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INTRODUCTION TO MATRIX ANALYTIC METHODS IN STOCHASTIC MODELING

by G. Latouche and V. Ramaswamy

A BOOK REVIEW

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Discrete and continuous-time Markov chains are the most common classes of stochastic processes used in modeling randomly evolving systems. A lot is known about the theoretical and computational aspects of these processes, see Kulkarni [1] and the bibliography therein. When the states of the Markov chains can be thought of as a pair (*level, phase*), we get the phase-type Markov chains. The main examples are: the quasi birth and death (QBD), or $M/G/1$ type or $G/M/1$ type, tree type processes, etc. For processes with this type of structure, the computational aspects become especially tractable, see Neuts [2] and [3]. The study of the properties, theoretical as well as algorithmic, of such processes is called the *matrix analytic method.* This can be thought of as the combination of matrix-geometric distribution and phase-type processes. This book is devoted to the study of the *matrix analytic method.* Although the book deals with general matrix analytic methods, there is more emphasis on QBD processes. The main feature of the book is the constant emphasis on probabilistic arguments, rather than matrix algebraic ones. Thus several iterative algorithms are developed by thinking of nth iteration as a transient analysis of a suitable process over the first n steps.

The book is divided into five sections. Section I contains a collection of several examples to which the general theory developed later can be applied. Section II discusses the method of phases: the phase type distributions (both discrete and continuous), and their properties; the renewal and point processes built by using the phase-type distributions. Section III is devoted to the well-known "matrix-geometric distribution." This material is discussed first with the standard birth and death processes and then extended to the QBDs. It is helpful in getting the reader thinking in the language of "levels" and "phases" within the levels. Section IV is the heart of the book: the algorithms. It discusses several numerical algorithms for the computation of the steady-state of the QBDs. The algorithms are well documented,

and their computational complexity clearly specified. I particularly liked the chapter on spectral analysis and the discussion on the implication of the caudal characteristic and the traffic intensity. Section V has a few short chapters describing how the earlier material can be extended to more general processes.

The book is well written and should become an additional useful resource book for researchers in this area. I do not think the authors intended it to be a textbook since the subject area is rather specialized, and also because there are no exercises or problem sets.

References

[1] Kulkarni, V.G., *Modeling and Analysis of Stochastic Systems*, Chapman Hall, London 1995.

[2] Neuts, M.F., *Matrix Geometric Solutions in Stochastic Models: An Algorithmic Approach*, Johns Hopkins University Press, Baltimore 1981.

[3] Neuts, M.F., *Structured Stochastic Matrices of $M/G/1$ Type and Their Applications*, Marcel Dekker, New York 1989.

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