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ON ORIENTATION CHANGES ACCOMPANYING SLIP AND TWINNING

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Abstract: The purpose of this note is to make a few observations on several analyses of lattice rotation by slip and by twinning in deformation.

SLIP

The lattice rotations accompanying slip are gradual; the amount of rotation being a continuous function of the shear strain. Several widely used analyses appear to predict different magnitudes and directions of rotation, but these differences are due only to the reference systems used. Although one analysis or another may appear to be appropriate for a given form of loading, this is because of the reference system natural to the geometry of loading rather than to the stress state producing slip.

Three analyses are considered below. All assume uniformly distributed slip. To facilitate comparison, only slip on a single system will be considered, although these basic analyses may be used under conditions of multiple slip.

The *Schmid "tension" analysis*[1,2] describes the lattice reorientation relative to *any physical line* in the material. If the initial and present values of the angle between this line and the slip direction are designated λ_o and λ, respectively,

$$\ell \sin \lambda = \ell_o \sin \lambda_o \qquad (1)$$

where ℓ_o and ℓ are the initial and final lengths of that line. If the strain along the line is tensile ($\ell > \ell_o$), the rate of rotation is given by

$$d\lambda/d\varepsilon = -\tan \lambda \qquad (2)$$

or

$$d\lambda/d\gamma = -\sin \lambda \cos \phi \qquad (3)$$

where $d\varepsilon = d\ell/\ell$, λ is the crystallographic shear strain and ϕ is the angle between the reference axis and the slip plane normal. The reference line approaches the slip direction; the axis of rotation being normal to both the slip direction and the reference line. This analysis is commonly applied to long single crystals deformed in tension, with the tensile axis serving as the physical reference line and the strain being that along the tensile axis.

Taylor "compression" analysis[3] describes the lattice rotation relative to *any physical plane* in the material. If the initial and final values of the angle between the reference plane normal and the slip plane normal are designated ϕ_o and ϕ, respectively,

$$h \sin \phi = h_o \sin \phi_o \qquad (4)$$

where h_o and h are the initial and final distances between two parallel reference planes. The rate of rotation is given by

$$\frac{d\phi}{d\varepsilon_h} = -\tan \phi \qquad (5)$$

or

$$\frac{d\phi}{d\gamma} = -\cos \lambda \sin \phi \qquad (6)$$

where $d\varepsilon_h = dh/h$, λ is the angle between the reference plane normal and the slip direction, and the sign of the shear strain γ is taken to be positive when ε_h is positive.

If the strain is compressive ($h < h_o$), the reference plane normal approaches the slip plane normal; the axis of rotation being lying in both the reference plane and slip plane. This analysis is commonly applied to the compression of flat wide crystals where the crystal faces in contact with the compression platens are the natural reference planes.

The difference in these two analyses is illustrated in the two dimensional illustration in Figure 1. Before

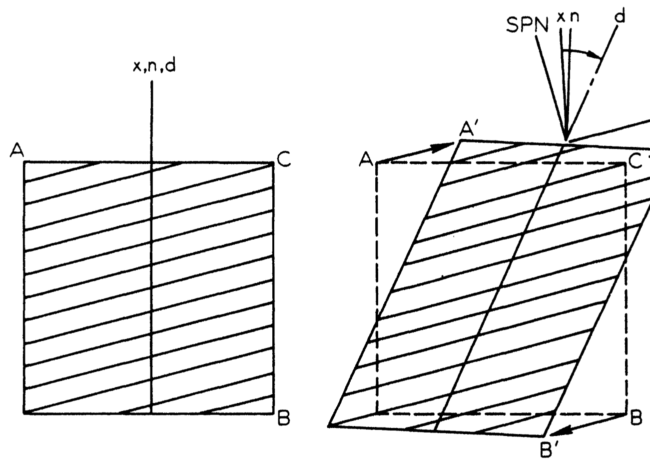

Figure 1. A schematic two-dimensional illustration of the different rotations due to slip. A physical line, d, which was initially parallel to x suffers one rotation. The normal to a physical plane, n, which was also initially parallel to x, suffers a different rotation.

deformation, the direction, d, and the normal, n, to the plane AC are both aligned with a crystallographic direction, x. After deformation, however, n and d are no longer parallel, having suffered different rotations.

It should be emphasized that the stress state causing slip is not intrinsic to either analysis; rather it is the geometry of the specimen and its constraints that make one or the other analysis more appropriate. Unfortunately, this has not been recognized in many theories of texture formation. The tensile axis of a long crystal will naturally remain parallel to its longest direction. During the compression of a flat crystal, the plane initially in contact with the compression platens will remain normal to the compression axis. However, one can conceive of tensile deformation of a short fat crystal where the Taylor "compression" analysis would be more appropriate, or the compression of a long thin crystal where the Schmid analysis would be more appropriate. Both analyses are special cases of a general treatment of finite strains by Chin *et al.*,[4] that covers multiple slip as well as single slip. Theories of texture development are concerned with the lattice rotation within grains of a polycrystal. If the grains are extended in a direction in which they are already very elongated, the Schmid analysis seems appropriate. Conversely, if the grains are already pancake-shaped and subjected to further compression, the Taylor analysis would be better. In many cases, however, the grains are more or less equiaxed, and are constrained on all sides and directions by neighboring grains. Neither analysis is quite appropriate.

The *mathematical analysis of rotations* describes the rotation of particles in a body with infinitesimal strain:

$$d\omega_{ij} = 1/2 \left(\frac{\partial u_i}{\partial x_j} - \frac{\partial u_j}{\partial x_i}\right) \qquad (7)$$

where ω_{ij} is the rotation about an axis a_3 normal to the i and j directions, u_i is the shear displacement parallel to i occurring over a distance x_j in the j direction. The conventional shear strain, defined by

$$d\gamma_{ij} = \frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \qquad (8)$$

If i and j are the slip direction and slip plane normal, respectively, $\partial u_j/\partial x_i = 0$ so,

$$d\omega = 1/2\ d\gamma \qquad (9)$$

and the rotation occurs about an axis normal to both the slip direction and the slip plane normal. An interesting aspect of this analysis is that the rotation is referred to no single external plane or direction. For this reason, and because its predictions are between those of the first two analyses, it appears to be more appropriate for equiaxed grains.

The axis of rotation predicted by the three analyses are shown in Figure 2 for slip on the (111) plane in the $[\bar{1}01]$ direction. The Schmid analysis predicts that a direction initially parallel [001] rotates toward the slip direction; the axis of rotation, a_1, being $[0\bar{1}0]$. The Taylor analysis predicts that the normal to a plane initially parallel to (001) rotates away from the slip plane normal about the axis $a_2 = [1\bar{1}0]$. Finally, according to the mathematical analysis, the lattice rotates about $a_3 = [\bar{1}01] \times [111] = [1\bar{2}1]$.

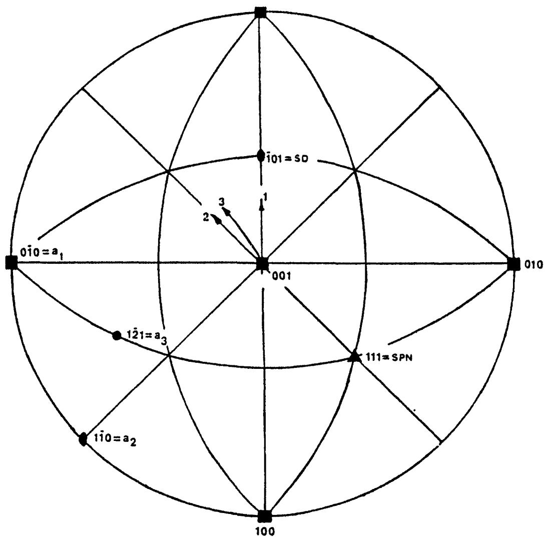

Figure 2. Stereographic representation of the three reference systems used to describe lattice rotation. Slip is on a single system, (111)[$\bar{1}$01]. The Schmid analysis describes rotation 1 of a physical direction initially parallel to [001] about an axis, a_1 = [0$\bar{1}$0]. The Taylor analysis describes rotation 2 of the normal to a physical plane initially perpendicular to (001) about an axis, a_2 = [1$\bar{1}$0]. The mathematical analysis describes rotation, 3, about and axis, a_3 = [1$\bar{2}$1].

Figure 3 shows the rotation caused by (111)[$\bar{1}$01] slip. Shown in Figure 3a are the reorientation of physical directions; in Figure 3b are the reorientations of normals to physical planes, and in Figure 3c the mathematical rotations. It may be noted that the directions of rotation predicted by the mathematical analysis are intermediate between those of the other two. The predicted rate of rotation is also intermediate. In fact, for the special cases where $\phi + \lambda = 90°$, Equations (3) and (6) reduce to $d\lambda/d\gamma = -\sin^2 \lambda$ and $d\lambda/d\gamma = -\cos^2 \lambda$, respectively. The average of these is $d\lambda/d\gamma = 1/2$ in accordance with Equation (9).

TWINNING

It is widely recognized that when a material undergoes twinning, the twinned regions undergo a large and discrete lattice reorientation relative to the untwinned regions. However, most workers have ignored the fact

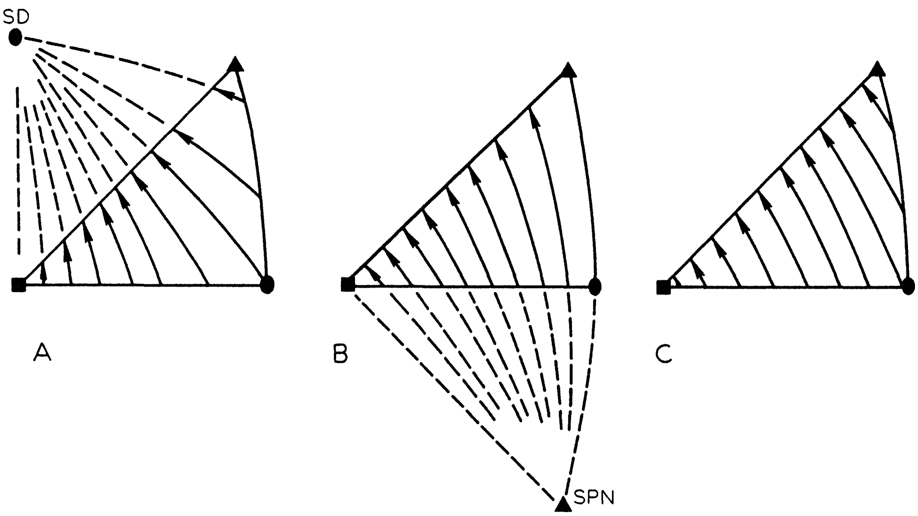

Figure 3. Illustration of the rotations described by the three analyses for single slip. 3A illustrates the rotation of a physical direction toward the slip direction. 3B shows the rotation of a normal to a physical plane. The mathematical rotations are represented in 3C.

that twinning also causes a gradual rotation of both the twinned and untwinned regions relative to the external coordinates. If the twinned regions are fine and uniformly distributed, this gradual rotation is a continuous function of the macroscopic shear strain,

$$\gamma = fS \tag{10}$$

where f is the volume fraction twinned and S is the discrete twinning shear strain. The magnitude of this rotation is exactly the same as that which would occur if the shear strain, γ, were produced by uniform slip with the same plane and direction of shear.

Consider the special case of a line initially at an angle λ_o to the twinning direction and $90° - \lambda_o$ to the twinning plane. Its angle, λ, to the twinning direction after twinning will be given by

$$\cot \lambda = \cot \lambda_o + fS \tag{11}$$

In the case of a fcc crystal, twinning on the $(111)[11\bar{2}]$ system, $S = \sqrt{2}/2$. A line initially, d, parallel to [001] in the matrix ($\lambda_o = 144° 44'$, $\cot \lambda_o =$

$-\sqrt{2})$, will be reoriented to a position λ = arc cot $(-\sqrt{2} + f\sqrt{2}/2)$. If the entire crystal is twinned (f = 1), the change of direction of this line $\lambda_o - \lambda$, is 19°28'. However, the [001] in the twin is reoriented by 70°32' relative to the [001] in the matrix. Thus the line initially parallel to [001] of the matrix is reoriented by 90° relative to the twinned lattice, becoming parallel to [$\bar{1}\bar{1}$0] of the twin. Similarly, a plane initially parallel to (001) will be reoriented to lie parallel to ($\bar{1}\bar{1}$0) of the twin (see Figure 4).

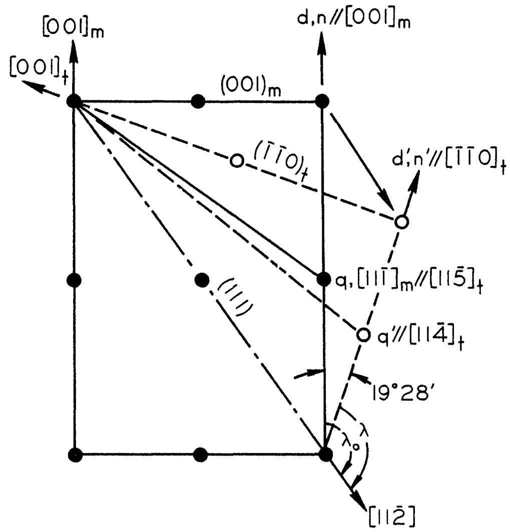

Figure 4. The lattice reorientations caused by [11$\bar{2}$](111) twinning in fcc. The plane of projection is (1$\bar{1}$0). A line, d, and a plane normal, n, initially parallel to [001] of the matrix, are rotated by 19°28' parallel with [$\bar{1}\bar{1}$0] of the twin. A line, q, initially parallel to [11$\bar{1}$] of the matrix and [11$\bar{5}$] of the twin is rotated to a position q' parallel to [11$\bar{4}$] of the twin.

Analogous rotation of (001) and [001] by 90° relative to the twinned lattice occur by (111)[11$\bar{2}$] twinning in bcc metals. In hcp metals the c-axis suffers a 90° rotation relative to twinned lattice after complete twinning on any of the {10$\bar{1}$2}<10$\bar{1}$1> systems.

It has been stated that in fcc metals (111)[11$\bar{2}$] twinning reorients the [11$\bar{1}$] direction to a [11$\bar{5}$] position.[5,6] It is true that the [11$\bar{5}$] direction of the twin is parallel to the [11$\bar{1}$] of the matrix. However, this is misleading. A physical line initially parallel to the

$[11\bar{1}]$ direction of the matrix is reoriented so that it is parallel to $[11\bar{4}]$ of the twin (see Figure 4). A long crystal with an initial tensile axis of $[11\bar{1}]$ would be converted by 100% twinning to a $[11\bar{4}]$ orientation rather than $[11\bar{5}]$.

CONCLUSION

In applying the concepts of lattice rotation produced by slip and twinning to predicting crystallographic textures, careful attention must be paid to the reference system used. Orientation change due to twinning have two components; one from the discrete reorientation of the twinning plus one from the gradual rotation due to the shear.

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