Textures and Microstructures, Vol. 33, pp. 111–123
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INFLUENCE OF THE TEXTURE ON THE Al–6%Mg ALLOY DEFORMATION

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The influence of the texture on material mechanical properties and deformation behaviour was widely discussed. (refer to Bunge, H.J. (1982). *Texture Analysis in Materials Science Mathematical Methods*). Butterworths, London. In this work elastic properties (Young's modulus) of cold rolled Al–6%Mg alloy sheet were estimated taking into account lattice preferred orientations, which can be described by the orientation distribution function (ODF). The ODF was reconstructed from pole figures measured by means of neutron diffraction and was approximated by normal distributions (Savyolova, T.I. (1994) *Zavodskaya Laboratoria* **50**, **N5**, 48–52). The method used for calculation is able to express explicitly the polycrystalline elastic property via the single crystal property and the texture parameters.

Stress–strain dependence (deformation curves) was measured by means of uniaxial tensile tests for Al–6%Mg alloy samples with different tensile axis directions. Samples for uniaxial tests were cut at different angles to the rolling direction. The conformity between experimental and computed results is discussed.

Keywords: Al-alloy; Texture; Young's modulus; Neutron diffraction

INTRODUCTION

The texture generated in the forming processes of constructional materials is to large extent the cause of anisotropic behaviour of these materials. Optimal engineering components design requires complete information about elastic and strength anisotropy in different stress states. Neglecting this factor would cause false conclusions both in product design and in strain measurement result treatment.

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The method for experimental strain evaluation in engineering components using neutron diffraction is in progress for last few years. The apparatus "Epsilon" by means of which it is possible to measure the complete deformation tensor is constructed in the Frank Laboratory of Neutron Physics at the Joint Institute for Nuclear Research (Dubna, Russia). This work was carried out during the first experiments at this apparatus. To determine correctly the residual stresses from the shifts of the diffraction peaks, it is necessary to know the elastic behaviour of the grains contributing to diffraction in a given direction, e.g. it is necessary to know directional dependence of elastic material properties.

In the present work the problem of polycrystalline elastic property calculation is solved with the help of the orientation distribution function (ODF). These properties can be described by fourth-rank tensors. Possibility of ODF using for polycrystalline physical property calculation by means of corresponding single crystal property averaging is substantiated by Bunge (1982). Because of the ambiguity of the ODF reproduction from experimental measured pole figures (PFs) we are considering ODF obtained by the normal distributions approximation. Possibility of using these distributions for describing polycrystalline texture is demonstrated in (Bucharova, 1993). In this work the analytical description of the ODF as a sum of normal distributions on the SO(3) rotation group is given. Then the polycrystalline elastic property description for cubic symmetry via corresponding single crystalline properties by means of fourth-rank tensors and normal distribution parameters is represented.

With the help of formulas obtained in this paper the calculation of Young's modulus of cold rolled Al–6%Mg alloy sheet was carried out. Calculated Young's modulus values are compared with the ones measured from uniaxial tensile tests. The advantages of the method proposed for calculations are discussed in this work.

THEORETICAL BACKGROUND

Tensor Representation for Single-crystal Elastic Properties

Elastic properties of single crystals can be represented by fourth-rank tensors. Because in this work the nontraditional form for their representation is used let us describe it in more detail.

Hooke's law can be formulated in either of the two forms:

$$\sigma_{ij} = C_{ijkl}\varepsilon_{kl}; \qquad \varepsilon_{ij} = S_{ijkl}\sigma_{kl}, \tag{1}$$

where C_{ijkl} is a stiffness tensor, S_{ijkl} is a compliance tensor. The tensors S_{ijkl} and C_{ijkl} obey the symmetry conditions. In case of cubic crystal symmetry there are only three independent constants: S_{1111}, S_{1122}, S_{2323}. In this case compliance single-crystal tensor can be represented in the form (Sirotin and Shaskolskaya, 1975):

$$\begin{aligned} S_{ijkl} = &\left(S_I^4 + 2S_I^{22}\right)\delta_{ij}\delta_{kl}/3 \\ &+ \left(S_I^4 - S_I^{22}\right)\left(\delta_{ik}\delta_{jl} + \delta_{il}\delta_{jk}\right)/3 + S_N^4 N_{ijkl}^0, \end{aligned} \tag{2}$$

where N_{ijkl} is a unit nonor of cubic symmetry. It has the following components in the crystallographic coordinate system:

$$N_{ijkl}^0 = \begin{cases} 1, & \text{for } i = j = k = l, \\ -\frac{1}{2}, & \text{for two pairs of equal indices,} \\ 0, & \text{for all other cases.} \end{cases} \tag{3}$$

The coefficients of expansion (2) can be found as follows:

$$\begin{aligned} &S_I^4 = (3S_{11} + 2S_{12} + S_{44})/5, \quad S_I^{22} = S_{12} - S_{44}/4, \\ &S_N^4 = (2S_{11} - S_{12} - S_{44})/5. \end{aligned} \tag{4}$$

The coefficient S_{mn} are connected with the S_{ijkl} by the rule:

$$\begin{aligned} &= S_{ijkl}; && m, n = 1, 2, 3, \\ S_{mn} &= S_{ijkl}/2; && m \text{ or } n = 4, 5, 6, \\ &= S_{ijkl}/4; && m \text{ and } n = 4, 5, 6. \end{aligned} \tag{5}$$

Similarly, C_{ijkl} are replaced by C_{mn}:

$$C_{mn} = C_{ijkl}. \tag{6}$$

Stiffness single-crystal tensor can be also represented by (2) with coefficients calculated with the help of formulas (4). In these formulas

C_{11}, C_{12}, $4C_{44}$ must be substituted instead of S_{11}, S_{12}, S_{44}. The directional property dependence can be represented by surface. If we cut a sample in the direction $\boldsymbol{h} = [hkl]$, where h, k, l are Miller indices, out of the material, then the reciprocal Young's modulus in this direction is given by:

$$E^{-1}(\boldsymbol{h}) = S_{ijkl}h_ih_jh_kh_l, \tag{7}$$

where S_{ijkl} is a single crystal compliance tensor. For instance, the stereographic projection of Young's modulus surface for aluminium single crystal is given on Fig. 1(a). Because the elastic properties can be described by fourth-rank tensors, the reciprocal Young's modulus is a fourth-rank surface. If one substitutes in (7) for compliance tensor S_{ijkl} the representation (2), one obtains:

$$E^{-1}(\boldsymbol{h}) = S_I^4 + S_N^4 N_{ijkl}^0 h_ih_jh_kh_l. \tag{8}$$

Surface representation for the unit nonor:

$$r(\boldsymbol{h}) = N_{ijkl}^0 h_ih_jh_kh_l = 1 - 5\left(h_1^2h_2^2 + h_2^2h_3^2 + h_1^2h_3^2\right) = nk_4^1(\boldsymbol{h}), \tag{9}$$

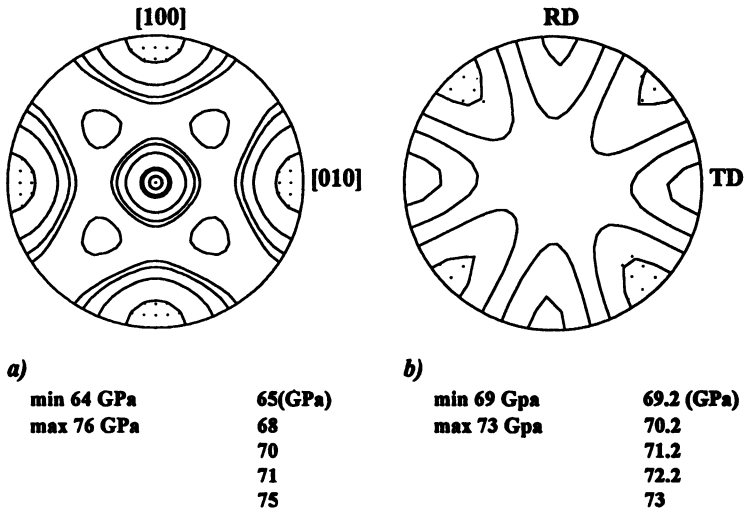

FIGURE 1 (a) Stereographic projection of Young's modulus surface (GPa) for aluminium single crystal; (b) Stereographic projection of Young's modulus surface for cold rolled Al–6%Mg alloy sheet.

where

$$k_4^1(\boldsymbol{h}) = \frac{1}{n}\left[1 - 5\left(h_1^2h_2^2 + h_2^2h_3^2 + h_1^2h_3^2\right)\right]$$

is a cubic spherical function, $n = \sqrt{16\pi/21}$.

Polycrystalline Elastic Properties

We use the approach developed by Bunge (1982) for the polycrystalline property calculations. Polycrystalline property $E(\boldsymbol{y})$ depends on the properties of the crystallites in the direction $\boldsymbol{y}$. In the crystallite of orientation g the crystal direction $\boldsymbol{h}$ may point in the sample direction $\vec{y}$. The average value of the property in the direction $\boldsymbol{y}$ over all orientations g that occur can be written in the following form:

$$\langle E(\boldsymbol{y})\rangle = \oint E(\boldsymbol{h})f(g)\,\mathrm{d}g, \tag{10}$$

where

$$f(g) = \frac{\mathrm{d}V(g)/V}{\mathrm{d}g} \tag{11}$$

is the ODF, which is given by the volume fraction of crystallites having the orientation g. Averaging first over all those orientations for which the crystal direction $\boldsymbol{h}$ falls in the sample direction $\vec{y}$ and then over the other orientations results in:

$$\langle E(\boldsymbol{y})\rangle = \frac{1}{4\pi}\oint E(\boldsymbol{h})A(\boldsymbol{h},\boldsymbol{y})\,\mathrm{d}\boldsymbol{h}, \tag{12}$$

where

$$A(\boldsymbol{h},\boldsymbol{y}) = \frac{1}{2\pi}\int_{\boldsymbol{h}\|\boldsymbol{y}} f(g)\,\mathrm{d}\psi \tag{13}$$

is general axis distribution function (Bunge, 1982). Since the direction $\boldsymbol{y}$ is constant during the integration, the $A(\boldsymbol{h},\boldsymbol{y})$ is the inverse PF of the

direction $\boldsymbol{y}$. For calculation of the average value $\langle E(\boldsymbol{y})\rangle$ in a sample direction $\boldsymbol{y}$ one thus needs only the inverse PF of this direction but not the complete ODF. This is the particular advantage of the surface property representation.

METHOD

Average value of reciprocal Young's modulus in direction $\boldsymbol{y}$ for cubic polycrystal taking into account (8), (9), (12) can be expressed as follows:

$$\langle E^{-1}(\boldsymbol{y})\rangle = S_1^4 + \frac{1}{4\pi} S_N^4 r(\boldsymbol{y}), \tag{14}$$

where

$$\langle r(\boldsymbol{y})\rangle = \oint A(\boldsymbol{h},\boldsymbol{y}) r(\boldsymbol{h})\, \mathrm{d}\boldsymbol{h}. \tag{15}$$

The ODF $f(g)$ which depends on orientation g can be developed in a series of generalized spherical harmonics (Bunge, 1982):

$$f(g) = \sum_{l=0}^{\infty} \sum_{m=1}^{M(l)} \sum_{n=1}^{N(l)} C_l^{mn} T_l^{mn}(g), \tag{16}$$

where $T_l^{mn}(g)$ are invariant with respect to crystal symmetry rotations as well as sample symmetry rotations. Similarly, the distribution function $A(\boldsymbol{h},\boldsymbol{y})$ can also be developed into a series of symmetrized spherical harmonics:

$$A(\boldsymbol{h},\boldsymbol{y}) = \sum_{l=0}^{\infty} \sum_{m=1}^{M(l)} \sum_{n=1}^{N(l)} \frac{4\pi}{2l+1} C_l^{mn} Y_l^{*m}(\boldsymbol{h}) Y_l^n(\boldsymbol{y}). \tag{17}$$

We are considering properties of cubic polycrystals which can be described by fourth-rank tensors. Therefore we are interested in the coefficients C_l^{mn} with $l \leq 4$:

$$\begin{aligned} A(\boldsymbol{h},\boldsymbol{y}) = 1 + k_4^1(\boldsymbol{h}) \{ & C_4^{10} Y_4^0(\boldsymbol{y}) + C_4^{12} [Y_4^2(\boldsymbol{y}) + Y_4^{2*}(\boldsymbol{y})] \\ & + C_4^{14} [Y_4^4(\boldsymbol{y}) + Y_4^{4*}(\boldsymbol{y})] \}. \end{aligned} \tag{18}$$

It is obtained after integration (15):

$$r(\vec{y}) = n\{C_4^{10} Y_4^0(y) + C_4^{12}[Y_4^2(y) + Y_4^{2*}(y)] + C_4^{14}[Y_4^4(y) + Y_4^{4*}(y)]\}. \tag{19}$$

The coefficients $C_4^{10}, C_4^{12}, C_4^{14}$ can be found from one measured PF $P_{h_i}(y)$:

$$C_4^{10} = \frac{1}{k_4^1(h_i)} \int_{S^2} P_{h_i}(y)\, Y_4^0(y)\, \mathrm{d}y, \tag{20}$$

$$C_4^{1k} = \frac{1}{k_4^1(h_i)} \int_{S^2} P_{h_i}(y)\left[Y_4^k(y) + Y_4^{k*}(y)\right] \mathrm{d}y \quad (k = 2, 4). \tag{21}$$

In this work we have used another method for ODF representation which is in progress now. This is the texture component method. Possibility of ODF approximation by normal distributions is justified by Savyolova (1984). According to this approach ODF can be represented in the form:

$$f(g) = \sum_{k=1}^{N} M_k f(g, g_{0k}, \varepsilon_k) = \sum_{k=1}^{N} M_k \sum_{l=0}^{\infty} (2l+1) \exp(-l(l+1)\varepsilon_k^2) \frac{\sin\left((l+\frac{1}{2})t_k\right)}{\sin(t_k/2)}, \tag{22}$$

where M_k is the weight of k_{th} component ($\sum_{k=1}^{N} M_k = 1$) g_{0k} is the "centre" of k_{th} component, ε_k is the width parameter, $\cos t_k = [\mathrm{Tr}(g_{0k}^{-1} g - 1)]/2$. Then the function $A(\boldsymbol{h}, \boldsymbol{y})$ can be written in the form:

$$A(\boldsymbol{h}, \boldsymbol{y}) = \sum_{k=1}^{N} M_k \sum_{l=0(2)}^{\infty} (2l+1) \exp(-l(l+1)\varepsilon_k^2) P_l(\cos\theta_k), \tag{23}$$

where $P_l(\cos\theta)$ are nonnormalized Legendre's polynoms, $\cos\theta_k = (\boldsymbol{h}, g_{0k}\boldsymbol{y})$. $A(\boldsymbol{h}, \boldsymbol{y})$ is symmetrized for cubic crystal symmetry and the

following expressions are obtained for $l \leq 4$:

$$A(\boldsymbol{h}, \boldsymbol{y}) = 1 + 4\pi \sum_{k=1}^{N} M_k \exp(-20\varepsilon_k^2) k_4^1(\boldsymbol{h}) k_4^1(g_{0k}\boldsymbol{y}), \tag{24}$$

$$r(\boldsymbol{y}) = 4\pi n \sum_{k=1}^{N} M_k \exp(-20\varepsilon_k^2) k_4^1(g_{0k}\boldsymbol{y}). \tag{25}$$

Finally, the Young's modulus for cubic polycrystals can be brought in the following form taking into account (14), (15), (24):

$$\langle E^{-1}(\vec{y}) \rangle = S_I^4 + nS_N^4 \sum_{k=1}^{N} M_k \exp(-20\varepsilon_k^2) k_4^1(g_{0k}\boldsymbol{y}). \tag{26}$$

To compare expression (26) for Young's modulus with the one obtained by Bunge (1982), we present the latter one:

$$\langle E^{-1}(\boldsymbol{y}) \rangle = \sum_{l=0}^{4} \sum_{n=1}^{N(l)} K_l^n Y_l^n(\boldsymbol{y}), \tag{27}$$

where

$$K_l^n = \frac{1}{2l+1} \sum_{m=1}^{M(l)} C_l^{mn} e_l^m. \tag{28}$$

Here C_l^{mn} are the ODF expansion coefficients on the generalized spherical harmonics, $Y_l^n(\boldsymbol{y})$ are the symmetrized spherical harmonics, e_l^m are the Young's modulus expansion coefficients on the symmetrized spherical harmonics.

EXPERIMENTAL

To demonstrate the method proposed above the Young's modulus for cold rolled Al–6%Mg sheet is calculated in accordance with (26). Experimental PFs (111), (200), (220) were measured at NSHR (Neutron Spectrometer of High Resolution) at the fast pulsed reactor IBR-2 at

the Joint Institute for Nuclear Research with help of time-of-flight technique (Feldmann *et al.*, 1991). The rolling direction (RD) coincides with the Z axis, the normal one (ND) with the X axis and the transverse one (TD) with the Y axis, where $\{X, Y, Z\}$ is the sample coordinate system.

Using these PFs the ten texture components $f(g_{0k}, \varepsilon_k, M_k)$ were reconstructed by means of the computer program **fit** with the help of the technique described by Helming (1993). Thus the multicomponent texture was generated due to rolling in the form:

$$f(g) = \sum_{k=1}^{9} f_k(g, g_{0k}, \varepsilon_k) + f_{10} M_{10}, \tag{29}$$

where f_k, $k = 1, \ldots, 9$ are the texture components which are normal distributions in the form (22) with the parameters presented in Table I; $f_{10} = 1$ is the isotropic distribution with $M_{10} = 0.342$. The orientation g_{0k} is here described by the Euler angles $g \equiv \{\alpha, \beta, \gamma\}$.

The experimental PFs and those recalculated by means of texture component method are given at Fig. 2. The good compliance between the experimental and the recalculated PFs can be seen. Besides, as also can be seen, the generated texture is not weak.

The Young's modulus surface was calculated according to (26) by taking into account ODF in the form (29) and single-crystal compliance constants (Frantchevich *et al.*, 1982) for aluminium $S_{11} = 15.7 \times 10^{-3}\,\mathrm{GPa}^{-1}$, $S_{12} = -5.7 \times 10^{-3}\,\mathrm{GPa}^{-1}$, $S_{44} = 35.1 \times 10^{-3}\,\mathrm{GPa}^{-1}$.

To check the calculations Young's modulus values were determined from the experimental uniaxial tensile tests. Samples for these tests were

TABLE I Texture component parameters

k	g_{0k}, deg.	ε_k	M_k
1	{307.09, 40.59, 32.2}	0.00695	0.09555
2	{59.94, 50.85, 43.77}	0.00699	0.09358
3	{45.62, 37.42, 63.68}	0.00647	0.08381
4	{134.15, 40.67, 23.77}	0.00615	0.08034
5	{332.19, 42.99, 2.73}	0.00786	0.07529
6	{22.70, 43.94, 2.30}	0.00748	0.07084
7	{224.63, 41.36, 64.77}	0.00537	0.06595
8	{235.81, 49.85, 50.60}	0.00302	0.04871
9	{123.18, 50.05, 37.22}	0.00313	0.04390

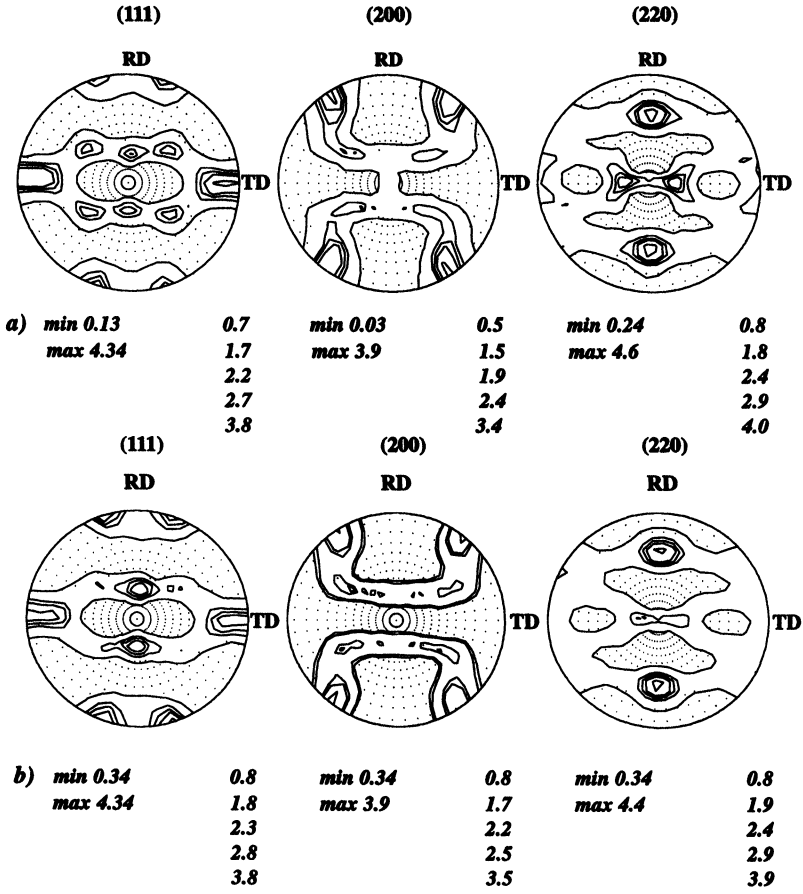

FIGURE 2 (a) Experimental pole figures for cold rolled Al–6%Mg alloy sheet measured with help of Neutron Spectrometer of High Resolution; (b) pole figures for the same material calculated with help of texture component method.

cut from the Al–6%Mg alloy sheet in the RD–TD plane with different angles to the RD. Measurements were carried out at the *TIRA-test* loading machine with help of a Tensile test metal EN 10002. The tensile diagrams for the different samples are given at Fig. 3(a). As can be seen from it the *TIRA-test* resolution is very high and even high enough to distinguish these diagrams in the elastic region.

The section of Young's modulus surface in the RD–TD plane and the measured Young's modulus values are presented at Fig. 3(b).

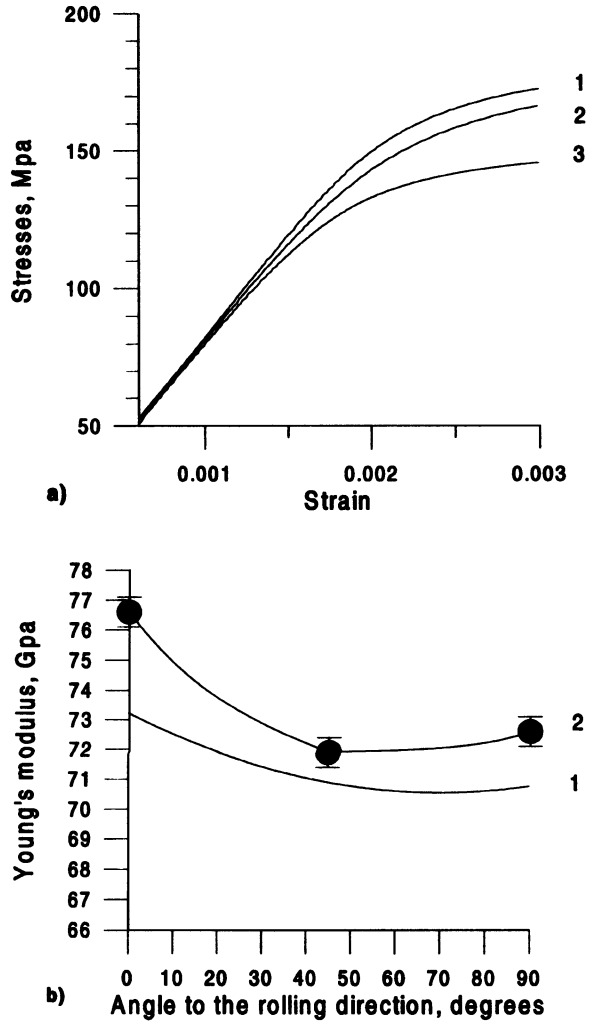

FIGURE 3 (for cold rolled Al–6%Mg alloy sheet) (a) Tensile diagram 1 is for the sample with the tensile axis along RD, 2 is for the sample with 45° to the RD tensile axis, 3 is for the sample with the tensile axis along TD; (b) Young's modulus dependences from the angle to the RD: 1 calculated; 2 experimental.

DISCUSSION

The reasonable compliance between the calculated and experimental Young's modulus can be seen from Fig. 3(b): the tendency of both

dependences is in good agreement. However, the Young's modulus directional dependence is not strong in spite of the fact that texture is not weak. This fact can be explained by the high cubic crystal symmetry. Nevertheless, due to the high *TIRA-test* apparatus resolution it is possible to differ tensile diagrams for samples with different tensile axis directions (see Fig. 3(a)).

The method proposed for cubic polycrystalline elastic modulus calculation has a number of advantages. Using this method it is possible to explicitly represent polycrystalline properties via the corresponding single-crystal elastic constants. Besides, the ODF representation as a sum of normal distributions (22) allows to express elastic polycrystalline properties via the normal distribution parameters $(\varepsilon_k, g_k, M_k)$ (26). These parameters are the texture characteristics (sharpness, texture component position, intensity) in contrast to ODF expansion coefficients on the generalized spherical harmonics which do not possess the descriptive information about texture. Using representation (26) it is possible to calculate the Young's modulus directional dependences from the texture parameters $M_k, g_{0k}, \varepsilon_k$. It cannot be done with help of (27).

We have to note also that there are a number of experimental Young's modulus values which are not in good agreement with the calculated ones (they are less than the calculated ones). We have not presented these values because of the following reasons. We obtained the expression for polycrystalline Young's modulus directional dependence under the assumption that the stresses at all crystallites making up the polycrystal are constant. This corresponds to the Reuss approximation (Bunge, 1982). As a rule, the real properties are higher than the ones calculated in Reuss approximation. That is why the experimental Young's modulus values which are less than the ones calculated with help of proposed method seem questionable. So, we are going to improve our calculations using other assumptions (Hill's, Voight's and others).

CONCLUSION

In this work the method for calculation of cubic polycrystalline elastic properties by taking into account ODF is proposed. The ODF is represented as a sum of the normal distributions on the SO(3) rotation group.

The Young's modulus values of cold rolled Al–6%Mg alloy sheet calculated with help of presented method in different directions are in good agreement with the experimentally measured ones.

The method proposed for computations has a number of advantages. It does not require a number of numerical calculations. Moreover, it enables to express the polycrystalline elastic properties descriptively via the single-crystal constants and normal distribution parameters which are the texture characteristics.

Acknowledgements

The authors are grateful to Dr. J. Schreiber from the Fraunhofer-Institute for Non-destructive Testing, G. Bokuchava from the Institute for Nuclear Research of RAS, Dr. K. Ullemeyer and V. Luzin from the Joint Institute of Nuclear Research for supplying the experimental equipment and support in texture and deformation measurements.

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