

Research Article

Performance Analysis of Backhaul-Aware User Association in 5G Ultradense Heterogeneous Networks

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In 5G ultradense heterogeneous networks, wireless backhaul, as one of the important base station (BS) resources that affect user services, has attracted more and more attention. However, a user would access to the BS which is the nearest for the user based on the conventional user association scheme, which constrains the network performance improvement due to the limited backhaul capacity. In this paper, using backhaul-aware user association scheme, semiclosed expressions of network performance metrics are derived in ultradense heterogeneous networks, including coverage probability, rate coverage, and network delay. Specifically, all possible access and backhaul links within the user connectable range of BSs and anchor base stations (A-BSs) are considered to minimize the analytical results of outage probability. The outage for the user occurs only when the access link or backhaul link which forms the link combination with the optimal performance is failure. Furthermore, the theoretical analysis and numerical results evaluate the impact of the fraction of A-BSs and the BS-to-user density ratio on network performance metric to seek for a more reasonable deployment of BSs in the practical scenario. The simulation results show that the coverage probability of backhaul-aware user association scheme is improved significantly by about 2× compared to that of the conventional user association scheme when backhaul is constrained.

1. Introduction

The rapidly growing data traffic brings more and more pressure to the wireless networks, which is predicted to increase by over ten thousand times in the next twenty years. Considering the deficiency in exciting wireless networks nowadays, heterogeneous networks are necessary to achieve the high capacity and massive connectivity requirements for 5G and beyond 5G (B5G) [1]. Therefore, the heterogeneous communication networks should be deeply investigated to realize the full benefits of each of them. To achieve higher coverage and rate, lots of flexible architectures are designed for terrestrial heterogeneous networks, such as ultradense heterogeneous networks [2], UAV-assisted heterogeneous networks [3], and massive MIMO heterogeneous networks [4]. These heterogeneous networks are composed of different types of base stations (BS), which are completely different in density, user association, transmit power, frequency range, and so on [5].

The 5G/B5G network would face great challenges, including higher capacity, lower latency, higher reliability, lower cost, higher spectrum efficiency, and more spectrum resource [6, 7]. To meet these increasing demands in wireless traffic, one of the core characteristics of future cellular network is the ultradense heterogeneous networks, where small base stations with low power consumption and low cost are ultradensely deployed in hotspots and can significantly improve the system capacity [8–10].

Recently, research on user association scheme in heterogeneous networks has been paid attention as an inseparable part of radio resource allocation, on account of user association which is important to improve the performance of the network [1, 3]. Meanwhile, the limited backhaul capacity of small cells is an important factor which should be considered in the ultradense heterogeneous networks. The backhaul capacity constraint becomes increasingly stringent in heterogeneous networks and has great impact on the overall

network performance. In other words, the capacity of the backhauling solution might be a bottleneck for the radio interface capacity of the small cells both wired and wireless [11]. Therefore, the performance of backhaul links should be considered when making the access decision.

Taking all these factors into consideration, the codesign of access and backhaul is necessary so as to make full use of the available resources of small base stations and increase the capacity of ultradense heterogeneous networks. However, few studies have analyzed the impact of limited backhaul on coverage probability and rate coverage.

In addition, most heterogeneous network analysis considers that all small base stations (SBSs) in the coverage area of the macro base station (MBS) are connected to the MBS [12, 13] or all BSs (including MBSs and SBSs) are directly backhauled to the core network [14]. However, in the ultradense network, a large number of SBSs connected to the MBS not only bring about frequent handovers and other problems but also increase the management complexity [15]. Therefore, it is beneficial to select a part of the SBSs called Anchor BSs (A-BSs) to have a wired backhaul and the rest of SBSs to wireless backhaul to the A-BSs.

In this paper, the performance indexes including coverage probability, rate coverage, and network delay are obtained based on the backhaul-aware user association scheme. Furthermore, we verify the performance gains in coverage probability and rate coverage compared with the conventional user association scheme. The main contributions of this paper are summarized as follows:

- (i) Semiclosed expressions of coverage probability, rate coverage, and network delay are derived for the backhaul-aware user association scheme in the ultradense heterogeneous networks. Users, BSs, and A-BSs are modeled as independent two-dimensional Poisson point processes (PPPs) which capture the irregular network structure
- (ii) In the range of BSs and A-BSs that the user can access, all possible access and backhaul links are considered to minimize the analytical results of outage probability. The outage for the user is defined that access link or backhaul link which forms the link combination with the optimal performance is interrupted in this paper
- (iii) The influence of the fraction of A-BSs and the BS-to-user density ratio on coverage probability, rate coverage, and network delay is analyzed to look for a more reasonable deployment of BSs in the practical scenario. Based on theoretic analysis and numerical results, the performance gains in coverage probability and rate coverage are validated, compared with the conventional user association scheme based on the minimum path loss

2. Related Works

Recently, stochastic geometry has been used extensively to model wireless backhaul network. In [16], a relevant back-

haul model was proposed and the delay performance of various backhaul technologies with different capabilities and characteristics was analyzed. A closed expression of outage probability of mmWave wireless backhaul in the presence of blockage was derived, and the impacts of various system parameters were investigated in [17]. In [18], the performance limits of ultradense cloud access network under limited backhaul was investigated. In [19], the total expected delay was derived by taking into account retransmissions over the wireless link, as well as the backhaul delay incurred from both wired and wireless backhaul. However, the authors [16–19] only concern the performance analysis of backhaul links but fail to consider the performance analysis of access links. In addition, only a few works (e.g., [17]) derive the coverage probability or rate coverage in closed form.

The performance analysis of backhaul link and further joint backhaul-access analysis are imperative for coverage and rate improvement in a wireless backhaul network. In [20], a tractable model for rate was proposed in self-backhauled mmWave cellular networks. However, the situation that the backhaul is limited is not considered in [20]. In light of this, in order to further improve network performance, joint backhaul-access analysis was derived in [21, 22]. In [21], a semiclosed-form expression was derived, which analyzed the ergodic throughput of the network where BSs have limited backhaul capacity. In [22], analytical expressions for coverage and average DL rate were derived in IBFD self-backhauling HetNets and the mathematical model introduced an end-to-end joint analysis of backhaul and access links. However, this paper is different from the works of [20–22] in the following aspects:

- (i) The derivation for the expressions of coverage probability and rate coverage in [20] was based on the traditional user association scheme. However, when users select to access to the nearest base station, there may be a situation that the performance of backhaul links is quite bad leading that users could not transfer data packets to A-BSs in spite of the performance of access link is well. To solve the problem, it is necessary to access to the tagged BS using the backhaul-aware user association scheme
- (ii) A backhaul-aware η -optimal biasing adjustment model was proposed for flexible coverage in [21]. It mainly focused on the optimization of the coverage for throughput improvement while matching the backhaul capacity after users having made accessing decisions, rather than the gains of coverage and rate for the joint backhaul-access analysis when users make accessing decisions
- (iii) Through joint analysis of access and backhaul links, analytical expressions for coverage were derived under different network geometries, respectively, in [22]. However, it is necessary to obtain the optimal combination of access and backhaul link for the user to minimize the outage probability and achieve the maximized gains in coverage and rate for the backhaul-aware user association scheme

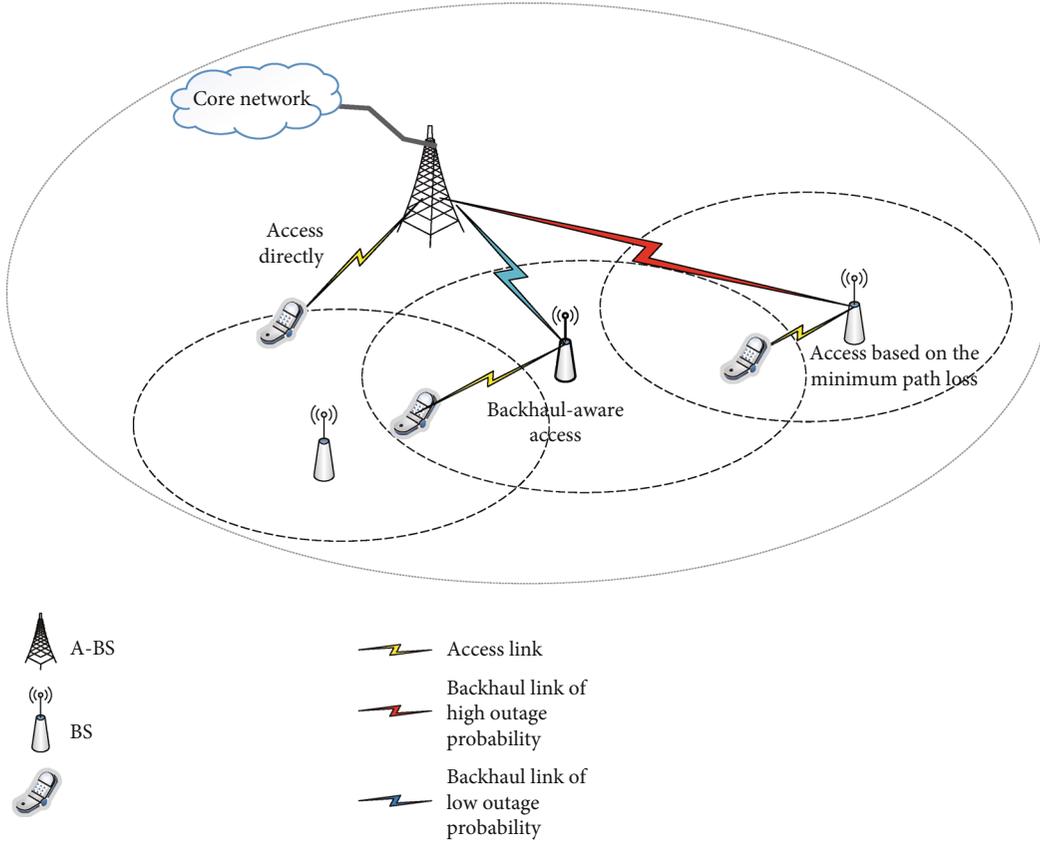


FIGURE 1: Wireless backhaul network with the A-BS providing the wireless backhaul to the associated BSs and access link to the associated users.

3. Mathematical Preliminaries

In this paper, log-normal random variables are used to model the shadowing effects caused by random blockages in ultra-dense network. We give the following lemmas which will be helpful to understand the analysis of the paper better.

Definition 1. The probability distribution of a log-normal random variable X is defined as

$$F_X(x) = \int_{t=-\infty}^x \frac{1}{\sqrt{2\pi}\sigma t} e^{-((\ln t - \mu)/\sigma)^2/2} dt, \quad (1)$$

where μ and σ are the mean and standard deviation of the variable's natural logarithm, respectively.

Lemma 2. Let $X \sim \ln N(\mu, \sigma^2)$, then $aX \sim \ln N(\mu + \ln a, \sigma^2)$, $a \in \mathbb{R}$.

Lemma 3. Let $X_j \sim \ln N(\mu_j, \sigma_j^2)$ which are independent log-normally distributed variables with varying μ_j and σ_j parameters, and $Y = \sum_{j=1}^n X_j$. Then the distribution of Y has no closed form expression but can be reasonably approximated by

another log-normal distribution Z with parameters [23].

$$\begin{aligned} \mu_Z &= \ln \left[\sum e^{\mu_j + \sigma_j^2/2} \right] - \frac{\sigma_Z^2}{2}, \\ \sigma_Z^2 &= \ln \left[\frac{\sum e^{2\mu_j + \sigma_j^2} (e^{\sigma_j^2} - 1)}{(\sum e^{\mu_j + \sigma_j^2/2})^2} + 1 \right]. \end{aligned} \quad (2)$$

4. System Model

4.1. The Network Model. The BSs and users are uniformly distributed in \mathcal{R}^2 according to independent PPPs denoted as Φ and Φ_u with densities λ and λ_u , respectively. A fraction ω of the BSs which are called A-BS henceforth wired backhaul and the rest of the BSs backhaul wirelessly to A-BSs.

In Figure 1, according to [17], it is probable that the outage probability of all possible backhaul links is high from the BS that a user selects to access based on the minimum path loss, leading that the traffic data could not transfer to A-BSs. To avoid this situation, users should access to the tagged BS based on the backhaul-aware user association scheme. In this way, the outage probability of the obtained access and backhaul link is defined as

$$P_{\text{out}} = 1 - \mathbb{P}\{\text{SNR}_a > T_a\} \mathbb{P}\{\text{SNR}_b > T_b\}, \quad (3)$$

where SNR_a and SNR_b denote the signal-noise ratio (SNR) of the access link and backhaul link, respectively, and T_a and T_b denote SNR coverage threshold of access link and backhaul link, respectively.

4.2. Propagation Assumptions. For analytic tractability, this paper uses the alpha plus beta model given in [24], which is based on the traditional free space path loss model and considers the log-normal shadowing. Therefore, the path loss associated with the transmission between any two nodes (user and BS, user and A-BS, or BS and A-BS) x_i and x_j can be expressed as

$$L(x_i, x_j)(\text{dB}) = \beta + 10\alpha \log \|x_i - x_j\| + \chi_N, \quad (4)$$

where $\|x_i - x_j\|$ is the distance between the i -th and j -th nodes and $\chi_N \sim N(0, \sigma^2)$. Let β be the path loss at a fixed small reference distance and α be the path loss exponent. Accordingly, for each of the LOS and NLOS cases, α , β , and σ^2 are altered.

Compared with low-frequency systems, in 5G ultradense networks, small-scale fading has little effect on the transmitted signal. It is mentioned a lot in the literature [24] that small-scale fading can be ignored in the analysis. However, in such networks, blockages and shadowing are more significant. Therefore, only considering shadowing and ignoring fading, the pdf of χ_N in (4) can be defined as

$$\chi_N \sim f_{\chi_N}(x; \mu_c, \sigma_c) = \frac{1}{x\sqrt{2\pi}\sigma_c} \exp\left(-\frac{(\log x - \mu_c)^2}{2\sigma_c^2}\right), \quad (5)$$

where the parameters μ_c and σ_c follow from [20] and $x > 0$.

Assume that all BSs are equipped with directional antennas with sector gain patterns. The antenna gain pattern of the BS is defined as a function of the angle θ

related to the steering angle as

$$G_b(\theta) = \begin{cases} G_{\max}, & \text{if } |\theta| \leq \theta_b, \\ G_{\min}, & \text{otherwise,} \end{cases} \quad (6)$$

where θ_b is the main lobe width or beam width. For simplicity, we assume that the antenna beams of the target access and backhaul links should be aligned; that is, the effective gain of the required access link and backhaul link is G_{\max} .

4.3. Access and Backhaul Load. It is assumed that the access and backhaul links share the same radio resource pool through orthogonal division, so the user rate depends on the user load on the BS and the BS load on the A-BS. Let $N_{u,b}$, $N_{u,a}$ and $N_{b,a}$, respectively, denote the number of users served by the tagged BS, users served by the tagged A-BS, and BSs associated with the tagged A-BS.

Since A-BS serves both users and BSs, it is assumed that the resources allocated to associated BSs that further serve their associated users are proportional to their average user load. The average number of users per BS can be expressed as $\kappa = \lambda_u/\lambda$, and the fraction of resources available for all the associated BSs at an A-BS is $\eta_{b,a} = \kappa N_{b,a}/(\kappa N_{b,a} + N_{u,a})$. Moreover, the fraction of resources available to the BS associated with the same A-BS is $\eta_{b,a}/N_{b,a}$, due to the fraction of resources allocated to the associated BSs at an A-BS assumed to be shared equally among the BSs, which is equivalent to the resource fraction used for backhaul by the corresponding BS. For each BS, it is assumed that access and backhaul capacity are shared equally among associated users, and the user rate is equal to the minimum of the access link rate and the backhaul link rate.

With the above described resource allocation model, the rate of a user is given by (7), as follows:

$$\text{Rate} = \begin{cases} \frac{B}{\kappa N_{b,a} + N_{u,a}} \log(1 + \text{SNR}_a), & \text{if associated with an A-BS,} \\ \frac{B}{N_{u,b}} \min\left(\left(1 - \frac{\kappa}{\kappa N_{b,a} + N_{u,a}}\right) \log(1 + \text{SNR}_a), \frac{\kappa}{\kappa N_{b,a} + N_{u,a}} \log(1 + \text{SNR}_b)\right), & \text{otherwise.} \end{cases} \quad (7)$$

4.4. SNR Analysis Model and Scheme. As stated before, any node can receive a signal through either LOS link or NLOS link. Leveraging the modeling of blockages from the fixed LOS probability model as was depicted in [25]; we consider two statistical models for every link. Assuming that the LOS area within a sphere was centered on the reference point with a radius of r_D . If the length of the LOS link is r , then if $r < r_D$, the probability of the link being LOS is p_{LOS} ; otherwise, it is 0. Similarly, the NLOS probability is denoted as p_{NLOS} . The parameters r and r_D are dependent on the deployment scenario of the network, and the values are based on the data accumulated by [25].

We now compute the SNR distribution taking both the LOS and NLOS links into consideration. The SNR can be formulated as

$$\gamma^i = \frac{PG_{\max} r^{-\alpha_i} \chi_N}{N_0}, \quad (8)$$

where $i \in \{\text{LOS}, \text{NLOS}\}$, P is the transmit power, r is the link length, α_i is the path loss exponent, and N_0 is the noise

power. The achievable SNR can be given as

$$\gamma_j = \gamma_j^{\text{LOS}} p_{\text{LOS}} + \gamma_j^{\text{NLOS}} p_{\text{NLOS}}, \quad (9)$$

where $j \in \{(u, a), (u, b), (b, a)\}$ represent the link from A-BS to associated user, from BS to associated user, and from A-BS to associated BS, respectively.

Considering the LOS scenario, the SNR distribution between the user and A-BS can be given as

$$\begin{aligned} F_{\gamma_{u,a}^{\text{LOS}}}(T_a) &= \mathbb{P}\left\{\frac{PG_{\max}\chi_N}{r^{\alpha_{\text{LOS}}}N_0} < T_a\right\} \\ &= \mathbb{P}\left\{\chi_N < \frac{T_a r^{\alpha_{\text{LOS}}}N_0}{PG_{\max}}\right\} \\ &= \frac{1}{2} \operatorname{erfc}\left(-\frac{\log(T_a r^{\alpha_{\text{LOS}}}N_0/PG_{\max}) - \mu_{u,a}^{\text{LOS}}}{\sqrt{2}\sigma_{u,a}^{\text{LOS}}}\right) \\ &= Q\left(\frac{\log(T_a r^{\alpha_{\text{LOS}}}N_0/PG_{\max}) - \mu_{u,a}^{\text{LOS}}}{\sigma_{u,a}^{\text{LOS}}}\right), \end{aligned} \quad (10)$$

where erfc is the complementary error function and Q is the cumulative distribution function of the standard normal distribution.

Using Lemma 2, the distribution of $\gamma_{u,a}^{\text{LOS}} p_{\text{LOS}}$ can be given as

$$F_{\gamma_{u,a}^{\text{LOS}}}(T_a) = Q\left(\frac{\log((T_a r^{\alpha_{\text{LOS}}}N_0)/PG_{\max}) - (\mu_{u,a}^{\text{LOS}} + p_{\text{LOS}})}{\sigma_{u,a}^{\text{LOS}}}\right). \quad (11)$$

Similarly, the SNR distribution in NLOS scenario and the distribution of $\gamma_{u,a}^{\text{NLOS}} p_{\text{NLOS}}$ can be characterized. The distribution of the total SNR $\gamma_{u,a}$ calculated using (5) has no closed form expression, because $\gamma_{u,a}^{\text{LOS}}$ and $\gamma_{u,a}^{\text{NLOS}}$ are two independent log-normally distributed variables. However, using Lemma 3, it can be approximated by another log-normal distribution with parameters $\mu_{u,a}$ and $\sigma_{u,a}^2$. Therefore, the distribution of the total SNR $\gamma_{u,b}$ and $\gamma_{b,a}$ can be characterized.

5. Analysis of Performance Metric

This is the main analysis section of the paper, in which network performance metrics based on the backhaul-aware user association scheme are derived in ultradense heterogeneous networks, including the coverage probability, rate coverage, and network delay.

5.1. Coverage Probability. Assume that the distances from the user to the tagged A-BS and tagged BS are, respectively r_1 and r_2 .

With the trigonometric formula and the scene, the distance from BS to the tagged A-BS can be given as

$$r_3 = \sqrt{r_1^2 + r_2^2 - 2r_1r_2 \cos \theta}. \quad (12)$$

First considering a simple situation, there are only one BS and one A-BS in the network. By the analysis of Section 4.4, when the user connects to the tagged A-BS directly, the SNR distribution can be given as

$$F_{\gamma_{u,a}}(T_a) = Q\left(\frac{\log((T_a r_1^{\alpha_i} N_0)/PG_{\max}) - \mu_{u,a}}{\sigma_{u,a}}\right). \quad (13)$$

Similarly, when the user connects to the tagged A-BS in two-hop links, the SNR distribution of backhaul link $F_{\gamma_{b,a}}$ (T_b) and the SNR distribution of access link from user to the tagged BS $F_{\gamma_{u,b}}(T_a)$ can be obtained.

Then, considering the actual scene, there are numerous A-BSs and BSs that a user may connect to, which are in the user connectable range.

Due to the assumption that the A-BSs are distributed uniformly in the plane as a homogeneous PPP, the probability of the number of A-BSs presenting in a two-dimensional area A can be expressed as $\mathbb{P}[M(A) = n_1] = ((\lambda\omega|A|)^{n_1}/n_1!)e^{-(\lambda\omega|A|)}$ and the pdf of distance r_1 can be expressed as $f_r(r_1) = 2r_1/R_1^2$, where R_1 is the range of A-BS that the tagged user can access.

Assume that N_1 is the maximum number of A-BSs that the tagged user can access in the two-dimensional area A ; the pdf of outage for a user associated with an A-BS can be given as

$$\begin{aligned} P_1(T_a) &= \sum_{n_1=0}^{N_1} \mathbb{P}[M(A) = n_1] \mathbb{P}(\gamma_{u,a} < T_a)^{n_1} \\ &= \sum_{n_1=0}^{N_1} \frac{(\lambda\omega\pi R_1^2)^{n_1}}{n_1!} \exp(-\lambda\omega\pi R_1^2) [F_{\gamma_{u,a}}(T_a)]^{n_1}. \end{aligned} \quad (14)$$

Therefore, the coverage probability for a user associated with an A-BS can be given as

$$\begin{aligned} S_{u,a}(T_a) &= \int_0^{R_1} \mathbb{P}[\text{SNR}_{u,a} > T_a | r] f_r(r) dr_1 \\ &= \int_0^{R_1} [1 - P_1(T_a)] \frac{2r_1}{R_1^2} dr_1. \end{aligned} \quad (15)$$

A user will connect to the tagged A-BS in two-hop links if and only if the performance of direct links from the tagged user to the whole A-BSs in the user-connectable range (SNR in this paper) is not good enough and falls below a certain threshold. Hence, when the tagged user connects to the corresponding A-BS through backhaul link, the pdf of outage can be given as (16), where R_2 is the range of BS that the tagged user can access and N_2 is assumed to be the maximum number of BSs that the tagged user can access in the two-dimensional area B .

$$\begin{aligned}
P_{\text{out}}(T_a, T_b) &= \sum_{n_1=0}^{N_1} \sum_{n_2=0}^{N_2} \mathbb{P}(M(A) = n_1) \mathbb{P}(M(B) = n_2) \\
&\cdot \mathbb{P}(\gamma_{u,a} < T_a)^{n_1} [1 - \mathbb{P}(\gamma_{u,b} > T_a) \mathbb{P}(\gamma_{b,a} > T_b)]^{n_1 n_2} \\
&= \sum_{n_1=0}^{N_1} \sum_{n_2=0}^{N_2} \frac{(\lambda \omega \pi R_1^2)^{n_1}}{n_1!} e^{-\lambda \omega \pi R_1^2} \frac{(\lambda(1-\omega) \pi R_2^2)^{n_2}}{n_2!} e^{-\lambda(1-\omega) \pi R_2^2} \\
&\cdot F_{\gamma_{u,a}}(T_a)^{n_1} \left[1 - \left(1 - F_{\gamma_{u,b}}(T_a) \right) \left(1 - F_{\gamma_{b,a}}(T_b) \right) \right]^{n_1 n_2}.
\end{aligned} \tag{16}$$

In the derivation of (16), we also consider the situation that a user associates with an A-BS. So, (16) is also the expression of the pdf of outage for a user connecting to the core network.

Therefore, the coverage probability can be given as

$$P_c(T_a, T_b) = \int_0^{R_1} \frac{2r_1}{R_1^2} dr_1 \int_0^{R_2} \frac{2r_2}{R_2^2} dr_2 \int_0^{2\pi} \frac{1}{2\pi} [1 - P_{\text{out}}(T_a, T_b)] d\theta. \tag{17}$$

5.2. Rate Coverage. Assume that the number of users served by the tagged BS and the number of BSs served by the tagged A-BS are independent of each other and the corresponding link SNRs. The analysis of load characterization is similar with [16] and is thus omitted.

With the analysis of Section 5.1, the pdf of outage for a user associated with a BS can be given as

$$\begin{aligned}
P_2(T_a) &= \sum_{n_2=0}^{N_2} \mathbb{P}(M(B) = n_2) \mathbb{P}(\gamma_{u,b} < T_a)^{n_2} \\
&= \sum_{n_2=0}^{N_2} \frac{(\lambda(1-\omega) \pi R_2^2)^{n_2}}{n_2!} e^{-\lambda(1-\omega) \pi R_2^2} \left[F_{\gamma_{u,b}}(T_a) \right]^{n_2}.
\end{aligned} \tag{18}$$

By the integration of (18), we can obtain the SNR distribution of access links from a user to the tagged BS

$$\begin{aligned}
S_{u,b}(T_a) &= \int_0^{R_2} \mathbb{P}[\text{SNR}_{u,b} > T_a \mid r] f_r(r_2) dr_2 \\
&= \int_0^{R_2} [1 - P_2(T_a)] \frac{2r_2}{R_2^2} dr_2.
\end{aligned} \tag{19}$$

When the performance of all the access links from a user to the tagged A-BS in the user-connectable range is terrible, we consider all possible paths of backhaul links to obtain the pdf of outage for backhaul link (20), shown as follows:

$$\begin{aligned}
P_3(T_a, T_b) &= \sum_{n_1=0}^{N_1} \sum_{n_2=0}^{N_2} \mathbb{P}(M(A) = n_1) \mathbb{P}(M(B) = n_2) \\
&\cdot [1 - \mathbb{P}(\gamma_{u,b} > T_a) \mathbb{P}(\gamma_{b,a} > T_b)]^{n_1 n_2} \\
&= \sum_{n_1=0}^{N_1} \sum_{n_2=0}^{N_2} \frac{(\lambda \omega \pi R_1^2)^{n_1}}{n_1!} e^{-\lambda \omega \pi R_1^2} \frac{(\lambda(1-\omega) \pi R_2^2)^{n_2}}{n_2!} \\
&\cdot e^{-\lambda(1-\omega) \pi R_2^2} \left[1 - \left(1 - F_{\gamma_{u,b}}(T_a) \right) \left(1 - F_{\gamma_{b,a}}(T_b) \right) \right]^{n_1 n_2}.
\end{aligned} \tag{20}$$

By the integration of (20), the SNR distribution for backhaul links can be given as

$$\begin{aligned}
S_{b,a}(T_b) &= \mathbb{P}(\text{SNR}_{b,a} > T_b) \\
&= \int_0^{R_1} \frac{2r_1}{R_1^2} dr_1 \int_0^{R_2} \frac{2r_2}{R_2^2} dr_2 \int_0^{2\pi} \frac{1}{2\pi} [1 - P_3(T_a, T_b)] d\theta.
\end{aligned} \tag{21}$$

Let A_w denote the event of the typical user associating with an A-BS, i.e., $P(A_w) = w$. Then, the rate coverage of a typical user in an ultradense network, which is described in the Section 4.3, for a rate threshold ρ is given by (22), where $\bar{\rho} = \rho/B$ and $v(x) = 2^x - 1$. (a) in (22) is obtained followed by invoking the independence among various loads and SNRs.

$$\begin{aligned}
R(\rho) \Delta &= \mathbb{P}(\text{rate} > \rho) = (1 - \omega) \\
&\cdot \mathbb{E} \left[S_{u,b} \left(v \left\{ \bar{\rho} N_{u,b} \frac{N_{b,a} + N_{u,a}/\kappa}{N_{b,a} + N_{u,a}/\kappa - 1} \right\} \right) \right. \\
&\cdot S_{b,a} \left(v \left\{ \bar{\rho} N_{u,b} (N_{b,a} + N_{u,a}/\kappa) \right\} \right) \\
&\left. + \omega \mathbb{E} \left[S_{u,a} \left(v \left\{ \bar{\rho} (N_{u,a} + \kappa N_{b,a}) \right\} \right) \right] \right) \\
&\stackrel{(a)}{=} (1 - \omega) \sum_{N_{u,b} \geq 1, N_{b,a} \geq 1, N_{u,a} \geq 0} K_t(\lambda(1-\omega), \lambda\omega, N_{b,a}) \\
&\cdot K_t(\lambda_u, \lambda, N_{u,b}) S_{u,b} \left(v \left\{ \bar{\rho} N_{u,b} \frac{N_{b,a} + N_{u,a}/\kappa}{N_{b,a} + N_{u,a}/\kappa - 1} \right\} \right) \\
&\times S_{b,a} \left(v \left\{ \bar{\rho} N_{u,b} (N_{b,a} + N_{u,a}/\kappa) \right\} \right) \\
&+ \omega \sum_{N_{b,a} \geq 0, N_{u,a} \geq 1} K(\lambda(1-\omega), \lambda\omega, N_{b,a}) K_t(\lambda_u, \lambda, N_{u,a}) \\
&\cdot S_{u,a} \left(v \left\{ \bar{\rho} (N_{u,a} + \kappa N_{b,a}) \right\} \right).
\end{aligned} \tag{22}$$

The rate coverage with mean load approximation for (22) using the analysis of load characterization in the appendix can be simplified by (23), shown as follows:

$$\begin{aligned}
\bar{R}(\rho) &= (1 - \omega) S_{u,b} \left(v \left\{ \bar{\rho} \left(1 + 1.28 \frac{\lambda_u}{\lambda} \right) \left(2 + 1.28 \frac{1-\omega}{\omega} \right) \right\} \right) \\
&\cdot S_{b,a} \left(v \left\{ \bar{\rho} \left(1 + 1.28 \frac{\lambda_u}{\lambda} \right) \frac{2 + 1.28(1-\omega)/\omega}{1 + 1.28(1-\omega)/\omega} \right\} \right) \\
&+ \omega S_{u,a} \left(v \left\{ \bar{\rho} \left(\frac{\lambda_u(1-\omega)}{\lambda\omega} + 1 + 1.28 \frac{\lambda_u}{\lambda} \right) \right\} \right).
\end{aligned} \tag{23}$$

Remark 4. As can be seen from the above expression (23), if we increase the fraction of A-BSs ω in the network, the probability of being served by an A-BS (the weight of the first term) will increase. With user and BS load per A-BS decreasing, the rate from an A-BS also increases with ω . Moreover, increasing ω also increases the backhaul rate of a user associated with a BS in the second term.

5.3. Network Delay. From the above derivation of rate coverage, the probability of a failure transmission for a typical user accessing to the tagged A-BS directly, a typical user accessing to the tagged BS, and backhaul to the tagged A-BS from typical BS is, respectively, given as $P_1(T_a)$, $P_2(T_a)$, and $P_3(T_a, T_b)$.

Using the analysis of [19], the pdf of the expected wireless delay for a typical user connecting to the tagged A-BS can be obtained by

$$t_1(T_a) = T_1 \sum_{i=1}^M \frac{1 - P_1(T_a)^i}{1 - P_1(T_a)}, \quad (24)$$

where M is the packet retransmission number from a typical A-BS to its serving user and T_1 is the time taken for a single packet transmission from a typical A-BS to its serving user.

Proof. The first failure occurs with probability $1 - P_s(r)$ and requires additional time T_1 . Given a first failure, the second failure occurs with probability $1 - P_s(r)$ and requires additional time T_1 . Continuing in this way, the expected delay is equivalent to

$$\begin{aligned} E[T] &= T_1 + (1 - P_s(r))[T_1 + (1 - P_s(r))(T + \dots)] \\ &= T_1 [1 + (1 - P_s(r)) + \dots + (1 - P_s(r))^{M-1}] \\ &= T_1 \left[1 - \frac{(1 - P_s(r)M)}{P_s(r)} \right]. \end{aligned} \quad (25)$$

Assume that the packet retransmission number from a typical BS to its serving user is also M and the time taken for a single packet transmission from a typical BS to its serving user is also T_1 . Similarly, the pdf of the expected wireless delay for a typical user connecting to the tagged BS can be obtained by

$$t_2(T_a) = T_1 \sum_{i=1}^M \frac{1 - P_2(T_a)^i}{1 - P_2(T_a)}. \quad (26)$$

If the backhaul does not interfere with the other transmissions, then the pdf of the expected backhaul delay is

$$t_3(T_a, T_b) = T_2 \sum_{i=1}^N \frac{1 - P_3(T_a, T_b)^i}{1 - P_3(T_a, T_b)}, \quad (27)$$

where N is the packet retransmission from a typical A-BS to the tagged BS and T_2 is the time taken for a single packet transmission from a typical A-BS backhaul to the tagged BS.

Remark 5. Noninterfering wireless backhaul simply adds a constant term to the expected delay that a single typical user connects to the core network. If the backhaul delay is large enough, the expected delay that users access to the tagged A-BS with the relay of the corresponding BS can always be higher than the expected delay that users access to the tagged A-BS directly. And if it is not too large, when the proportion of A-BS ω is higher, the expected delay that users access to the tagged A-BS directly will eventually be smaller than the expected delay that users access to the tagged A-BS with the relay of the corresponding BS.

Therefore, using (24), (26), and (27), the total expected network delay for the static case in noninterfering wireless backhaul is formulated as

$$\begin{aligned} E[T] &= \frac{\omega \lambda_u^2}{\lambda} \int_0^{R_1} t_1(T_a) \frac{2r_1}{R_1^2} dr_1 + \frac{(1 - \omega) \lambda_u^2}{\lambda} \int_0^{R_2} t_2(T_a) \frac{2r_2}{R_2^2} dr_2 \\ &\quad + (1 - \omega) \lambda_u \int_0^{R_1} \frac{2r_1}{R_1^2} dr_1 \int_0^{R_2} \frac{2r_2}{R_2^2} dr_2 \int_0^{2\pi} \frac{1}{2\pi} t_3(T_a, T_b) d\theta. \end{aligned} \quad (28)$$

6. Numerical Results

Numerical simulations are performed to verify the accuracy of the analytical results and to show how system parameters influence the performance. Simulation parameters are listed in Table 1, some of which refer to [25, 26].

6.1. Coverage Probability. Figure 2 gives the simulation result of SNR coverage probability under different proportions of A-BS for $\lambda = 2000/\text{km}^2$. As can be seen from Figure 2, the validity of the analysis is guaranteed by the good match of analytical curves to simulation results. Coverage probability is minimum when the fraction of A-BSs is about 1/3 and either a smaller or a larger fraction of A-BSs would make the coverage probability larger. Hence, using the backhaul-aware user association scheme, in order to improve the performance of coverage in UDN, the density of A-BSs should be much smaller or larger than the density of BSs.

Figure 3 presents coverage probability versus BS (A-BS) density under different SNR threshold for $\omega = 1/2$. Coverage probability increases with BS density because the typical user is more closed to the tagged BS. As can be seen, for different SNR thresholds, increasing BS density will lead to saturation of coverage probability in the sense that the gains of coverage probability obtained by having larger BS density vanish beyond a certain value of BS density. When BS density is larger than $3000/\text{km}^2$, coverage gain brought by further increasing BS density is very small.

Figure 4 gives the comparison of coverage probability between this paper and reference papers under different BS densities for $\omega = 1/4$. The user association scheme in reference papers [20–22] is that the typical user associates with the closest BS and the BS associates with the closest A-BS for backhaul link. Moreover, [21] makes the optimization of flexible coverage after users having associated with BSs and A-BSs and the definition of coverage probability and rate

TABLE 1: Simulation parameters.

Notation	Parameter	Value
P	BS transmit power	BS: 30 dBm
α	Path lose exponent	Access: LOS = 2.0, NLOS = 3.3 Backhaul: LOS = 2.0, NLOS = 3.5
β	Path loss at 1 m	70 dB
G_{\max}	Antenna gain	18 dB
N_0	Noise power	Thermal noise +10 dB noise figure
r_D	Radius of the bounded region	200 meters
M, N	Retransmission number	3
T_1	Access time	1
T_2	Backhaul time	0.5

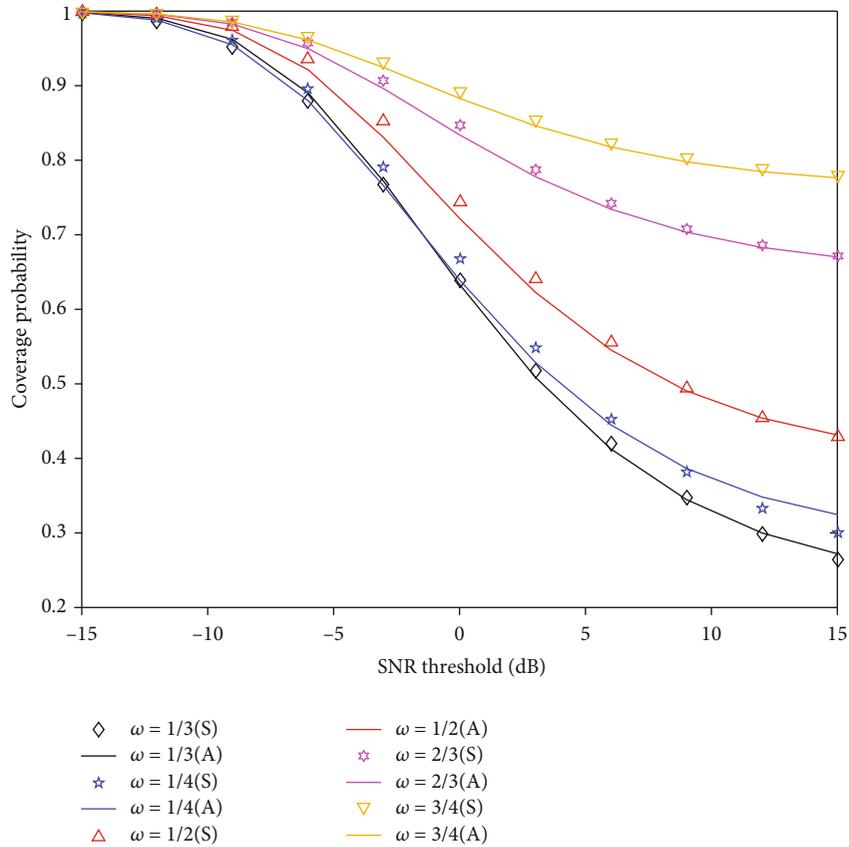


FIGURE 2: Simulation result of SNR coverage probability under different proportion of A-BS.

coverage in [22] is different. However, in this paper, when the typical user selects a BS to access, we consider all the possible access and backhaul links which are in the user connectable range of BSs and A-BSs to maximize the coverage probability of the user. Therefore, as can be seen in Figure 4, for the same SNR threshold and BS density, the coverage probability using the backhaul-aware user association scheme of this paper is higher than those using the user association schemes of reference papers.

6.2. Rate Coverage. Figure 5 presents the comparison of rate coverage between this paper and reference papers under different user-to-BS density ratio for $B = 2$ GHz, $\lambda = 1000/\text{km}^2$, and $\omega = 1/2$. As can be seen in Figure 5, the rate coverage of this paper is larger than those of reference papers, especially when the rate threshold is large, due to the improvement of the user association scheme. In addition, since there is a limitation for user connectable BS or A-BS range, when the rate threshold is lower than 10^8 bps, the gain of improving the

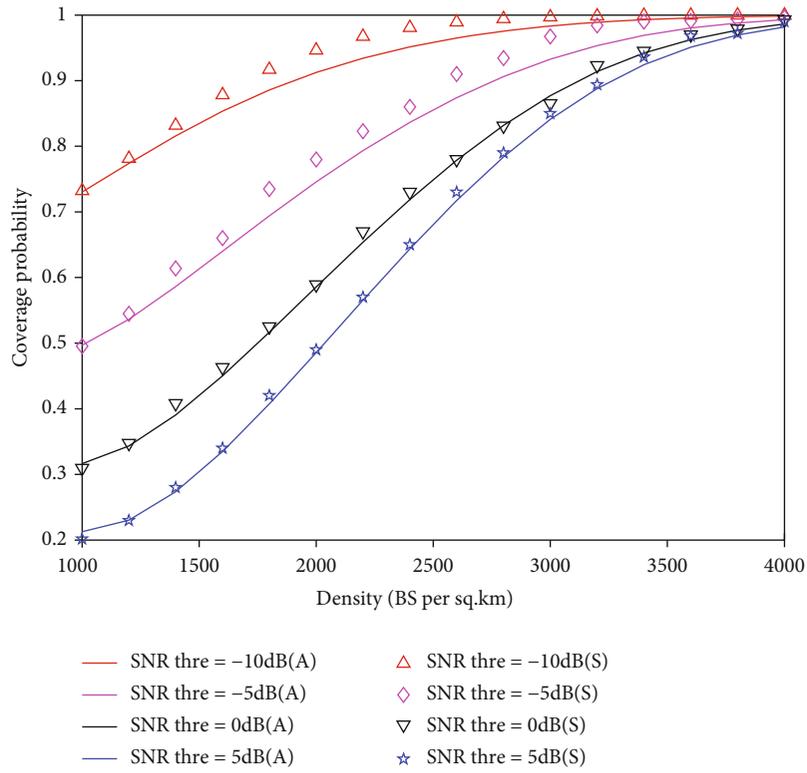


FIGURE 3: Relation between coverage probability and BS density under different SNR threshold.

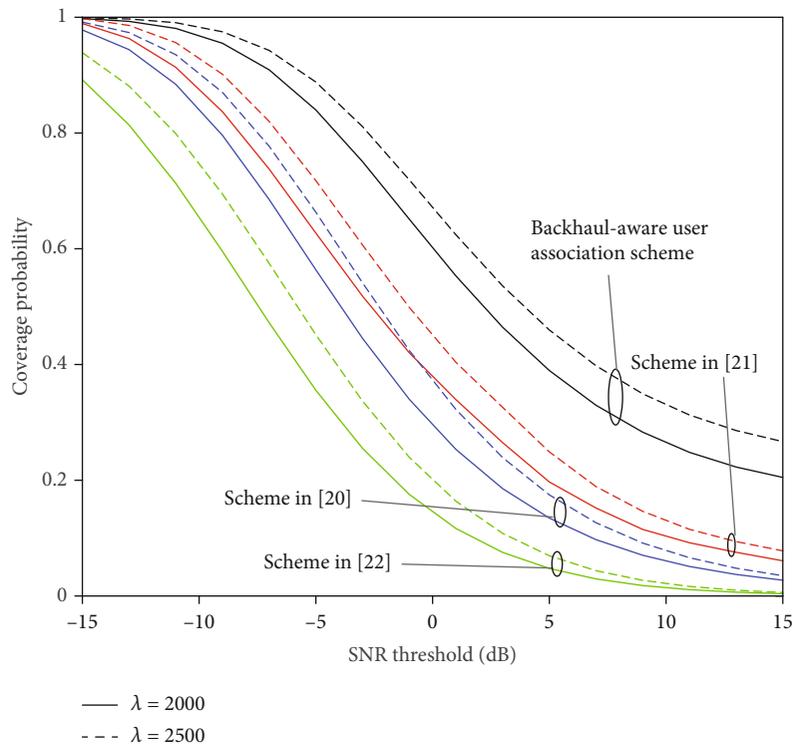


FIGURE 4: Comparison of coverage probability between this paper and reference papers under different BS densities.

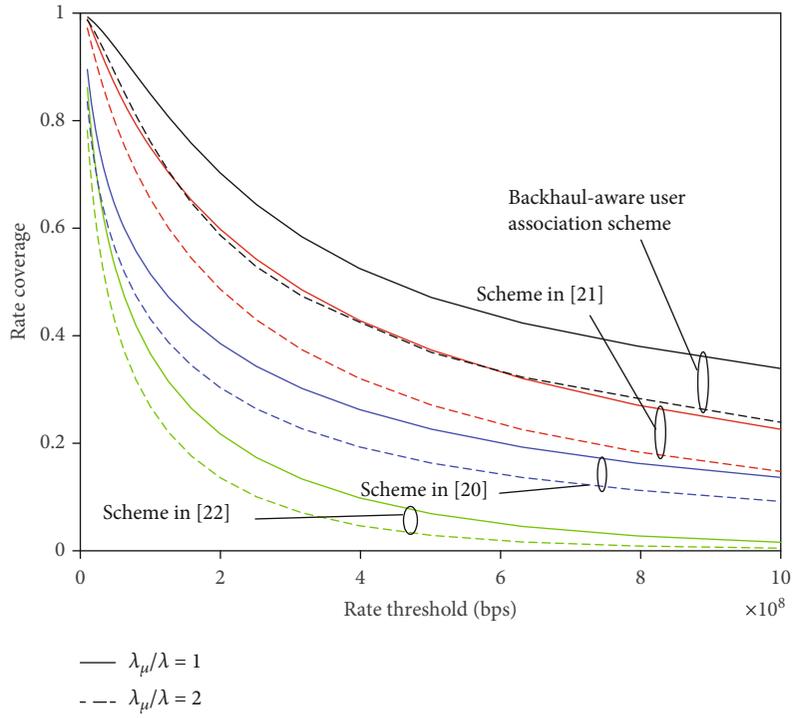


FIGURE 5: Comparison of rate coverage between this paper and reference papers under different user-to-BS density ratios.

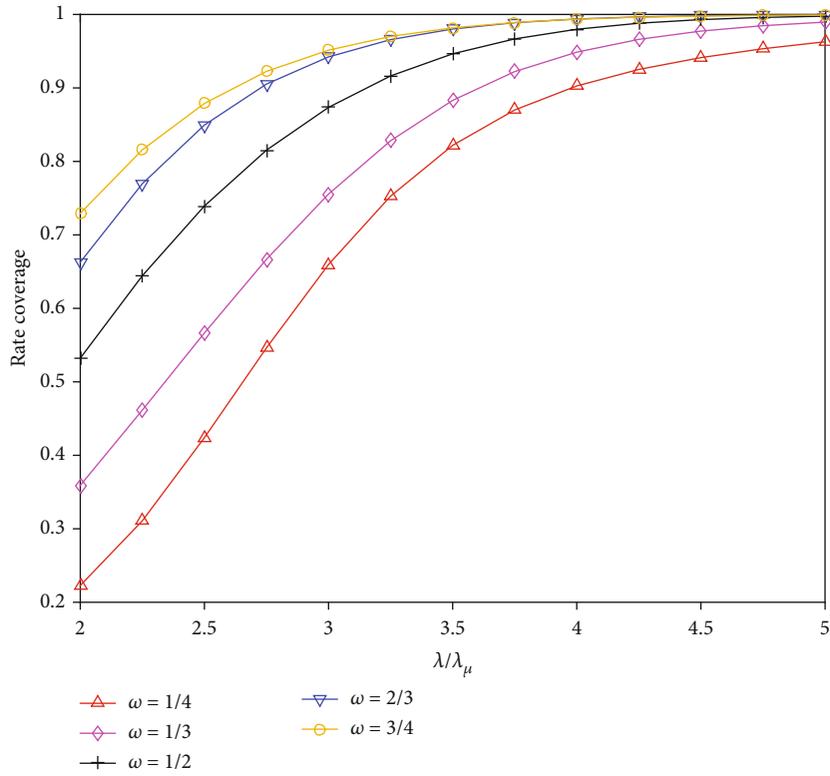


FIGURE 6: Relation between rate coverage and BS-to-user density ratio under different fraction of A-BSs.

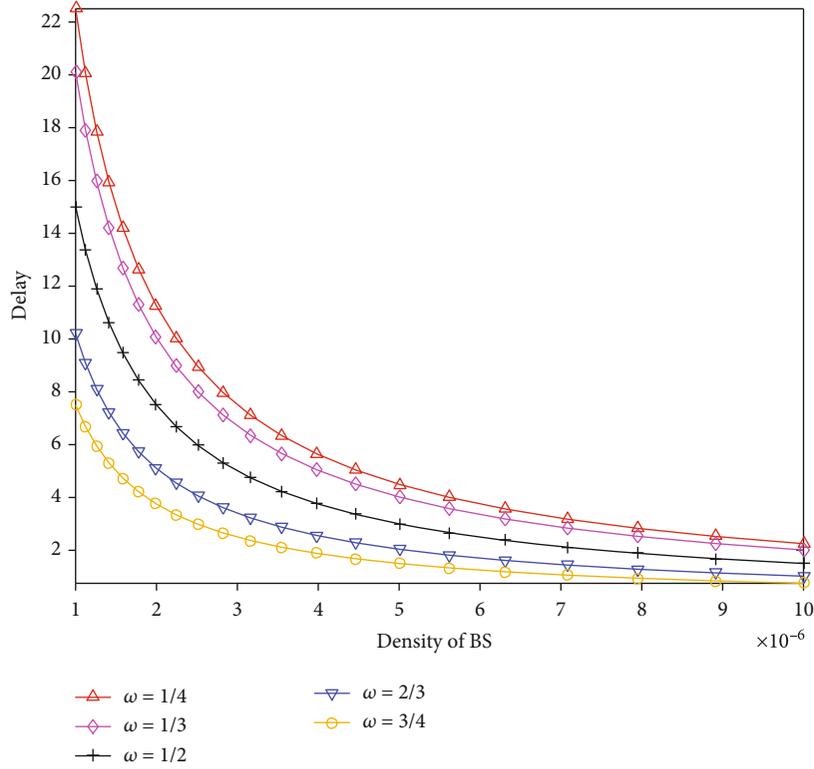


FIGURE 7: Relation between delay and BS density under different fraction of A-BSs.

user association scheme in rate coverage is not obvious and the performance of rate coverage is even worse compared with the reference papers [20–22].

The effect of BS-to-user density ratio on rate coverage is investigated in Figure 6 for $B = 2$ GHz, $\lambda_u = 500/\text{km}^2$ and $\text{thre} = 10^9$ bps. As can be seen in Figure 6, for a fixed ω , increasing λ/λ_u will lead to saturation of rate coverage in the sense that the gain of rate coverage obtained by having larger BS density vanish beyond a certain value of λ/λ_u . In addition, the rate coverage would increase with the increasing fraction of A-BSs.

6.3. Network Delay. Figure 7 presents network delay versus BS density under different fractions of A-BSs for $\lambda_u = 2 \times 10^{-5}/\text{km}^2$. As can be seen, for a fixed density of BSs and A-BSs, the network delay decreases with increasing fraction of A-BSs in ultradense network, since the increase of ω leads that the proportion of users which access to the tagged A-BS directly increases and the backhaul delay decreases. As the density of the deployed BSs increases, the network delay gradually converges to a value. In addition, the improvement of coverage probability would reduce the number of retransmission, so the performance of network delay is expected to be improved for the backhaul-aware user association scheme.

7. Conclusion

Based on the backhaul-aware user association scheme, semi-closed expressions of coverage probability, rate coverage, and network delay are derived in ultradense heterogeneous net-

work when backhaul capacity is limited. To maximize the analytic results of coverage probability, all possible access and backhaul links in the range of BSs and A-BSs that users can access are considered. Besides, the influence of the fraction of A-BSs and the BS-to-user density ratio on network performance metrics is evaluated to look for a more reasonable deployment of BSs in the practical scenario. According to the theoretical results, the performance of coverage probability and rate coverage would be highly improved compared with the conventional user association scheme based on the minimum path loss. Simulation results also demonstrate significant performance gains of the backhaul-aware user association scheme in SNR coverage probability and rate coverage.

Appendix

The load characteristic analysis is as follows.

The probability mass function (PMF) of the number of users $N_{u,b}$ associated with the tagged BS is

$$K_t(\lambda_u, \lambda, n) = \mathbb{P}(N_{u,b} = n), \quad n \geq 1, \quad (\text{A.1})$$

where

$$K_t(a, b, n) = \frac{3.5^{3.5}}{(n-1)!} \frac{\Gamma(n+3.5)}{\Gamma(3.5)} \left(\frac{a}{b}\right)^{n-1} \left(3.5 + \frac{a}{b}\right)^{-(n+3.5)}, \quad (\text{A.2})$$

and the gamma function $\Gamma(x) = \int_0^{\infty} e^{-t} t^{x-1} dt$. The corresponding mean is $\bar{N}_{u,b} = 1 + 1.28\lambda_u/\lambda$. The number of users $N_{u,a}$ served by the tagged A-BS follows the same distribution as those in a typical BS.

When the typical user is served by the A-BS, the PMF of the number of BSs $N_{b,a}$ served by the tagged A-BS is

$$K(\lambda(1-\omega), \lambda\omega, n) = \mathbb{P}(N_{b,a} = n), \quad n \geq 0, \quad (\text{A.3})$$

where

$$K(a, b, n) = \frac{3.5^{3.5} \Gamma(n+3.5)}{(n)! \Gamma(3.5)} \left(\frac{a}{b}\right)^n \left(3.5 + \frac{a}{b}\right)^{-(n+3.5)}. \quad (\text{A.4})$$

The corresponding mean is $\bar{N}_{b,a} = (1-\omega)/\omega$. In the scenario that the typical user associates with a BS, the PMF of the number of BSs $N_{b,a}$ associated with the tagged A-BS is given by

$$K_t(\lambda(1-\omega), \lambda\omega, n) = \mathbb{P}(N_{b,a} = n), \quad n \geq 1. \quad (\text{A.5})$$

The corresponding mean is $\bar{N}_{b,a} = 1 + 1.28(1-\omega)/\omega$.

The proofs follow along the similar lines of [20] and are thus omitted.

Data Availability

The data used to support the findings of this study are available from the corresponding author upon request.

Conflicts of Interest

The authors declare that there are no conflicts of interest regarding the publication of this paper.

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