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ON A UNIQUE SOLUTION OF A DIFFERENTIAL EQUATION IN VECTOR DISTRIBUTION

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Let us consider a differential equation in vector distribution

$$\dot{x}(t) = f(x,u,t) + b(x,u,t)\dot{u}(t), \quad x(t_0) = x_0, \tag{1}$$

where $x \in R^n$, functions $f(x,u,t) \in R^n$ and $b(x,u,t) \in R^{n \times m}$ are piecewise continuous in x, u, t, and $u(t) = (u_1(t), \ldots, u_m(t)) \in R^m$ is a bounded variation function which is non-decreasing in the following sense: $u(t_2) \geq u(t_1)$ as $t_2 \geq t_1$, if $u_i(t_2) \geq u_i(t_1)$ for all $i = 1, \ldots, m$.

A solution to equation (1) is defined as a vibrosolution [1], i.e., as a unique limit of absolutely continuous solutions to pre-limiting equations (1) corresponding to absolutely continuous non-decreasing approximations of function $u(t)$. Solutions to pre-limiting equations (1) are regarded as conventional solutions of ordinary differential equations with discontinuous right-hand sides in the sense of Filippov [2]. A vibrosolution is expected to be a function discontinuous at discontinuity points of the function $u(t)$.

Theorem 1: *Let the following conditions hold:*
(1) *functions $f(x,u,t)$, $b(x,u,t)$ $\partial b(x,u,t)/\partial x, \partial b(x,u,t))/\partial t$ are piecewise continuous domains are locally connected;*
(2) *functions $f(x,u,t), b(x,u,t)$ satisfy the one-side Lipschitz condition in x* [2].

A unique vibrosolution to equation (1) exists *if and only if* an $n \times m$-dimensional system of differential equations in differentials

$$\frac{d\xi}{du} = b(\xi, u, x), \quad \xi(\omega) = z, \tag{2}$$

is solvable inside a cone $K = \{u \in R^m : u_i \geq \omega_i, i = 1, \ldots, m\}$ with arbitrary initial values $\omega \in R^m$, $\omega \geq u(t_0)$, $z \in R^n$, and $s \geq t_0$.

Since a vibrosolution is a discontinuous solution to equation (1), it is helpful to design an equation with a measure which enables us to compute jumps of a vibrosolution at discontinuity points of function $u(t)$. This completely determines the behavior of a vibrosolution.

Theorem 2: *Let the conditions of Theorem 1 hold. Then a solution of an equivalent equation with a measure*

$$dx(t) = f(x,u,t)dt + b(x,u,t)du^c(t) + \sum_{t_i} G(x(t_i-), u(t_i-), \Delta u(t_i), t_i) d\chi(t - t_i), \; x(t_0) = x_0,$$

coincides with a vibrosolution of equation (1), *if* $G(z,v,u,s) = \xi(z,v,v+u,s) - z$, *where*

$\xi(z, v, u, s)$ is a solution to system (2); $u^c(t)$ is a continuous component of a nondecreasing function $u(t)$, $\Delta u(t_i) = u(t_i +) - u(t_i -)$ is a jump of a function $u(t)$ at t_i, t_i are discontinuity points of a function $u(t)$, and $\chi(t - t_i)$ is the Heaviside function.

References

[1] Krasnosel'skii, M.A., Pokrovskiy, A.V., *Systems with Hysteresis*, Springer-Verlag, Berlin 1989.

[2] Filippov, A.F., *Differential Equations with Discontinuous Right-Hand Sides*, Kluwer, New York 1988.