f_{G_{\xi_5,\nu_5,\nu_3,PR}}(y) dy dt.$$  

(B.4)

Using the result of (B.2) for the inner integral of (B.4) and the PDF of RV $g_m$, we have the next result:

$$\Phi_2 = \int_0^{\infty} \omega \ e^{-\omega t} \left( \frac{\lambda_2 (v_5 + v_6 t)}{\lambda_2 + \lambda_2 (v_5 + v_6 t)} - \lambda_2 \lambda_4 \lambda_5 \sum_{p=0}^{M} \left( \frac{p}{M} \right) \frac{(-1)^p}{\lambda_2 v_3(p) - \lambda_2 v_4(p)} \right) \ln \left( \frac{\lambda_5 + \lambda_2 (v_5 + v_6 t) v_3(p)}{\lambda_5 v_3(p) + (v_5 + v_6 t) v_4(p)} \right) \right) dt$$

$$= \lambda_2 \lambda_4 \lambda_5 \sum_{p=0}^{M} \left( \frac{p}{M} \right) \frac{(-1)^p}{\lambda_2 v_3(p) - \lambda_2 v_4(p)} \int_0^{\infty} e^{-\omega t} \ln \left( \frac{\lambda_5 + \lambda_2 (v_5 + v_6 t) v_3(p)}{\lambda_5 v_3(p) + (v_5 + v_6 t) v_4(p)} \right) dt.$$  

(B.5)

We solve sequentially the integrals $I_1, I_2$, and $I_3$ in (B.5) as

$$I_1 = 1 - \frac{\lambda_2 \lambda_4 \lambda_5}{\lambda_2 v_6} e^{(\lambda_2 + \lambda_2 v_6) \Omega/(\lambda_2 v_6)} \Gamma \left( 0, \frac{\lambda_5 + \lambda_2 v_3(\Omega)}{\lambda_2 v_6} \right),$$  

(B.6)

$$I_2 = \lambda_2 \lambda_4 \lambda_5 \sum_{p=0}^{M} \left( \frac{p}{M} \right) \frac{(-1)^p}{\lambda_2 v_3(p) - \lambda_2 v_4(p)} \int_0^{\infty} e^{-\omega t} \ln \left( \frac{\lambda_5 + \lambda_2 (v_5 + v_6 t) v_3(p)}{\lambda_5 v_3(p) + (v_5 + v_6 t) v_4(p)} \right) dt$$

$$= \lambda_2 \lambda_4 \lambda_5 \sum_{p=0}^{M} \left( \frac{p}{M} \right) \frac{(-1)^p e^{(\lambda_2 + \lambda_2 v_6) \Omega/(\lambda_2 v_6)}}{\lambda_2 v_3(p) - \lambda_2 v_4(p)} \int_0^{\infty} \frac{(-1)^p}{\lambda_5 v_3(p) - \lambda_5 v_4(p)} \Gamma \left( 0, \frac{\lambda_5 + \lambda_2 v_3(\Omega)}{\lambda_2 v_6} \right).$$  

(B.7)
where \( \Gamma(u, v) \) is the upper incomplete Gamma function ([42], eq. 8.350.2).

Substituting (B.6), (B.7), and (B.8) into (B.5), the probability \( \Phi_2 \) is analyzed for the case \( e = 1 \) as (26) in Lemma 2. Hence, the lemma is proven completely.

**Data Availability**

The data used to support the findings of this study are included within the article.

**Conflicts of Interest**

The authors declare that there is no conflict of interest regarding the publication of this paper.

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