

Flipping Modules to Minimize Maximum Wire Length*

KYUNRAK CHONG

Computer Science Department, HongIk University, Seoul, Korea

SARTAJ SAHNI

Computer and Information Sciences Department, University of Florida, Gainesville, FL 32611

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We show that obtaining the optimal orientations of modules to minimize the length of the longest wire is NP-hard. If each module is permitted only two possible orientations, this can be done in linear time. When all four orientations are permissible and wires are restricted to connect modules whose separation is bounded by some constant, the problem also can be solved in linear time.

Key Words: *Module flipping; Maximum wire length; Complexity*

1. INTRODUCTION

Both circuit performance and routability can be affected by flipping modules while keeping the module placement fixed. In performance sensitive applications, it is desirable to flip modules so as to minimize the maximum estimated wire length (MEL). When performance is not so critical, routability can be improved by flipping modules so as to minimize the estimated total wire length (ETL).

The problem of minimizing the ETL by flipping modules as well as by rotating modules has been studied earlier by Yamada and Liu [7], Libeskind-Hadas and Liu [6], Ahn [1], and Chong and Sahni [2]. The starting point of all these studies is an already placed module set. When modules can be flipped, it is assumed that they can be flipped about their horizontal and/or vertical axes. Hence there are four possible orientations of a module under flipping (Figure 1). The initial or reference orientation is denoted by ϕ . h , v , and b , respectively, denote the orientation that results from flipping about the horizontal axis, vertical axis, and both axes. For rotation, rotations of 0, 90, 180, and 270

degrees are permitted and it is assumed that these rotations do not result in module overlaps.

Let ETLF (ETLR) denote the problem of flipping (rotating) modules so as to minimize the estimated total wire length. In both [7] and [6], the Euclidean distance between wire end points is used to estimate wire length. Both ETLF and ETLR are shown to be NP-hard in [6]. The proofs of [6] are easily modified for the case when wire length is estimated using the Manhattan distance between the wire end points. Further, the proof of ETLF holds even if flips are restricted to be made along the vertical (horizontal) axis. Ahn [1] has provided an alternate NP-hard proof for ETLF using the Manhattan metric. This proof shows that ETLF is NP-hard even when we have a single row of modules and modules may be flipped only along the horizontal axis. Yamada and Liu [7] propose an analytical method to obtain sub-optimal solutions for ETLF. This algorithm is shown (experimentally) to be competitive with hill-climbing and simulated annealing algorithms for ETLF. In [6], Libeskind-Hadas and Liu propose neural network formulations for ETLF and ETLR.

Chong and Sahni [2] show that ETLF is linearly solvable for the case when the modules are arranged as a matrix in which wires connect only pairs of modules that are in adjacent columns. They show that ETLF is polynomially solvable for standard cell

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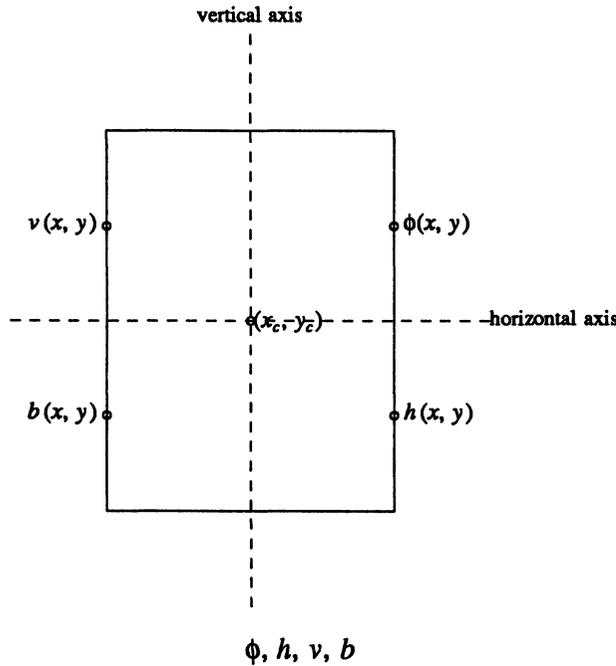


FIGURE 1

designs in which wires connect modules in adjacent columns and either the number of module columns is two or the number of modules dependent on any other module is bounded by some constant. Chong and Sahni [2] also evaluate a simple greedy heuristic that attempts to minimize ETL by flipping modules. Experimental results reported by them indicate this heuristic is superior to the neural network approach of [6].

In this paper, we restrict ourselves to the problem of minimizing MEL. We consider both the case MEL4 of minimizing the maximum estimated wire length when each module has four possible orientations and MEL2 when the number of possible orientations per module is at most two. We show that MEL4 is NP-hard even when all modules lie in a single row (or a single column). MEL4 can be solved

in linear time when modules are in a single row and the wires are restricted to connect modules whose separation is bounded by some constant. We also show that MEL2 is solvable in $O(n \log n)$ time where n is the number of modules. This is so, independent of the placement of the n modules on a two dimensional surface. The wire lengths are required to be polynomial in n . This is really no restriction.

2. COMPLEXITY OF MEL4

In this section we shall show that determining module orientations that minimize the maximum estimated wire length is NP-hard when each module is permitted one of four orientations $\{\phi, v, h, b\}$. I.e., problem MEL4 is NP-hard. Our proof actually shows that MEL4 is NP-hard even when all modules lie in a single row. The reduction we provide uses the known NP-complete problem [4] 4 color which is formally stated below.

Definition [4 color]

Input: An undirected graph $G = (V, E)$

Output: 'yes' iff G is four colorable \square

We show how, for every instance G of 4 color, one can construct, in polynomial time, a single row instance, $M(G)$, of MEL4 with the property that from the length of the longest wire in the solution to $M(G)$ one can determine in $O(1)$ time whether or not G is four colorable. So, MEL4 is NP-hard.

Suppose we have a pair of unit width modules that are connected by a pair of wires as in Figure 2(a). In this figure, the two wires are labeled 1 and 2, respectively. The distance between the two modules is t , the difference in y -coordinate of the two points labeled 1 and 2 in each module is r , and the four wire end points are equidistant from the nearest end of the vertical edge of the module they are on. The two wire connection of Figure 2(a) is called an (r, t) connection. Note that r uniquely deter-

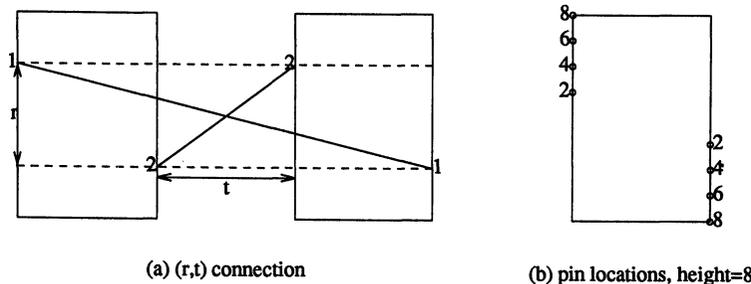


FIGURE 2

mines the position of the wire end points. These are on opposite vertical edges and located $\frac{r}{2}$ units from the center. Figure 2(b) shows a module whose height is 8. Possible pin positions on each vertical edge are marked. These are one unit apart. So, there are 9 positions on each edge. The pin pairs used in an (r, t) connection for $r = 2, 4, 6,$ and 8 are labeled by the r value.

Let the module orientations of Figure 2(a) be (ϕ, ϕ) and let $l(f, g)$ be the Manhattan length of the longer wire when the left module has orientation f and the right one has orientation $g, f \in \{\phi, \nu, h, b\}, g \in \{\phi, \nu, h, b\}$. We obtain the following values for l .

$$\begin{aligned} l(\phi, \phi) &= l(\nu, \nu) = l(h, h) = l(b, b) = r + t + 2 \\ l(\phi, \nu) &= l(\nu, \phi) = l(h, b) = l(b, h) = r + t + 1 \\ l(\phi, h) &= l(\nu, b) = l(h, \phi) = l(b, \nu) = t + 2 \\ l(\phi, b) &= l(\nu, h) = l(h, \nu) = l(b, \phi) = t + 1 \end{aligned}$$

For any undirected graph G , we construct a single row instance $M(G)$ of MEL4 that has n modules of unit width and height $2n$ where n is the number of vertices in G . The modules are placed in a single row with the right edge of a module one unit from the left edge of the next. Let the modules be M_1, \dots, M_n left to right. Module M_i denotes vertex i of G . For each edge $(i, j), i < j,$ in G we establish two wires between M_i and M_j . The distance between the right edge of M_i and the left edge of M_j is $2(j - i) - 1$. The two wires constitute a $(2(n - j + i + 1), 2(j - i) - 1)$ connection between modules M_i and M_j . An example is given in Figure 3. Notice that with this type of wire connection, the

maximum wirelength is $r + t + 2 = 2(n - j + i + 1) + 2(j - i) - 1 + 2 = 2n + 3$ and this occurs iff there are two modules with identical orientations that have an (r, t) connection between them.

Theorem 1: Single row MEL4 is NP-hard.

Proof: For any instance G of 4 color, construct an instance $M(G)$ of MEL4 as described above. This construction is easily done in polynomial time. If G is four colorable then consider a four coloring C of G using the four colors ϕ, h, ν, b . If a vertex u of G is assigned color $c \in \{\phi, h, \nu, b\}$ in C , then the module corresponding to u is assigned the orientation c . Using these orientations for the modules of $M(G)$, there is no pair of modules with an edge between them for which both modules in the pair have the same module orientation. Hence, the maximum wire length is less than $2n + 3$.

Next suppose $M(G)$ has a set of module orientations for which the maximum wire length is less than $2n + 3$. Then every pair of modules that have a wire between them is such that the module orientations are different. Color vertex u of G with color c iff the module that corresponds to u in $M(G)$ has orientation c . With this coloring, no pair of adjacent vertices of G is assigned the same color. Hence, G is four colorable. So, G is four colorable iff $M(G)$ has an orientation set with maximum wire length less than $2n + 3$. Since 4 color is NP-complete, MEL4 is NP-hard. \square

3. POLYNOMIALLY SOLVABLE CASES OF MEL4

The proof of the preceding section is readily adapted to the case when all modules lie in a single column.

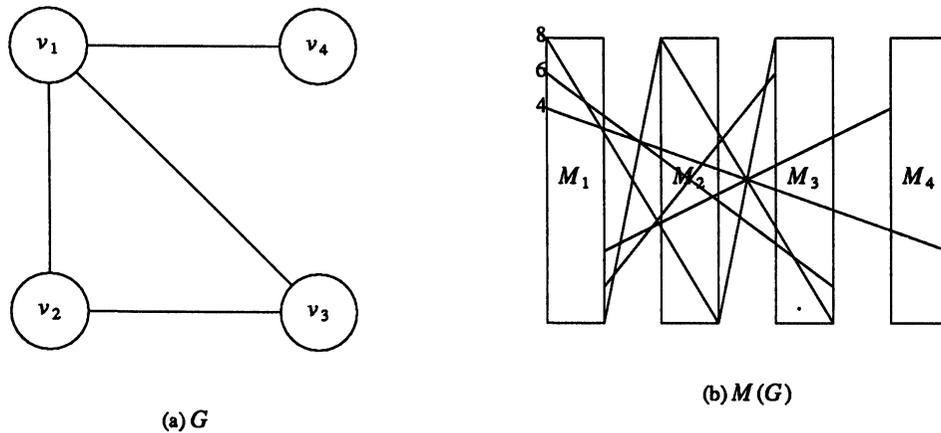


FIGURE 3

So, the only possibilities for polynomially solvable cases are those in which the distance between connected modules is bounded by some constant k . Note that in the construction of the preceding section the instances of MEL4 that are created have wire connections between modules that are as far as $n - 1$ modules apart. For example, there may be a connection between modules 1 and n . In this section we shall show that if inter module connections are restricted to those modules that are at most k apart, then MEL4 can be solved in time linear in the number of modules and connections for a single row of modules. While the complexity is linear in the instance size, it is exponential in k . We expect this as the problem is NP-hard for unrestricted k . Our algorithm for the single row case does not require that modules have the same width or the same height. It only requires that the modules be ordered left to right as in a row.

For an n module single row instance I of MEL4, we construct a dag (directed acyclic graph) with $n + 2$ columns of vertices. Column zero contains the single vertex s and column $n + 1$ contains the single vertex t . The vertices in column i of the dag correspond to module i of I . Each vertex in column i of G has the format $(i, f_{i-k}, f_{i-k+1}, \dots, f_{i+k})$ where $f_a \in \{\phi, h, \nu, b\}$ gives the orientation of the module a which is $|i - a|$ units from module i . If $i - a < 0$ then f_a is the orientation of a module to the right of module i ; otherwise it is the orientation of a module to the left of module i . For example, for module 4 of an 8 module instance with $k = 2$, the vertices have the format $(4, f_2, f_3, f_4, f_5, f_6)$. If $i = 1$ and $k = 2$, then the format is $(1, -, -, f_1, f_2, f_3)$ as there are no modules to the left of module 1. The total number of vertices in the constructed dag is less than $1024n + 2$ as each f_i has at most four possible values.

The edges of the dag are obtained in the following way. From vertex s of column zero, there is an edge to each vertex in column 1. These edges have zero weight. From each vertex in column n , there is an edge to vertex t . The remaining edges connect a vertex in column i to one in column $i + 1$, $1 \leq i < n$. There is an edge between two vertices x and y in columns i and $i + 1$, respectively, if all modules that are common to the vertex labels x and y have the same orientation in x as in y . The weight of this edge is the length of the longest wire connecting to module M_i when the orientation of all connected modules are as given by the label.

As an example, consider module 4 of the 8 module $k = 2$ example above. The vertices in column 4 have the format $(4, f_2, f_3, f_4, f_5, f_6)$. Those in col-

umn 5 have the format $(5, g_3, g_4, g_5, g_6, g_7)$. There is an edge between $x = (4, f_2, f_3, f_4, f_5, f_6)$ and $y = (5, g_3, g_4, g_5, g_6, g_7)$ iff $f_3 = g_3$, $f_4 = g_4$, $f_5 = g_5$, and $f_6 = g_6$. The weight of this edge is the length of the longest wire connecting to module M_4 under the assumption that modules M_2, \dots, M_6 , respectively, have the orientations f_2, \dots, f_6 . Note that all wires from M_4 terminate in one of the modules M_2, M_3, M_5 , and M_6 as $k = 2$. Hence this weight is easily determined.

From the construction, one easily sees that each s to t path of the dag uniquely defines a set of module orientations for I . Furthermore the maximum estimated wire length for this set of orientations is the maximum weight on the path. The reverse is also true. i.e., for every orientation set, there is an s to t path in the dag with maximum edge weight equal to the maximum estimated wire length for the orientation set. Hence, by finding an s to t path for which the maximum edge weight is minimum, one obtains the solution of the MEL4 instance I . Such a path can be found in $O(n)$ time using dynamic programming as in [5]. This coupled with the fact that the dag can be constructed from I in $O(n + m)$ time where m is the number of wires in I implies that single row MEL4 can be solved in $O(n + m)$ time. Note that the number of edges in the dag is $O(n)$.

For MEL4 instances with m rows and n columns a dag can be similarly constructed. The modules are first ordered by column and then within columns by rows. Let M_1, M_2, \dots, M_{mn} be the modules in this order. The dag has $mn + 2$ vertex columns with column i representing M_i , $1 \leq i \leq mn$. Column zero has the vertex s and column $mn + 2$ has the vertex t . Vertex labels now contain $2km + m$ module orientations. So the total number of vertices is $O(mn4^{2km+m})$ and the number of edges is $O(mn4^{2km+m})$. Hence the complexity becomes $O(mn4^{2km+m})$.

4. POLYNOMIAL ALGORITHM FOR MEL2

Suppose we have n modules placed at arbitrary locations on a two dimensional surface and that each module has one of two possible orientations. Let a_i and b_i be the two possible orientations for module i . a_i and b_i will generally be in the set $\{\phi, h, \nu, b\}$. However, our formulation doesn't require this. All we require is that for every set of orientations of the n modules, the maximum wire length is bounded by some polynomial function $p(n)$ of n and that in the optimal orientation the maxi-

imum wire length is an integer. An instance I which satisfies these restrictions is an instance of MEL2.

Suppose we can solve, in time $O(t(n))$, the problem P1: given an instance I of MEL2, find an orientation set for which the maximum wire length is $\leq k$. If there is no orientation set that satisfies this length requirement, then return the answer 'nil'. Then MEL2 can be solved in $O(t(n)\log n)$ time by a binary search process that looks for the smallest k in the range $[1, p(n)]$ for which the answer to the restricted problem P1 is not 'nil'.

To solve P1 for any k , we construct an instance C of the 2CNF* problem such that C is satisfiable iff the answer is P1 is not 'nil'. Furthermore, every set of truth assignments to the variables of C that satisfies C (i.e., C is true for this assignment) defines a set of module orientations for which the maximum wire length is $\leq k$.

C is constructed in the following way. For each module M_i in an instance I of P1, we shall have one variable x_i , $1 \leq i \leq n$. We shall use the interpretation x_i is true iff module M_i has orientation a_i (so, x_i is false iff module M_i has orientation b_i). For each wire (M_i, M_j) in P1 (i.e., the wire connects modules M_i and M_j) we examine the four possible values for the pair (f_i, f_j) where $f_i \in \{a_i, b_i\}$ and $f_j \in \{a_j, b_j\}$ and determine which (if any) of these results in the length of this wire exceeding k . Each pair (f_i, f_j) that does this results in a clause of the 2CNF instance C that is being constructed. Table 1 gives the clause constructed for each (f_i, f_j) that results in the wire length exceeding k .

We readily see that the 2CNF instance C has the properties stated earlier. Further, the number of clauses in C is $O(m)$ where m is the number of wires in I and the time to construct C is $O(n + m)$. Since the solution to the 2CNF problem can be found in $O(n + m)$ time [3], P1 can also be solved in $O(n + m)$ time. Hence MEL2 can be solved in $O((n + m)\log n)$ time. When m is $O(n)$, this is $O(n \log n)$.

5. CONCLUSIONS

We have shown that flipping modules about their horizontal and/or vertical axes so as to minimize

*In the 2CNF problem we are given a conjunctive normal form formula with n variables and m clauses. Each clause has at most two literals and we are to determine a truth assignment for which the formula is true.

TABLE 1
Clause for a module pair orientation (f_i, f_j)
that results in wire of length $> \kappa$

(f_i, f_j)	Clause
(a_i, a_j)	$(\bar{x}_i + \bar{x}_j)$
(a_i, b_j)	$(\bar{x}_i + x_j)$
(b_i, a_j)	$(x_i + \bar{x}_j)$
(b_i, b_j)	$(x_i + x_j)$

the maximum wire length is NP-hard. In fact, this is so even when all modules lie in a single row. Our proof requires that long wires (i.e., wires connecting modules that are $O(n)$ apart) be present. When long wires are not permitted, the problem can be solved in linear time for a single row of modules. When the number of module orientations is limited to two, the maximum wire length can be minimized in $O(n \log n)$ time irrespective of the module placement.

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Biographies

KYUNRAK CHONG is an Assistant Professor of Computer Science at HongIk University. He obtained his Ph.D. in Computer Science from the University of Minnesota in 1991. His research interests include algorithms for VLSI design.

SARTAJ SAHNI is a Professor of Computer and Information Sciences at the University of Florida and a Fellow of the IEEE. He received his B.Tech. (Electrical Engineering) degree from the Indian Institute of Technology, Kanpur, and the M.S. and Ph.D. degrees in Computer Science from Cornell University. Dr. Sahni is a co-editor of the *Journal of Parallel and Distributed Computing* and is on the editorial boards of *IEEE Transactions on Parallel and Distributed Systems*, *IEEE Parallel and Distributed Technology: Systems and Applications*, and *Computer Systems: Science and Engineering*.



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