Journal of Applied Mathematics and Stochastic Analysis, 11:2 (1998), 217-218.

GENERALIZED OPTIMAL STOPPING PROBLEMS AND FINANCIAL MARKETS

by Dennis Wong

A BOOK REVIEW

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(Received 1997; Revised February 1998)

The aim of this book is to present the mathematical tools underlying the valuation of American-style derivative securities, i.e. contracts that can be exercised prior to their maturity date at the option of the holder. The book is divided into 6 chapters: the first 2 provide background mathematical results, the next 4 present applications to the valuation of derivative securities.

Chapter 1 contains standard material on stochastic processes, martingales and stochastic calculus. Some of the topics, such as the martingale representation theorem or the Girsanov change of measure, are essential for later development. Others, such as the discussion of Levy processes, appear less relevant. Chapter 2 covers key results from the Theory of Optimal Stopping. The results are presented in a condensed form with short but explicit proofs. The presentation is informative and easy to follow. Much of the discussion is centered around the Dirichlet and Snell Envelopes. These notions are extended to the case in which the stopping region is a subset of the time domain. The classic notions and their properties extend in a natural way to this setting.

The application to finance is mainly based on the martingale approach to valuation (risk neutralization procedure). The foundations of the approach for the complete market case are detailed in Chapter 3. That material has now become standard as it is presented in a number of books (see for example Karatzas and Shreve (1988)). Some of the roots of this procedure can be found in an early article of Cox and Ross (1976).

Chapter 4 presents a very short introduction to options. The classification adopted distinguishes (*i*) options with constrained exercise times, (*ii*) options with constrained portfolios and (*iii*) path-dependent options. Although the reasons behind this classification are understood it appears odd in certain respects. Indeed, constraints on the exercise time or path-dependency are properties of the option payoff, i.e. of the option contract. Constrained portfolios are usually properties of the financial market in which one operates, and not of the option contract itself. Thus under (*ii*) the author

really means to study valuation when the financial market fails to be complete. Also American options which can be exercised at any time before maturity are classified under (*i*) even though, strictly speaking, there is no restriction on the exercise region in this case. Path-dependent options are never studied in following chapters. One may also note that the definitions of some of the contracts discussed in this chapter, such as "classic" European Call and Put options, are nonstandard.

Chapter 5 examines valuation when the exercise region is restricted. The presentation follows a natural pattern. The notion of fair price is defined. More detailed motivation could have been provided by drawing the link to the concept of "absence of arbitrage opportunities". The relation between valuation and optimal stopping is then discussed. The value of an American-style derivative security is related to the Snell envelope of the discounted payoff where discounting is a the locally risk free rate and the expectation is under the equivalent martingale measure (risk neutral measure). An alternative characterization in terms of variational inequalities is also given. Special cases of European and American derivatives such as options are then discussed. A short discussion of the Early Exercise Premium (EEP) is also presented. Overall the analysis in this chapter remains at a very general level, i.e. providing general representation formulas for derivative security prices. One can regret that well known special cases, such as the model with constant coefficients, in which more explicit solutions are available are not mentioned. There is also an abundant literature on the valuation of American options which would have been worth quoting. Notable references which are absent include van Moerbeke (1976), Kim (1990), Jacka (1991), Carr, Jarrow and Myneni (1992), and Myneni (1992). The last section of this chapter examines Gittens indices and their relation to the EEP. This result is not well known and is potentially interesting. The reader would have benefited from a discussion about the potential uses of this representation for option valuation.

Chapter 6 concludes with a brief exposition of a valuation method for European-style derivatives when portfolios are constrained. The bulk of the material is a summary of the paper by Cvitanic and Karatzas (1992). The value of a K-hedgeable derivative is defined as the minimum cost for which the derivative can be super-replicated. This value is shown to equal the value of the derivative in an auxiliary unconstrained financial market with suitably adjusted interest rate and stock returns. Although the mathematics involved are interesting some readers will undoubtedly wonder about the foundations of this valuation method. Indeed, when markets fail to be complete the introduction of new derivatives will, in general, have an effect on the underlying securities prices. Even if these feedback effects are ignored it is not obvious that the price of the derivative can be computed without deriving investors' demand functions and imposing market clearing conditions. A mention of these difficulties inherent in the process of valuation in frictional markets would have been welcomed.

Overall I enjoyed reading this monograph. It can serve as a useful complementary reading in a course on the valuation of derivative securities.

Generalized Optimal Stopping Problems and Financial Markets
by **Dennis Wong**
Publisher: Addison Wesley Longman
Publication Year: 1996
ISBN 0269-3674 $xi + 114$ pp.
Price: $40.33 (paperback)