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ON THE CRYSTALLOGRAPHIC BASIS OF YIELD CRITERIA

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Consideration of crystallographic texture has lead to a much better understanding of anisotopic yielding behavior than yield criteria postulated without regard to deformation mechanisms. Continuum anisotropic yield criteria have been developed to simulate the results of calculations based on the crystallographic nature of slip. These criteria, which involve high stress exponents, describe actual forming behavior much better than the quadratic yield criterion postulated by Hill. Today it is possible to calculate anisotropic yielding behavior directly from texture data, although for metal forming analyses, some other means must be used to characterize the strain-hardening behavior.

KEY WORDS: Yield criteria anisotropy crystallographic slip pencil glide texture strain hardening.

INTRODUCTION

A yield criterion is simply a mathematical expression of the combination of stress components that is satisfied at the initiation of plastic flow. For those interested in crystals, Schmid's law (1931) is the simplest yield criterion. It says slip (yielding) will start when the shear stress, τ_{nd}, in a potential slip direction on a potential slip plane reaches a critical value, $\pm\tau_C$.

$$\tau_{nd} = \pm\tau_C. \tag{1}$$

ISOTROPY

More familiar to most engineers are two isotropic yield criteria. One is the Tresca (1864) or maximum shear stress criterion. Expressed in terms of principal stresses with the convention $\sigma_1 \geq \sigma_2 \geq \sigma_3$,

$$\sigma_1 - \sigma_3 = Y, \tag{2}$$

where Y is the yield strength in a tension test. The other is the von Mises criterion (1913)

$$(\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2 + (\sigma_1 - \sigma_2)^2 = 2Y^2. \tag{3}$$

Although widely accepted, neither has a fundamental basis and the little experimental data tends to lie between the predictions of the two.

CRYSTALLOGRAPHIC BASIS

The work of Taylor (1938) and of Bishop and Hill (1951) laid the foundations for an upper-bound method of calculating the shape of the yield surface for randomly oriented polycrystals from a knowledge of the slip systems. In this approach, every grain within the polycrystal is assumed to undergo the same strains (as referred to an external coordinate system) and it is assumed that the shape change occurs with the minimum expenditure of energy. To simulate isotropy, calculations are made for all possible grain orientations and averaged. Details of how this approach can be used to calculate yield loci are given elsewhere (Hosford, 1993). The method has been applied to fcc metals deforming by {111}<1$\bar{1}$0> slip (which is equivalent to bcc metals deforming by {1$\bar{1}$0}<111> slip) and to bcc metals deforming by <111>-pencil glide. Points on the resulting isotropic yield loci (Figure 1) lie between the Tresca and von

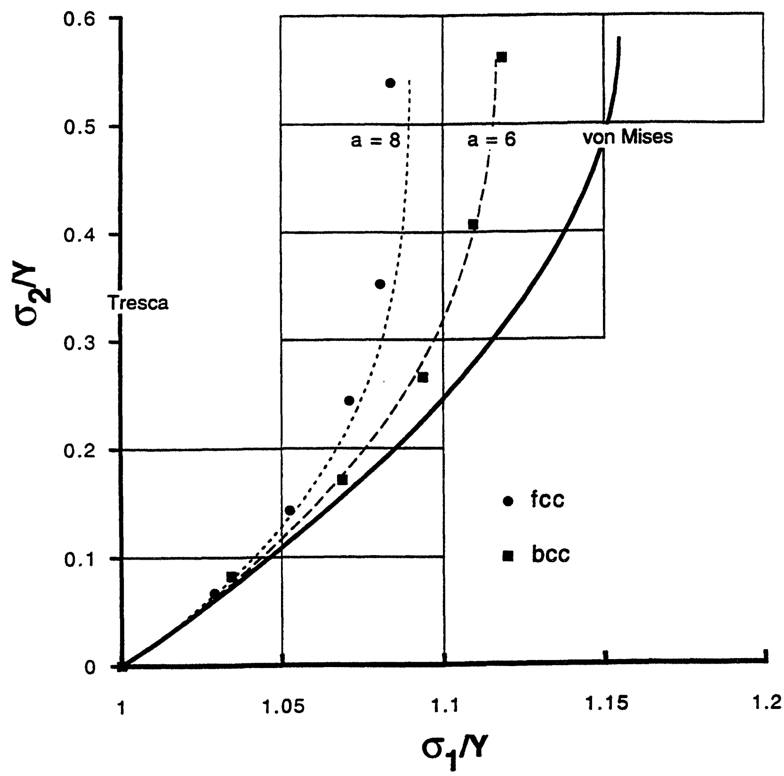

Figure 1 Representative Section of Isotropic Yield Loci with $\sigma_3 = 0$. Tresca and von Mises are compared with calculations based on the {111}<110> slip for fcc and <111>-pencil glide for bcc. Note the σ_1/Y scale is expanded relative to the σ_2/Y scale for ease of comparison. The dash and dotted lines represent eq. 6 with exponents a = 8 suggested for fcc and a = 6.

Mises predictions. A more critical test of yield criteria is the predicted relation between the strains resulting from yielding and the stress state. Figure 2 shows the dependence of the stress ratio, $\alpha = (\sigma_2 - \sigma_3)/(\sigma_1 - \sigma_3)$ on the ratio of plastic strains $\rho = \varepsilon_2/\varepsilon_1$.

The von Mises criterion (eq. 3) can be expressed as

$$(\sigma_2 - \sigma_3)^4 + (\sigma_3 - \sigma_1)^4 + (\sigma_1 - \sigma_2)^4 = 2Y^4. \tag{4}$$

Similarly the Tresca criterion (eq. 2) can be expressed as

$$|\sigma_2 - \sigma_3|^1 + |\sigma_3 - \sigma_1|^1 + |\sigma_1 - \sigma_2|^1 = 2Y^1. \tag{5}$$

Furthermore the predictions of a generalized criterion (Hosford, 1972)

$$|\sigma_2 - \sigma_3|^a + |\sigma_3 - \sigma_1|^a + |\sigma_1 - \sigma_2|^a = 2Y^a, \tag{6}$$

approach those of Tresca as a $\rightarrow \infty$.

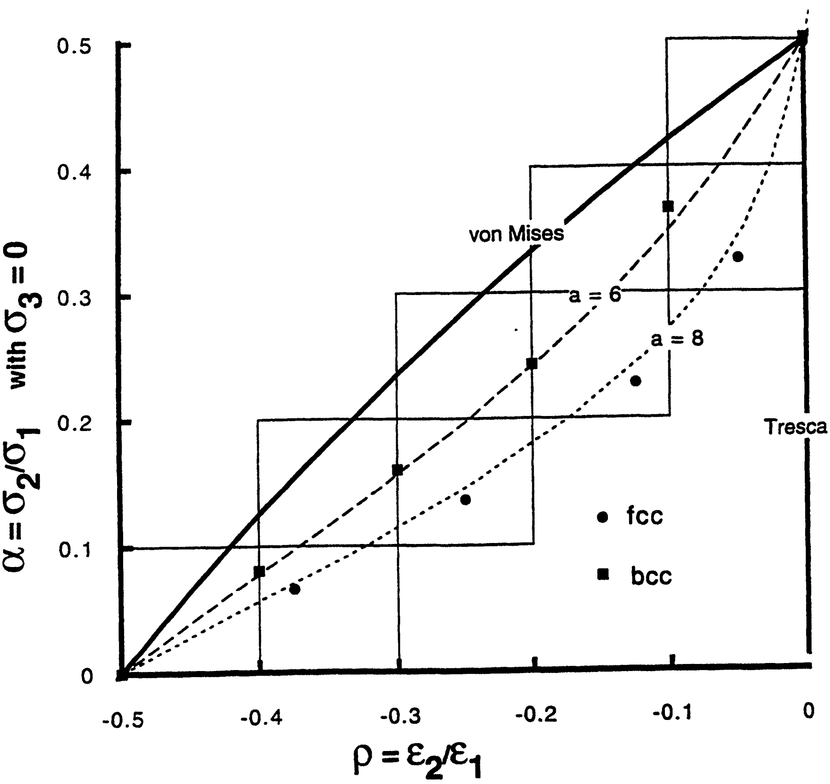

Figure 2 Stress Ratios, $\alpha = \sigma_2/\sigma_1$, for Plane Stress ($\sigma_3 = 0$) and Corresponding Ratios of Plasic Strains, $\varepsilon_2/\varepsilon_1$. The solid lines represent the flow rules for the von Mises and Tresca criteria. The points are from calculations based on the crystallography of slip. The dashed and dotted lines are from the flow rules for eq. 6.

Yielding behavior intermediate between von Mises and Tresca can be modeled by equation 6 with either $1 \leq a \leq 2$ or $4 \leq a < \infty$. The higher range of the exponent a, is more convenient. The calculations based on the model of Taylor and of Bishop and Hill are well represented by a = 8 for fcc {111}<1$\bar{1}$0> slip (Hosford, 1979) and a = 6 for bcc <111>-pencil glide (Logan and Hosford, 1980). Figures 1 and 2 show the predictions of eq. 8 and its flow rules for a = 6 and a = 8. These provide much better fits to the calculations based on fcc {111}<1$\bar{1}$0> slip and bcc <111>-pencil glide than either Tresca or von Mises.

ANISOTROPY - CONTINUUM APPROACH

Polycrystalline materials usually have crystallographic textures and therefore the yielding behavior is anisotropic. The first complete and reasonable anisotropic yield criterion is that postulated by Hill (1948) without regard to deformation mechanism:

$$F(\sigma_y - \sigma_z)^2 + G(\sigma_z - \sigma_x)^2 + H(\sigma_x - \sigma_y)^2 + 2L\tau_{yz}{}^2 + 2M\tau_{zx}{}^2 + 2N\tau_{xy}{}^2 = 1. \qquad (7)$$

where F, G, H, L, M, and N are the constants that describe the anisotropy and x, y and z are the principal symmetry axes (orthogonal axes of two-fold symmetry). This criterion is a generalization of the Mises yield criterion. For the special case of plane-stress ($\sigma_z = \tau_{yz} = \tau_{zx} = 0$) loading, it simplifies to:

$$R(\sigma_y - \sigma_z)^2 + P(\sigma_z - \sigma_x)^2 + RP(\sigma_x - \sigma_y)^2 + (2Q+1)(R+P)\tau_{xy}{}^2 = P(1+R)X^2, \qquad (8)$$

where R, P and Q are the ratios of lateral contraction strains in tension tests along the x, y, and 45° directions respectively. For planar isotropy (P = R = Q) and in-plane loading this further simplifies to

$$\sigma_y{}^2 + \sigma_x{}^2 + R(\sigma_x - \sigma_y)^2 = (1 + R)Y^2, \qquad (9)$$

Woodthorpe and Pierce (1970) reported that some aluminum alloy sheets in which R, P and Q were all less than unity had yield strengths in balanced biaxial tension higher than in uniaxial tension. Because this "anomalous" behavior was not consistent with eqs. 8 or 9, Hill (1979) proposed a further generalization

$$\begin{aligned} &f|\sigma_2 - \sigma_3|^m + g|\sigma_3 - \sigma_1|^m + h|\sigma_1 - \sigma_2|^m + \\ &a|2\sigma_1 - \sigma_2 - \sigma_3|^m + b|2\sigma_2 - \sigma_3 - \sigma_1|^m + c|2\sigma_3 - \sigma_1 - \sigma_2|^m = 1, \end{aligned} \qquad (10)$$

where a, b, c, f, g, h, and m are constants and the 1, 2, and 3 axes are the principal symmetry axes. Hill suggested four simplifications of this criterion but only one of the four is free of mathematical limitations (Zhu *et al.*, 1987). For this case (a = b = f = g = 0) the criterion can be simplified to:

$$(1 + 2R)|\sigma_1 - \sigma_2|^m + |2\sigma_3 - \sigma_1 - \sigma_2|^m = 2(R + 1)Y^m, \qquad (11)$$

where Y is the yield strength for tension tests in all directions in the 1–2 plane the strain ratio R has the same value for all directions in the 1–2 plane. With this criterion, the exponent, m, must be adjusted for sheets having different values of R.

ANISOTROPY - CRYSTALLOGRAPHIC APPROACH

The approach of Taylor and of Bishop and Hill can also be used to calculate anisotropic yield loci of textured metals. Behavior is averaged over the orientations present in the texture. A large number of such calculations were made for textures having rotational symmetry about the 3-axis, assuming deformation by {111}<110> slip in fcc metals (Hosford, 1979) and <1111>-pencil glide in bcc metals (Logan and Hosford, 1980). Predictions were made of the strengths for several characteristic loading paths (biaxial tension, plane-strain tension with $\varepsilon_2 = 0$ and $\sigma_3 = 0$, and plane strain with $\varepsilon_3 = 0$ and $\sigma_3 = 0$.) as well as the strength and strain ratios in uniaxial tension. Figure 3 show calculated ratios of strength for several loading paths to the strength in uniaxial tension plotted as a function of the calculated R values. Each point is for one texture. The general trends clearly do not coincide with the predictions of the original Hill theory (a = 2).

Examination of such yield-locus calculations lead to the proposal of an anisotropic criterion of the form

$$R|\sigma_2 - \sigma_3|^a + P|\sigma_3 - \sigma_1|^a + RP|\sigma_1 - \sigma_2|^a = P(R + 1)X^a = R(P + 1)Y^a \qquad (12)$$

with a = 8 for fcc metals and a = 6 for bcc metals. Here R and P have the same meanings as above, X and Y are the yield strengths in x- and y-direction tension tests. It should be noted that this criterion is both a generalization of the non-quadratic isotropic yield criterion (eq. 6) and Hill's original anisotropic yield criterion (eq. 8). It is also specialization of Hill's general yield criterion (eq. 10) with a = b = c = 0. For even integer exponents, no absolute magnitude signs are required in equation 12. For planar isotropy (P = R) and in-plane loading ($\sigma_z = 0$)this further simplifies to

$$\sigma_1^a + \sigma_2^a + R(\sigma_1 - \sigma_2)^a = (1 + R)Y^a, \qquad (13)$$

The curves in Figure 3 correspond to this equation with several exponents. Lower-bound calculations (Galdos and Hosford, 1990), based on assuming the same stress state is assumed in each grain instead of the same strain state resulted in similar conclusions. Again, exponents of about a = 8 and about a = 6 gave the best fits for fcc {111}<110> slip and bcc <111>-pencil glide respectively.

One shortcoming of this criterion, like that of Hill's 1979 criterion, is that it can be used only for stress states in which the principal stress axes and the principal symmetry axes coincide (i.e. shear stresses $\tau_{23} = \tau_{31} = \tau_{12} = 0$). Barlat and Lian (1989), however, proposed a generalization which will accommodate shear stresses. In the notation used here it can be expressed as,

$$2Y^a = a|K_1 + K_2|^m + a|K_1 - K_2|^m + (2 - a)|2K_2|^m,$$

where $K_1 = (\sigma_x + h\sigma_y)/2$, $K_2 = \{[\sigma_x - h\sigma_y)/2]^2 + p\tau_{xy}^2\}^{1/2}$ and $m = 8$. (14)

The constants, a, h, and p can be found from the strain ratios in tension tests. Lege *et al.* (1989) used this equation to fit the yield loci calculated for textured aluminum from measured texture data using a Bishop-and-Hill type analysis for fcc. Figure 5 is an example. In all cases the best fit was found with an exponent of about 8. For more general stress states, Barlat and coworkers (1991) later proposed a more complex six component yield criterion.

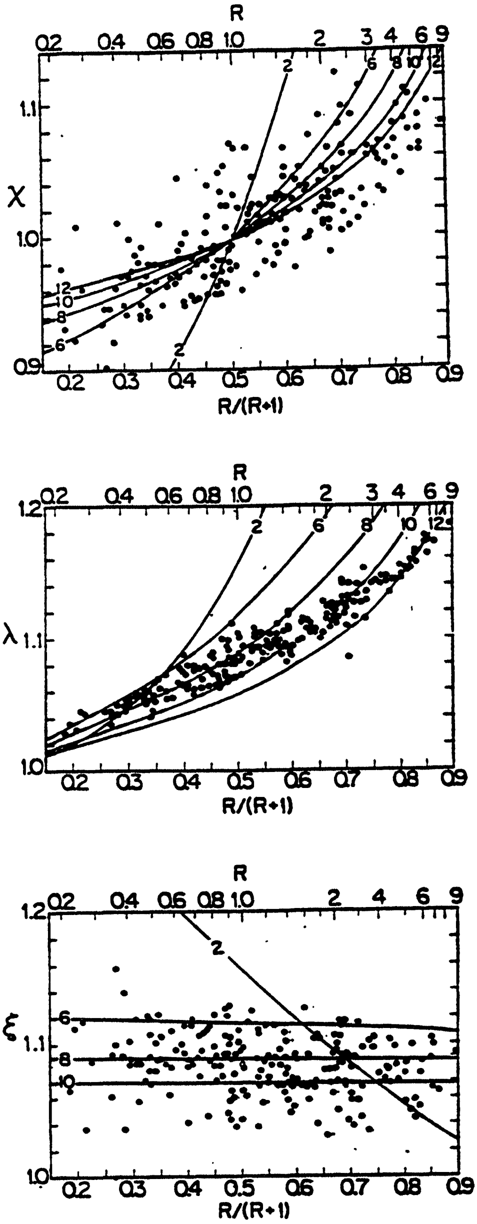

Figure 3 Ratios of Strengths for Several Loading Paths Plotted as a Function of R-Values. The points were calculated for textures with rotational symmetry about z for fcc metals using the Bishop and Hill approach.

A. χ is the ratio of biaxial strength $\sigma_x = \sigma_y$ with $\sigma_z = 0$ to the uniaxial strength. B. λ is the ratio of plane-strain strength, σ_x, with $\varepsilon_y = 0$ and $\sigma_z = 0$ to the uniaxial strength. C. ξ is the ratio of plane-strain strength, $2\sigma_x$, with $\varepsilon_z = 0$ and $\sigma_z = 0$ to the uniaxial strength. The continuous curves are predictions of eq. 13 with several values a. A value of a $\approx$ 8 fits the points much better than a = 2 (original Hill theory). From Hosford (1979).

Another, more approximate, way has been suggested of accommodating loading conditions in which the principal stress axes do not coincide with the symmetry axes (Hosford, 1985). This approach uses a yield criterion in which the material constants are expressed along the principal stress axes. For loading with $\sigma_3 = 0$ and the 1-axis at an angle of θ,

$$R_\theta \sigma_2^{\,a} + R_{\theta+90}\sigma_1^{\,a} + R_\theta R_{\theta+90}(\sigma_1 - \sigma_2)^a = R_{\theta+90}(R_\theta + 1)Y_\theta^{\,a}. \qquad (15)$$

Although this criterion violates the principle of normality, it does offer a simple way of approximating the yielding behavior under off-axis loading.

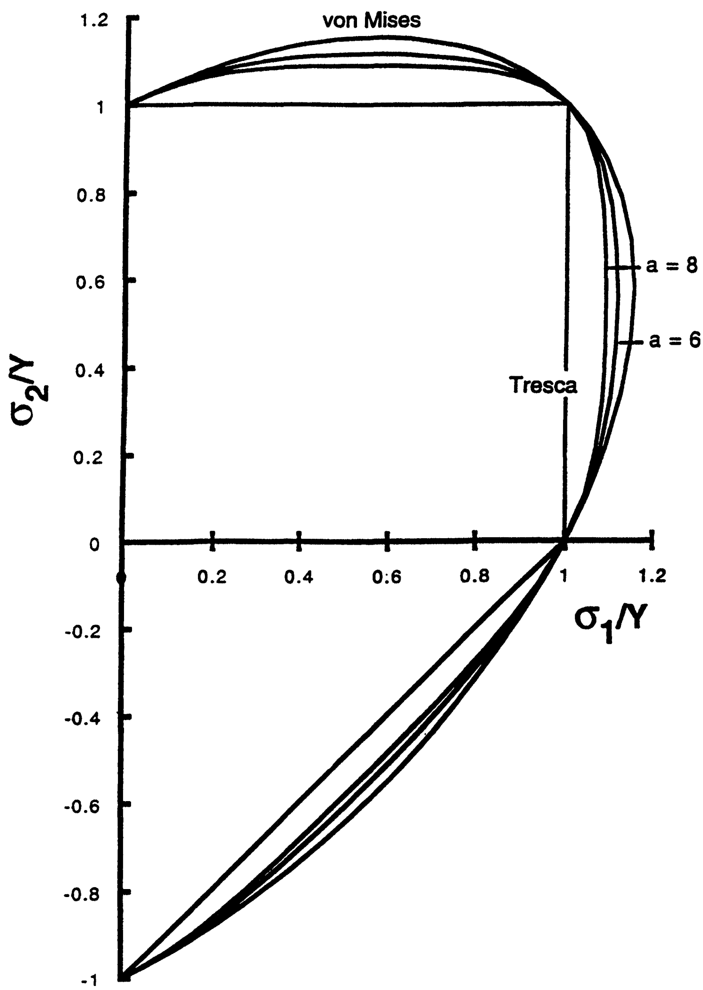

Figure 4 The ($\sigma_Z = 0$) Yield Loci Predicted by Equation 6 for Several Values of a. The calculations, based on the model of Taylor and of Bishop and Hill are well represented by a = 8 for fcc {111} <1$\bar{1}$0> slip and a = 6 for <111> pencil glide.

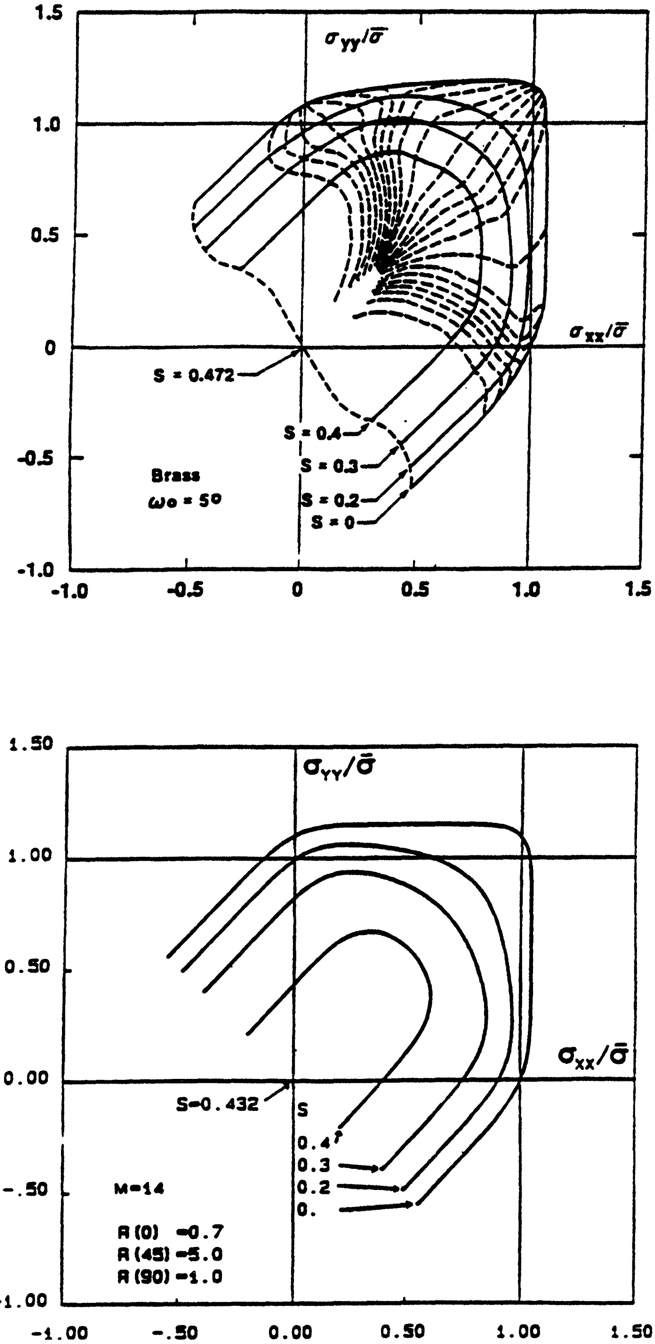

Figure 5 (A) Comparison of the (σ_z = 0) Yield Locus Calculated for Textured Aluminum from Measured Texture Data using the a Bishop-and-Hill Type Analysis and (B) the Fit of Equation 16 with m = 8, a = 1.24, h = 1.15 and p = 1.02. From Barlat and Lian (1989)

This criterion can be used to predict the angular variation of yield strength from a knowledge of the angular variation of strain ratio, R.

$$Y_{\theta 2}/Y_{\theta 1} = \{[R_{\theta 1+90}(R_{\theta 1} + 1)]/[R_{\theta 2+90}(R_{\theta 2} + 1)]\}^{1/a} \tag{16}$$

Strain hardening must be considered to compare offset yield strengths (stress levels measured at the same tensile strain, rather at than the same plastic work). With power-law hardening, $\bar{\sigma} = k\bar{\varepsilon}^n$, the ratio of the stress levels at any fixed offset strain, $\sigma_{\theta 2}/\sigma_{\theta}$, is related to the strain ratios by

$$(\sigma_{\theta 2}/\sigma_{\theta})^a = \{[R_{\theta 1+90}(R_{\theta 1} + 1)]/[R_{\theta 2+90}(R_{\theta 2} + 1)]\}^{n+1}. \tag{17}$$

In figure 6, ratio of σ_{90}/σ_0 is plotted as a function of $\{[P(R + 1)]/[R(P + 1)]\}^{n+1}$ and σ_{45}/σ_0 is plotted as a function of $\{[2P(R + 1)]/[(Q + 1)(R + P)]\}^{n+1}$ for a wide range of metals. While there is a great deal of scatter, it is clear that an exponent of 8 approximates the data much better than an exponent of 2.

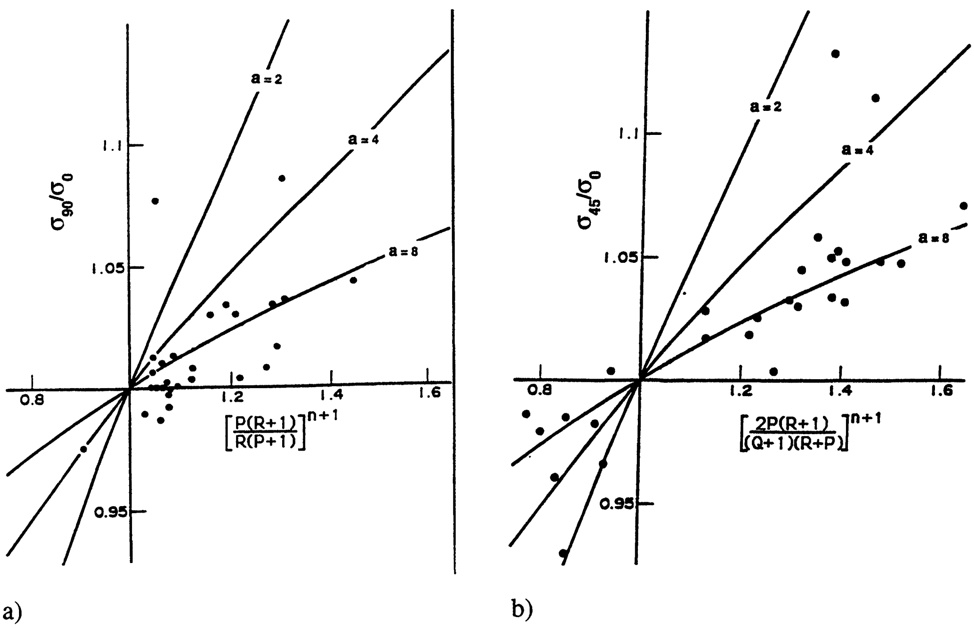

Figure 6 a, b Comparison of Experimental Offset-Yield Strength Ratios, σ_{90}/σ_0 and σ_{45}/σ_0 with Strain-Ratio Measurements. The solid lines are the predictions of eq. 16. Despite the scatter is clear that the data are represented better by a = 8 than a = 2 (original Hill theory). The data, from Meuleman (1980), represent a wide range of both fcc and bcc metals.

APPLICATION TO METAL-FORMING ANALYSES

One of the most important uses of yield criteria is to predict sheet metal forming behavior. Comparing actual and predicted behavior therefore can be used to test the theories. Several examples are the limiting drawing ratio in cupping, the shape of forming limit diagrams.

A critical paper by Whiteley (1960) first showed that the limiting drawing ratio (LDR) in deep drawing of flat-bottom cylindrical cups depends on the normal anisotropy, $\bar{R}$. Figure 7 compares the R-dependence of LDR, calculated (Logan *et al.*, 1987) using the high exponent criterion (eq. 13) with the experimentally determined dependence (Meuleman, 1980). Clearly the predictions based on a = 8 represent the experimental trends much better than those calculated with R = 2.

Recently the approximate high-exponent criterion (eq. 16) was used to calculate the variation of earing with texture (Logan, 1995). Calculations for a = 2 and a = 8 are shown in Figure 8. The experimental data are from Wilson and Butler (1962). Again the experimental data are best represented by a = 8.

In sheet metal forming, failure occurs by localized necking if the strains in the plane of the sheet are too large. A graphical representation of the strain levels at which such failure occur is called a forming limit diagram. The left-hand side of these diagrams fit the strain combinations that correspond to a constant level of thinning of the sheet. However, the right-hand side was not understood until Marciniak and Kuczinski (1967)

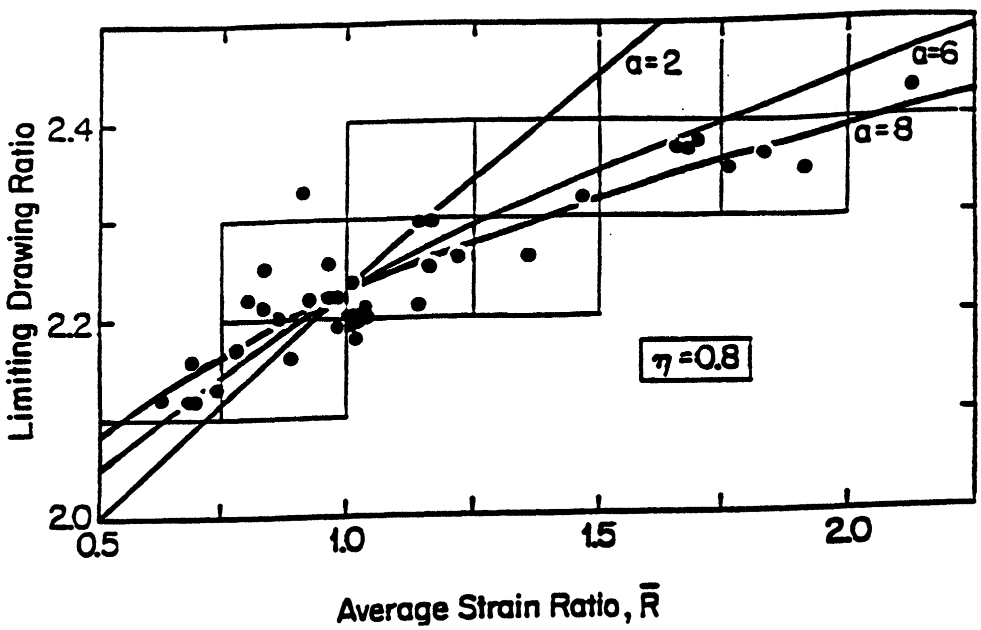

Figure 7 Comparison of the Calculated and Experimental Determined Dependence of the Limiting Drawing Ratio on $\bar{R}$. From Logan *et al.* (1987).

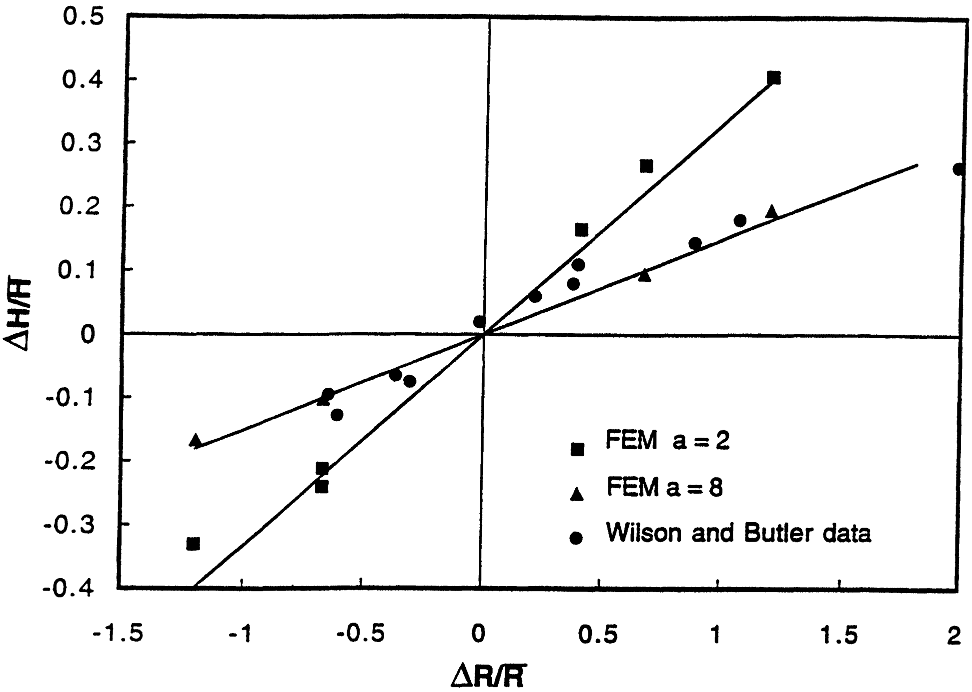

Figure 8 The Variation Ear Height with $\Delta R/\bar{R}$. $\Delta R = (R_0 + R_{90} - 2R_{45})/2$ and $\bar{R} = (R_0 + R_{90} + 2R_{45})/4$. The finite element calculations for a = 2 and a = 8 are based on eq. 16. Adapted from Logan *et al.* with data from Wilson and Butler (1962).

proposed that the final localized neck grows gradually out of an initial defect. They outlined a procedure for calculating the shape of right-hand side based on this assumption. Such calculations, (Marciniak and Kuczinski, 1967) and Parmar and Mellor, 1978) initially seemed to give reasonable results until they were applied to anisotropic sheets using Hill's quadratic yield criterion (eq. 9) which predicted a large effect of the $\bar{R}$ on the level of the FLD (Figure 9a). However, no such effect was experimentally observed. When the high exponent criterion (eq. 13) was used to make such calculations (Graf and Hosford, 1990), the predicted effect of the $\bar{R}$ on the level of the FLD disappeared (Figure 9b). Padwal and Chaturvedi (1993) also found good agreement with experiment.

Padwal (1993) made finite difference calculations of the strain distribution in bulge testing using Woo's method (1965). The agreement between theory and experimental measurements of Ilahi *et al.* (1981) was much better when the calculations were made with the high exponent criterion (eq. 13) than with the Hill's quadratic form. The comparison of the calculated and measured distribution of thickness strain (Figure 10) is one example.

Results of finite element calculations of metal forming often depend greatly on the yield criterion assumed in the calculations. Recently, D. Zhou and R. H. Wagoner (1991) explored the use of several yield criteria in finite element calculations and found that

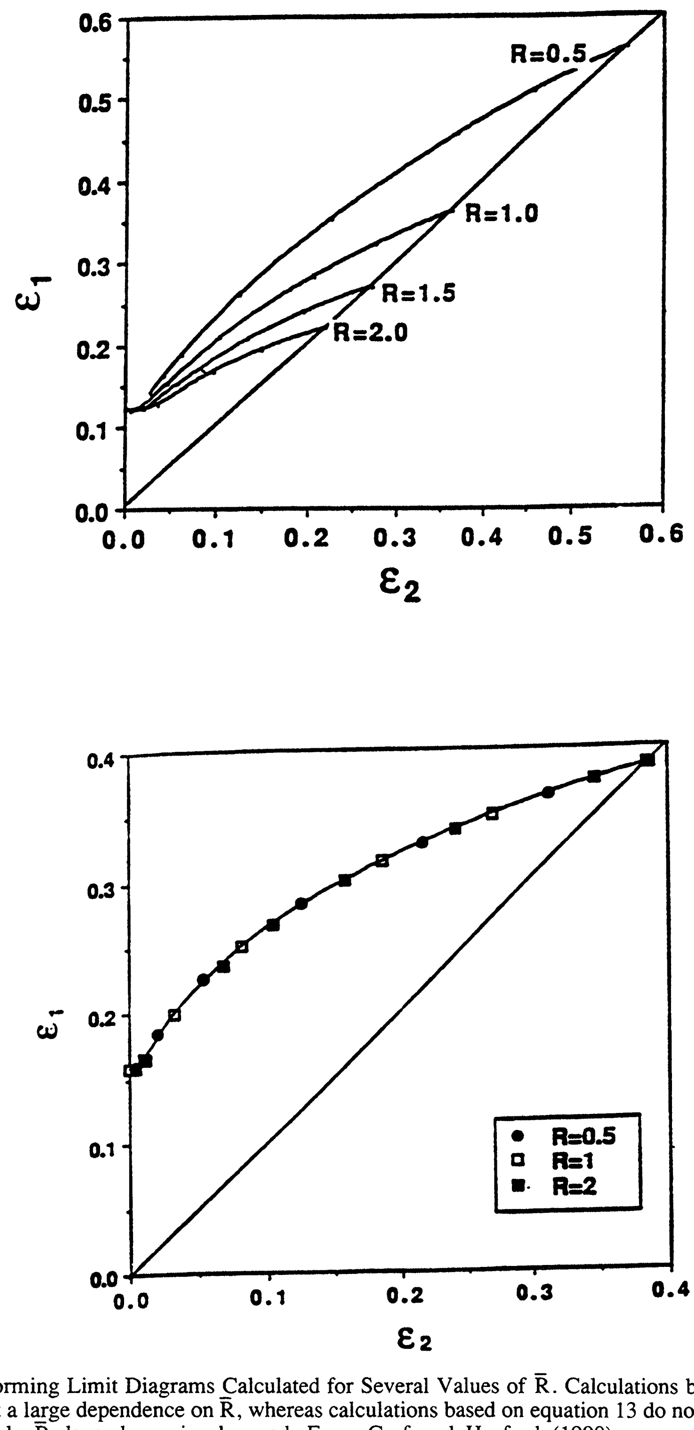

Figure 9 Forming Limit Diagrams Calculated for Several Values of $\bar{R}$. Calculations based on equation 9 (A) predict a large dependence on $\bar{R}$, whereas calculations based on equation 13 do not. Experimentally no appreciable $\bar{R}$-dependence is observed. From Graf and Hosford (1990).

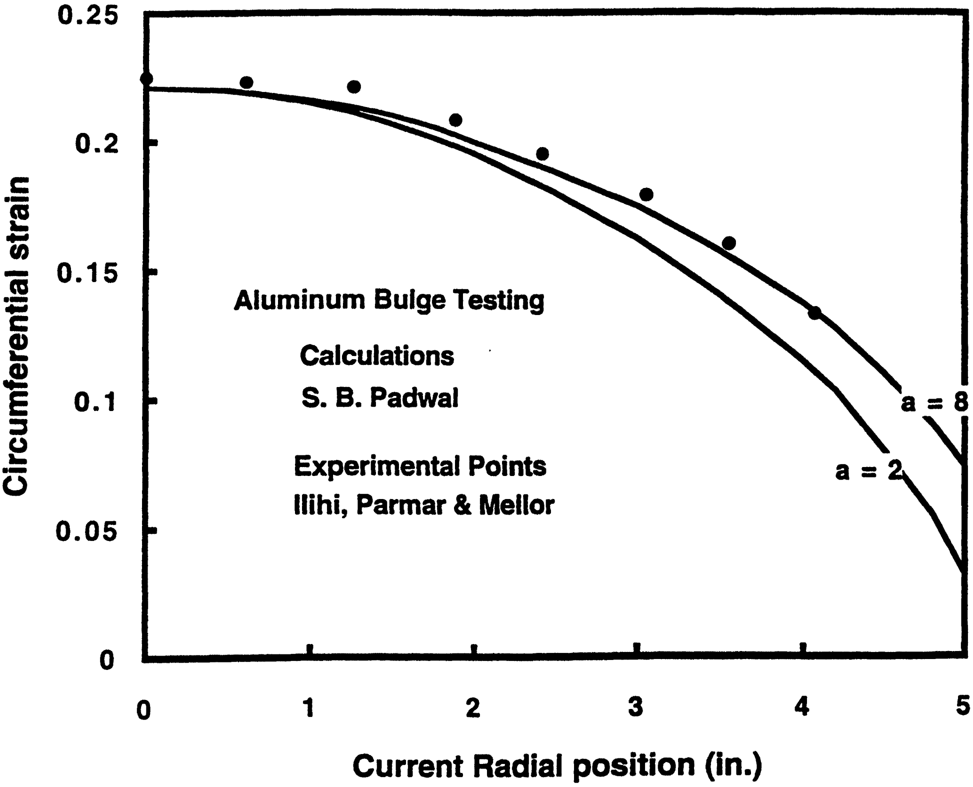

Figure 10 Thickness Strains during Hydraulic Bulge Testing. The thickness variation calculated using eq. 13 agrees much better with the measurements that the calculations based on eq. 9. Adapted from Padwal (1993).

the results were very sensitive to the assumed yield criterion as well as the assumed friction coefficient. They made finite element calculations of the strain distribution in stretching a sheet over a hemispherical dome and adjusted the friction coefficient in the calculations to obtain agreement with experiment. The value of friction coefficient required for agreement was more reasonable with the high exponent criterion than with the quadratic criterion, although in both cases the value seems high.

DIRECT USE OF TEXTURE DATA IN FORMING CALCULATIONS

It is possible to use x-ray textural data directly to predict forming behavior, without a continuum yield criterion. Instead of deducing a continuum yield function from crystallographic considerations and a few mechanical measurements (yield strengthened and R-values.) Barlat (1987) and Barlat and Richman (1987) have successfully calculated the shapes of forming limit diagrams directly from ODF textural data. However, in addition to the texture data, the work-hardening and strain-rate hardening behavior had to be measured or assumed. While this approach is intellectually satisfying, its industrial use in the near future would seem to be limited.

CONCLUSIONS

In analyses of many metal forming operations, the anisotropic yielding behavior of textured metals is an important input. Choice of the anisotropic yield criterion has a strong affect on the results of calculations. A high-exponent criterion seems to give much more reasonable predictions than Hill's quadratic criterion. While in principle, anisotropic behavior can be calculated from texture data, it is usually simpler to assume a continuum criterion and evaluate the constants from mechanical tests.

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