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SELECTION OF AN EXPRESSION FOR THE HYDRODYNAMIC DRAG ON A PARTICLE IN A MAGNETIC SEPARATOR

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The results of a numerical solution of equations of the motion of a paramagnetic particle in the working gap of a magnetic separator for various values of magnetic induction (from 0.2 to 2 T) and the particle diameter (from 10 μm to 1 mm) show that along the particle trajectory various types of flow modes: e.g. laminar, transitional and turbulent can be present. It is also shown that some of the well-known formulae approximating the experimental dependence of the hydrodynamic drag coefficient on Reynolds number Re (the so-called standard hydrodynamic drag curve) as a step-smooth function, do not ensure the condition of continuity of the hydrodynamic drag on the boundaries of the range of values Re for which these formulae were obtained. One of the variants of approximation of the standard hydrodynamic drag curve for the case where along the same trajectory various types of regimes of flow-around a particle are present, is proposed.

Keywords: Magnetic separation; Hydrodynamic drag; Reynolds number; Stokes law

INTRODUCTION

In magnetic separation the main force determining the process of separation is the magnetic force acting on a particle in the working gap.

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However, in order to estimate parameters of this process it is necessary to determine the complete forces: the hydrodynamic drag and the gravitational force. In many cases we can neglect the influence of the gravitational force (small enough sizes of particles or for small difference of the densities of particles and medium flow), whereas the hydrodynamic drag force plays as important role as the magnetic force. To determine the hydrodynamic drag force of spherical particles Stokes formula is often used. It is applicable to the values of Reynolds numbers $\mathrm{Re} < \mathrm{Re}_1$ where Re_1 is assumed to be equal to 0.3; 0.5 or 1 depending on the level of acceptable error (for example, with $\mathrm{Re} = 1$ the error of the Stokes formula is about 10%). In order to extend the range of solved problems it is necessary to use the hydrodynamic drag estimation formula and the equation of the motion of particles applicable not only in the Stokes regime but also in a wider range of values of Reynolds number. Such formulae and equations of motion will be considered below.

THEORETICAL MODEL

Equations of motion of a paramagnetic particle in the working gap of a magnetic filter (separator) with cylindrical ferromagnetic collectors were obtained in [1]. The hydrodynamic drag force of a particle was determined by the Stokes formula. In a more general case when the hydrodynamic drag depends on the particle flow regime (the Stokes transitional or Newton regime) we can obtain the following equations:

$$\tau\left(\frac{\mathrm{d}^2 r}{\mathrm{d}t^2} - r\left(\frac{\mathrm{d}\theta}{\mathrm{d}t}\right)^2\right) = -\frac{V_m}{\tilde{r}^3}\left(\frac{A}{\tilde{r}^2} + \cos 2\theta\right) - N_\mathrm{d}(V_{\mathrm{p},r} - V_{\mathrm{f},r}) + V_g \cos(\theta - \beta); \quad (1)$$

$$\tau\left(r\frac{\mathrm{d}^2\theta}{\mathrm{d}t^2} + 2\frac{\mathrm{d}r}{\mathrm{d}t}\frac{\mathrm{d}\theta}{\mathrm{d}t}\right) = -\frac{V_m}{\tilde{r}^3}\sin 2\theta - N_\mathrm{d}(V_{\mathrm{p},\theta} - V_{\mathrm{f},\theta}) - V_g \sin(\theta - \beta), \quad (2)$$

where r and θ are, respectively, the radius and the angle of polar system of coordinates, the center of which is coincides with the center of the cross section of the cylindrical collector; t is time; $\tilde{r} = r/r_\mathrm{w}$; r_w is the radius of the cylindrical ferromagnetic collector;

$$\tau = \frac{\rho_p d_p^2}{18\eta};$$

$$V_g = \frac{g d_p^2}{18\eta}(\rho_p - \rho_f);$$

$$V_m = \frac{\mu_0(\kappa_p - \kappa_f) d_p^2}{18\eta} \frac{M H_0}{r_w};$$

$N_d = C_d / C_{d,0}$; C_d, $C_{d,0}$ are the coefficients of the hydrodynamic drag of a particle in the general case and for the Stokes regime respectively; d_p is the diameter of a particle; g is the acceleration of gravity; β is the angle between axis x (from this axis the polar angle θ is measured) and the vector $\vec{g}$ (the axis x is parallel to the vector $\vec{H}_0$); M is the magnetization of the ferromagnetic cylinder; H_0 is the strength of the magnetic field;

$$A = \frac{\mu_w - \mu_f}{\mu_w + \mu_f};$$

μ_w and μ_f are the magnetic permeabilities of the cylinder and the medium, respectively, μ_0 is the magnetic constant (permeability of free space); κ_p and κ_f are the volume magnetic susceptibilities of the particle and the medium, respectively, ρ_p and ρ_f are the densities of particles and carrying medium, respectively, η is the dynamic viscosity of the carrying medium; $\vec{V}_p$, $\vec{V}_f$ are the velocity vectors of the particle and of the carrying medium; $V_{p,r}$, $V_{f,r}$ are the radial components of vectors $\vec{V}_p$ and $\vec{V}_f$ respectively, $V_{p,\theta}$, $V_{f,\theta}$ are the tangential components of vectors $\vec{V}_p$ and $\vec{V}_f$ respectively.

Expressions $V_{f,r}(r, \theta)$ and $V_{f,\theta}(r, \theta)$ are considered to be known functions. Assuming a model of the potential flow around the cylinder by a fluid with the velocity vector of the unpertubered flow $\vec{V}_0$ normal to the axis of the cylinder, we obtain:

$$V_{f,r} = -V_0\left(1 - \frac{1}{\tilde{r}^2}\right)\cos(\theta - \alpha);$$

$$V_{f,\theta} = V_0\left(1 + \frac{1}{\tilde{r}^2}\right)\sin(\theta - \alpha),$$

where α is the angle between the axis x and the vector $\vec{u}_0 = -\vec{V}_0$.

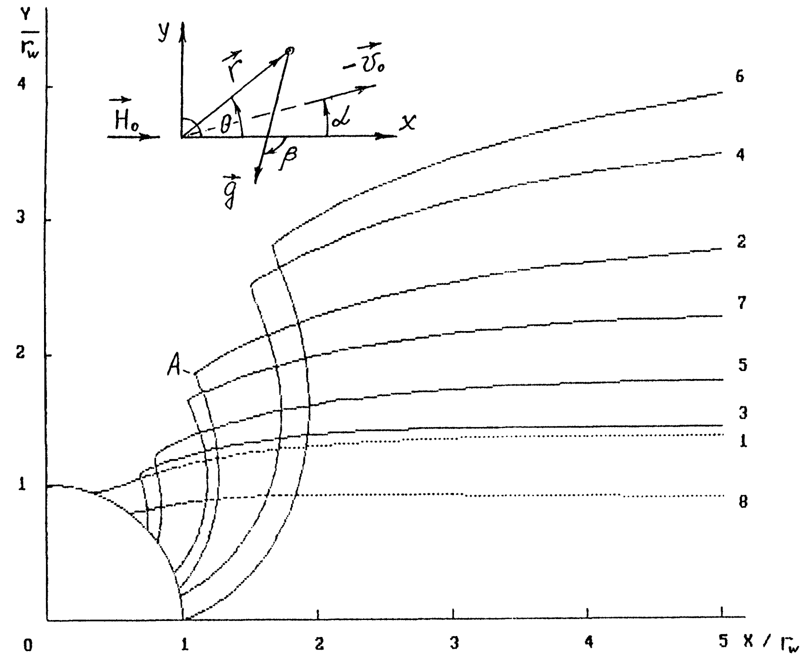

FIGURE 1 The limiting trajectories of motion of paramagnetic particles in the vicinity of a magnetized cylindrical collector.

The plus and minus signs before the terms on the right-hand sides of formulae (1) and (2) and also before V_0 in the two last equations correspond to the origin and the direction of measurement of angles θ, α, β, which are shown in Fig. 1.

DISCUSSION OF THE RESULTS

In order to analyse the effect of technological parameters (H_0, V_0) of separation and of the size of particles on the hydrodynamic flow regime numerical integration of the equation system (1) and (2) was carried out for a number of variants of the initial data assuming that particles have a spherical shape.

Values of the magnetic field strength H_0, of the unpertubered medium flow velocity V_0, diameter of the ferromagnetic collector $d_w = 2r_w$ and the diameter of a particle d_p for these variants are shown in the Table I.

TABLE I The values of variables used in the calculations

Number of variant	1	2	3	4	5	6	7	8	9	10	11
H_0, 10^5 A/m	1.59	3.98	7.96	7.96	7.96	7.96	15.9	15.9	1.59	15.9	15.9
V_0, m/s	0.10	0.10	0.02	0.02	0.10	0.10	0.02	0.10	0.02	0.10	0.10
d_p, μm	100	100	10	30	30	100	10	10	100	500	1000
d_w, mm	1	1	1	1	1	1	1	1	1	2	4

Values of other determining parameters in all the variants were kept constant: specific magnetic susceptibility of particles $\chi_p = 1.5 \times 10^{-6}\,\text{m}^3/\text{kg}$ ($\kappa_p = \chi_p \rho_p$); density of particles $\rho_p = 4.7 \times 10^3\,\text{kg/m}^3$; density of carrying medium $\rho_f = 10^3\,\text{kg/m}^3$; dynamic viscosity of medium $\eta = 10^{-3}\,\text{N} \times \text{s/m}^2$; saturation magnetic induction of a ferromagnetic collector $B_s = 2.15\,\text{T}$.

Magnetic permeability μ_w was determined for given values of H_0 using known experimental dependencies. The so-called longitudinal configuration of a collector [2], in which vector $\vec{V}_0$ is parallel with the direction of vector $\vec{H}_0$ ($\alpha = 0$) was considered. Initial conditions for numerical integration of Eqs. (1) and (2) were determined by defining a position of the origin $x = x_0$; $y = y_0$ ($\theta_0 = \text{arctg}\,(y_0/x_0)$; $r_0 = \sqrt{x_0^2 + y_0^2}$) and components of vector $\vec{V}_p$ of the particle velocity at the origin: $V_{p,r}$ and $V_{p,\theta}$. These values were assumed to be equal to the corresponding components of the velocity vector of the medium flow at the origin. With a fixed value of x_0 and different values of y_0 different trajectories were obtained. From them the limiting one, which differentiates a set of trajectories of particles capture and a set of trajectories along which particles are carried away by the flow medium, was selected.

Figure 1 shows the calculation results of the limiting trajectories of particles, and Fig. 2 shows the calculation results of the relative velocity $V_{p,f}/V_0$ as a function of the relative distance r/r_w from the axis of a collector, where $V_{p,f} = |\vec{V}_p - \vec{V}_f|$. The numbering of curves shown in Figs. 1 and 2 corresponds to variants in Table I.

Velocity $V_{p,f}$ is the determining quantity when calculating the hydrodynamic drag force:

$$\vec{F}_d = -\tfrac{1}{2} C_d \rho_f \left|\vec{V}_{p,f}\right| \vec{V}_{p,f} S_m;$$

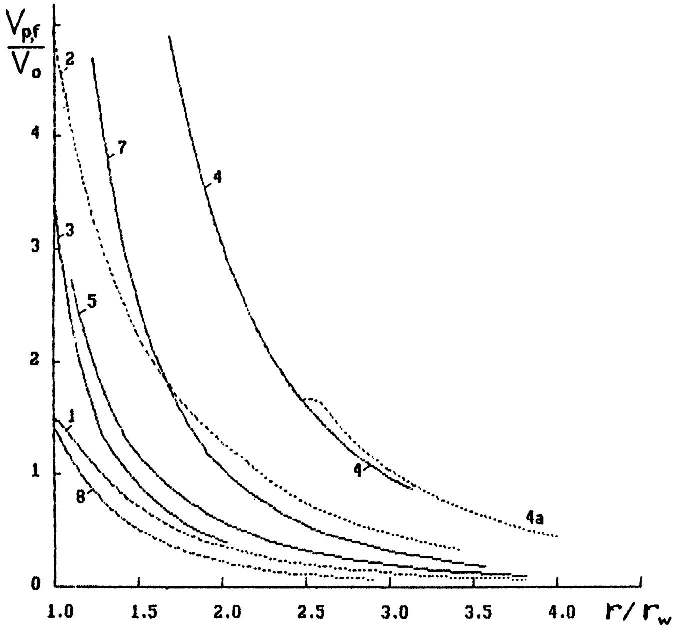

FIGURE 2 Distribution of relative velocity $V_{p,f}/V_0$ of particle motion along their limiting trajectories ($V_{p,f}$ is the velocity of particle motion relative to the flow of medium).

where $C_d = C_d(\mathrm{Re})$; Re is the Reynolds number;

$$\mathrm{Re} = \frac{\rho_f |\vec{V}_{p,f}| d_p}{\eta}; \tag{3}$$

and S_m is the cross-sectional area of a particle.

Velocity $V_{p,f}$ can exceed many times velocity V_0 of the unpertubered medium flow. For example, for variants 2, 4 and 7 the ratio $V_{p,f}/V_0$ near collector is 4.9, 19.1 and 9.0 respectively.

In the initial stage of the trajectory the flow condition of a particle can be laminar, while when approaching the collector, as a result of the increasing velocity $V_{p,f}$, the transitional or turbulent condition can be reached. In order to assess the flow condition the value of Reynolds number is usually used [7].

Results of calculations of the Reynolds number using Eq. (3), along the trajectory of particles (i.e. depending on the relative distance r/r_w)

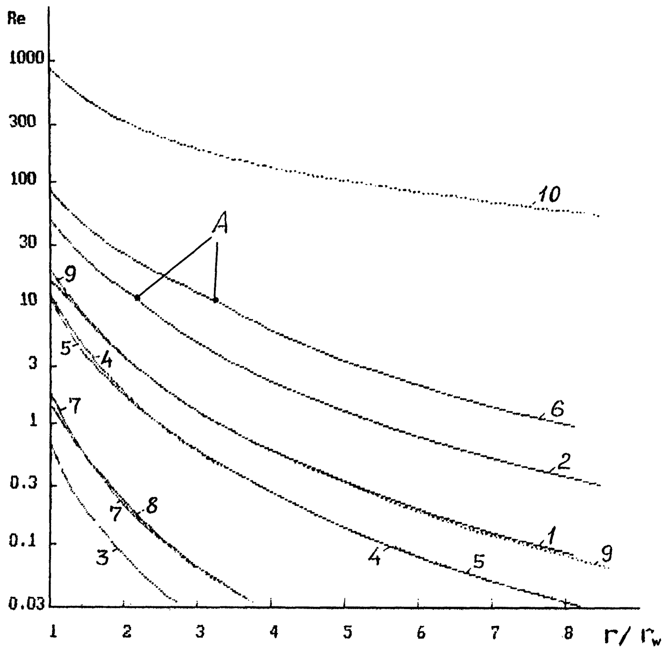

FIGURE 3 Distribution of the Reynolds number along the limiting trajectories of particles.

are shown in Fig. 3, where the numbering of curves corresponds to the variants from Table I. It can be seen from these graphs that the Reynolds number Re can vary substantially for different trajectories, and for motion along a fixed trajectory. Point A on lines 2 and 6 corresponds to the equilibrium point on the trajectory $y(x)$ (Fig. 1). And when the trajectory is passing through point A it can curve sharply.

The results show that the velocity $V_{\mathrm{p,f}}$ of the particle movement relative to the medium practically does not depend on velocity V_0. It can be seen, for example, comparing curves 1 and 9 in Fig. 3, taking into account that the dependence $\mathrm{Re}(\tilde{r})$ differs from the dependence $V_{\mathrm{p,f}}(\tilde{r})$ by a factor $(\rho_{\mathrm{p}} d_{\mathrm{p}})/\eta$. For variants 1 and 9 the values of velocity V_0 differ by a factor of five with other conditions constant. However, dependencies $\mathrm{Re}(\tilde{r})$, and therefore dependencies $V_{\mathrm{p,f}}(\tilde{r})$, for variants 1 and 9, practically coincide. We can make similar conclusions when comparising curves 4 and 5 and also 7 and 8 in Fig. 3.

It is usually assumed [3–7] that for calculations of the hydrodynamic drag of a spherical particle the Stokes formula (expressed in terms of factor C_d):

$$C_d = 24/\mathrm{Re} \tag{4}$$

is applicable in the range of Reynolds numbers $\mathrm{Re} \leq 1$, although with $\mathrm{Re} = 1$ the error of the Stokes formula is about 10%. It is possible to use the generalised experimental dependence $C_d(\mathrm{Re})$, the so-called standard hydrodynamic drag curve [5–7,10]. However, approximation of this curve by a single formula with a small error in a wide range of Reynolds factor will be a complicated and bulky expression. In [10] various approximations of the curve $C_d(\mathrm{Re})$ were considered and in particular the following expressions [11] were assumed:

$$C_d = \begin{cases} 24/\mathrm{Re}, & \mathrm{Re} \leq 2; \\ 18.5/\mathrm{Re}^{0.6}, & 2 < \mathrm{Re} \leq 500; \\ 0.44, & \mathrm{Re} > 500. \end{cases} \qquad \begin{matrix} \\ (5) \\ (6) \end{matrix}$$

The same expressions are given in [9], but in contrast to [11] the transition from Eq. (4) to Eq. (5) is supposed[1] to occur at $\mathrm{Re} = 0.2$ rather than at $\mathrm{Re} = 2$.

In [8] for evaluation of the hydrodynamic drag of spherical particles various approximations of the experimental dependence $C_d(\mathrm{Re})$ for different intervals $10^n < \mathrm{Re} < 10^{n+1}$ $(n = -1, 0, 1, 2)$ are used. The Stokes formula is considered applicable in the range $\mathrm{Re} < 0.1$, while for $0.1 < \mathrm{Re} < 1$ the following expression is suggested:

$$C_d = C_{d,0}(0.947 + 0.1538\mathrm{Re} + 0.003763/\mathrm{Re}),$$

where $C_{d,0} = 24/\mathrm{Re}$.

This equation becomes equal to the Stokes formula if the expression in the parentheses is assumed to be equal to unity.

[1] Apparently, the value of $\mathrm{Re} = 0.2$ in [9] was given by mistake, since the value of C_d calculated under Eq. (5) at $\mathrm{Re} = 0.2$, is approximately 60% lower than the value under the standard drag curve, whereas with $\mathrm{Re} = 2$, Eqs. (4) and (5) give almost equal values (17% and 15% below the standard curve respectively).

More often another type of a step-smooth function $C_d(\mathrm{Re})$ is used:

$$\text{for } \mathrm{Re} \leq 1 \qquad C_d = C_{d,0};$$

$$\text{for } 1 < \mathrm{Re} < 1000 \quad C_d = C_{d,0}(1 + \mathrm{Re}^{2/3}/6) \tag{7}$$

$$\text{or } C_d = C_{d,0}(1 + 0.15\mathrm{Re}^{0.683}). \tag{8}$$

Equation (7) is known as the Klatchko formula (see for example [3,10]); Eq. (8) is given in [4,10]. Transition from the Stokes formula to (7) and (8) at $\mathrm{Re} = 1$ the function $C_d(\mathrm{Re})$ changes in a step. That is why the results of calculation using such step-smooth dependencies have singularities of artificial character which are absent in reality.

For example, the dependence of the velocity of particle motion relative to the carrying medium on parameter r/r_w considered above (see Fig. 2.) has "a plateau", i.e. a section of almost constant value for those values of r, which correspond to a section of trajectory with Reynolds number close to unity. Curve 4a in Fig. 2 was obtained from the calculation of variant 4 using the Stokes formula for $\mathrm{Re} < 1$ and using Eq. (8) for $\mathrm{Re} > 1$. Curve 4 in Fig. 2 was obtained for this variant using Eq. (4) for $\mathrm{Re} < 0.3$ and the formula

$$C_d = C_{d,1} - 0.48/\mathrm{Re}^2 \quad \text{for } 0.3 < \mathrm{Re} < 920, \tag{9}$$

where $C_{d,1}$ is the value of C_d, calculated using Eq. (8).

When these formulae are used the discontinuity between two different dependencies of $C_d(\mathrm{Re})$ at point $\mathrm{Re} = 0.3$ is eliminated. Also the error in $C_d(\mathrm{Re})$ decreases in comparison with Eq. (8).

The upper limit of the Re range in which Eq. (9) was used is determined from the condition that $C_d(\mathrm{Re})$ reaches the value corresponding to the Newton's regime: $C_d \approx \mathrm{const} = 0.44$. $\mathrm{Re} = 920$ corresponds to this condition. Figure 4 shows the relative error ΔC_d as a function of the Re number for different formulae $C_d(\mathrm{Re})$ in relation to experimental data $C_{d,e}(\mathrm{Re})$:

$$\Delta C_d = (C_d - C_{d,e})/C_{d,e}.$$

The experimental data were taken from [8] where they are given in a tabular form. Lines 1, 2, 3, 4 and 5 in Fig. 4 show errors of Eqs. (4), (5), (7), (8) and (9) respectively. When using Eqs. (4) and (9) connecting at the point $\mathrm{Re} = 0.3$, the error ΔC_d does not exceed 3% for $0.1 < \mathrm{Re} < 100$

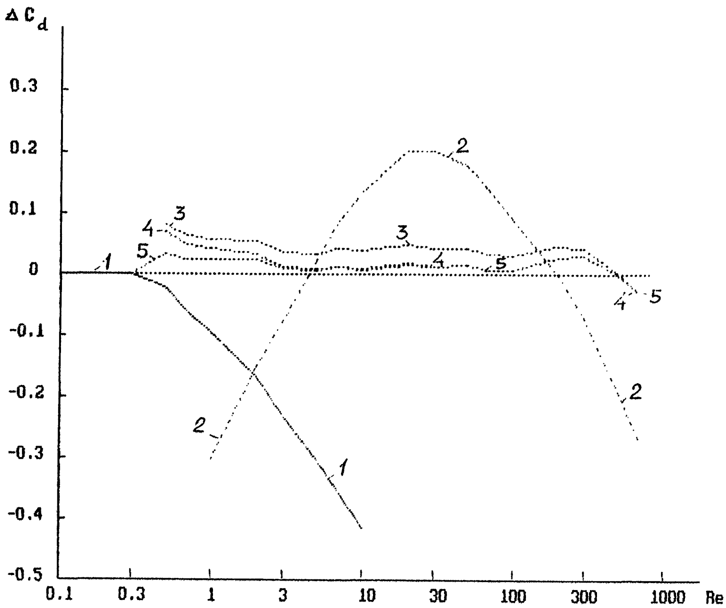

FIGURE 4 The relative errors $\Delta C_d = (C_d - C_{d,e})/C_{d,e}$ of the hydrodynamic drag coefficient, determined using various formulas, as a function of Reynolds number. Curves 1, 2, 3, 4 and 5 correspond to Eqs. (4), (5), (7), (8) and (9), respectively.

and 7% for $0.1 < Re < 1000$. We should note that the results of calculation expressed in Figs. 1–3 (except for curve 4a in Fig. 2), were obtained using the dependencies (4) and (9) connecting at the point $Re = 0.3$.

The calculation shows that for given values of the fluid viscosity, magnetic susceptibility of particles and other determining parameters the turbulent flow condition takes place for $d_p = 1$ mm and magnetic induction $B_0 = 2$ T (variant 11), and in the case, when $B_0 < 2$ T, for $d_p > 1$ mm. With the increase of the magnetic susceptibility the limiting values of the diameter of a particle and of the magnetic field strength at which the turbulent condition develops decrease.

CONCLUSIONS

An analysis of several formulae of the coefficient of the hydrodynamic drag for a spherical particle has been carried out and a modification of one of the expressions, in the form of Eq. (9), was proposed. The error of this expression, relative to the experimental data in the range $0.3 < \mathrm{Re} < 1000$ does not exceed 7%, while for $\mathrm{Re} < 0.3$ the Stokes formula is used. The distribution of the velocity of motion of particles relative to the carrying medium has been calculated and the corresponding Reynolds numbers along the trajectory of motion, for a number of characteristic combinations of the separation conditions, have been determined.

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