INTRODUCTION TO STOCHASTIC CALCULUS APPLIED TO FINANCE

by Damien Lambertson and Bernard Lapeyre

Translated by N. Rabeau and F. Manton

A BOOK REVIEW

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To get a brief glimpse at the subject matter of this delightful monograph, consider the following “classical” problem of pricing a European call option on a stock, which is a contract giving the holder the right, but not the obligation (hence the term “option”), to purchase, at a specified time in future, one share of the underlying stock at a predetermined price. Although the early attempts of solving this problem went at least back to the classical works by Arrow and Debreu on equilibrium analysis, it is in the celebrated paper by Black and Scholes (*Journal of Political Economy*, 1973) and a further extension and more rigorous mathematical treatment by Merton that brought a definite answer to this problem, using the powerful and yet beautiful tools from Martingale and Stochastic Calculus, thus leading to a Nobel prize in Economics in 1997. Roughly speaking, the idea is to first assume that the price of the stock follows a geometric Brownian motion, and then to show, by using the martingale representation theorem and Girsanov theorem, that one can *exactly* replicate, with probability one, the payoff of the option by only trading the stock continuously and using the conventional money market account. Therefore, the price of the option must be the necessary money needed for such a replication. Undoubtedly, this is a brilliant example of applied mathematics, as it solves an important practical problem by using non-trivial mathematics.

After a brief reviewing of some necessary mathematical backgrounds in the first three chapters, the Black-Scholes model is presented in Chapter 4. The chapter begins with a description of the model based on the geometric Brownian motion; then moves on to the Girsanov theorem transforming the original model to “risk-neutral” world so that, after proper discounting, the original geometric Brownian motion becomes a martingale; and finally use the martingale representation theorem to show

that the European options can be replicated with probability one. A generalization of the model to pricing of American options, involving optimal stopping, is also discussed in the last section. Although the chapter is only about fifteen pages, the exposition is clear, and all the essential mathematics has been revealed.

Chapter 5 concerns about the connection between the Black-Scholes model and the partial differential equations (PDE's). It begins with a discussion of the Feynman-Kac formula in Theorem 5.1.7, leading to the Black-Scholes PDE in equation (5.10). Afterwards it shifts the gear to the numerical solution of the PDE by a finite difference method, and then extend the method to numerical solution of American options, based on the results of the authors on variational inequalities. The chapter ends up with a brief mention of an alternative numerical procedure, the binomial tree method. This is my most favorite chapter in the book, giving a unobscured presentation of the theory of the PDE's, the numerical ways of implementation, as well as a latest cutting edge treatment of the subject based on the authors' own research.

Extensions of the Black-Scholes model to interest rate options and models with possible jumps are studied in Chapters 6 and 7, respectively. After defining some basic terminologies of interest rate, the classical models based on spot rates, such as Vasicek and Cox-Ingersoll-Ross models, are presented, as well as a brief discussion of the alternative no-arbitrage model, Heath-Jarrow-Morton model, are discussed in Section 6.2. A model incorporating possible Poisson jumps, a very interesting case of incomplete markets, is studied in Chapter 7, based on an approach of minimizing a mean square error.

Simulation issues are discussed in the last chapter, indicating how to simulate a Brownian motion and a Brownian motion with Poisson jumps. Since the chapter is relatively short (only about 12 pages), readers may also consult other sources for a more complete treatment. Some good references are Boyle, Broadie, and Glasserman (1997, *Journal of Economic Dynamics and Control*, pp. 1267-1321), and Broadie and Glasserman (ibid, pp. 1323-1352).

I have only one minor criticism. Inevitably in a book covering a broad area such as theory of option pricing, readers will also find some that they wish had been developed more completely. For example, in view of its increasing importance, a more detailed treatment of the Heath-Jarrow-Morton model, instead of two and half pages at the current setting, would definitely be a plus. In addition, a discussion of incomplete markets in general, credit risks as well as a systematic treatment of exotic options would be desirable. Given the book having a title involving "Finance", not just options, even some optimal investment models, such as mean variance analysis, Merton's model and Kelly criterion, seem to be not totally out of the context. However, it should be recognized that this enlargement perhaps would significantly increase the pages of the book, and readers eager for additional knowledge can also be referred to relevant monographs and research papers.

In summary, this is a marvelous book of introduction to stochastic models in option pricing. One great attraction of the book is that it summarizes, with a careful treatment of the mathematics and a friendly and concise exposition (thanks also to the fantastic translation by Rabeau and Manton), the basic mathematical theory of option pricing in five chapters (Chapters 5-10) with slightly more than 100 pages, thus making this book an ideal one for somebody with good stochastic background wanting to grasp an up-to-date knowledge of this fascinating field. The authors should be congratulated for their excellent work, and I enthusiastically recommend the book.

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