

## GENERALIZATION OF THE ORIENTATION DISTRIBUTION FUNCTION: DETERMINATION OF CRYSTALLINE ORIENTATIONS IN ANY SECTION OF THE SPECIMEN

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*Abstract:* A generalization of the O.D.F. representation has been proposed in such a way that its calculation is possible in any axes system of a material provided that is known in one particular system. Using an appropriate rotation matrix  $T(a,b,c)$  it is possible to compute crystallographic orientations  $(hkl)[uvw]$  in any macroscopic plane of the material. This calculation method leads also to a direction distribution function

$\int_{\pi}^{[u_1v_1w_1]} F(\psi, \alpha) d\Omega$  which gives the angular distribution of the crystallographic direction  $[u_1v_1w_1]$  in a macroscopic plane of the material.

This analysis has been applied to the representation of texture and of slip directions in planes at 45 degrees from the directions RD, ND and TD of a mild steel deformed in equi-biaxial expansion.

### 1. INTRODUCTION

The quantitative analysis of the crystalline texture can easily be obtained by the three dimensional analysis of the orientation distribution function (O.D.F.).<sup>1,2</sup> This function is in practice established from experimental pole figures determined in easily accessible planes of the material, for example the rolling plane. The crystalline orientations in the three-dimensional representation are then characterized with respect to the classical directions RD (rolling direction), ND (normal direction) and TD (transverse direction).

In numerous problems, however, it is essential to represent this texture with respect to directions not directly accessible to experiment. This is so, for instance, in the case of the mechanisms of deformation of metals which may

advantageously be described in maximum shear planes at 45 degrees from the applied stresses.

For this reason we have generalized the O.D.F. representation in such a way that its calculation is possible in any cartesian axes system of the material provided that is known in one particular system. As an example, this analysis has been applied to the representation of texture and of slip directions in planes at 45 degrees from the directions RD, ND and TD of a mild steel deformed in equibiaxial expansion.

## 2. CRYSTALLITE ORIENTATION

The orientation of a crystallite with respect to any cartesian referential  $K_\pi$  connected with the material is completely defined by the rotation  $g_{c\pi}(\psi, \theta, \Phi)$  which brings the referential  $K_\pi$  into coincidence with the crystallite referential  $K_c$  (Fig. 1):

$$K_c = g_{c\pi} \cdot K_\pi \quad (1)$$

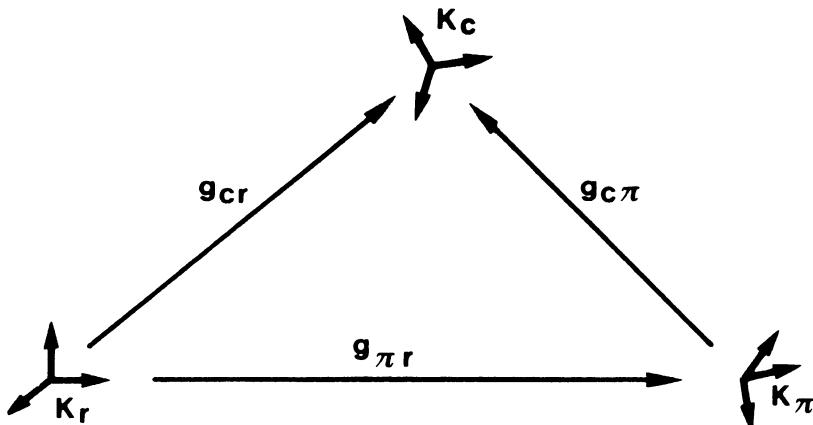


Fig. 1. Rotation in any axes system.

This defines the function of distribution of the crystallites  $\pi F(g_{c\pi})$  which expresses the probability that a crystallite will have an orientation described by the rotation  $g_{c\pi}$  in the system  $K_\pi$ . This function, which has good properties of continuity irrespectively of the system  $K_\pi$  considered, can be developed in an absolutely and uniformly convergent series of the generalized spherical functions  $Y_\ell^{mn}(g_{c\pi})$  defined in the appendix, i.e.:

$$\pi F(g_{c\pi}) = \sum_{\ell=0}^{\infty} \sum_{m=-\ell}^{+\ell} \sum_{n=-\ell}^{+\ell} \pi F_{mn}^\ell Y_\ell^{mn}(g_{c\pi}) \quad (2)$$

The coefficients  $\pi F_{mn}^\ell$  of the distribution function are quite generally complex numbers of the type  $\pi F_{mn}^\ell = \pi F_{mn}^{\ell R} + i \pi F_{mn}^{\ell I}$  and can be calculated on the basis of knowledge of the pole figures  $\{hkl\}_j$  determined experimentally in the plane  $\pi$  of the material of which the normal corresponds to the  $\hat{z}$  axis

of the chosen system  $K_\pi$ . In fact the density of poles  $\pi q_j(\chi, \eta)$  at a point on a figure of this kind expresses the integration of the distribution function  $\pi F(g_{c\pi})$  about the reciprocal vector  $\vec{r}_j(hkl)$  (Fig. 2).

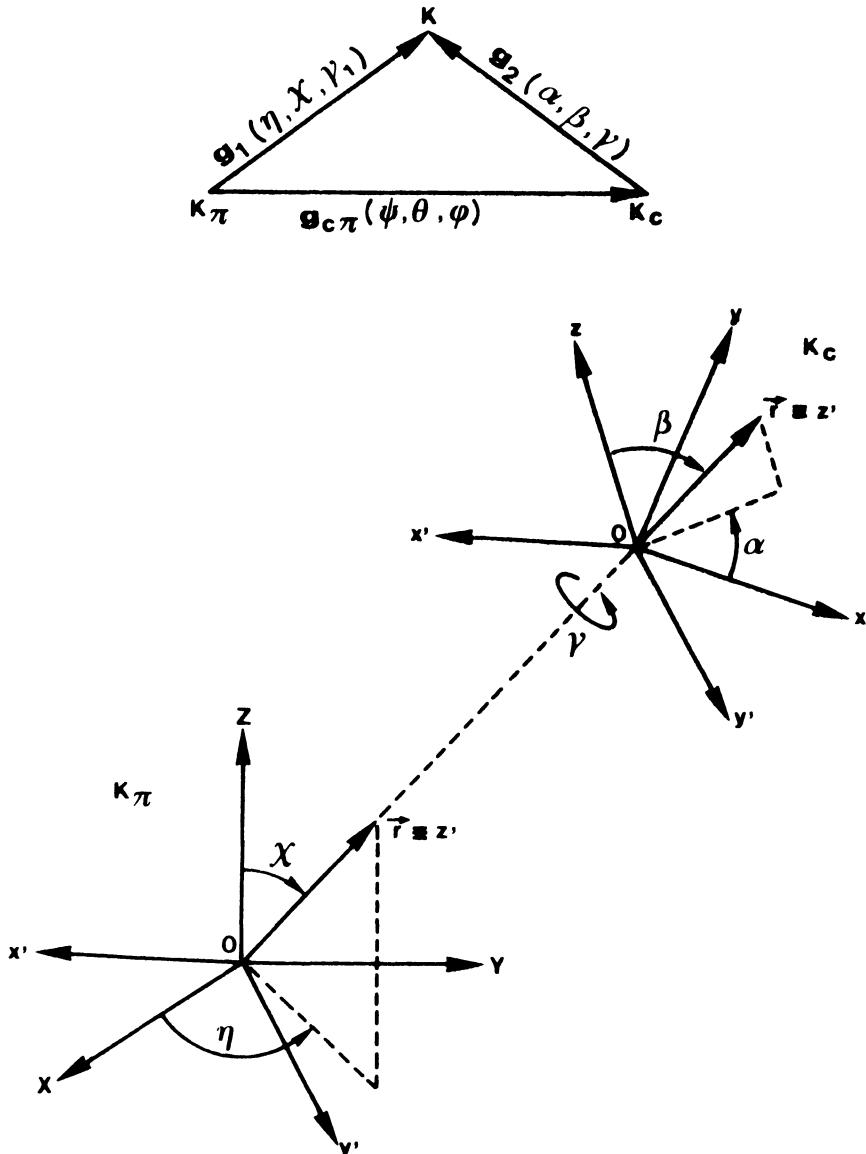


Fig. 2. Relation between classical pole figure and its three dimensional representation.

In the plane  $\pi$  of the material, the density of poles  $\pi q_j(\chi, \eta)$  will thus be expressed by the relationship:

$$\pi q_j(\chi, \eta) = \frac{1}{2\pi} \int_0^{2\pi} F(g_{c\pi}) d\gamma \quad (3)$$

The resolution of the rotation  $g_{c\pi}$  into two partial rotations according to polar angles  $(\eta, \chi)$  and crystallographic angles  $(\alpha, \beta)$  so that  $g_{c\pi} = g_{jc}^{-1} \cdot g_{j\pi}$  with  $g_{j\pi} = (\eta, \chi, 0)$  and  $g_{jc} = (\alpha, \beta, \gamma)$  makes it possible to effect, by generalization of the theorem of addition of spherical functions<sup>3,4</sup> (see appendix), the integration of the relationship (3) which becomes:

$$\pi q_j(\chi, \eta) = \sum_{\ell} \sum_m \pi Q_m^{j\ell} Y_m^{\ell}(\chi, \eta) \quad (4)$$

with

$$\pi Q_m^{j\ell} = \sqrt{\frac{2}{2\ell + 1}} \sum_n \pi F_m^{\ell} n \overline{Y_n^{\ell}(\beta, \alpha)} \quad (5)$$

Using the properties of orthonormalization of spherical harmonics, the coefficients  $\pi Q_m^{j\ell}$  can be calculated on the basis of the relationship (4). These coefficients  $\pi Q_m^{j\ell}$  introduced into the relationship (5) lead to determination of the coefficients  $\pi F_m^{\ell}$  and, therefore, to the function of distribution of crystallites  $\pi F(g_{c\pi})$  in the  $K_\pi$  system.

The determination of the distribution function  $\pi F(g_{c\pi})$  in the  $K_\pi$  system allows an immediate identification of crystallographic planes and directions in the  $\pi$  plane of the material of which the normal corresponds to the axis  $\partial z$  of the referential  $K_\pi$ . In fact the distribution function  $\pi F(g_{c\pi})$  may be interpreted in terms of preferential orientations  $(hk\ell)[uvw]$ ,  $(hk\ell)$  representing the crystallographic planes parallel to the plane  $\pi$  considered,  $[uvw]$  giving the directions of these crystallographic planes parallel to the axis  $\partial x$  of the referential  $K_\pi$ . Matrix relationships between the Euler angles  $\psi, \theta, \phi$  and these orientations  $(hk\ell)[uvw]$  are:

$$\begin{pmatrix} h \\ k \\ \ell \end{pmatrix} = T(\psi, \theta, \phi) \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \quad (6)$$

$$\begin{bmatrix} u \\ v \\ w \end{bmatrix} = T(\psi, \theta, \phi) \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$$

where  $T(\psi, \theta, \phi)$  is the matrix of rotation  $g_{c\pi}$  in the space  $R^3$ .

Taking into account the law of Friedel which imposes  $\ell$  even, the calculation of the coefficients  $\pi F_m^{\ell}$  calls, in the most general case, for knowledge of  $(2\ell + 1)$  pole figures in the plane  $\pi$ . The crystallographic symmetries and those of the  $\pi$  plane chosen allow to reduce this number of pole figures: in the case of mild steel sheets (cubic symmetry), two pole figures determined experimentally in the rolling plane are sufficient for calculating the distribution function in the system RD, ND, and TD.

In the case of any section  $\pi$  of the material, however, the determination of the function  $\pi F(g_{c\pi})$  by the experimental method described above calls for precise and time-consuming preparation of the samples (machining of the section  $\pi$  of the material, thinning the samples to obtain the periphery of the

pole figures in transmission), consequent use of the texture goniometer (establishment of a large number of pole figures according to the crystallographic symmetries and those of the  $\pi$  plane) and a time-consuming computer treatment of the data.

These difficulties disappear automatically by the generalization of the three dimensional representation proposed.

In fact, as we shall show, it is possible to calculate the function  $\pi F(g_{c\pi})$  quite generally from the knowledge of the distribution function in a particular easily accessible referential  $K_p$ .

In this  $K_p$  system, the distribution function of crystal-line orientations is in fact expressed by:

$$pF(g_{cp}) = \sum_{\ell} \sum_{m} \sum_{n} pF_m^{\ell} \cdot Y_{\ell}^{mn}(g_{cp}) \quad (7)$$

where the rotation  $g_{cp}$  is defined by the relationship:

$$K_c = g_{cp} \cdot K_p \quad (8)$$

To obtain the distribution function in any cartesian referential  $K_{\pi}$  it is necessary to know the relationships existing between the coefficients  $\pi F_m^{\ell}$  of the system  $K_{\pi}$  and the coefficients  $pF_m^{\ell}$  of the system  $K_p$ .

For this purpose it is necessary to define the rotation  $g_{\pi p}$  with Euler angles  $(a, b, c)$  which allows the referential  $K_p$  to coincide with the referential  $K_{\pi}$ , i.e.:

$$K_{\pi} = g_{\pi p} \cdot K_p \quad (9)$$

Between the three rotations  $g_{c\pi}$ ,  $g_{cp}$  and  $g_{\pi p}$  defined in (1), (8) and (9) we have the relationship

$$g_{cp} = g_{c\pi} \cdot g_{\pi p} \quad (10)$$

whereby it is possible, by generalizing the theorem of addition of spherical functions (see appendix), to establish the relationship

$$\pi F_m^{\ell} = \sum_{\mu} pF_{\mu n}^{\ell} \frac{Y_{\ell}^{\mu m}(g_{\pi p})}{Y_{\ell}^{\mu \mu}(o)} \quad (11)$$

### 3. REPRESENTATION OF CRYSTALLOGRAPHIC DIRECTIONS

Knowledge of the distribution function in the particular referential  $K_p$  of the material also allows to determine the angular distribution of a crystallographic direction  $[u_1 v_1 w_1]$  with respect to a referential direction  $R_0$  in any section  $\pi$  of the material (Fig. 3).

Let us consider a particular direction  $R(\alpha)$  of the  $\pi$  section making an angle  $\alpha$  with the reference direction  $R_0$ . The density of crystallographic planes  $(hk\ell)$  parallel to the  $\pi$  section with crystallographic direction  $[u_1 v_1 w_1]$  parallel to the direction  $R(\alpha)$  will be defined by the distribution

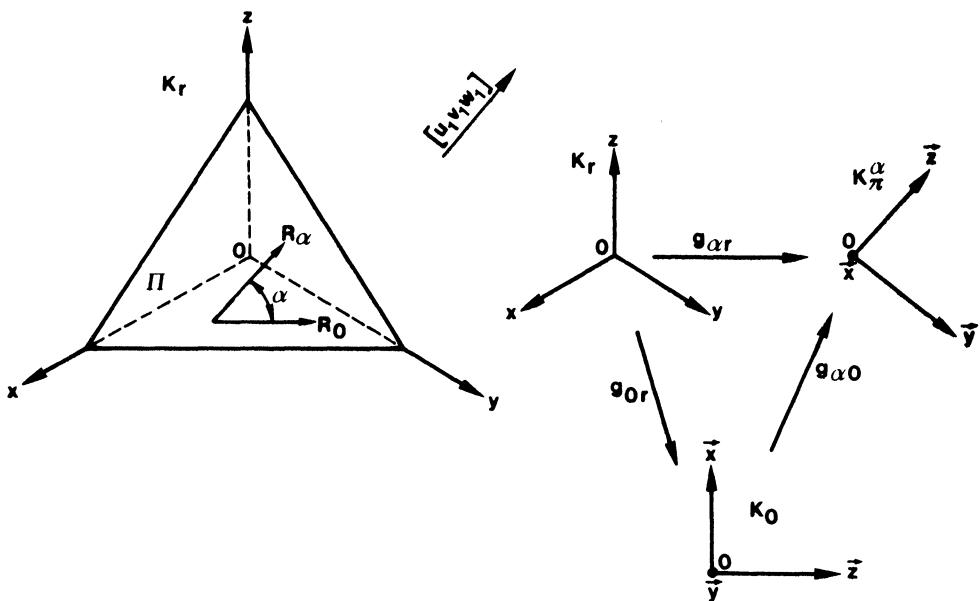


Fig. 3. Crystallographic directions representation.

function  $\frac{\alpha}{\pi}F(\psi, \theta_1, \phi_1)$  with  $\theta_1, \phi_1$  constant and  $\psi \in [0, 2\pi]$ ; this function being defined in the referential  $K_{\pi}^{\alpha}$  with axis  $\vec{ox}$  normal to the section  $\pi$  and with axis  $\vec{oz}$  parallel to the direction  $R(\alpha)$ . In this case, the angles  $\psi, \theta_1, \phi_1$  and the crystallographic orientations  $(hk\ell)[u_1v_1w_1]$  will be connected by the matrix relationships:

$$\begin{aligned} \begin{pmatrix} h \\ k \\ l \end{pmatrix} &= T(\psi, \theta_1, \phi_1) \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} \\ \begin{bmatrix} u_1 \\ v_1 \\ w_1 \end{bmatrix} &= T(\psi, \theta_1, \phi_1) \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \end{aligned} \quad (12)$$

Now, the angular distribution, in the section plane  $\pi$  of the material, of a particular crystallographic direction  $[u_1v_1w_1]$  will be completely defined by the function

$\frac{[u_1v_1w_1]}{\pi}F(\psi, \alpha)$  obtained by examining all the directions  $R(\alpha)$  contained in the section, that is to say in the interval  $\alpha \in [0, \pi]$ . The value of this function will therefore be:

$$\frac{[u_1v_1w_1]}{\pi}F(\psi, \alpha) = \frac{\alpha}{\pi}F(\psi, \theta_1, \phi_1) \text{ with } \alpha \in [0, \pi]; \psi \in [0, 2\pi] \quad (13)$$

$\theta_1, \phi_1$  are constant and depend on the crystallographic direction examined (relationship 12).

The function  $\frac{[u_1v_1w_1]}{\pi}F(\psi, \alpha)$  can be calculated by the method of rotations described previously, on the basis of

knowledge of the coefficients  $pF_m^l$  of the distribution function of crystalline orientations  $pF(g_{cp})$  in the particular referential  $K_p$ . In fact, if  $g_{ap}$  is the rotation connecting the referentials  $K_p$  and  $K_\pi^\alpha$  in such a manner that  $K_\pi^\alpha = g_{ap} \cdot K_p$ , from relationships (2) and (11) the function  ${}^{[u_1 v_1 w_1]} \pi F(\psi, \alpha)$  can be written in the form:

$${}^{[u_1 v_1 w_1]} \pi F(\psi, \alpha) = \sum_l \sum_m \sum_n \frac{\alpha}{\pi} F_m^l \cdot Y_l^{mn}(\psi, \theta_1, \phi_1) \quad (14)$$

$$\text{with } \frac{\alpha}{\pi} F_m^l = \sum_\mu p F_\mu^l \frac{Y_l^{\mu m}(g_{ap})}{Y_l^{\mu \mu}(0)}$$

The rotation  $g_{ap}$ , moreover, can be resolved into two partial rotations  $g_{op}$  and  $g_{ao} \equiv (0, \alpha, 90)$  such that

$$g_{ap} = g_{ao} \cdot g_{op} \quad (15)$$

The rotation  $g_{op}$  thus defined allows to bring the referential  $K_p$  into coincidence with a referential  $K_o$  with axis  $\vec{Oy}$  perpendicular to the plane examined and with axis  $\vec{Oz}$  parallel to the reference direction  $\vec{Ro}$ .

The generalization of the theorem of addition of spherical functions allows then to express the coefficients  $\frac{\alpha}{\pi} F_m^l$  according to the relationship:

$$\frac{\alpha}{\pi} F_m^l = \frac{1}{[Y_l^{mm}(0)]^2} \sum_\mu \sum_s p F_\mu^l \cdot Y_l^{\mu s}(g_{op}) \cdot Y_l^{sm}(g_{ao}) \quad (16)$$

This expression of the coefficients  $\frac{\alpha}{\pi} F_m^l$  introduced into the relationship (14) leads to the explicit form of the function  ${}^{[u_1 v_1 w_1]} \pi F(\psi, \alpha)$ , i.e.:

$${}^{[u_1 v_1 w_1]} \pi F(\psi, \alpha) = \sum_{l=0}^{\infty} \sum_{m=-l}^{+l} \sum_{n=-l}^{+l} D_{mn}^l Y_l^{mn}(0, \alpha, \psi) \quad (17)$$

with

$$D_{mn}^l = \frac{1}{[Y_l^{mm}(0)]^2} \sum_{\mu=-l}^{+l} \sum_{s=-l}^{+l} p F_\mu^l \cdot Y_l^{\mu s}(g_{op}) \cdot Y_l^{ns}\left(\frac{\pi}{2}, \theta_1, \phi_1\right).$$

#### 4. APPLICATION TO THIN SHEETS OF MILD STEEL

##### 4.1 O.D.F. Representation in Rolling Plane

In the case of mild steel sheets, the determination of the distribution function  $\pi F(g_{c\pi})$  in the planes  $\pi_r$ ,  $\pi_t$ ,  $\pi_n$  perpendicular, respectively, to the axes ND, RD and TD calls only for knowledge of two pole figures in these planes; this results from the cubic symmetry and from the fact that these planes are planes of symmetry of the particular rolling referential  $K_p$  (RD, ND, TD).

Thus, the experimental determination, in the rolling plane, of pole figures  $\{200\}$  and  $\{211\}$  allows to calculate the distribution function  $pF(g_{cp})$  in the system  $K_p$  from relationships (7), (4) and (5). In this case, the coefficients  $pF_{mn}^{\lambda}$  are real and the distribution function becomes:

$$pF(g_{cp}) = \sum_{\lambda=0}^{\infty} \sum_{m=-\lambda}^{+\lambda} \sum_{n=-\lambda}^{+\lambda} pF_{mn}^{\lambda R} \cdot Y_{\lambda}^{mn}(g_{cp}) \quad (18)$$

$$\lambda = 0, 2, 4 \dots; m = 0, 2, 4 \dots; n = 0, 4, 8 \dots$$

Figure 4 represents the two sections  $\phi = 0^\circ$  and  $\phi = 45^\circ$  of the distribution function  $pF(g_{cp})$  calculated for a steel

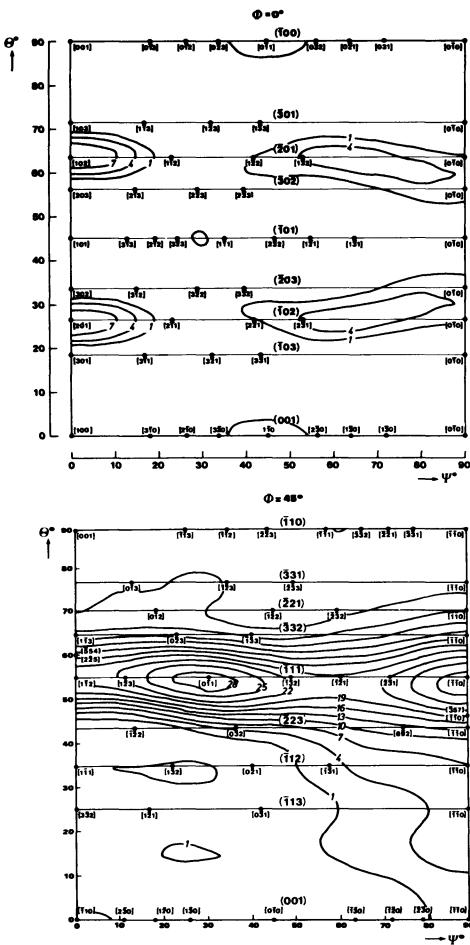


Fig. 4. Three dimensional representation; section  $\phi = 0^\circ$  and  $\phi = 45^\circ$ .

for drawing applications which has been previously deformed by equibiaxial expansion ( $\epsilon = 0.098$ ); the principal orientations of this steel are parts of the fibre texture  $\langle 111 \rangle // ND$

with  $(\bar{1}\bar{1}1)[\bar{1}\bar{1}0]$  and  $(\bar{1}\bar{1}1)[1\bar{1}2]$  as major components.

**4.2. Representation of the O.D.F. in the Planes at  
45 Degrees from the Directions of the Sheet  
(RD, ND, TD)**

From the calculation of the coefficients  $pF_{mn}^{\ell R}$  of the distribution function  $pF(g_{cp})$  it is possible to obtain, by the rotation method described previously, the pole figures and the distribution function  $\pi F(g_{c\pi})$  in any  $\pi$  section of the sheet.

As an example, the crystalline orientations in the octahedral planes 123 and in the planes 13 at  $45^\circ$ , respectively, from the directions RD, ND and TD and from the directions RD and ND (Fig. 5) have been determined for the preceding steel.

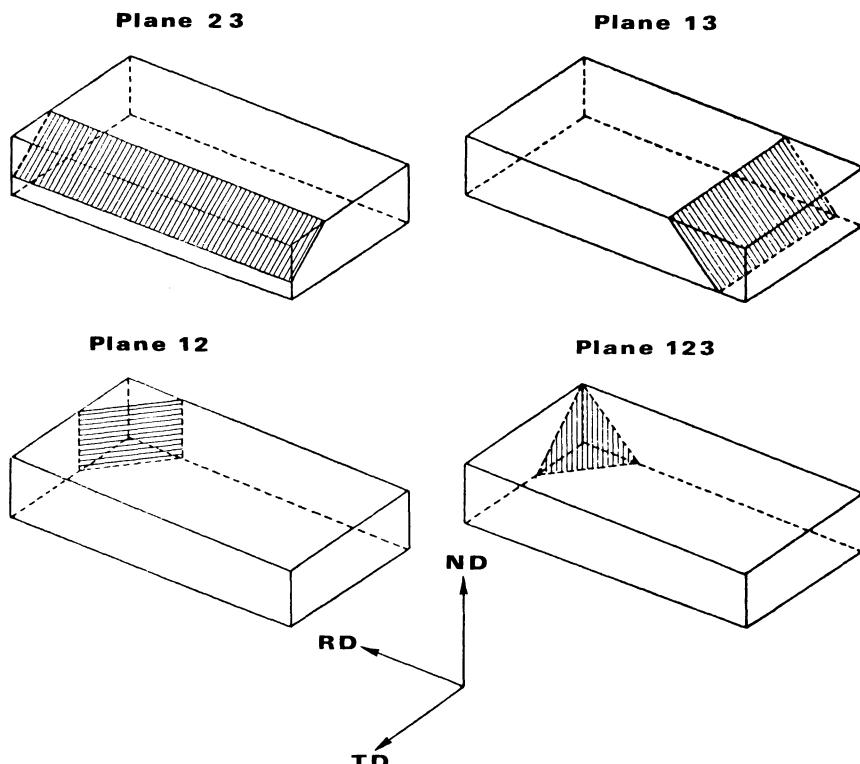


Fig. 5. Definition of planes 12, 13, 23 and 123.

The method that has been developed requires, for calculation of the coefficients  $\pi F_{mn}^{\ell}$  by means of relationship (11), knowledge of the rotation  $g_{\pi p}$  connecting the referential  $K_\pi$  to the rolling referential  $K_p$ .

As regards the octahedral planes 123, there are in fact four sets of planes at  $45^\circ$  from the RD, ND and TD directions (Fig. 6); the corresponding referentials  $K_\pi$  will be defined by the rotations  $g_{\pi p}$  with Euler angles:

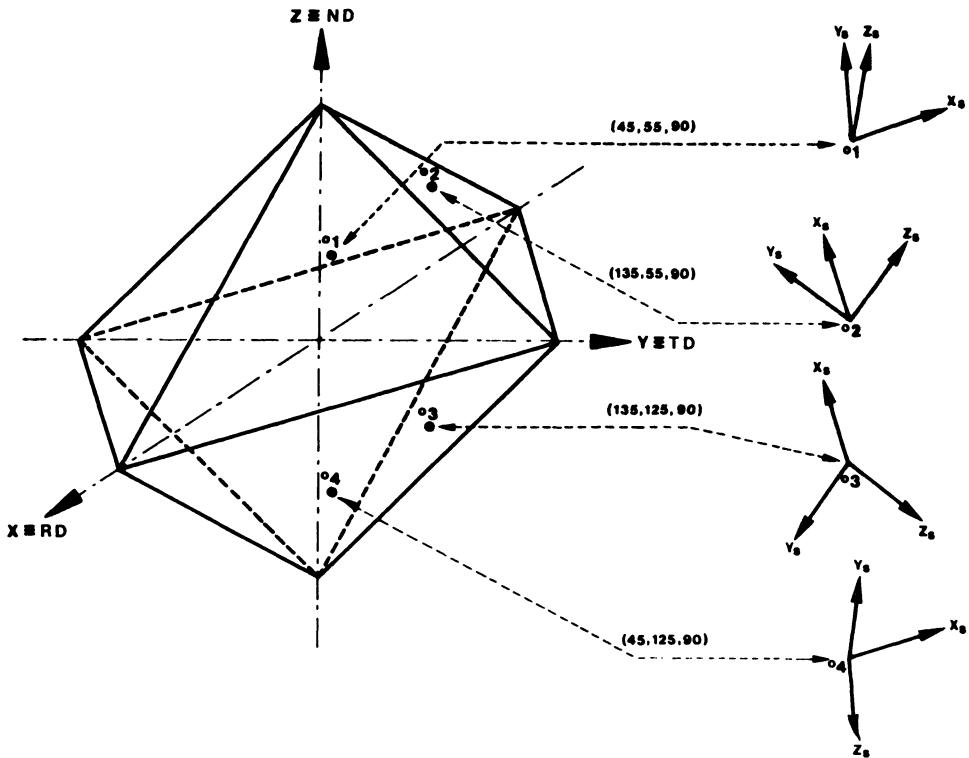


Fig. 6. Equivalent octahedral planes 123.

$$(a, b, c) \equiv (|180\beta - 45|, |180\delta - 55|, 90) \text{ with } \beta = 0, 1 \text{ and} \\ \delta = 0, 1$$

The coefficients  $\frac{\beta\delta}{\pi} F_{mn}^l$  calculated on the basis of these rotations (see relationship 11) are complex numbers:

$$\frac{\beta\delta}{\pi} F_{mn}^l = (-1)^{m\beta} [F_{mn}^{lR} + i(-1)^{\beta+\delta} F_{mn}^{lI}] \\ l = 0, 2, 4, \dots; m = 0, 1, 2, \dots; n = 0, 4, 8, \dots \quad (19)$$

Each of the four octahedral planes is therefore characterized by a separate series of coefficients  $\frac{\beta\delta}{\pi} F_{mn}^l$  of the same absolute value:

$$|\frac{\beta\delta}{\pi} F_{mn}^l| = \sqrt{(F_{mn}^{lR})^2 - (F_{mn}^{lI})^2} \quad \forall \beta, \delta \quad (20)$$

Since the four octahedral planes are geometrically equivalent, it is appropriate to define mean coefficients

$$\overline{F_{mn}^l} = \frac{1}{4} \sum_{\beta, \delta} \frac{\beta\delta}{\pi} F_{mn}^l \quad (21)$$

or, taking into account the expression of the coefficients given in (19),

$$\overline{\pi F_m^l} = F_m^l R \quad (22)$$

$$l = 0, 2, 4, \dots; m = 0, 2, 4, \dots; n = 0, 4, 8, \dots$$

The pole figures {110} calculated by means of the relationships (4) and (5) on the basis of the four separate series of coefficients  $\beta_\pi^l \overline{\pi F_m^l}$  are given in Fig. 7.

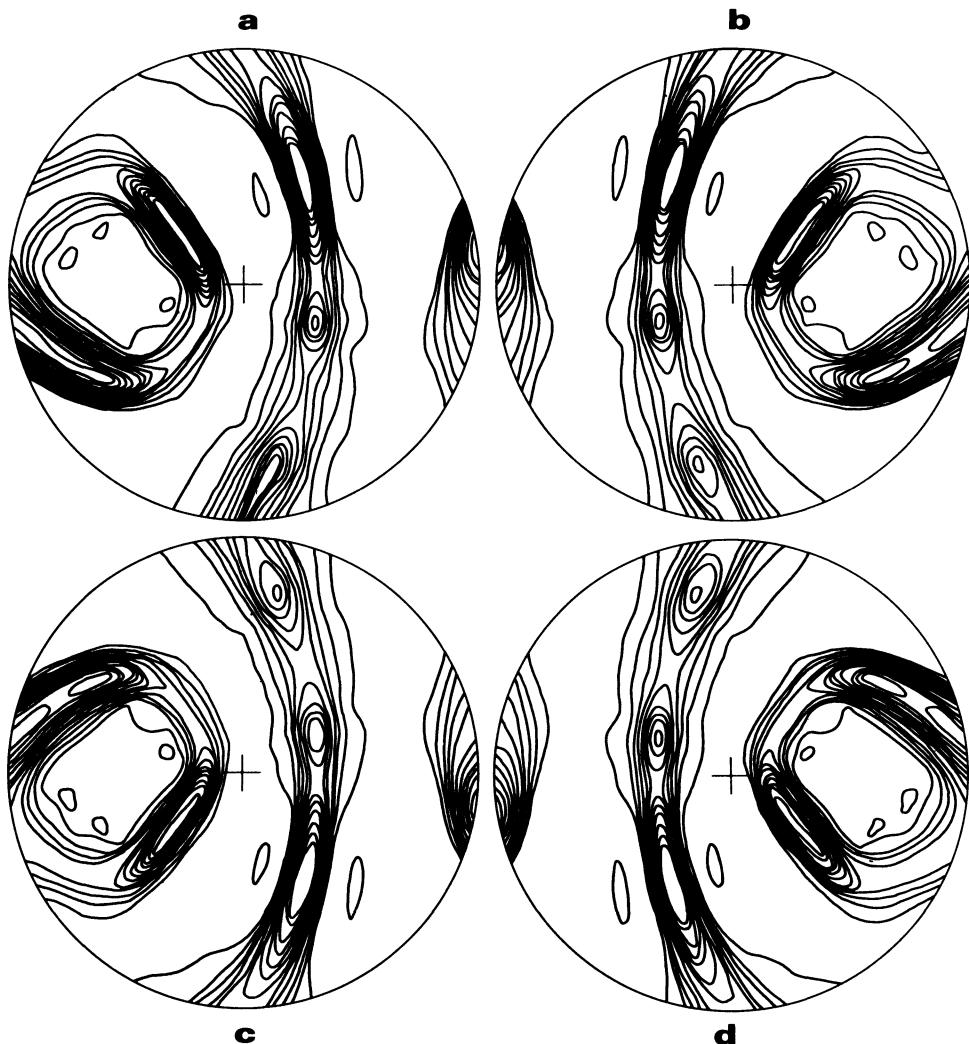


Fig. 7. Partial pole figure {110} computed in 123 planes.  
 a = (45, 55, 90); b = (135, 55, 90); c = (135, 125, 90); d = (45, 125, 90).

The method that has been developed, of course, makes it possible to recover, by the rotation  $g_{\pi p} = (135, 55, 90)$  the pole figure {110} determined experimentally by means of the

Siemens-Lücke goniometer in the section 123 of the sheet obtained by machining (Fig. 8).

### Plane 123

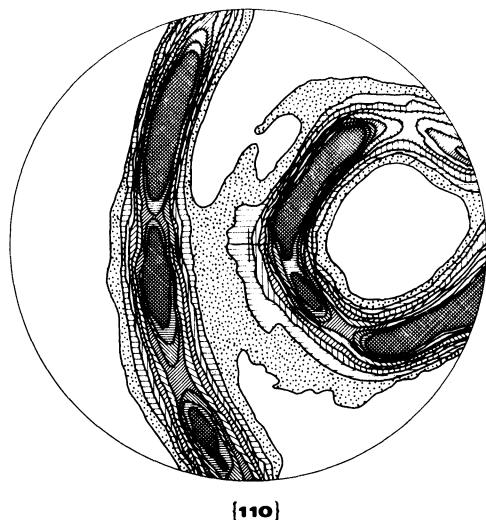


Fig. 8. Experimental pole figure in 123 planes.

By use of the mean coefficients  $\pi F_{mn}^l$ , the mean pole figure is obtained which represents the contributions of the four equivalent octahedral planes (Fig. 9). This means pole figure, which is perfectly symmetrical with respect to the axes  $\hat{O}x$  and  $\hat{O}y$  of the referential  $K_\pi$ , is obviously found again in the four planes considered.

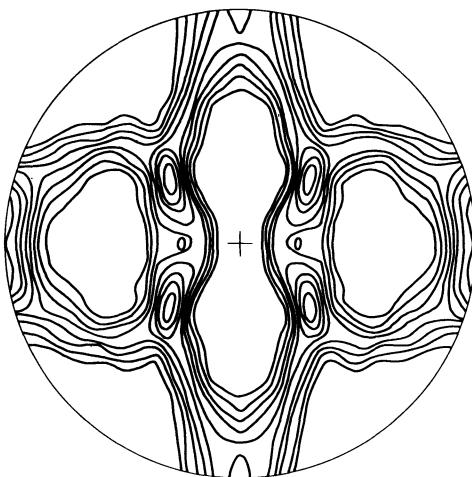


Fig. 9. Mean pole figure  $\{110\}$  computed in 123 planes.

Figures 10 and 11 show the principal sections  $\phi = 0^\circ$  and  $\phi = 45^\circ$  of the distribution functions of the crystalline orientations with respect to the four octahedral planes 123; these functions have been calculated by means of the relation-

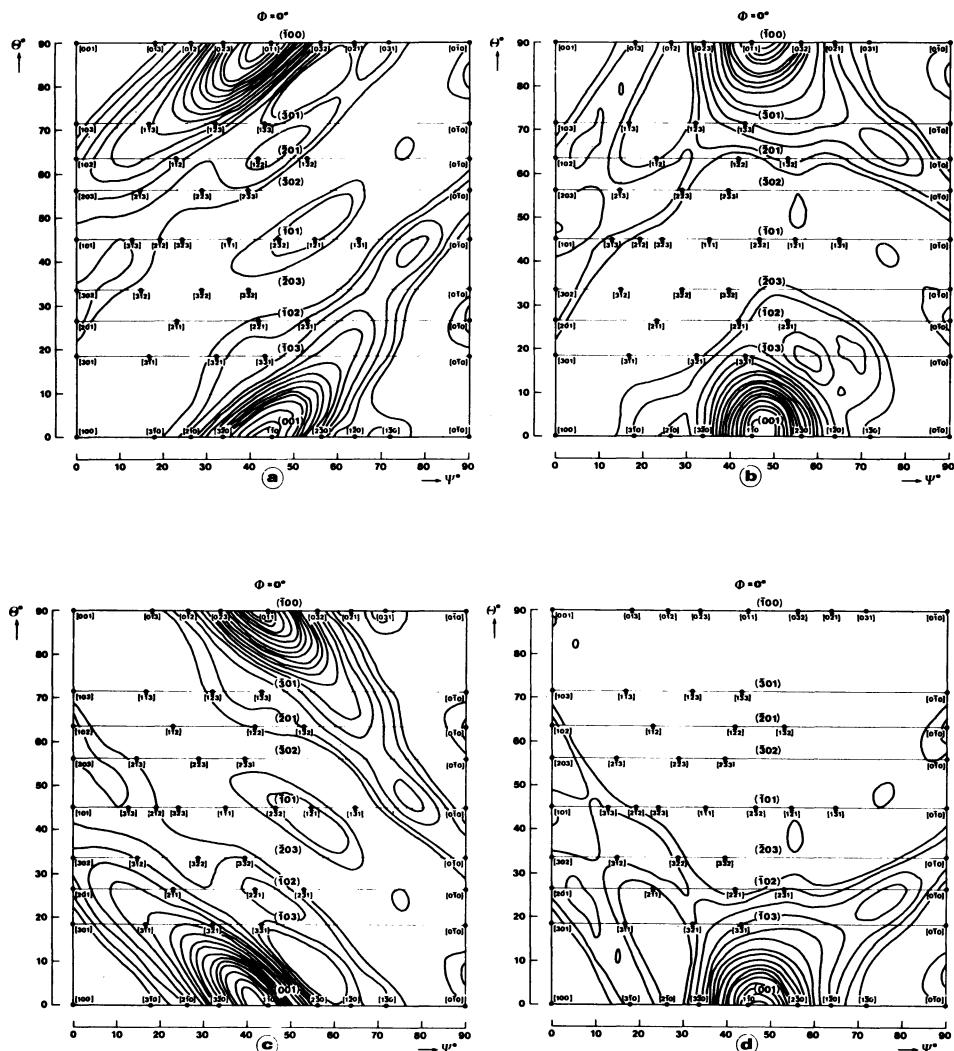


Fig. 10.  $\phi = 0^\circ$  section of the three dimensional representation computed from partial distribution functions; octahedral planes 123. a = (45, 55, 90); b = (135, 55, 90); c = (135, 125, 90); d = (45, 125, 90).

ship (2) on the basis of the four separate series of coefficients  $\beta_{mn}^{\ell} F_m^{\ell}$  given in (19). The sections  $\phi = 0^\circ$  and  $\phi = 45^\circ$  of the distribution function corresponding to the mean coefficients  $\pi_{mn}^{\ell} F_m^{\ell}$  are referred to in Fig. 12. This function has the advantage of giving the mean of the different distribution functions corresponding to the four equivalent octahedral planes, as shown in Fig. 13 for  $(\bar{1}13)$  and  $(100)$  planes distribution described by the sections  $(\psi, \theta, \phi) \equiv (\psi, 25, 45)$  and  $(\psi, 0, 0)$ . The fibre texture  $\langle 111 \rangle / ND$  of mild steel sheets is then obviously described in the octahedral planes 123 by  $(5\bar{1}\bar{1})$ ,  $(7\bar{1}\bar{1})$ ,  $(010)$  and  $(122)$  texture components, i.e., crystallographic planes located at about  $55^\circ$  from the direction  $\langle 111 \rangle$ .

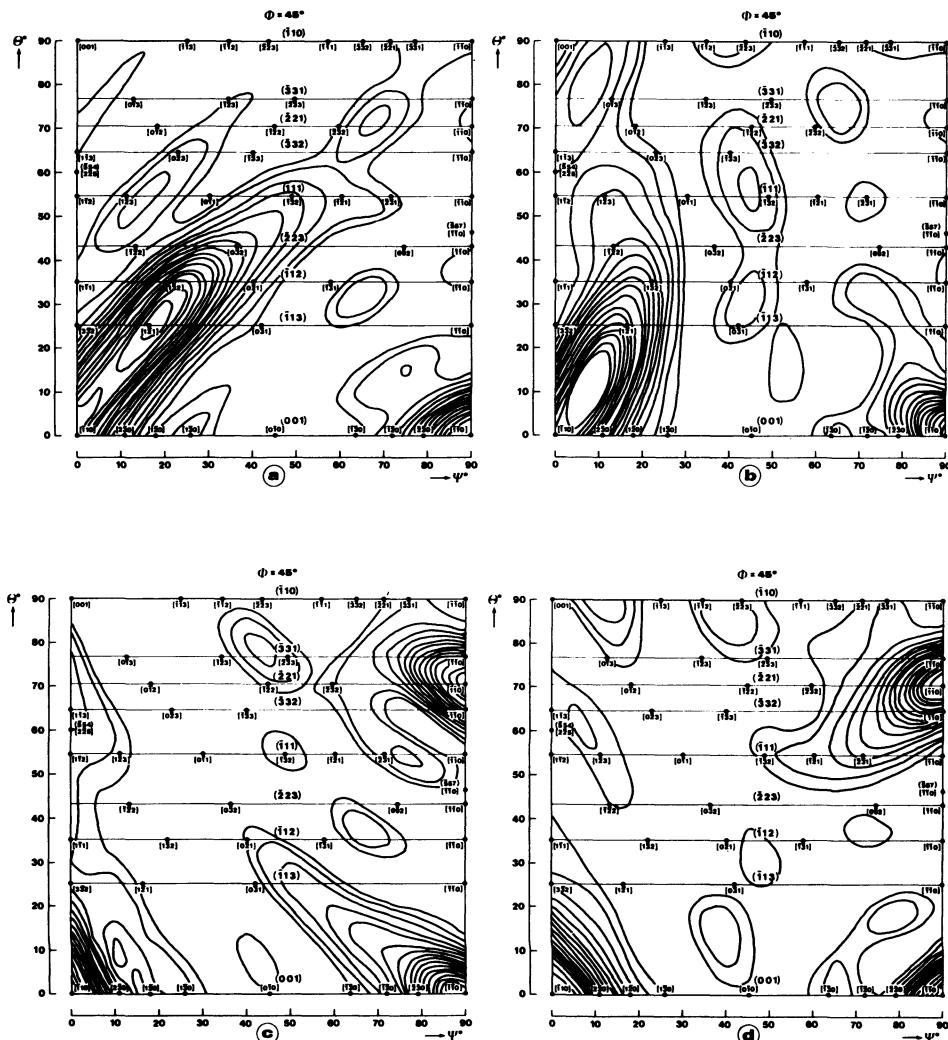


Fig. 11.  $\phi = 45^\circ$  section of the three dimensional representation computed from partial distribution functions; octahedral planes 123. a = (45, 55, 90); b = (135, 55, 90); c = (135, 125, 90); d = (45, 125, 90).

For planes 13, there are only two separate referentials defined by the rotations  $g_{\pi p}$  with Euler angles  $(a, b, c) \equiv (0, 45 + \beta 90^\circ, 0)$  with  $\beta = 0, 1$  (Fig. 14). In this case, the coefficients  $\frac{\beta}{\pi} F_{mn}^l$  are real numbers

$$\frac{\beta}{\pi} F_{mn}^l = (-1)^{m\beta} \pi F_{mn}^{lR} \quad \beta = 0, 1 \quad (23)$$

$$l = 0, 2, 4, \dots; m = 0, 1, 2, \dots; n = 0, 4, 8, \dots$$

for which there is a corresponding series of mean coefficients  $\pi F_{mn}^l$  also subject to the relationship (22).

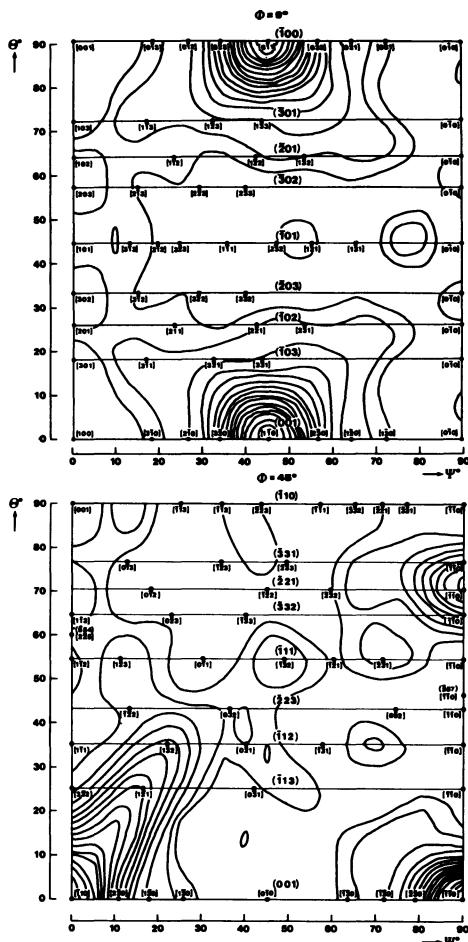


Fig. 12.  $\phi = 0^\circ$  and  $\phi = 45^\circ$  sections of the mean distribution function; octahedral planes 123.

The pole figures  $\{110\}$  calculated on the basis of the coefficients  $\frac{\beta}{\pi} F_{mn}^l$  are given in Fig. 15. The application of the rotation  $g_{\pi p} = (0, 135, 0)$  makes it possible to calculate the pole figure determined experimentally in the section 13 of the sheet obtained by machining (Fig. 16).

The pole figure calculated on the basis of the mean coefficients  $\frac{\beta}{\pi} F_{mn}^l$  represents the contributions of the two geometrically equivalent planes 13 (Fig. 17). Figure 18 contains the principal sections  $\phi = 0^\circ$  and  $\phi = 45^\circ$  of the partial and mean distribution functions corresponding to the plane 13.

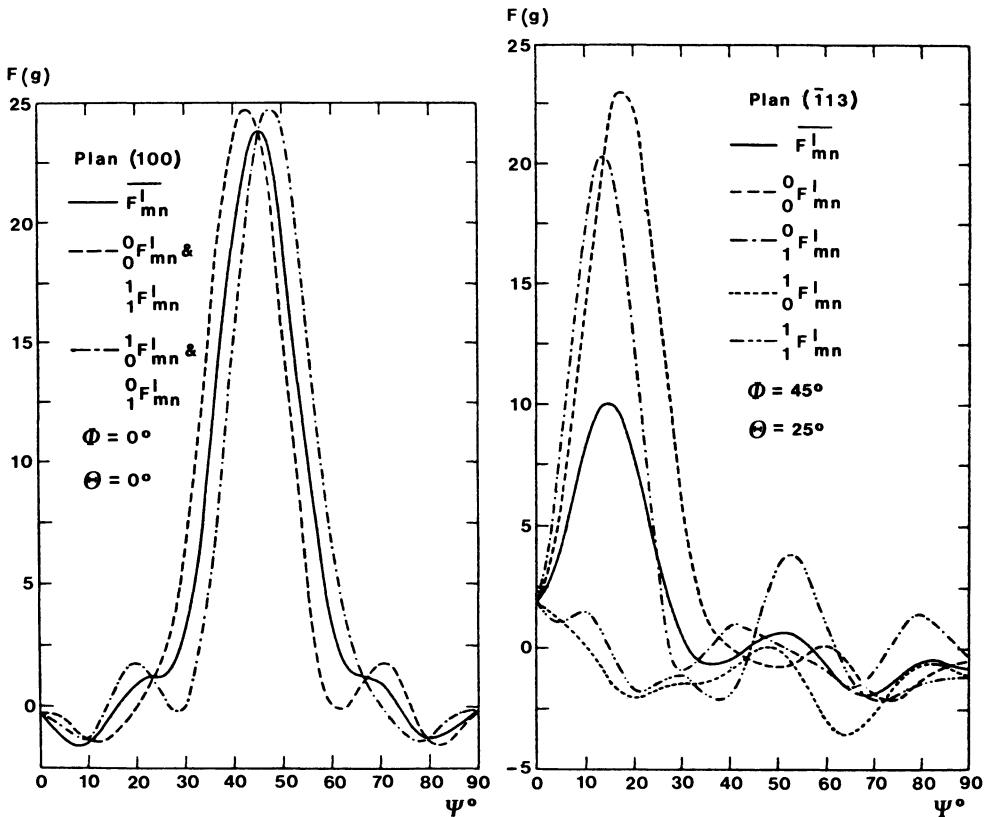


Fig. 13. Distribution of crystallographic planes (113) and (100) in the octahedral planes 123.

#### 4.3. Representation of the Directions $\langle 111 \rangle$ in the Planes at 45 Degrees from the Directions ND, RD, TD

The angular distribution of the crystallographic directions  $\langle 111 \rangle$  in the planes 12, 23 and 31 of the material has been examined by means of the function  ${}^{[u_1 v_1 w_1]} \pi F(\psi, \alpha)$ . In this case, the angles  $\theta_1$  and  $\phi_1$  defined by the relationship (12) are respectively  $\theta_1 = 55^\circ$  and  $\phi_1 = 45^\circ$ , the rotation  $g_{op}$  moreover being defined, according to the section plane considered, by the Euler angles:

$$\begin{aligned} \text{plane 12: } g_{op} &\equiv (135, 0, 0) & \vec{R}_o // \text{ND} \\ \text{plane 23: } g_{op} &\equiv (0, 90, 45) & \vec{R}_o // \text{RD} \\ \text{plane 31: } g_{op} &\equiv (90, 90, 135) & \vec{R}_o // \text{TD} \end{aligned}$$

The calculation of the function  ${}^{[u_1 v_1 w_1]} \pi F(\psi, \alpha)$  on the basis of the relationship (17) makes it necessary to take into consideration only the real parts  $D_{mn}^{\ell R}$  of the complex coefficients  $D_{mn}^{\ell}$  with  $\ell, m$  even and  $n = 4k$ ; these conditions correspond to the calculation of the mean distribution function.

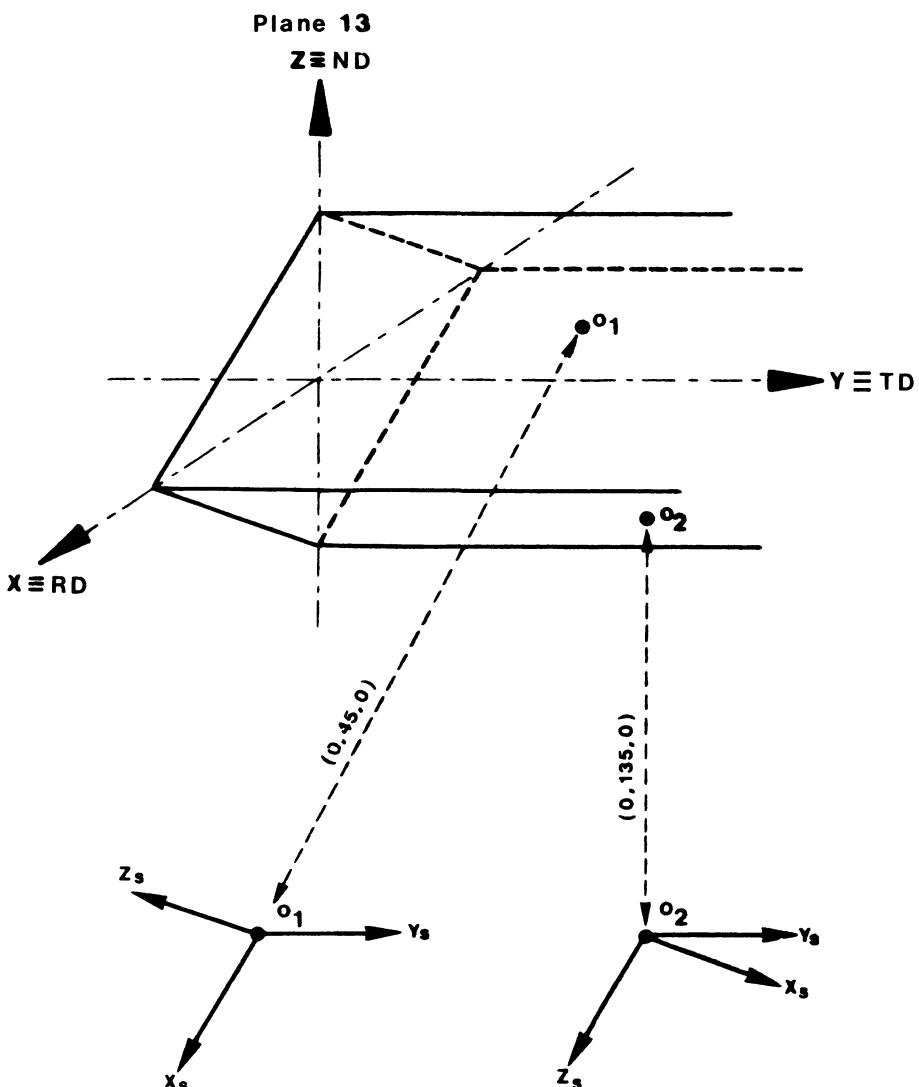


Fig. 14. Equivalent planes 13.

The isodensity curves of the function  $\pi F(\psi, \alpha)$  in the three planes 12, 23, 31 of the material are represented in Fig. 19. For plane 12 a strong maximum of the directions [111] in the reference direction  $R_o$  chosen parallel to ND is then observed; this maximum corresponds to the fibre texture  $<111>/\text{ND}$ . Moreover, textural components  $(011)[111]$  also appear with less strength at  $72^\circ$  from the reference direction  $R_o$ .

As regards the planes 23 and 31, the distribution of the directions [111] is practically the same with a maximum between  $25$  and  $30^\circ$  from the reference direction  $R_o$  (RD for the plane 23 and TD for the plane 31) corresponding to a crystallographic plane half-way between  $(1\bar{2}3)$  and  $(0\bar{1}1)$ .

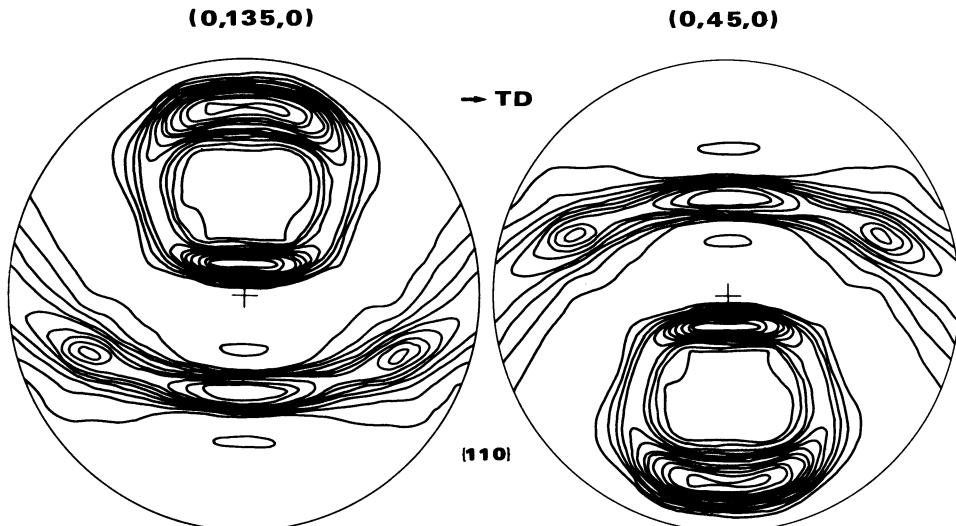


Fig. 15. Partial pole figures {110} computed in planes 13.

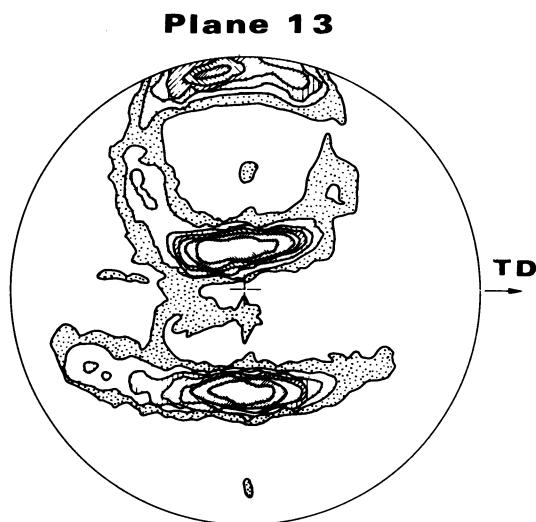


Fig. 16. Experimental pole figure in planes 13.

This similarity of the distribution of the directions [111] in the plane 23 and 31 results from the equibiaxiality of the deformation for the stretched sample studied ( $\epsilon = 0.098$ ), since, during equibiaxial expansion the maximum shear stresses are equal in the two planes 23 and 31.<sup>5</sup> Then, as the whole deformation of a.c.c. material results from gliding in <111> directions, it may be assumed that during deformation the crystals will undergo rotation<sup>6</sup> of which one of the aspects will be that of equalizing the distribution of the slip systems <111> in the planes 23 and 31.

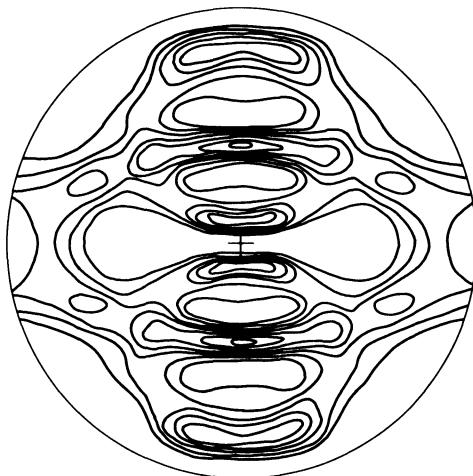


Fig. 17. Mean pole figure {110} computed in planes 13.

## 5. CONCLUSIONS

From the knowledge of the orientation distribution function in a particular system of the material, it is possible by defining the appropriate rotation matrix  $T(a,b,c)$  to compute crystallographic orientations  $(hkl)[uvw]$  in any macroscopic plane of the material. This calculation method leads also to a direction distribution function  $\int_{-\pi}^{\pi} F(\psi, \alpha) d\psi$  which gives the angular distribution of the crystallographic direction  $[u_1v_1w_1]$  in a macroscopic plane of the material.

This method has been applied to crystalline orientation distribution analysis in different planes of a low carbon steel sheet stretched in expansion.

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## APPENDIX

### Generalized spherical functions

#### Definition

$$Y_l^{mn}(g) = Y_l^{mn}(\psi, \theta, \phi) = \frac{1}{2\pi} Z_l^{mn} (\cos \theta) e^{im\psi} e^{in\phi}$$

where  $Z_l^{mn}$  is a generalization of the associated legendre functions.<sup>1</sup>

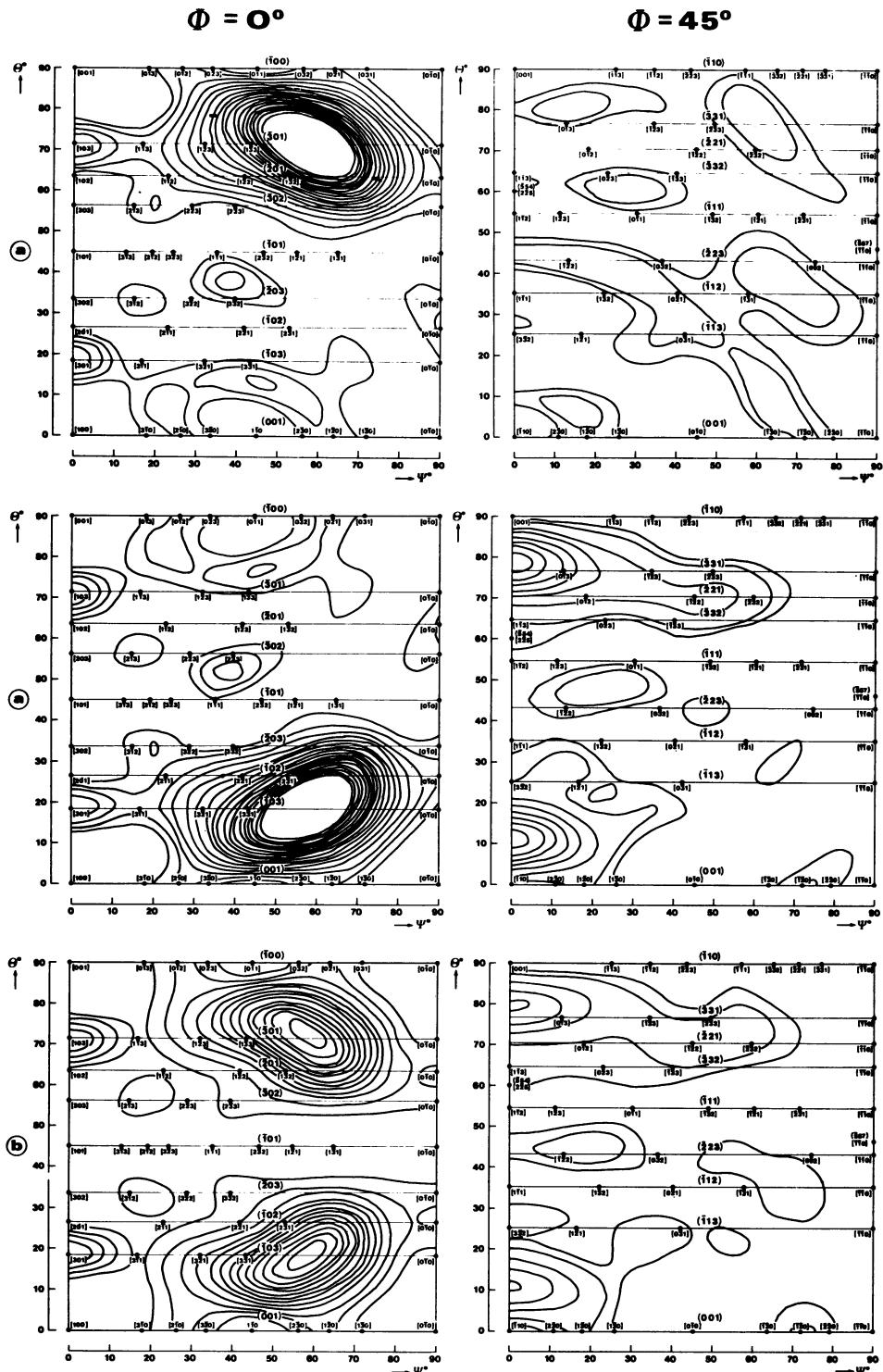


Fig. 18. Three dimensional representation:  $\phi = 0^\circ$  and  $\phi = 45^\circ$  sections in planes 13. (a) Partial distribution function. (b) Mean distribution function.

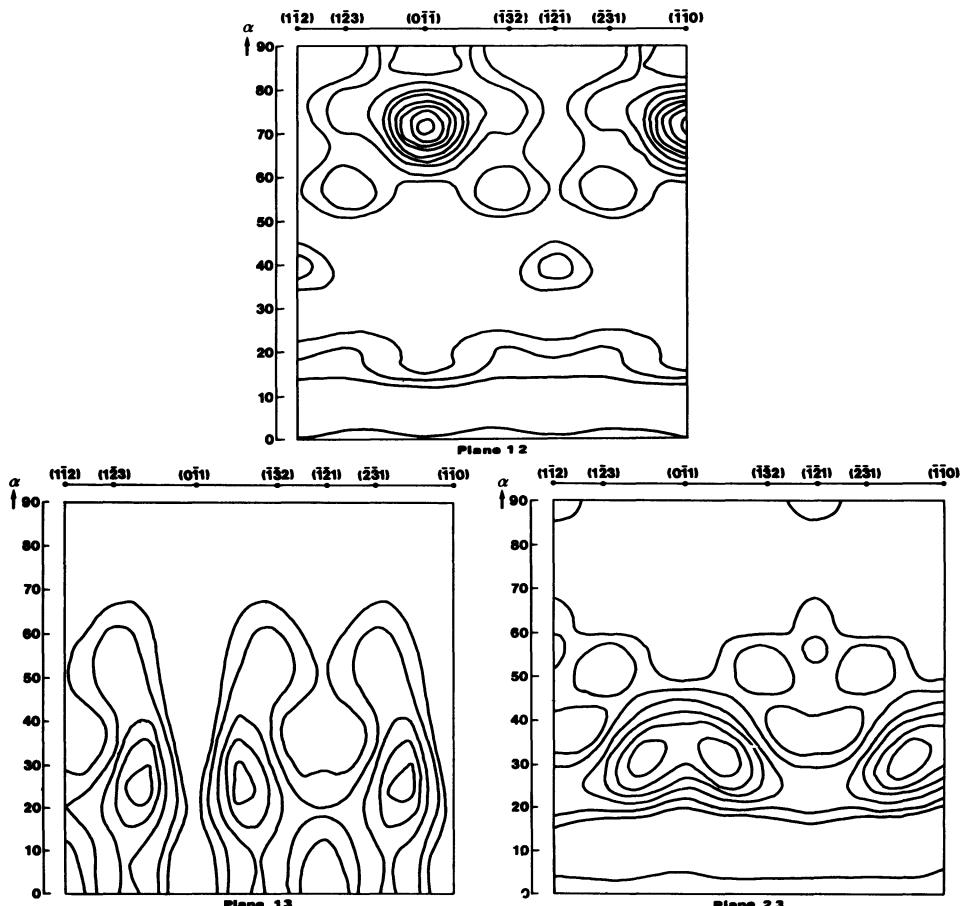


Fig. 19. Representation of the  $\langle 111 \rangle$  directions in planes 12, 13 and 23.

Orthonormalization relationship  $g \in [0, 2\pi] \times [0, \pi] \times [0, 2\pi]$

$$\int_g Y_\ell^{mn}(g) \overline{Y_\ell^{m'n'}(g)} dg = \delta_{\ell\ell'} \delta_{mm'} \delta_{nn'},$$

where  $\overline{Y_\ell^{mn}}$  is the conjugated function of  $Y_\ell^{mn}$ .

#### Additivity theorem

If on the one hand the two systems  $K_1$  and  $K_2$  are related together by the  $g(\psi, \theta, \phi)$  rotation and on the other, to a  $K$  system, by  $g_1(\psi_1, \theta_1, \phi_1)$  and  $g_2(\psi_2, \theta_2, \phi_2)$  rotations with  $g \equiv g_2^{-1} g_1$ ,  $(g, g_1, g_2)$  rotations are connected by:

$$Y_\ell^{mn}(o) Y_\ell^{mn}(g) = \sum_{s=-\ell}^{+\ell} Y_\ell^{ms}(g_1) \overline{Y_\ell^{ns}(g_2)}$$

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